CHAPTER 3

A Discussion of Some Measured Wave Data

by

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ABSTRACT

The significant wave height and peak period as derived from the spectral analysis of 171 measured wave records taken in the ocean off Western Head, Nova Scotia are compared to the more classical parameters derived from individual wave heights and by the Tucker method. The highest surface elevation and the maximum wave height occurring in the records are compared to values predicted by Cartwright and Longuet-Higgins (1956), Goda (1970), and Longuet-Higgins (1952).

1. Introduction

The Canadian Wave Climate Study has been in operation since 1968, as a joint venture between the Department of Public Works and the Department of the Environment. The present study was patterned after the pioneer one conducted earlier by the National Research Council of Canada in Lake Superior and the Gulf of St. Lawrence. The purpose of the effort was to obtain data in direct support of specific engineering projects, and to provide general coverage of large areas of the coastline. This has led to the maintenance by the study during the past two years of in excess of twenty-five wave recording stations, each recording for 20 minutes every 3 hours.

Because of the large quantity of wave records being collected, it became necessary to employ digital computing techniques if the analysis was to keep pace with the field program. Accordingly, a suitable recording and digitizing system, similar in general philosophy to the one employed earlier by the National Research Council, was developed. The main difference in the new system was its orientation towards the production of a permanent file of wave profiles on digital magnetic tape in addition to an analysis for wave parameters.

At that point it became necessary to make a decision as to the analysis technique to be employed. It soon became obvious that the most economical and practical method was to compute the spectrum of each 20 minute wave record using the fast fourier transform algorithm developed by Cooley and Tukey. The reasons for this were, firstly, that the signals recorded often contained some low and high frequency noise outside the wave frequencies. This made it necessary to digitally low pass and, in some cases, even band pass filter the data before reliable results could be obtained from a

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computer algorithm for picking off crests, troughs, zero crossings, etc., for a traditional analysis. Secondly, the computer algorithm developed to pick off the crests, etc., and perform the traditional analysis, proved to be slightly more expensive than the fourier transform. Adding to this the cost of filtering, the traditional analysis became prohibitively expensive. The spectral analysis of a 20 minute wave record presently costs the study about 32 cents.

A drawback of adopting this technique as our standard analysis procedure was that it has not been that commonly used. Consequently, many applications and design techniques are based on parameters that are not directly derived from the spectrum. The zero crossing period, for example, has no spectral equivalent. Design may require the most probable maximum crest to trough wave height or the significant wave height defined as the mean of the highest one-third of the waves. Present theory permits derivation of these quantities directly from the specturm only when the spectrum consists of a single narrow band of frequencies.

The work reported here was undertaken to gain some insight, for ourselves and users of our data unfamiliar with the spectrum approach, into the empirical relations between the spectral parameters and the more classical ones. In particular, we are interested in this comparison for routinely gathered wave data featuring a range of spectral widths. Also because design techniques often require information on extreme wave heights or maximum water elevations, we required to compare observed values with those predicted by theory.

2. Wave Data

The wave data presented here was collected off Western Head, Nova Scotia, between May 1970 and July 1971. The wave measuring device employed was the Dutch waverider accelerometer buoy. It was moored at a point where the water depth was approximately 20 fathoms, as shown in Figure 1. The water surface height was telemetered to the navigation light station at Western Head and recorded as an F.M. signal on a commercial quality stereo tape recorder.

In selecting the records to be used in the analysis, care was taken to obtain a reasonably uniform distribution of spectral widths. The lowest value measured in the 15 months was 0.41, the highest 0.91. The distribution was fairly uniform from 0.5 to 0.8 with a few values between 0.41 and 0.5 and between 0.8 and 0.9. In all, 171 records were used in the exercise.

The data was first passed through our standard system of programs to obtain the spectral parameters. In computing the spectrum the records were broken into 8 sections, each section consisting of 1,024 samples of water surface height. The time step between samples was chosen to give about 60 estimates of spectral density in the frequency band corresponding to 2 to 20 second period waves. The spectral density was then computed as the mean of the 8 values at each frequency.

