CHAPTER 129

TIDAL WAVES IN SCHEMATIC ESTUARIES

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ABSTRACT

An investigation of tidal waves in schematic estuaries has been carried out simultaneously for hydraulic and hydrodynamic-numerical (HN) models Thereby a far-reaching agreement of results is obtained for geometrically simple shapes. In these cases hydraulic investigations can be replaced equivalently by HN-computations.

INTRODUCTION AND GENERAL VIEW

The knowledge of the tidal caused motion processes in the near shore area, especially in the estuaries, is of great importance for coastal engineering and navigation. The quantitative determination of these processes, e g. finding out of water levels and current velocities depending on space and time, can be done by direct measurements in the natural area In recent years for this purpose also hydraulic and hydrodynamic-numerical (HN) models are used which simulate the natural conditions The application of such models requires e g. much less effort and costs and gives, further on, the possibility to study the effect of prospected coastal engineering structures On these grounds model techniques succeeded in the last years on a large scale Especially, the HN-methods belong to the most effective tools of dynamical oceanography.

The following results on the propagation of tidal waves in estuaries are obtained by means of hydraulic and HN-models It was a main intention thereby to find out by a systematic investigation of geometrical simple estuarine shapes the degree of agreement and the possibilities and restrictions of the two principally different methods. It appears, that in this way a senseful mutual completion of both methods could be reached. Now, the more expensive hydraulic model experiments will be carried out only in those complicated cases, for which theoretical solutions are not yet available On the other hand, HN-investigations can relieve the hydraulic model testing plants of time-consuming routine experiments and be very useful if the constructional equipment of the laboratory do not allow the study of certain questions e g. the influence of the Coriolis force The investigations which result from a co-operation of the Bundesanstalt fur Wasserbau Hamburg and the Institut fur Meereskunde der Universitat Hamburg, are sponsored by the Deutsche Forschungsgemeinschaft (Vo 153/1/2) The whole study will be reported later on We are very thankful to W Wulzinger for his assistance in the hydraulic model investigations

The hydraulic processes in a tidal estuary without a steady inflow are mainly influenced by two aspects

(a) Geometry of the estuary

(b) Shape of the incoming tidal wave

In the following only the effect (a) is discussed Fig. 1 shows the considered schematic estuaries The types B and E are developed from tidal rivers at the German coast By this time the comparison between the two models was realized for the types A and D.

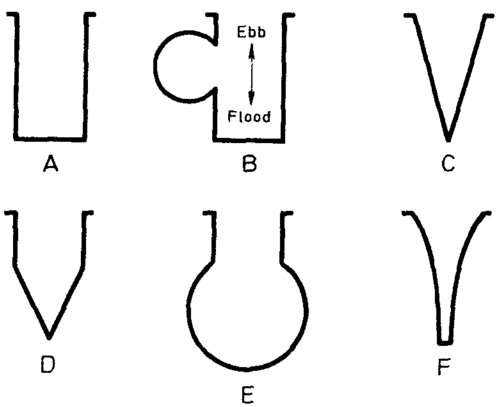


Fig 1 Types of Estuaries

In all cases a uniform M_2-tide with a mean amplitude of 1,5 m has been used

Actually, the HN-models produce vertically integrated horizontal velocities It is intended, furtherly, to consider the vertical dimension too.

THE NUMERICAL MODEL

The foundation of the mathematical model is, that flow movements (especially instationary tidal waves which are of interest here) take place according to the well-known fundamental equations of hydrodynamics. Therefore, the mathematical treatment of these equations gives the possibility, in principal, to determine the motion processes in a certain area The mathematical model has to be adapted extensively to natural conditions

In general, due to the nonlinearity of the hydrodynamic differential equations, an analytic solution is not available Therefore the application of numerical methods is necessary, e g. the HN-difference method (HANSEN [1]) This requires the application of electronic computers In the last years, the HN-method was proved for many natural sea areas (Technical Report [4], SUNDERMANN [2]) The following differential equations have been used

$$(1)\qquad \begin{aligned} &(1\,1) && \frac{\partial u}{\partial t}+\frac{r}{h+\zeta}u\sqrt{u^2+v^2}-A_H\,\Delta u-fv+g\frac{\partial\zeta}{\partial x} = 0\\ &(1\,2) && \frac{\partial v}{\partial t}+\frac{r}{h+\zeta}v\sqrt{u^2+v^2}-A_H\,\Delta v+fu+g\frac{\partial\zeta}{\partial y} = 0\\ &(1\,3) && \frac{\partial\zeta}{\partial t}+\frac{\partial}{\partial x}\Big((h+\zeta)u\Big)+\frac{\partial}{\partial y}\Big((h+\zeta)v\Big) = 0 \end{aligned}$$

