

CHAPTER 12

RESEARCHES ON THE DEFORMATION OF WAVE SPECTRA IN INTERMEDIATE WATER AREA BY CALCULATION

by

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ABSTRACT

In order to devise more rational approach to predicat the wave features in intermediate water area, energy spectra in deep water area are to be changed to histograms, and every stripe of the histogram is to represent an elementary small amplitude wave. The deformation of such elementary waves over intermediate water area can be calculated theoretically by energy flux equation. Spectra in intermediate area can be worked out by summing up the wave energies deformed by shoaling, bottom friction and refraction.

Calculation are carried out by computer. The peak of spectra will never change when it propagates from deep water to intermediate water area if the bottom contours are parallel to the shore line. Tangible process for using this approach to practical engineering problems is pending investigation.

1. INTRODUCTION

From a practical point of view, the semi-empirical relations for wave forecasting can be classified by two methods: the S-M-B method and the P-N-J method. Both methods utilize the wave height

distribution function derived theoretically by Longuet-Higgins, that is, the distribution of wave height is a Gamma distribution in a state of narrow band-width spectrum. They are related to the deep water. However, the transformation of irregular ocean waves over the water area where they feel the bottom is a complicated process which is not yet fully understood. One method of treating the problems is to represent the actual wave system by the superposition of elementary sinusoidal wave series of different height, period, phase and direction. Such a system would have a two-dimensional energy spectrum. When the waves propagate over the intermediate water area, they are deformed by bottom friction, shoaling, refraction and percolation, etc. As an overall effect, the deformation of wave spectrum results that the wave height can no longer be defined by the special type of Gamma distribution such as the Rayleigh distribution.

Traditional forecasting methods in shallow water were roughly due to Thijsse-Schijf (1949) and Bretschneider (1954) in two entirely different approaches. Nevertheless, they were done by making use of the deep water forecasting relationships originally developed by S-M-B, and establishing a numerical procedure for computing the wave in shallow water (Tang, 1970), without regarding the complicated component waves.

A new approach of calculating the shallow water waves was first introduced by Bretschneider (1963) according to the shallow water spectrum. Also Karlsson (1969) calculated the shallow water waves from shallow water spectra on the effect of refraction only. In this paper a reasonable way has been made to investigate the deformation of wave spectra in intermediate water area by assuming the deep water spectra being already known. A special type of topographical configuration with constant bottom slope and parallel contour lines was adopted. Comparison of traditional forecasting method and the following method was made.

2. FUNDAMENTAL ASSUMPTIONS

The basic assumptions needed in this development are:

- (1) The waves in deep water are assumed to be fully arisen, and have been existing for quite a long time. Consequently, the velocity dispersion is not necessary to be considered during calculation.
- (2) Pierson-Moskowitz's one-dimensional spectrum (P-M spectrum) is to be adopted for calculation, i.e.

$$S(\sigma) = \alpha g^2 \sigma^{-5} \exp\left[\beta\left(\frac{g}{U\sigma}\right)^4\right] \quad (1)$$

where

$$\alpha = 0.0081$$

$$\beta = -0.74$$

g = gravity acceleration constant

U = Wind velocity

σ = angular frequency ($\sigma = \frac{2}{T}$), T = wave period

- (3) The coordinate system to be used in this development is that shown in Fig. 1. The bottom slope is denoted by "s", water depth by "d" and wave direction by "θ". Whereas, the shallow water spectrum is a function of (x, y, z, σ, θ, d).
- (4) Assumes that the shore line is straight and that the slope of sea bottom is uniform. So the contour lines are parallel to the shore line.
- (5) The width of the fetch area of deep water waves is supposed to be considerably large, and the points of interest are located in the middle of the fetch width. Accordingly, the effect of angular spreading is not essential in this case.
- (6) Each component wave which represents a stripe of histogram is small amplitude wave and its deformation in shallow water area is able to be calculated by following equation of energy flux.

$$(E G b)_1 - (E G b)_2 = - \frac{\partial (E G b)}{\partial x} = P_d b \quad (2)$$

where G = group velocity
 b = width between orthogonals
 P_d = rate of energy dissipation between section 1 and 2
 E = total wave energy per unit area
 x = wave propagates direction

Neglecting the effect of infiltration, P_d is mainly due to internal and bottom friction.