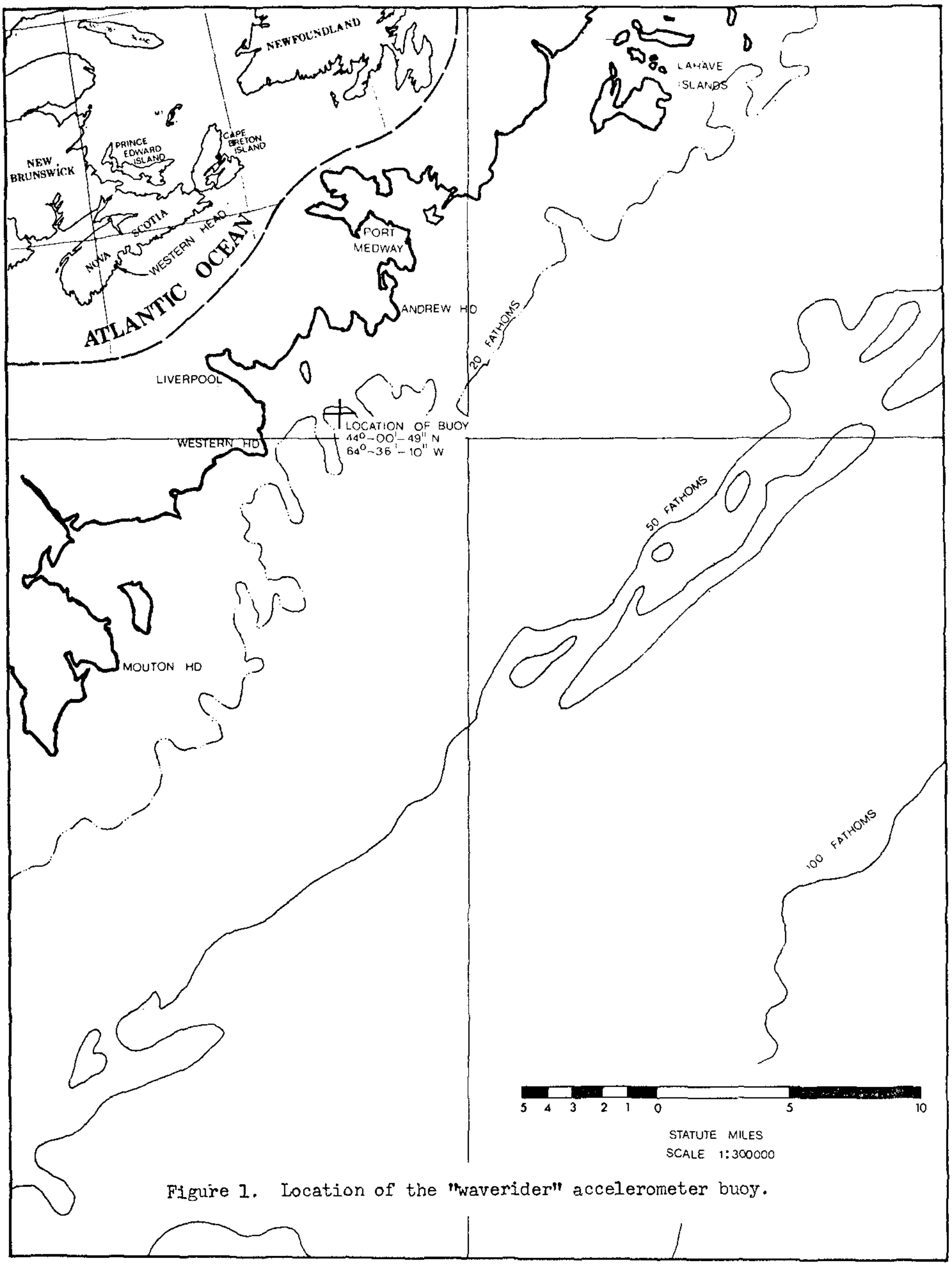

Figure 1. Location of the "waverider" accelerometer buoy.

Secondly, the data was filtered with a sharp cut-off phase shiftless digital band-pass filter which effectively removed frequencies above 0.7 and below 0.025 Hz. Algorithms were then developed to pick off crests, troughs, zero crossings, etc., and compute from these the non-spectral parameters discussed in the following section.

3. Experimental Results

3.1 Significant Wave Height

Figure 2 shows the comparison between the significant wave height defined as $4 m_0^{1/2}$ and $H_{1/3}$, the mean of the highest 1/3 of the crest to trough waves. Here m_0 is defined as the zeroth moment of the spectrum integrated between frequencies corresponding to periods of 2 and 20 seconds. The crest to trough wave is defined as the difference in height between a crest and the preceeding trough.

The slope of the least squares line is 0.8. The spectrum has generally yielded a significant wave height of the order of 25 percent greater than $H_{1/3}$. The cluster of points in the lower left corner of the plot is contributed by the records with the smaller values of spectral width. As would be expected the scatter is considerably less and this data would be better fitted by a line of steeper slope.

Figure 3 shows the comparison of the significant wave heights as derived by the Tucker and spectrum methods. The second highest crest and lowest trough were used for the Tucker analysis in this case. The scatter has slightly increased but general agreement is improved as the slope of the least squares line is 0.87.

Figure 4 shows the comparison of the mean of the highest 1/3 of the zero upcrossing waves and the significant wave height from the spectrum. The zero upcrossing wave was calculated as the difference in height between the highest surface elevation between a zero upcrossing and the following downcrossing and the the lowest surface depression between the previous zero downcrossing and upcrossing. The slope of the least squares line is 0.94 and the correlation coefficient is 1.00.

The rather good agreement should not be surprising. The slope of the spectrum for most of the records considered would be a steep rise at the low frequency end to a peak followed by a fall at a rate something like f^{-5}. The integral under the spectrum would be relatively insensitive to the presence of higher frequencies because of the f^{-5}. By calculating the zero upcrossing waves as defined, the effect of small waves carried on large waves has essentially been ignored. It would seem, therefore, that both analyses have been determined by waves with frequencies in the neighbourhood of

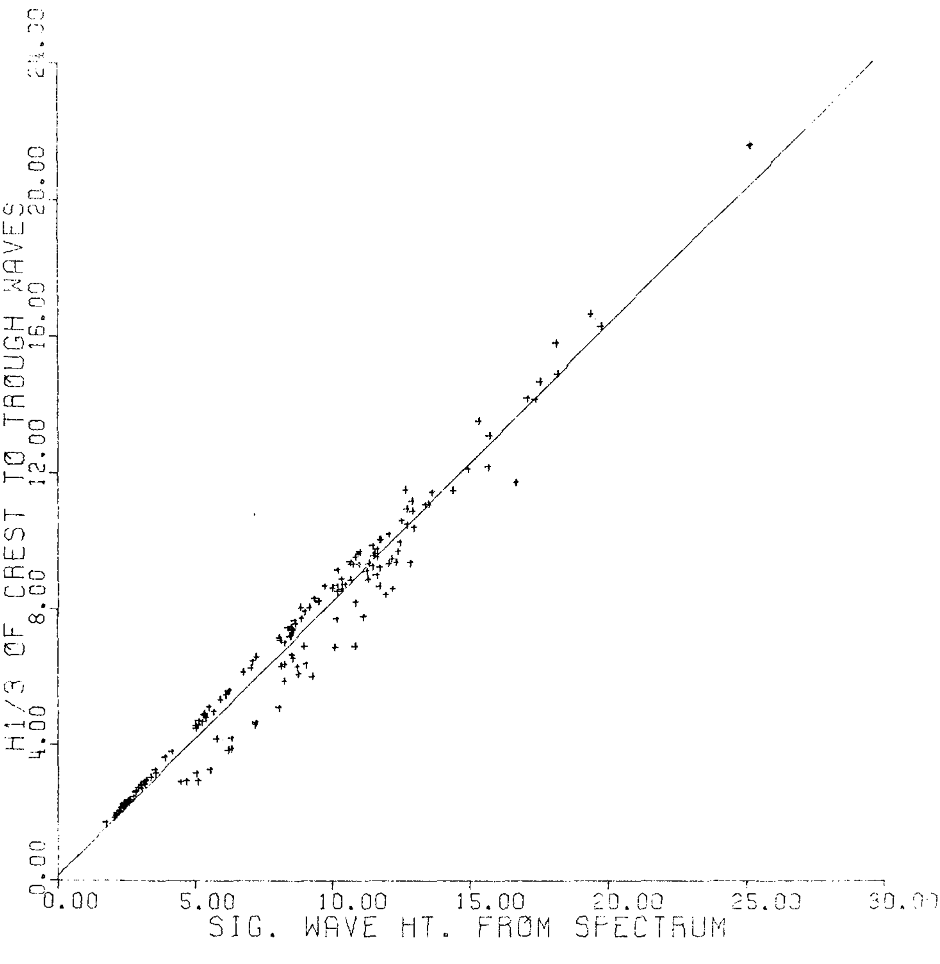

Figure 2. H 1/3, in feet, of crest to trough waves plotted against the significant wave height, in feet, as obtained from integration under the spectrum. The correlation coefficient is 0.99 and the slope of the least squares line is 0.80.