The first two equations are motion equations and 1 3 is the continuity equation The system is two-dimensional For this a cartesian system of coordinates has been used The x-axis is directed to East, the y-axis to North, the z-axis indicates the vertical direction (see Fig 2)

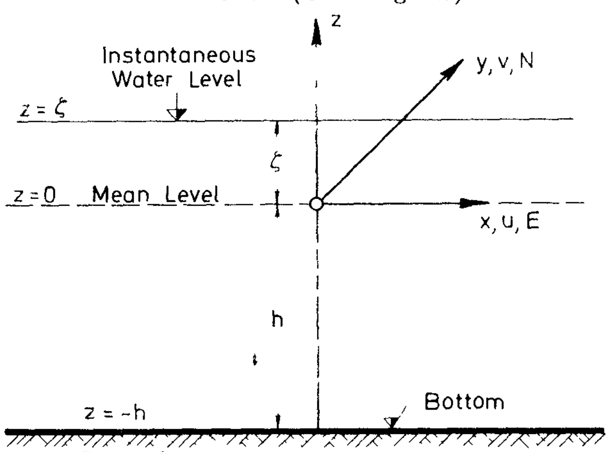


Fig 2 System of Coordinates

Herein denotes

- u,v — components of the vertical averaged current velocity $\mathfrak{w}$ in the x- or y-direction, respectively
- ξ — water level = distance from the mean level
- h — mean water depth = distance between the mean water level and bottom
- t — time
- A_H — horizontal turbulence coefficient
- f — Coriolis parameter
- g — gravitational acceleration
- r — friction factor

$$(2)\quad \begin{cases} (2.1) & \mathfrak{w}_n = 0 & \text{along the coastline} \\ (2.2) & \xi(t) = A\cos(\sigma t - \varkappa) & \text{at the entrance of the estuary} \end{cases}$$

The boundary conditions are given in the equations (2) 2.1 signifies that no normal component of the velocity appears at the coast. In 2.2 A denotes the amplitude, σ the frequency and $\varkappa$ the phase of the incoming tidal wave Geometry, A and $\varkappa$ must be given

The initial conditions are

$$(3)\quad u = v = 0, \qquad \xi = 0$$

The water level ξ is included in the total depth $H = h + \xi$ and the bottom friction is given by a quadratical law Therefore, the system (1) becomes nonlinear

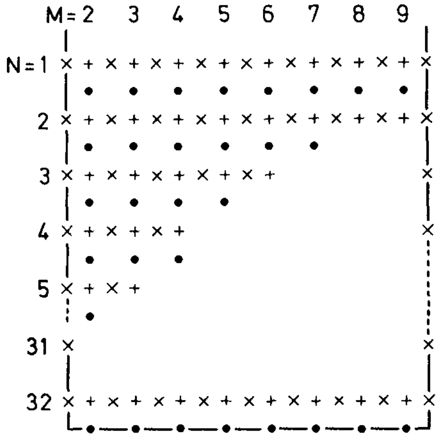
As shown by computation with and without horizontal turbulence term, this effect can be neglected for the model types A, C, D and F ($A_H = 0$) The Coriolis parameter has been taken as $f = 0$, because the hydraulic model installations did not allow to realize this effect A numerical experiment has shown however the influence of the earth's rotation on the distribution of current velocities, while the water level nearly did not change

With these assumptions it is sufficient to use instead of (1) the one-dimensional equations for the types A,C,D,F

$$(4)\quad \begin{cases} (4.1) & \dfrac{\partial u}{\partial t} + \dfrac{r}{h+\xi} u|u| + g\dfrac{\partial \xi}{\partial x} = 0 \\ (4.2) & \dfrac{\partial \xi}{\partial t} + \dfrac{\partial}{\partial x}\big((h+\xi)u\big) = 0 \end{cases}$$

Because the used calculation method was utilized mainly for natural areas, the hydraulic model was fitted with a corresponding scale.