3. SOME CONSIDERATIONS OF ONE-DIMENSIONAL SPECTRA IN INTERMEDIATE WATER AREA

The amount of friction losses was first given by Hough (1896) basing on the small amplitude wave theory. He found that

$$\begin{aligned} P_d &= \frac{k}{2\pi} \int_{-d}^0 \int_0^L \mu \left[\left(\frac{\partial w}{\partial x} - \frac{\partial u}{\partial z} \right)^2 - 4 \left(\frac{\partial u}{\partial x} \frac{\partial w}{\partial z} - \frac{\partial u}{\partial z} \frac{\partial w}{\partial x} \right) \right] dx dz \\ &= \frac{\pi^2 \mu \beta H^2}{2 T^2 (\sinh kd)^2} \left[1 + \frac{2k}{\beta} \sinh 2kd + \dots \right] \end{aligned} \quad (3)$$

where u and w are components of fluid velocity in x and z direction respectively; $k = \frac{2\pi}{L}$, L : wave length, μ : viscosity of fluid, $\beta = \left(\frac{\sigma}{2\nu} \right)^{\frac{1}{2}}$, $\sigma = \frac{2\pi}{T}$, $\nu = \frac{\mu}{\rho}$, T : period, ρ : unit mass of fluid, H : wave height.

3.1 Situation of predominant wave direction perpendicular to shore line

If the wave direction is perpendicular to shore line, every component wave front remains parallel to the contour lines when it propagates into shallow water. Then the width between any two orthogonals are constant. According to Eq. (2), we have

$$\frac{\partial (E G)}{\partial x} = G \frac{\partial E}{\partial x} + E \frac{\partial G}{\partial x} = - P_d \quad (4)$$

The total wave energy E per unit surface area is given by

$$E = \frac{1}{8} \rho g H^2 \tag{5}$$

which is transmitted across the unit area with group velocity G. The group velocity changes from deep water into shallow water according to

$$G = n c = \frac{1}{2} \left(1 + \frac{2 kd}{\sinh 2 kd} \right) \frac{gT}{2\pi} \tanh kd \tag{6}$$

where c is the wave celerity and n is the transmission coefficient.

From Fig. 1, we see

$$d = \frac{L_0}{2} - sx \tag{7}$$

where L_0 is wave length in deep water, s is bottom slope, both d and s are positive values. We obtain following formulas from Eq. (7)

$$dx = -\frac{1}{s} d(d) = -\frac{1}{s} L_0 \ 2n \ d\xi = -\frac{gT^2}{2\pi s} \frac{1}{k_s} d\xi \tag{8}$$

where $\xi = \frac{d}{L}$, $k_s^2 = \frac{2 (\cosh 2\pi\xi)^2}{\sinh 4\pi\xi + 4\pi\xi}$, which is the so-

called shoaling coefficient.

By using Eq.(3), (5), (6), (8) together with changing variables, Eq. (4) can be expanded to yield

$$\frac{dH}{H} = J \ d\xi \tag{9}$$

where

$$J = -\frac{\pi (\sinh 4\pi\xi - 2\pi\xi \cosh 4\pi\xi + 2\pi\xi)}{(\cosh 2\pi\xi)^2 \{ (\sinh 2\pi\xi)^2 + 2\pi\xi \tanh 2\pi\xi \}} + \frac{\sqrt{2} \pi \nu^{\frac{1}{2}} \sigma^{\frac{3}{2}}}{g s (\sinh 2\pi\xi)^2} + \frac{8 \pi \nu \sigma^3}{g^2 s} (\coth 2\pi\xi)^2 \tag{10}$$

By integrating Eq.(10) and setting the lower limit of integration

in deep water where $\xi_0 = d/L_0 = \frac{1}{2}$ and the upper limit in shallow water of any depth, we obtain

$$H/H_0 = \exp \left(\int_{\xi_0}^{\xi} J d\xi \right) \quad (11)$$

where H_0 is the wave height in deep water.