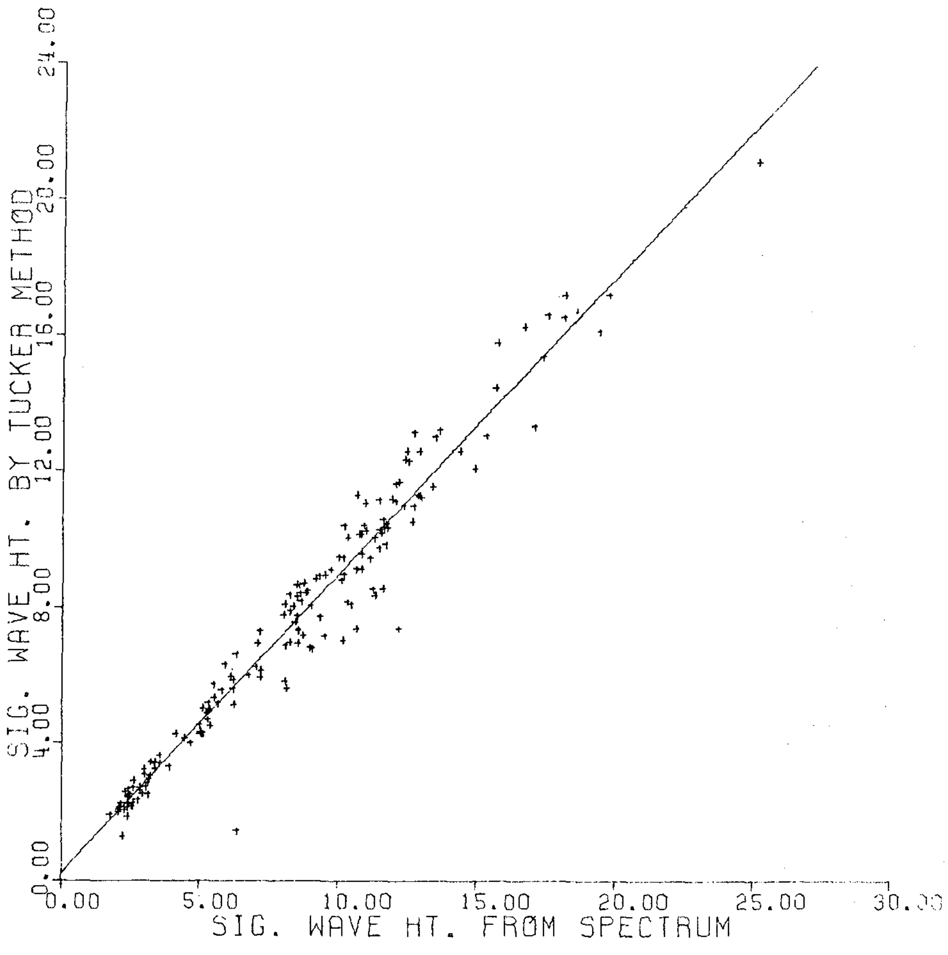

Figure 3. Significant wave height in feet as obtained by the Tucker method plotted against significant wave height as obtained from integration under the spectrum. The correlation coefficient is 0.98 and the slope of the least squares line is 0.87.