For the difference method the use of a horizontal grid system is indispensable. Fig 3 illustrates the two-dimensional grid system of a rectangular estuary.



Constants

$\Delta x = \Delta y = 500$ m	Net distance
$\Delta t = 27{,}9$ sec	Time step
$h = 15$ m	Depth
$A_H = 10^{8} cm^{2} s^{-1}$	Turbulence coefficient
$f = 1{,}2\ 10^{4} s^{-1}$	Coriolis parameter (North Sea)
$r = 0{,}003$	Friction factor

$+ = \zeta$ - Calculation point

$x = u$ " "

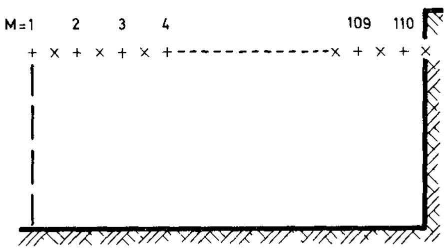
$. = v$ " "

$A = 150$ cm	Amplitude of the tide at the entrance of the estuary
$\sigma = 28{,}984$ °/h	Angular velocity of the M_2-tide
$\varkappa = 90^{o}$	Tidal phase at the entrance of the estuary

Fig 3 Grid System for 16 km Estuary (Type A)

Steady flow conditions occur in the two-dimensional consideration after about 12 periods

The one-dimensional treatment is based on a 55 km canal (Type A) The grid system and the constants are given in Fig 4



Constants

Δx = 500 m Net distance

Δt = 30 sec Time step

h = 15 m Depth

r = 0,003 Friction factor

+ = ζ - Calculation point

x = u - " "

A = 162,5 resp 164 cm Amplitude of the tide at the entrance of the estuary

σ = 28,984 °/h Angular velocity of the M_2-tide

$\varkappa$ = 90° Tidal phase at the entrance of the estuary

Fig 4 Grid System for 55 km Estuary (Type A)

In the HN-model, the amplitude of the incoming tidal wave was adapted to the value measured in the hydraulic model The steady state was reached after about 5 periods

All programs are written in FORTRAN IV

As an example, in Fig. 5 the tidal and the velocity curves are given for 3 selected points The tidal wave increases to the end of the estuary The velocity curves show the typical steep gradient in the flood branch and the smoother distribution of the ebb

THE HYDRAULIC MODEL

The hydraulic model of type A is 55 m long, 4 m wide, the mean water depth amounts 0 15 m Fig 6 shows the model installation. The entrance of the model has a fixed bed with artificial roughness. The estuary itself was filled by sand which was not moved by the current In this part of the model the roughness was simulated by artificial 1-cm-ripples This kind of roughness corresponded with the assumed friction factor r = 0 003 in the HN-model

According to natural conditions, the following scales were appointed the extensions of length and width 1 1000, depths 1 100, e.g the distortion is tenfold The whole hydraulic data were reduced by Froude. The tidal period is 7 45 minutes in the model. The water levels were measured with mechanical and electrical water gages and the current velocities with micro-current meters (VOLLMERS [3])