Since the power spectrum density for a particular frequency is proportional to the square of wave height, i.e., $S(\sigma) d\sigma \sim H^2$. The ratio of shallow water spectrum density, $S(\sigma)$, to the deep water spectrum density, $S_0(\sigma)$, results in

$$\frac{S(\sigma)}{S_0(\sigma)} = \exp \left[2 \int_{\xi_0}^{\xi} J d\xi \right] \quad (12)$$

Using P-M spectrum, the shallow water one-dimensional frequency spectrum in this case is written as follows:

$$S(\sigma) = 0.0081 g^2 \sigma^{-5} \exp \left[-0.74 \left(\frac{g}{U\sigma} \right)^4 \right] \cdot \exp \left[2 \int_{\xi_0}^{\xi} J d\xi \right] \quad (13)$$

Each stripe of histogram of shallow water spectrum based on Eq.(13) at the place where $d/L_{op} = 0.2$ (L_{op} is the deep water wave length of optimum frequency wave) were plotted smoothly in Fig. 2. The dash line was given by Tsuchiya and Inoue (1961), which was based on the wave decay due to bottom friction in the uniform slope sea bottom. It gave

$$\begin{aligned} \ln \left(\frac{H}{H_0} \right) &= -4\pi^2 \sqrt{\pi} \left(\frac{\nu}{g^2 T^3} \right)^{\frac{1}{2}} \int_{\xi}^{\xi_0} \\ &\frac{\tanh 2\pi\xi + 2\pi\xi (\operatorname{sech} 2\pi\xi)^2}{(1 + 4\pi\xi \operatorname{csch} 4\pi\xi) \tanh 2\pi\xi (\sinh 2\pi\xi)^2} d\xi \\ &- \frac{\ln \left[(1 + 4\pi\xi \operatorname{csch} 4\pi\xi) \tanh 2\pi\xi \right]}{2(1 + 4\pi\xi_0 \operatorname{csch} 4\pi\xi_0) \tanh 2\pi\xi_0} \end{aligned} \quad (14)$$

Eq. (13) can be computed by numerical procedure to yield significant wave height. Fig. 3 shows the corresponding variation of the ratio of the shallow water significant height, H_s , to the deep water

significant wave height, H_{0s} , for various values of d/L_{op} .

3.2 Situation of predominant wave direction inclined to shore line

In the case of wave direction inclined to the shore line, the waves are affected by refraction that tend to align the their fronts to the depth contours.

Eq. (2) to be written as

$$b \frac{\partial(EG)}{\partial x} + (EG) \frac{\partial b}{\partial x} = -P_d b \tag{15}$$

By utilizing Snell's law directly, following equation is obtained

$$b = \frac{b_0}{\cos \theta_0} \sqrt{1 - (\tanh 2\pi\xi \sin \theta_0)^2} \tag{16}$$

where b_0 is the width between two orthogonals in deep water, and θ_0 is the angle between the deep water wave and the contour line.

Substituting Eq. (3), (5), (6) and (16) in Eq.(15), we obtain

$$\frac{\partial(EG)}{\partial \xi} + (EG) f(\xi, \theta_0, \sigma) = 0 \tag{17}$$

where

$$f(\xi, \theta_0, \sigma) = \frac{2\pi \tanh 2\pi\xi (\sin \theta_0 \operatorname{sech} 2\pi\xi)^2}{(\sin \theta_0 \tanh 2\pi\xi)^2 - 1} - \frac{4\sqrt{2} \pi \nu^{\frac{1}{2}} \sigma^{\frac{3}{2}}}{g s} \left[1 + \frac{4\sqrt{2} \nu^{\frac{1}{2}} \sigma^{\frac{3}{2}}}{g} (\cosh 2\pi\xi)^2 \right] \tag{18}$$

Integrating Eq. (17) to give

$$\frac{EG}{(EG)_0} = \exp\left[-\int_{\xi_0}^{\xi} f(\xi, \theta_0, \sigma) d\xi\right] \tag{19}$$

The symbol "0" refers to the case of deep water. Hence

$$\frac{H^2}{H_0^2} = \frac{S(\sigma)}{S_0(\sigma)} = K_s^2 \exp\left[-\int_{\xi_0}^{\xi} f(\xi, \theta_0, \sigma) d\xi\right] \tag{20}$$

where $K_s^2 = \frac{G_0}{G} = C_0/2nc$

Using P-M spectrum, the shallow water spectrum in this case is to be the following form

$$S(\sigma) = 0.0081 g^2 \sigma^{-5} \exp\left[-0.74 \left(\frac{g}{U\sigma}\right)^4\right] \cdot K_s^2 \exp\left[-\int_{\xi_0}^{\xi} f(\xi, \theta_0, \sigma) d\xi\right] \quad (21)$$

For $S = 0.05$, $U = 30$ Knots, $\theta_0 = \pi/12$, the relationship between H_s/H_{os} and d/L_{op} is shown in Fig. 4

4. DEFORMATION OF DIRECTIONAL SPECTRUM IN INTERMEDIATE WATER AREA AND ITS COMBINATION

In the foregoing sections we use the one-dimensional frequency spectrum in deep water to compute the frequency spectrum in shallow water. And assume that every component wave serves is a simple sinusoidal wave. In this section, we will take a more reasonable approach, namely the directional spectrum is to be used to calculate the deformation in shallow water, and then combine the shallow water directional as well as frequency spectrum to give the wave characteristics.