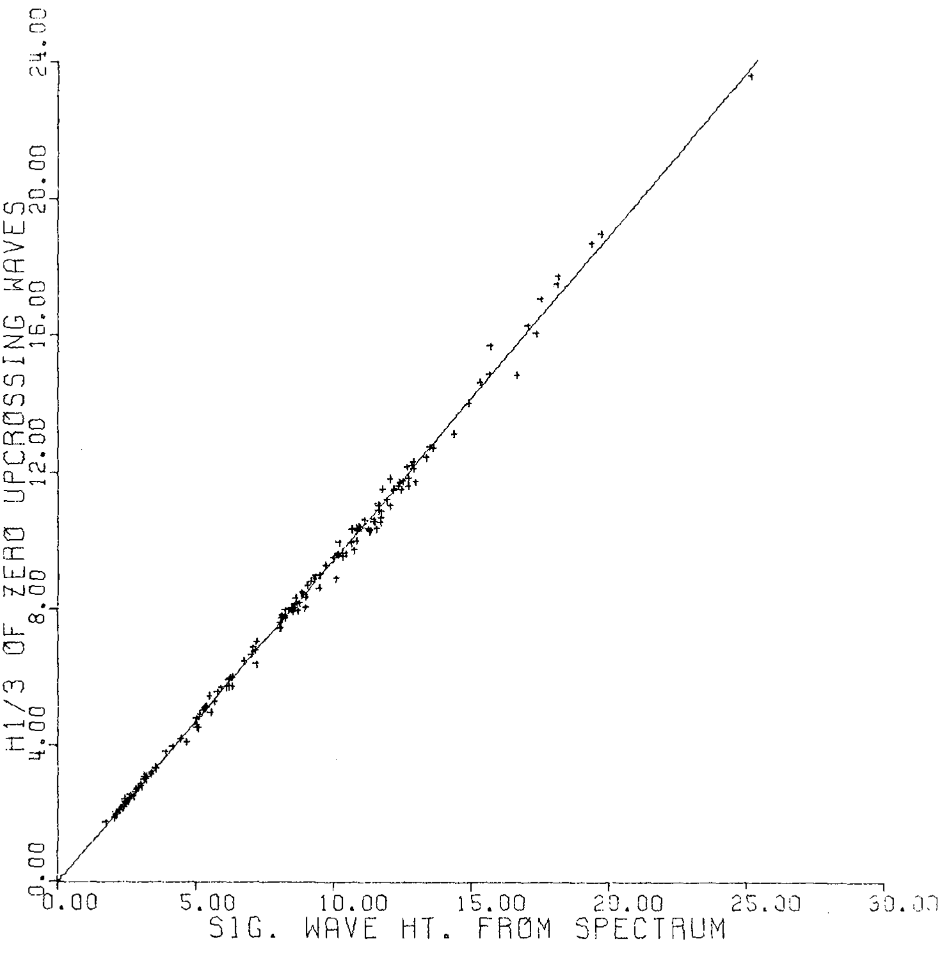

Figure 4. H 1/3 in feet of zero upcrossing waves plotted against the significant wave height as obtained from integration under the spectrum. The correlation coefficient is 1.00 and the slope of the least squares line is 0.94.

the peak of the spectrum. Consequently, the statistics of the zero upcrossing waves should be rather similar to those of the crest to trough waves in the case of a wave record featuring a single narrow band of frequencies. This is in part borne out by the observations that zero upcrossing waves are Rayleigh distributed irregardless of spectral width. These comments, of course, do not necessarily apply to a double peaked spectrum.

3.2 Wave Period

Figures 5, 6 and 7 are the comparisons between the peak period of the spectrum, and the zero crossing, significant crest to trough and significant zero upcrossing periods respectively. The peak period of the spectrum is defined as the inverse of the frequency at which the maximum spectral density occurred. The zero crossing period was computed in the usual manner. The significant crest to trough period is the mean of the periods of the highest one-third of the crest to trough waves. The significant zero upcrossing period is the same parameter for the zero upcrossing waves.

The correlation coefficients of 0.89 to 0.98 came as a surprise as various people had commented to us that these correlations did not exist. In retrospect it is clear that for the type of wave records analysed they must exist. If the type of waves encountered are those associated with more or less fully developed wind generated seas, then the spectrum will usually be of the single peaked f^{-5} type discussed earlier. If this is the case, the waves in the neighbourhood of the peak of the spectrum will essentially determine when the surface crosses zero. Lacking strong phase coherence, the other waves simply do not possess enough amplitude to overcome the large waves very often.

Since most of the wave records analysed here had significant wave heights in excess of 5 feet, the fully developed condition usually existed and, therefore, the high correlation. If lower significant wave heights had been considered, more double peaked spectra might have been encountered with a resulting decrease in the correlation.

3.3 Height of the Maxima of Surface Elevation

A comprehensive theory has been developed for the statistical behaviour of the maxima of surface elevation by Cartwright and Lonquet-Higgins (1956). The maxima are defined as the heights of the crests above the zero line. The theory shows that the statistical distribution of these maxima when normalized by $m_0^{1/2}$ is a function

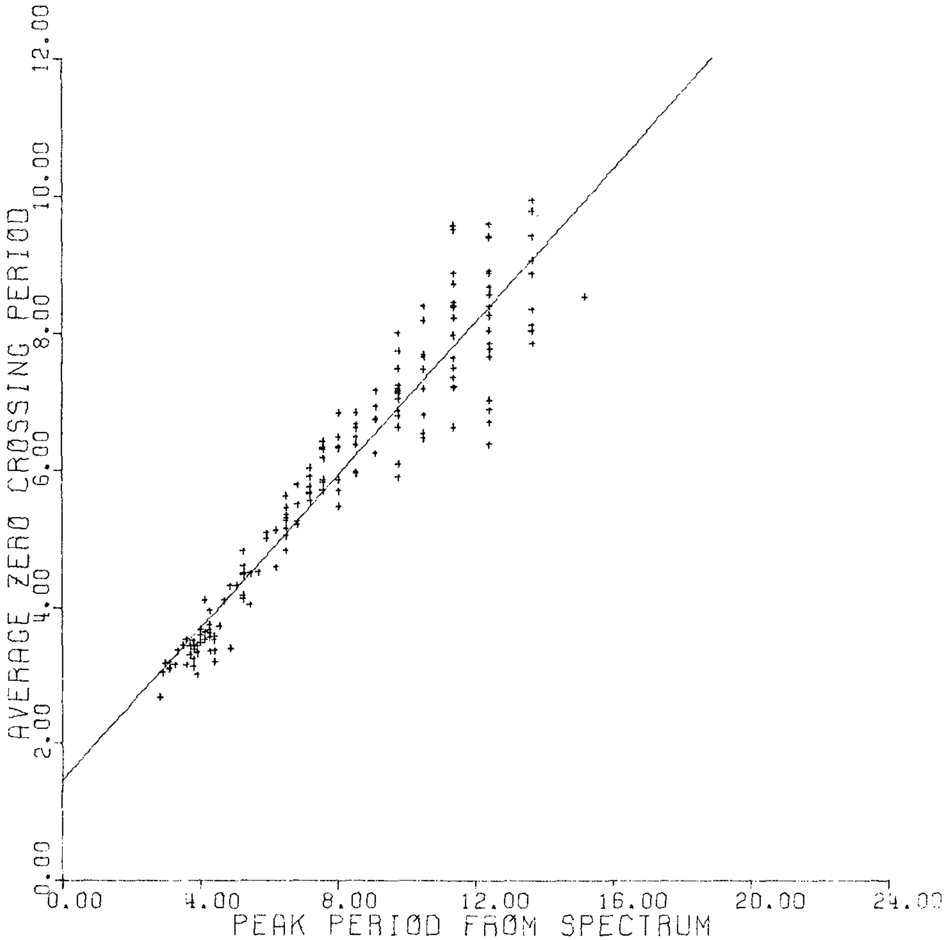

Figure 5. Average zero upcrossing period in seconds plotted against the peak period of the spectrum. The correlation coefficient is 0.96 and the slope of the least squares line is 0.56.