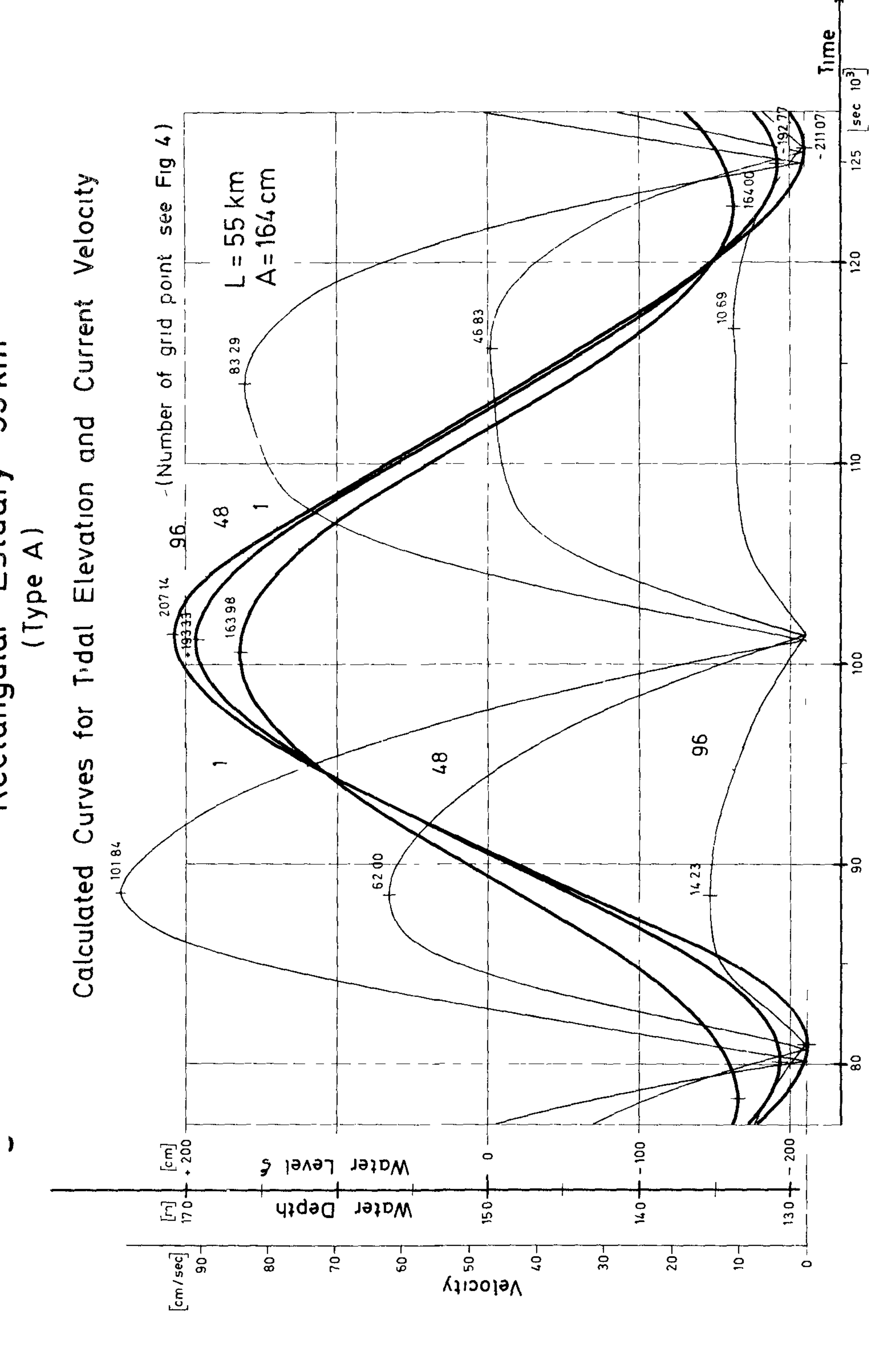

(Type A)
Calculated Curves for Tidal Elevation and Current Velocity
(Number of grid point see Fig 4)
L = 55 km
A = 164 cm
96
48
1
101 84
62 00
14 23
207 14
+193 33
163 98
83 29
46 83
10 69
164 00
-192 77
-211 07
Time
[sec 10³]
80
90
100
110
120
125
Water Level ξ
[cm]
+200
0
-100
-200
Water Depth
[m]
170
150
140
130
Velocity
[cm/sec]
90
80
70
60
50
40
30
20
10
0

Fig. 6 Model Installation

The first tests were carried out in a 16 m long rectangular basin (e.g. 16 km in natur), but it was not possible to get clear results because the model was too short and therefore the differences for the speed are too small for measuring.

The measurement of the hydraulic data yields good results in the long canal (55 km in nature) of the Type A. Fig. 7 demonstrates side by side the measured and calculated water levels for different points during one tide. It was necessary to draw the curves side by side, because the good agreement shows nearly no differences between measurement and calculation. The tidal curve increases, as it is well-known, at the end of the canal and the time ratio between flood and ebb will be displaced. The tidal range increases from 3.28 to 4.18 m, the time ratio, tide low water to tide high water and tide high water to tide low water decreases from 1 at the entrance to about 0.80 at the end of the canal.