For practical use, the directional spectrum may be obtained from the Pierson-Moskowitz's spectrum for a fully developed sea and the directional function such as that obtained in the Stereo Wave Observation Project (SWOP), or that used by Pierson, Neumann and James (1955).

The directional function used by P-N-J and later adopted in the so-called DSA forecasting method has the form

$$\theta(\theta) = \begin{cases} \frac{2}{\pi} \cos^2 \Omega & |\Omega| < \frac{\pi}{2} \\ 0 & |\Omega| > \frac{\pi}{2} \end{cases} \quad (22)$$

where $\Omega = \theta - \psi_0$, θ is the travel direction of any component wave, ψ_0 is the predominant direction of the combined waves."

We designate the incident angle in deep water of predominant direction by ψ_0 , as shown in Fig. 5. For any component wave of angular frequency σ_i and travel direction θ_j , the directional spectrum in deep water is then given by

$$S_o(\sigma_i, \theta_j) = 0.0081 g^2 \sigma_i^{-5} \exp[-0.74 (\frac{g}{U \sigma_i})^4] \cdot \frac{2}{\pi} \cos^2(\theta_j - \psi_0) \quad (23)$$

whereas the corresponding shallow water directional spectrum is obtained according to Eq. (20) as follows

$$S(\sigma_i, \theta_j') = S_o(\sigma_i, \theta_j) K_s^2 \exp[-\int_{\xi_0}^{\xi} f(\xi, \theta_j, \sigma_i) d\xi] \quad (24)$$

Shallow water frequency spectrum is written as

$$S(\sigma_i) = \int_0^{2\pi} S(\sigma_i, \theta_j') d\theta_j' \quad (25)$$

where θ_j' is the refracted direction angle in shallow water which was originally denoted by θ_j in deep water.

From Snell's law

$$d\theta_j' = \frac{\tanh 2\pi\xi \cos \theta_j}{\sqrt{1 - (\tanh 2\pi\xi \sin \theta_j)^2}} d\theta_j \quad (26)$$

Consequently, we finally obtain the frequency spectrum of shallow water as follows

$$S(\sigma_i) = \int_{\alpha}^{\beta} 0.0081 g^2 \sigma_i^{-5} \exp[-0.74 (\frac{g}{U \sigma_i})^4] \frac{2}{\pi} \cos^2(\theta_j - \psi_0) \cdot K_s^2 \exp[-\int_{\xi_0}^{\xi} f(\xi, \theta_j, \sigma_i) d\xi] \frac{\tanh 2\pi\xi \cos \theta_j}{\sqrt{1 - (\tanh 2\pi\xi \sin \theta_j)^2}} d\theta_j \quad (27)$$

where $\alpha = -\frac{\pi}{2} + \psi_0$, $\beta = \frac{\pi}{2}$ because the component waves which direct toward offshore should be excluded here.

The double integrals were evaluated by using finite difference method for a single angular frequency. Fig. 6 and Fig. 7 show the deformation of frequency spectra at various values of d/L_{op} for

$\Psi_0 = 0^\circ$ and $\Psi_0 = \frac{\pi}{6}$ respectively. Fig. 8 shows the comparison of frequency spectrum deformation obtained from considering directional spectrum to that of neglecting direction function, with the value of $d/L_{op} = 0.2$ and $\Psi_0 = 0$. Fig. 9 shows the relationship between the relation of significant wave height and the values of d/L_{op} .

5. COMPARISON IN RESULT OF FOREGOING METHOD AND THE TRADITIONAL METHOD

5.1 Traditional method

In order to compare the two method at the same basic conditions, Preison-Moskowitz's spectrum was adopted to calculate the deep wave height and period in the case of the fully arising spectrum in $U = 30$ knots, we have

$$H_0 = 4 \sqrt{E} = 5.31 \text{ m} \tag{28}$$

$$T_{op} = 11 \text{ sec}$$

For bottom slope $s = 0.002$, the shore line locates far from the origin of coordinate system (Fig. 1) at the distance of $x = 47,000$ m.