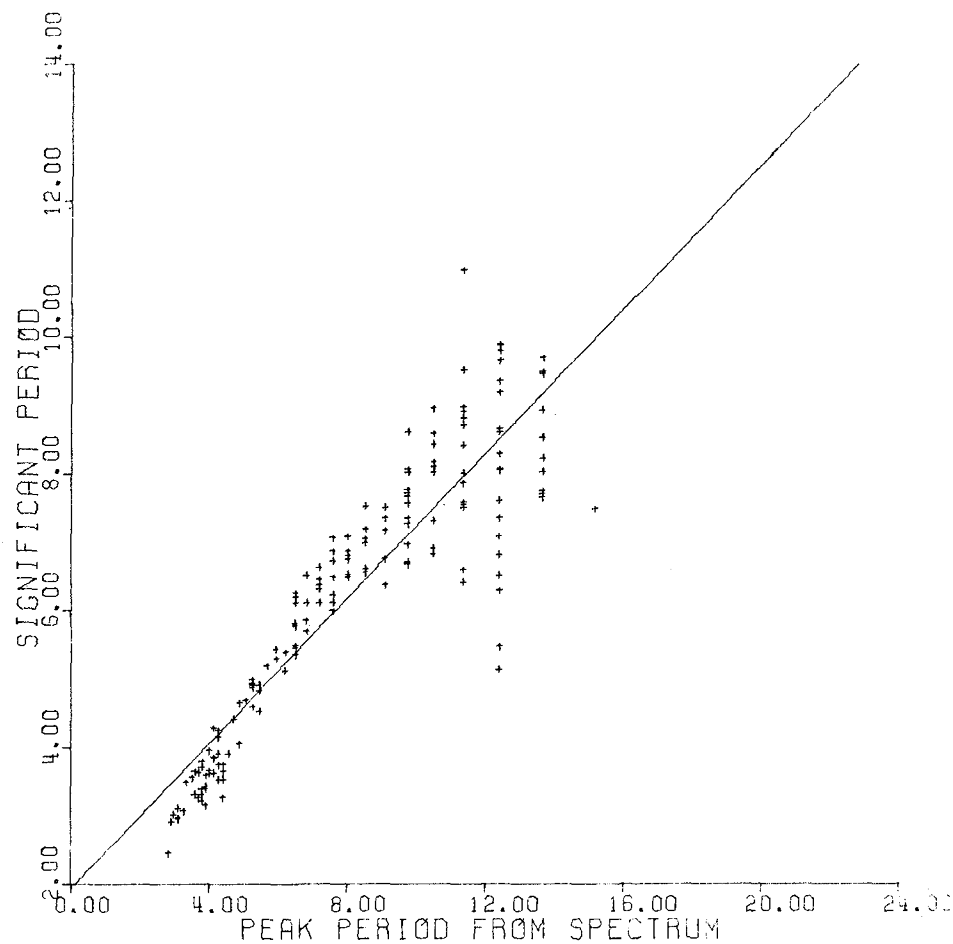

Figure 6. Significant period in seconds of the crest to trough waves plotted against the peak period of the spectrum. The correlation coefficient is 0.90 and the slope of the least squares line is 0.53.

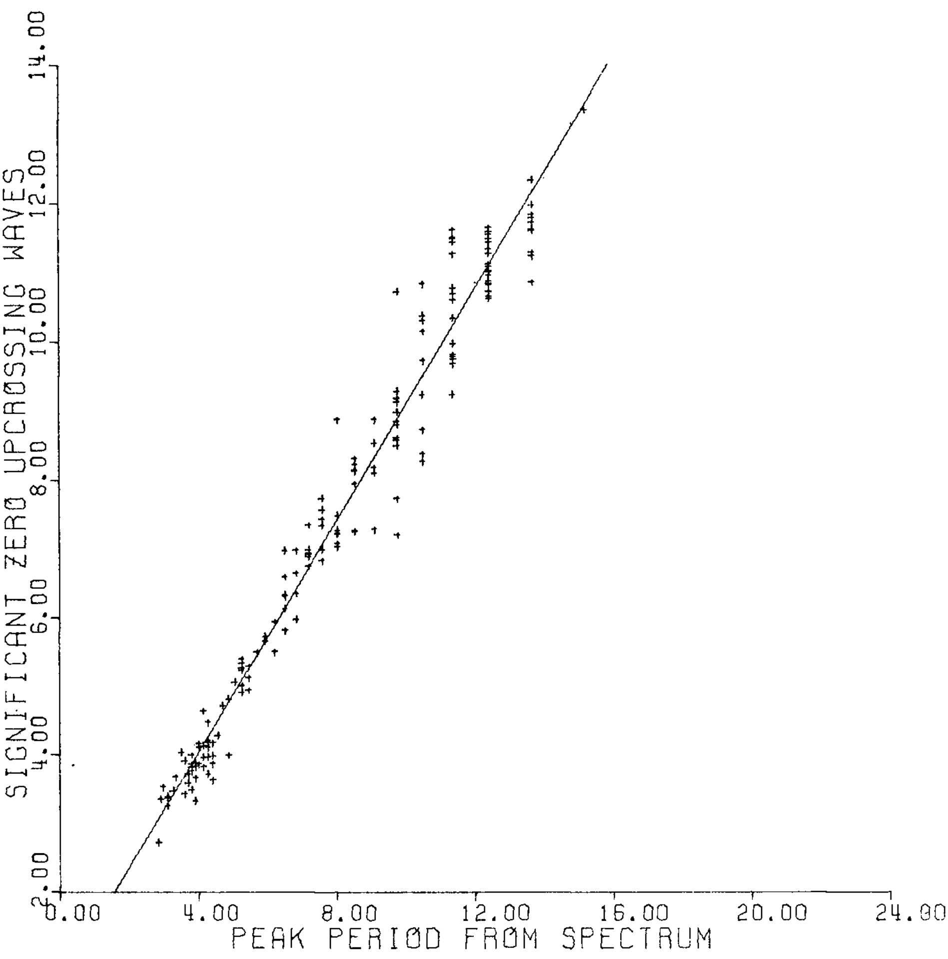

Figure 7. Significant period in seconds of the zero upcrossing waves plotted against the peak period of the spectrum. The correlation coefficient is 0.98 and the slope of the least squares line is 0.84.

only of the spectral width. $m_0^{1/2}$ is the square root of the integral under the spectrum and the spectral width is defined as:

$$\varepsilon^2 = \frac{m_0 m_4 - m_2^2}{m_0 m_4}$$

where m_n is the n^{th} moment of the spectrum taken about zero.