The current velocities were measured at different water depths.

Due to the restriction of the HN-model to vertically averaged velocities, the comparison between the two models was possible only for mean conditions. Fig. 8 shows this comparison for a mean calculated and measured curve. The agreement is good as well in shape as in order of magnitude.

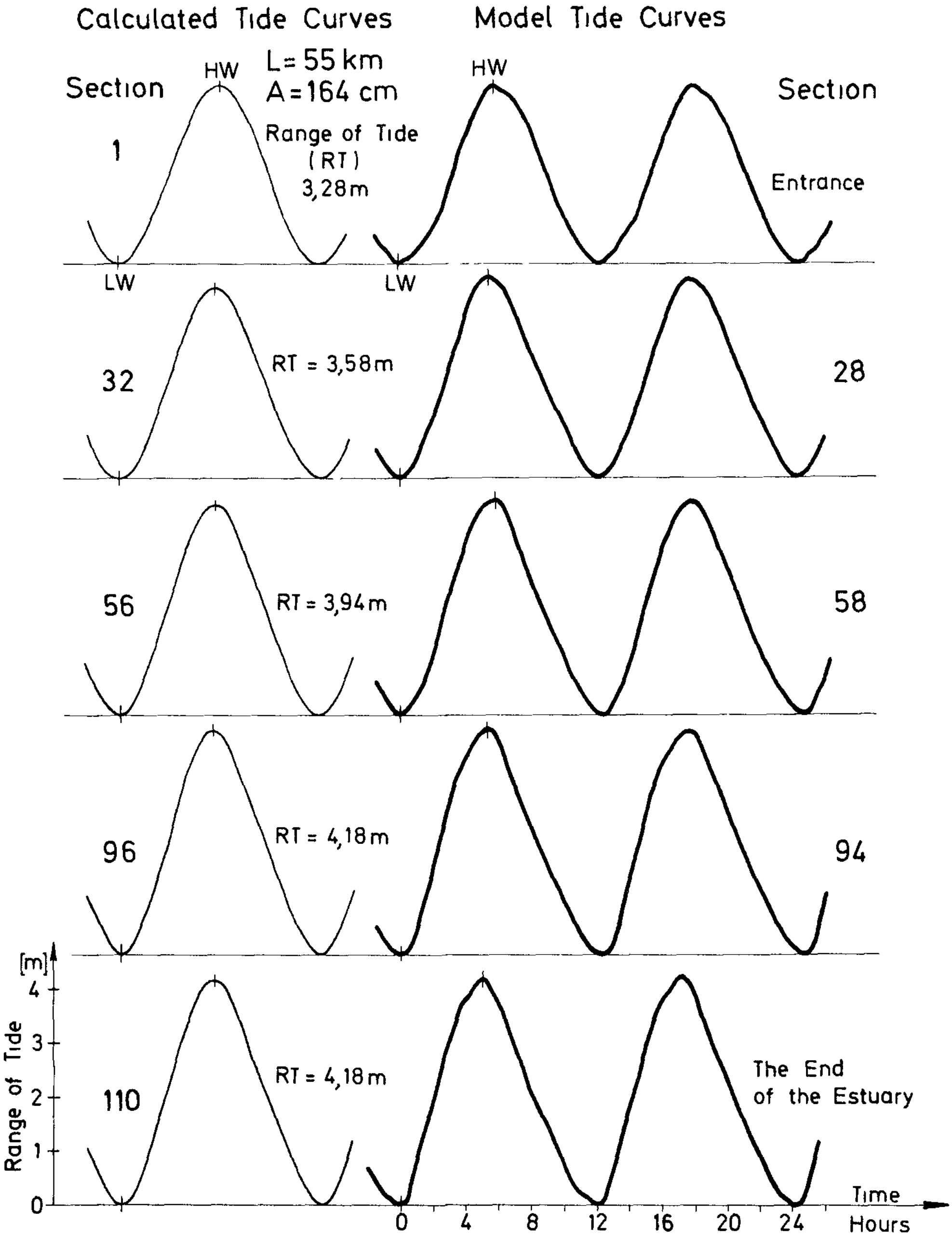


Fig 7 Rectangular Estuary 55 km

(Type A)