The traditional method for calculating shallow water waves based on the equation shown by

$$H = K_s K_{ra} K_d H_0 \tag{29}$$

where K_s is the shoaling coefficient; K_{ra} is the refraction coefficient and can be obtained by drawing the refraction diagram with straight shore and parallel contours; K_d is the decay coefficient, here we consider friction loss only.

A numerical procedure is utilized by subdividing the whole distance x into small segments; in each increment Δx , the depth is considered constant; and numerical calculations are performed

step by step. The significant wave height at the depth $d=10$ m is computed to be $H = 2.42$ m, here we assumed the bottom friction coefficient $f = 0.01$.

5.2 The abovementioned method

As discussed in previous sections, the parameter d/L is a variable according to the wave period. Since wave length L is the function of period, the same value of d/L does not denote the same point in the shallow water. Hence, in practical computation, one must change the Eq.(27) into the parameter of depth d instead of relative depth d/L . Through long term of derivations, we obtain

$$\begin{aligned}
 S(\sigma) = & \int_{-\frac{\pi}{2}+\psi_0}^{\frac{\pi}{2}} 0.0081 g^2 \sigma^{-5} \exp[-0.74 \left(\frac{g}{U\sigma}\right)^4] \frac{2}{\pi} \cos^2(\theta_j - \psi_0) \\
 & \cdot \exp\left\{ \int_{\text{asl}}^d \left\{ \frac{2\pi (\sin \theta_j \sinh \frac{2\pi d}{L})^2}{[1 - (\tanh \frac{2\pi d}{L} \sin \theta_j)^2] [L_0 (\sinh \frac{2\pi d}{L})^2 + 2\pi d]} \right. \right. \\
 & - \frac{8\pi \left(\frac{1}{2} \sinh \frac{4\pi d}{L} - \frac{\pi d}{L} \cosh \frac{4\pi d}{L} + \frac{\pi d}{L}\right)}{(\sinh \frac{4\pi d}{L} + \frac{4\pi d}{L}) [L_0 (\sinh \frac{2\pi d}{L})^2 + 2\pi d]} + \frac{4\sigma^3 \nu \beta}{g^2 S} (\coth \frac{2\pi d}{L})^2 \\
 & \left. \left. \cdot \left(1 + \frac{4\pi d}{L\beta} \sinh \frac{4\pi d}{L}\right) \right\} d(d) \right\} \frac{\tanh \frac{2\pi d}{L} \cos \theta_j}{\sqrt{1 - (\tanh \frac{2\pi d}{L} \sin \theta_j)^2}} d\theta_j \quad (30)
 \end{aligned}$$

At the depth $d = 10$ m, the spectral density is shown in Fig. 10. We find that $H_{1/3} = 4\sqrt{E} = 1.51$ m.

6. SUMMARY AND CONCLUSION

- (1) From the computation as shown in Fig. 6 and Fig. 7, the peak of spectrum will never change when wave propagates over the intermediate water area where the contour lines are parallel to the shore line. However, if the configuration of sea bottom is complex, the results will be different.

- (2) The wave theory adopted herein was the linear wave theory. The foregoing method is adequate for the range of $d/L \geq 0.2$.
- (3) The foregoing approaches did not take the effect of wind in transformation area into consideration, it can only be adequate to swell in the present state.
- (4) In actual engineering problems, numerical method calculating spectra deformation on irregular sea bottom can be devised basing on abovementioned principles. However, it would not be accuracy until the caustic effect of wave rays has satisfactory explanation.