From the distribution the following relations are developed:

$$\mu_2' = 2 - \varepsilon^2$$

$$\overline{\eta}_{max} = \sqrt{2}\left\{ \left(\ln\left((1-\varepsilon^2)^{\frac{1}{2}} N\right)\right)^{1/2} + \frac{\gamma}{2}\left(\ln\left((1-\varepsilon^2)^{1/2} N\right)\right)^{-1/2} \right\}$$

where μ_2' is the second moment of the normalized maxima, η taken about zero, $\overline{\eta}_{max}$ is the expected mean of the highest maximum in the sample, and N is the No. of waves in the sample.

This leads to:

$$\frac{\overline{\eta}_{max}}{(\mu_2')^{\frac{1}{2}}} = \frac{\left[\ln\left((1-\varepsilon^2)^{1/2} N\right)\right]^{\frac{1}{2}} + \frac{\gamma}{2}\left[\ln\left((1-\varepsilon^2)^{\frac{1}{2}} N\right)\right]^{-1/2}}{(1-\frac{1}{2}\varepsilon^2)^{\frac{1}{2}}}$$

which describes the ratio of the maximum in a sample to the root-mean-square as a function of spectral width.

Figure 8 shows a comparison of the above theoretical relations with the observed values for the first 100 maxima in each of the 171 wave records. The records were grouped according to spectral width and then the means and standard deviations in the means were plotted as the error bars. The smooth curve in each case is the expected theoretical value.

In the upper plot only the first point shows significant departure of $(\mu_2')^{\frac{1}{2}}$ from the theoretical value. Both the first and last points were determined by only a few records. These records were characterized by significant wave heights small enough so that the signal to noise ratio of the recording system might have been important. Little weight can therefore be given to the departure.

The middle plot describing the normalized expected maximum in 100 waves as a function of spectral width shows good agreement with the theory with no significant departures.

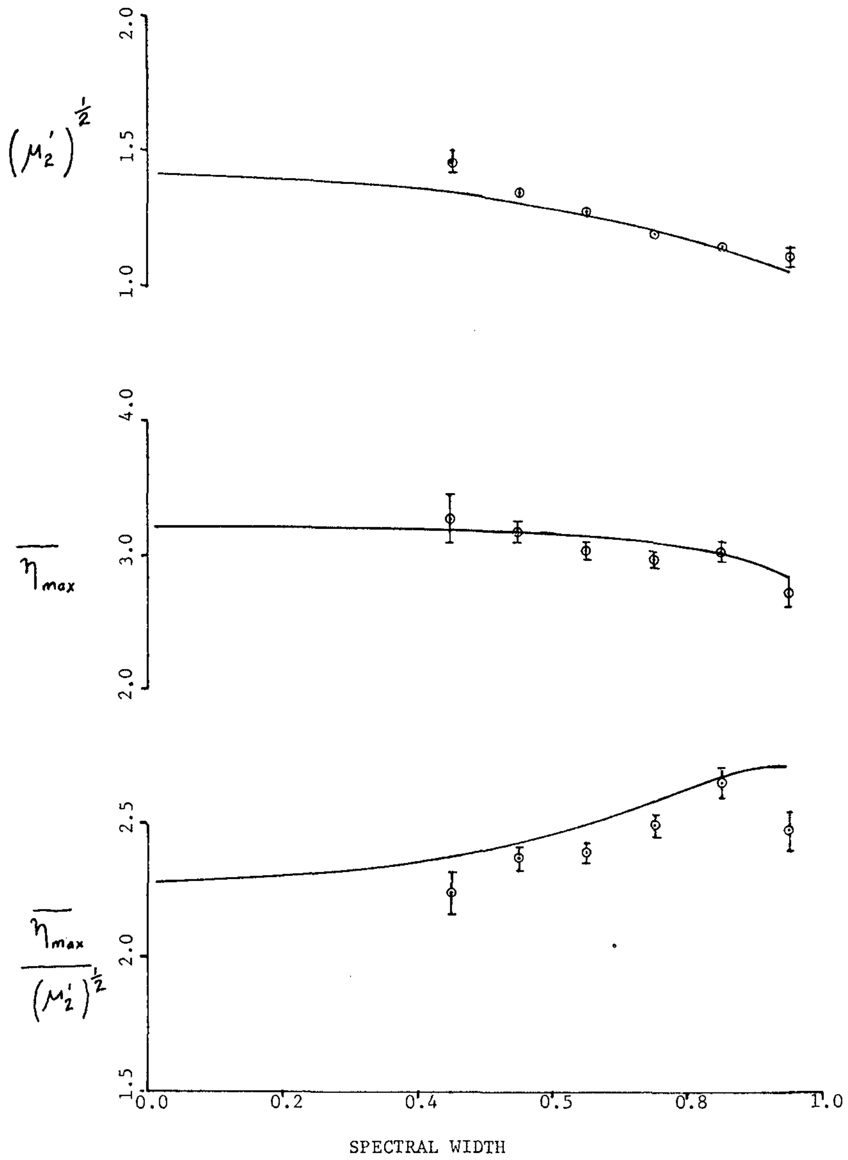

FIGURE 8. Parameters of Cartwright and Longuet-Higgins (1956) plotted against spectral width. The error bars represent the mean and the standard deviation in the mean of the observed values. The continuous lines are the values predicted by the theory.

The lower plot shows the mean of the ratio of the largest in the sample to the root-mean-square as a function of spectral width. The points corresponding to the smallest and largest values of the absissa are in doubt as mentioned earlier. Although it would appear that the other observed points tend to take a slightly lower value than the theoretical, the agreement must be considered good as the largest deviation is approximately 4%.

Figure 9 is a comparison of the experimentally observed distribution of the highest maximum in a train of 100 zero upcrossing waves compared to the theoretical distribution as developed by Goda (1970) from the theories of Cartwright and Longuet-Higgins (1956) and Davenport (1964). It is expected that the records for which the signal to noise ratio was poor would in this case also tend to yield too small values of the highest maximum. This might account for the excess of low values. However, the conclusion reached by Goda in his simulation study that the observed concentration was narrower than the theoretical would not seem to be justified here. The number of observed values in excess of 4 is greater than the theory predicts.