The characteristic shows, that acceleration and deceleration are considerable and oscillations are small during the flood The distribution curve is more filled and the oscillations are greater during ebb stream

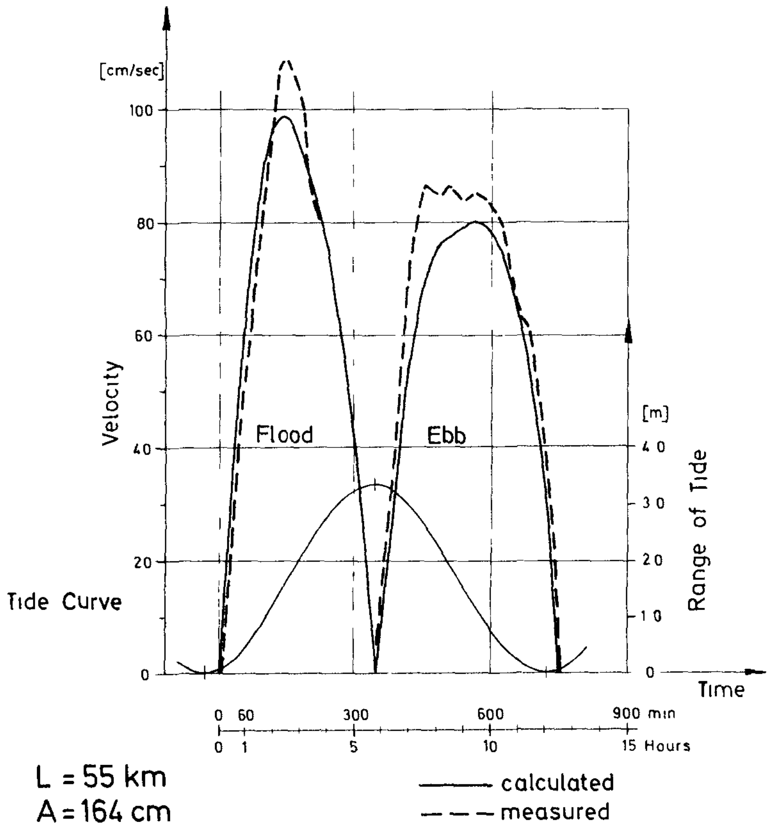


Fig 8 Typical velocity curve

The mean current velocities, averaged over cross section 5 (see Fig 4) for different water depths, are drawn in Fig. 9. The agreement with the corresponding calculated curve is satisfactory.

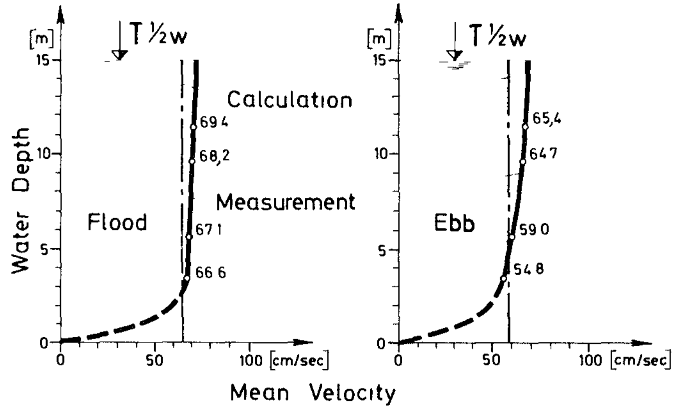


Fig. 9 Typical vertical distribution of velocity

Fig 10 shows the tidal ranges along the canal In order to make also a comparison between other canal sizes (16 km and 50 kilometer), the calculated curves are listed with another amplitude The differences between calculation and measurement for the 55-km canal are small With regard to the model scale, only differences of 0 3 to 0 6 mm are present. 50 % of these data are already inside of the accuracy of measurement

SUMMARY

It was shown by a systematic comparison between the hydraulic and the mathematical model, that the water levels and the mean current velocities for the investigated estuarine forms are in a fair agreement In these cases the one-dimensional method was sufficient