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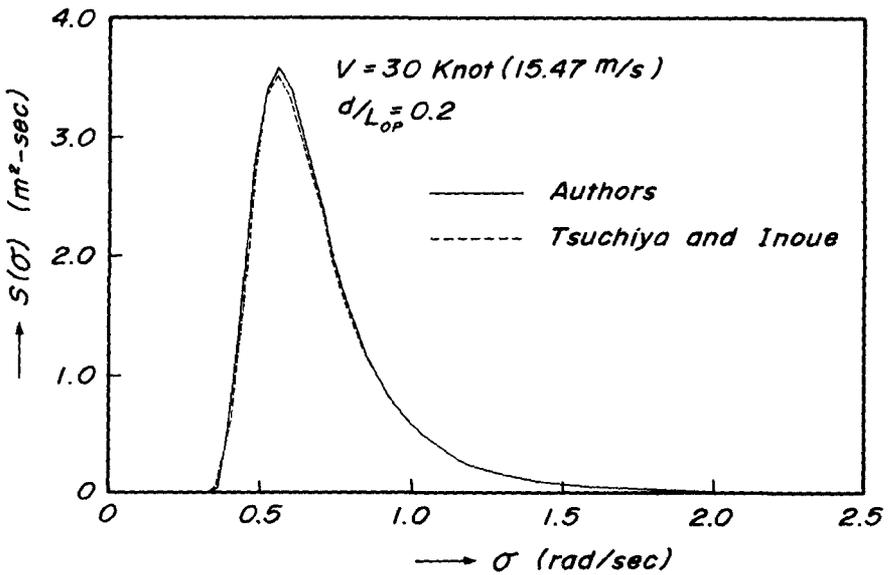
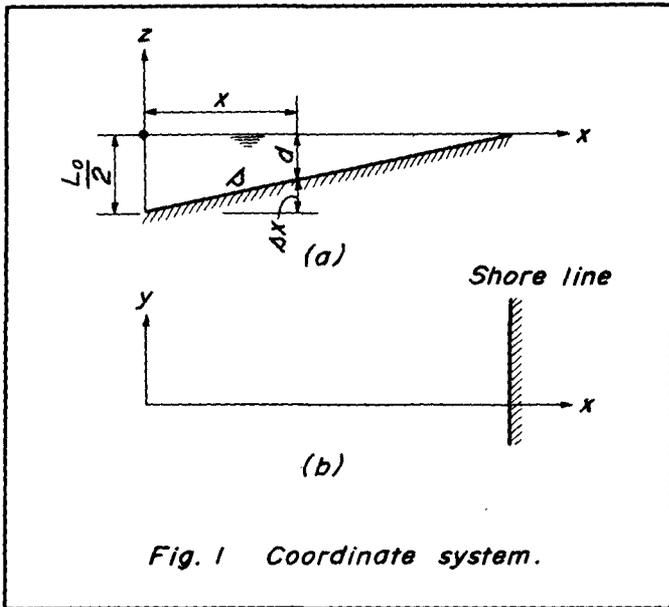


Fig. 2 Shallow water frequency spectrum.

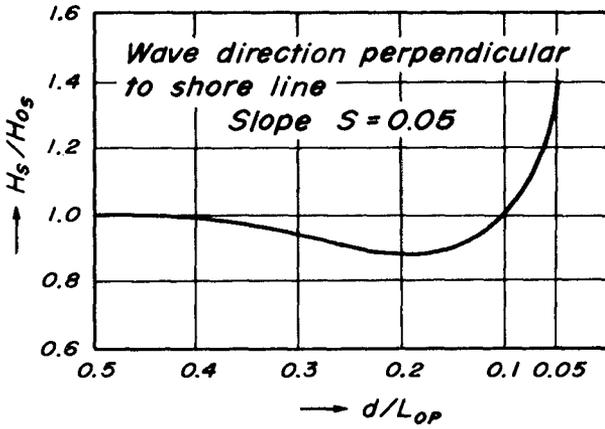


Fig. 3 Ratio of significant wave heights.