The theoretical curve is given by $P(\eta_{max}) = \eta_{max}\, \xi\, e^{-\xi}$

where, $\xi = N_0\, e^{-\frac{\eta_{max}^2}{2}}$ $\quad N_0 = 100$

3.4 Maximum Wave Amplitude

Figure 10 shows plots of the ratio of the maximum wave amplitude to the root-mean-square wave amplitude as a function of spectral width for the first 100 waves of each record. The upper plot is for zero upcrossing waves and the lower for crest to trough waves. The horizontal line corresponds to the expected value of 2.28 proposed by Longuet-Higgins (1952) for 100 waves. In the case of the crest to trough waves the observed values increase with spectral width as expected. The value for the small spectral widths is less than the expected theoretical value by about 3 to 5%. In the case of the zero upcrossing waves the observed value appears to be nearly independent of spectral width as would be expected under the assumption that the statistics of the zero upcrossing waves are similar to crest to trough waves with a small value of spectral width. The value is also slightly lower than the predicted 2.28 however.

3.5 Runs of Waves greater than $H_{1/3}$.

A run of waves of length j is said to have occurred when j waves in a row have exceeded some predetermined value. Goda (1970) gives the probability of a run of waves greater than $H_{1/3}$ and of length j occurring as:

$$P_1\ (j) = p^{j-1}Q$$

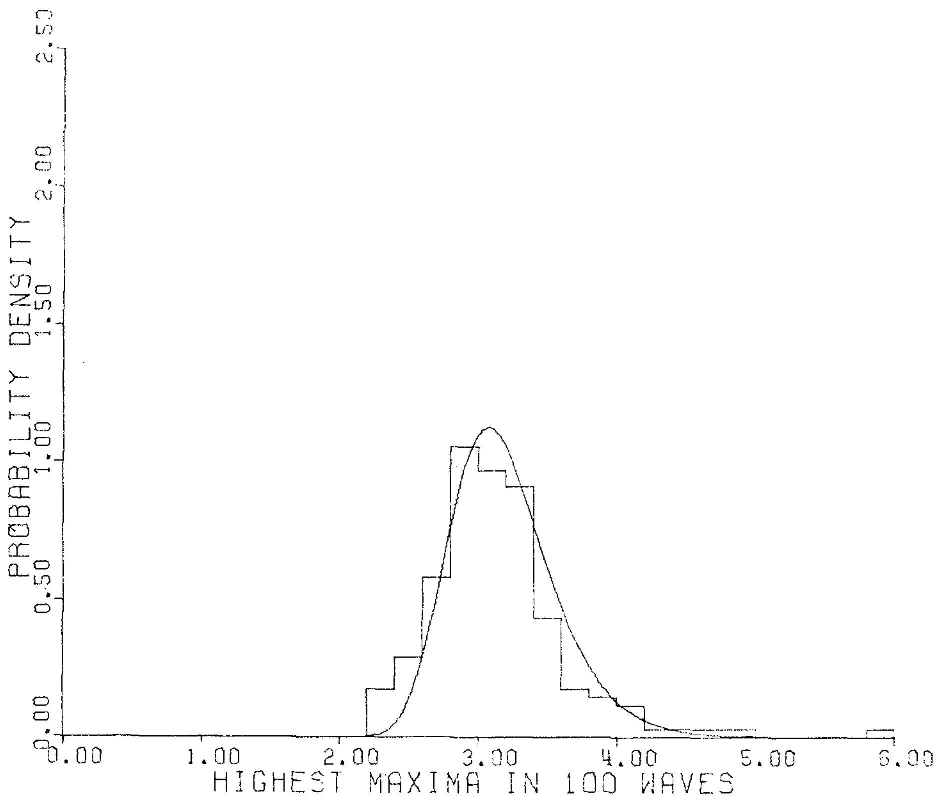

Figure 9. Observed and theoretical values of the highest maxima (normalized) in 100 zero upcrossings plotted gainst the probability density. The bar graph is the observed data and the smooth curve the predicted probability density function.

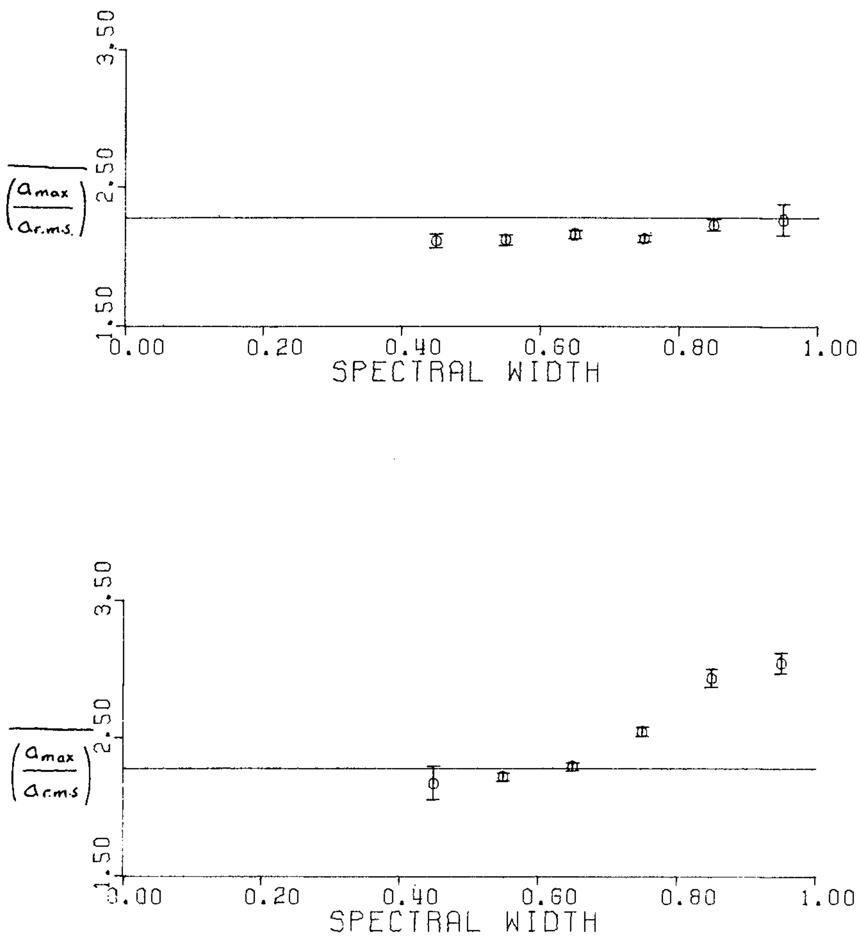

Figure 10. The observed ratio of the highest wave amplitude in a train of 100 waves to the root-mean-square wave amplitude plotted against spectral width. The top plot is for zero upcrossing waves and the bottom plot is for crest to trough waves. The horizontal line represents the value of 2.28 predicted by Longuet-Higgins (1952) for a narrow band spectrum.