For the types B and E, however, the cross extension place an important role. Therefore the corresponding HN-model must be considered two-dimensionally

Finally, further investigations will include the vertical dimension, in order to study the shear stress distribution at the bottom

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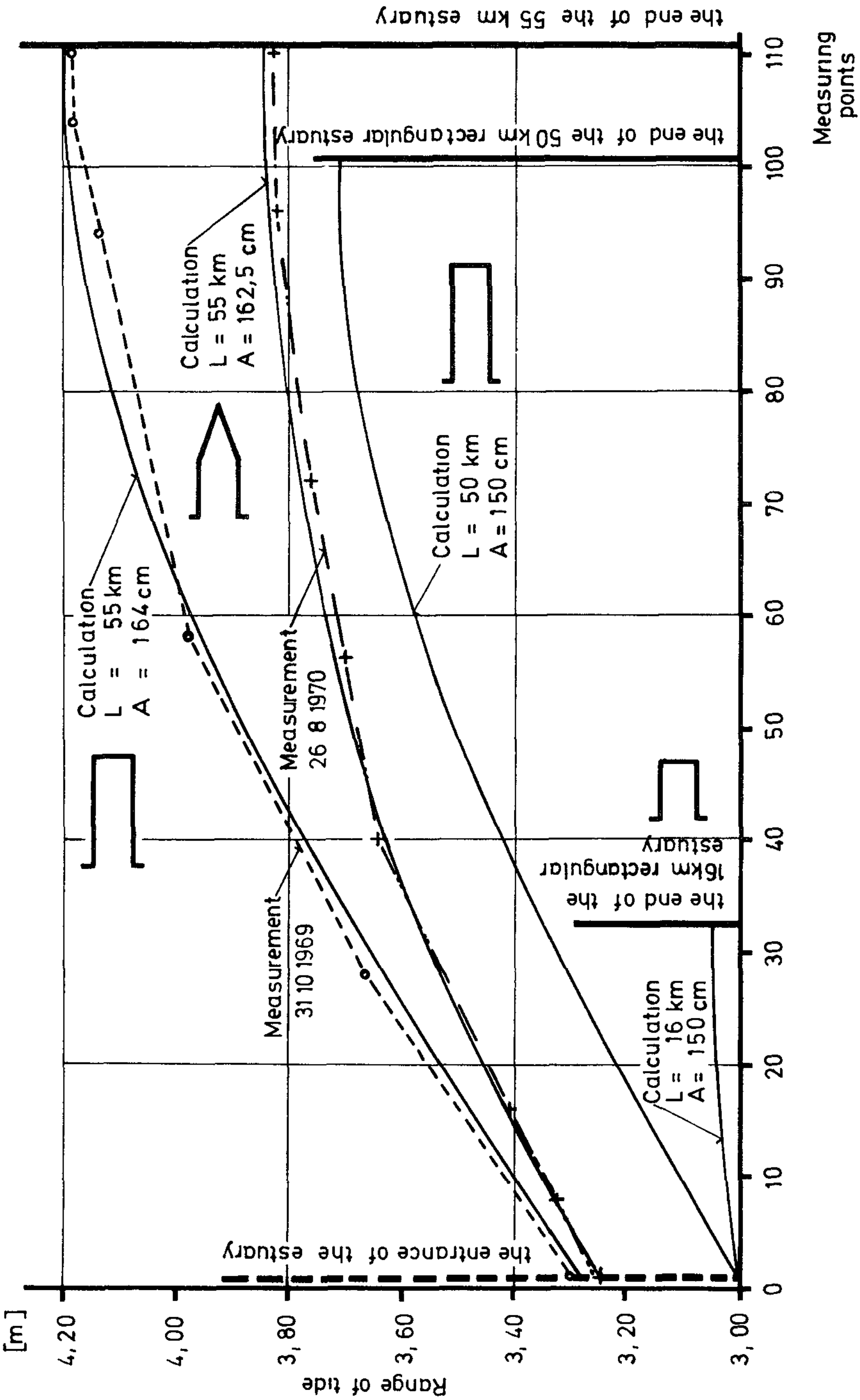

Fig 10 The Range of Tides in Relation to Different Types of Estuaries

Model (Measurement) and Calculation