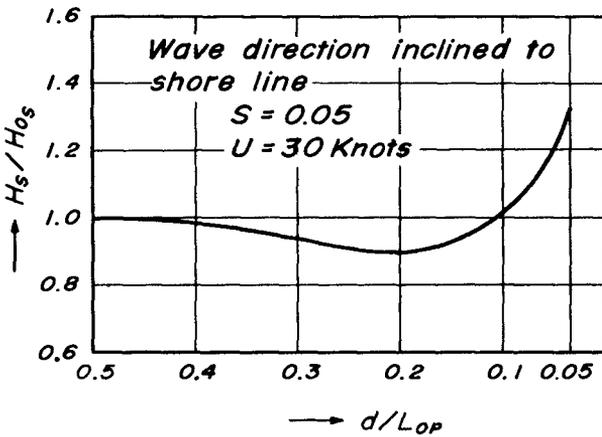


Fig. 4 Ratio of significant wave heights

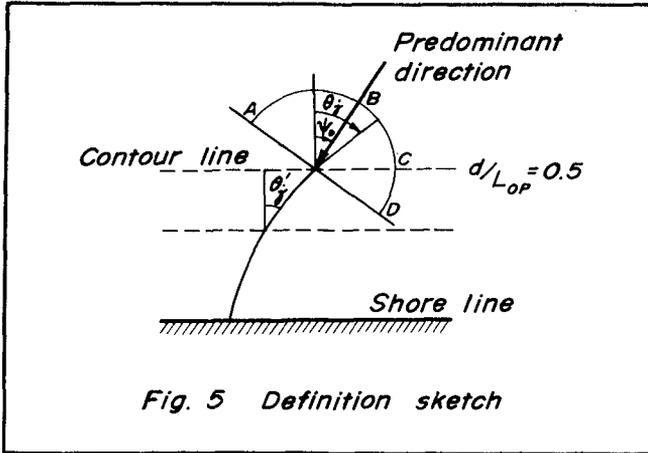


Fig. 5 Definition sketch

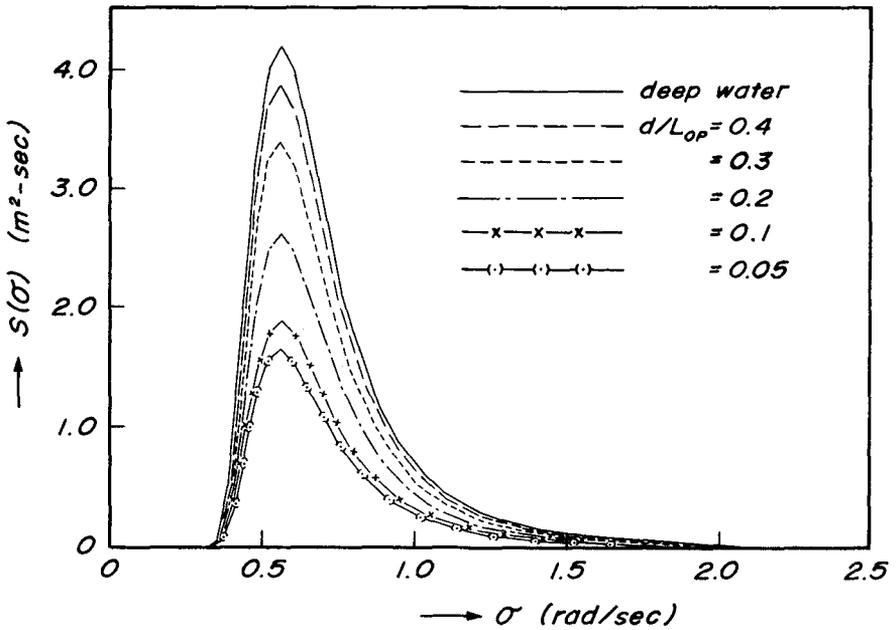


Fig. 6 Deformation of frequency spectra for $\psi_0 = 0^\circ$

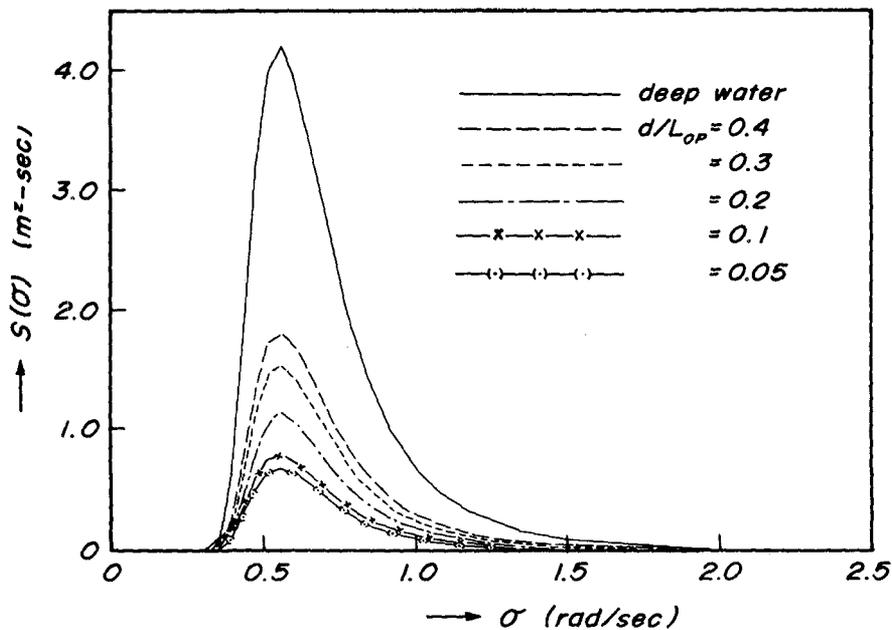


Fig. 7 Deformation of frequency spectra for $\psi_0 = \frac{\pi}{6}$