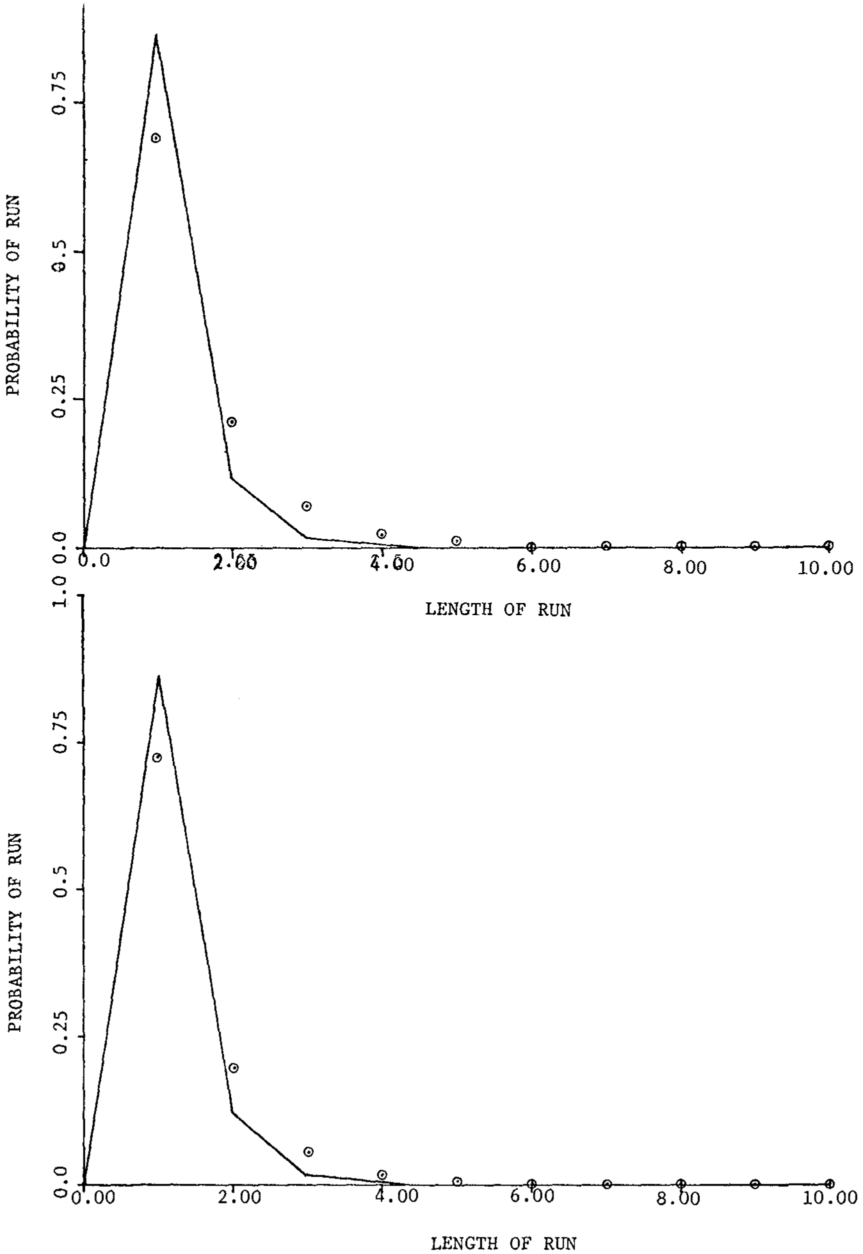

FIGURE 11. The observed and theoretical values of the probability of a run of waves greater than H 1/3 versus the length of the run. The top plot is for zero crossing waves and the bottom plot is for crest to trough waves.

where P is the probability of a wave exceeding $H_{1/3}$ (=0.134) and Q = 1-P.

Figure 11 shows a comparison of the observed results for zero upcrossing waves (top plot) and crest-to-trough waves (bottom plot). The line is the theoretical value and the points the observed probabilities. The results are very similar. Both tend to show runs of waves tend to occur much more frequently than the simple theory suggests.

4. Conclusions

The significant wave height computed from the integral under the spectrum most nearly represents the mean of the highest one-third of the zero upcrossing waves and is independent of spectral width for the type of records analyzed here.

There are strong correlations between the zero crossing period, significant period and significant zero upcrossing period respectively and the peak period at least in the case of fully developed seas featuring a singly peaked spectrum with an f^{-5} behaviour.

The theories of Cartwright and Longuet-Higgins predict very well the values of the ratio of the largest in a sample of maxima to the root-mean-square as a function of spectral width for values of spectral width between 0.5 and 0.9, the largest observed deviation being about 4%. The theoretical distribution presented by Goda (1970) for the maximum water surface height in 100 zero upcrossings fits the observed distribution reasonably well. However, the deficiency of high values found by Goda in his simulation experiment has not occurred here.

Runs of waves greater than $H_{1/3}$ and of lengths greater than 1 seem to occur in some cases several times more frequently than indicated by simple theory based on the probability that the event will occur at one trial.

References

Cartwright, D.E. and Longuet-Higgins, M.S. 1956, Proceedings of the Royal Society, A, 237, 212.

Davenport, A.G. 1964, Proc. Inst. Civil Engineering, 28, 187.

Goda, Y. 1970, Report of the Port and Harbour Research Institute, 9, No. 3.

Longuet-Higgins, M.S. 1952, Journal of Marine Research, XI, No. 3, 245.