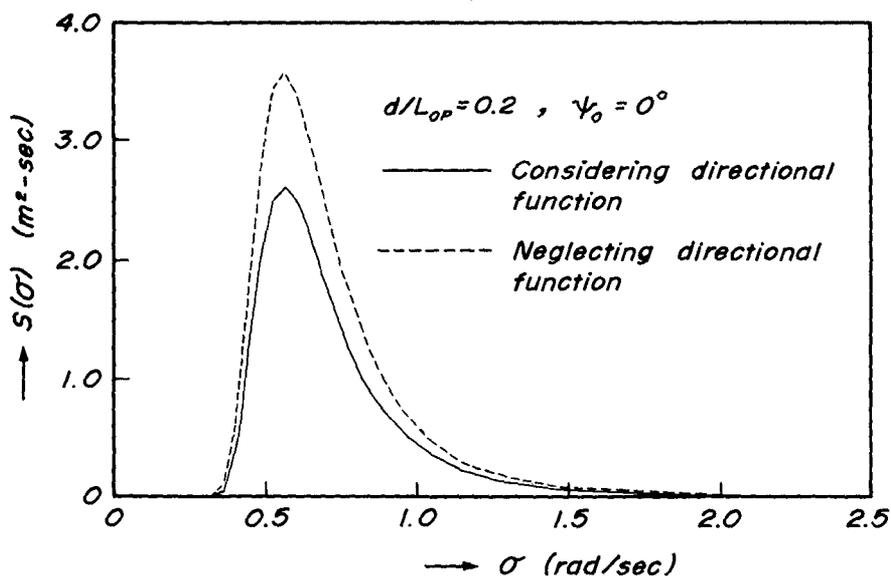


Fig. 8 Comparison of deformation of frequency obtained by different approaches.

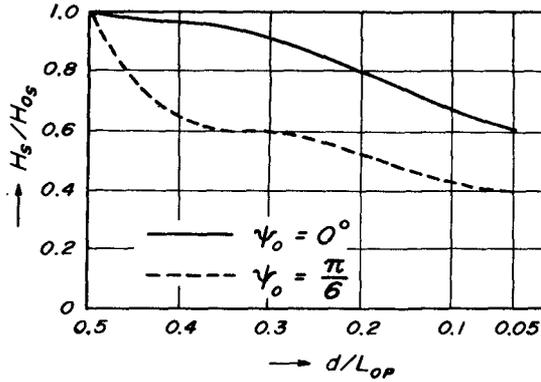


Fig. 9 Ratio of significant heights obtained from directional spectrum.

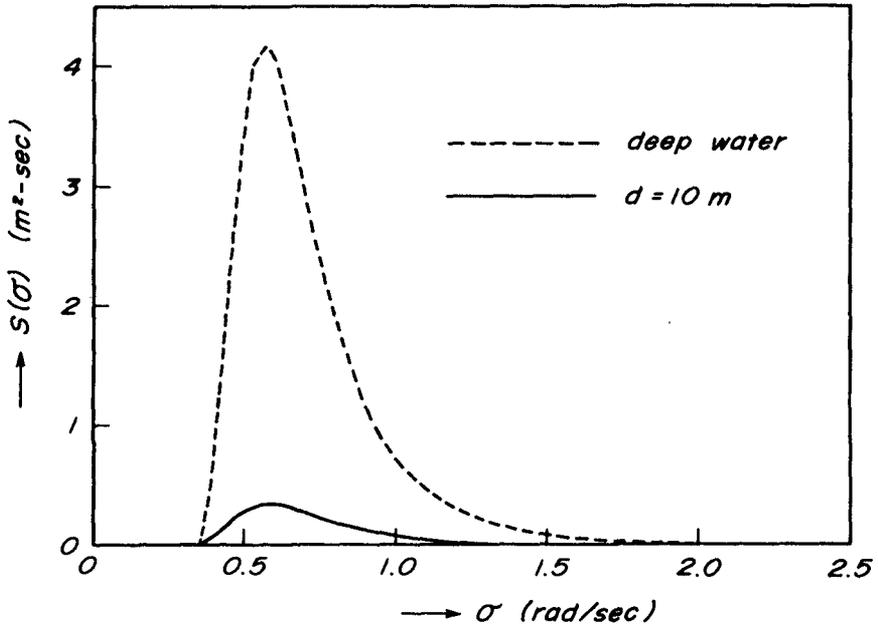


Fig. 10 Computed frequency spectral density.