CHAPTER 113

FLUSHING PATTERN OF NON-REACTIVE EFFLUENTS

by

Adel M. Kamel, AM.ASCE Associate Professor of Civil Engineering Louisiana State University, Baton Rouge, La. USA.

ABSTRACT

In south central Louisiana non-reactive liquid effluents are introduced into man-made relatively straight prismatic canals which are comparatively narrow and have brackish To study the flushing pattern of liquid effluents water introduced into those canals (or estuaries), a one-dimensional numerical model is considered for a simplified system consisting of a long straight gently sloping reach with a sinusoidal tidal variation at the mouth of the estuary, sinusoidal tidal veriation with a phase lag and an attenuated amplitude at the upstream end, a variable inflow hydrograph, and a variable inflow or outflow. For this system, the continuity and momentum equations are solved numerically by an explicit finite difference scheme. The output of the model describes the spatial and temporal variations in flow velocity (also in water depth and discharge) from which the flushing pattern is obtained for a liquid effluent introduced at any time during the tidal cycle at any section along the estuary. The numerical model is applied to Charenton drainage canal in south central Louisiana and good agreement is obtained between the velocities and stage elevations predicted from the model and recorded in the field. An IBM 360/65 computer is utilized.

INTRODUCTION

In south central Louisiana non-reactive liquid effluents mainly from sugar mills are introduced into man-made relatively straight prismatic canals which are comparatively narrow and have brackish water. Effluents introduced oscilate up and down the canals with tidal movement and are carried slowly to the Gulf of Mexico with the net flow.

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The study of the flushing pattern of effluents introduced into those estuaries is of interest in the selection of the location of new outfalls and the time of release of effluents The flushing pattern would depend on the flow velocity and the density of the effluent. The presence of tidal movement in the estuaries under consideration (for a 1.5 ft tide at the mouth of estuary, Gulf of Mexico, tidal variation of more than 1.0 ft is obtained about 6 miles upstream) result in unsteady flow which makes the evaluation of flow velocity based on steady state conditions no longer feasible. Resort to the evaluation of the flow by either empirical methods or continuous field measurements are also not feasible because of the lack of generality in the first and the incredible amount of work and expense in the latter.

Hydraulic and analog studies could be employed for predicting the flushing pattern in estuaries. However, such models provide results that are unique to the particular tidal reach for which they are applied. Application of those models to another tidal reach would require the construction of new hydraulic or analog model. On the other hand a general purpose numerical model would be applicable to any tidal reach in the area as long as the characteristics of the reach under consideration (such as length, roughness coefficient, shape of cross section of flow, tidal amplitude and phase angle, etc.) are provided as input to the numerical model.

Techniques for one-dimensional numerical modeling of both natural and artificially induced transient flows in rivers and estuaries have been developed by investigators such as (1,2,4,5,8,10,11,12, and 18)*. Recently work has begun on two-dimensional numerical modeling of tidal flows such as the work of (3,6,7,13, and 17). Those techniques involve the solution of the shallow water equations for the appropriate initial and boundary conditions.

This paper constitutes the results of a pilot study on the numerical simulation of the flushing pattern of a non-reactive liquid effluent (no decay or absorption of effluent across flow boundaries) which has a density similar to that of the estuarine water. The estuaries considered are

*Numerals in brackets refer to similarly numbered items in Appendix I-References.

comparatively narrow and straight with brackish waters. The numerical model is applied to Charenton drainage canal (figure 1) in south central Louisiana and good agreement is obtained between predicted and measured water stage elevations and velocities in the canal.

ANALYSIS

Simulation of Spatial and Temporal Velocity Changes.- The estuaries considered are sectionally homogeneous making a one-dimensional presentation adequate for the study of the flushing pattern and concentration distribution of liquid effluents having density similar to that of the receiving waters. The hydromechanics of the estuaries considered could be studied by developing a one-dimensional numerical model for a simplified system consisting of a long straight gently sloping reach with a sinusoidal tidal variation at mouth of estuary, a sinusoidal tidal variation with phase lag and amplitude attenuation at the upstream end, an inflow hydrograph, and lateral inflow or outflow.

The equations of continuity and of motion for one-dimensional unsteady flow of homogeneous density in a straight gently sloping prismatic tidal reach are expressed as.

 $\frac{*\partial E}{\partial t} + \frac{\mu}{\partial x} + u \frac{\partial H}{\partial x} + u \frac{\partial H}{\partial x} = 0 \qquad -1 - \frac{1}{2}$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + \sigma \frac{\partial H}{\partial x} + \sigma q = 0 \qquad -2-$$

Equations 1 and 2 are the familiar forms of the shallow water equations and are valid under the following assumptions 1 the effect of the coriolis force and the wind stress term are negligable, 2. In the direction normal to the axis of the channel the surface particles lie transversely on a horizontal line. Since the reach section is prismatic in form, it follows that the width B would be a function of depth only, 3. hydrostatic pressure prevails at any point and a uniform velocity distribution exists over any cross section.

*Symbols used are defined in Appendix II-Notation.

Further assuming that the coefficient of roughness for unsteady flow is the same as that for steady flow and can be expressed by the Manning's equation, the friction slope term in equation 2 could be expressed as

$$g_{e} = \frac{g_{k}}{(r_{e})^{4/3}} u |u| -3-$$

Subistituting equation 3 into 2, the later becomes,

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + g \frac{\partial E}{\partial x} + \frac{g k}{(R)^{4/3}} u \left| u \right| + \frac{g u}{A} = 0 -4-$$

Equations 1 and 4 are two simultaneous quasi-linear partial differential equations with two dependent variables u and E and two independent variables x and t. To solve equations 1 and 4 numerically three methods were considered namely, characteristics, explicit, and implicit. Results obtained from the explicit method only are reported herein.

The computational scheme employed in this study is based on operating with finite differences by using a fixed rectangular net in the x-t plane. In this case it is not necessary to calculate the values of the coordinates x,t of the net points themselves. The reach of length L is divided into N equal sections of length Δx . The time interval Δt is selected to satisfy the Courant condition for empirical stability,

$$\Delta t = \frac{\Delta x}{Y(|u|+c)}$$

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The boundary conditions of water surface elevation were selected since in canals considered gaging stations are usually operated by the Corps of Engineers and supply continuous record of E. The computational procedure starts by assuming the initial condition (values of u and E at t = t) along the reach length and advancing the solution one step to time $t_1 = \Delta t$. Now that all values of u and E are known at each point in the grid at time $t_1 = \Delta t$, the same procedure is used to advance the solution from $t_1 = \Delta t$ to $t_2 = 2 \Delta t$. In this case the values of u and E

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obtained at t1 = Δ t are considered as initial conditions.

Simulation of Flushing Pattern of Index Particles. - The differential equationsdescribing the movement of index particles released at any time at any location along the reach is expressed as

 $\frac{\mathrm{d}x}{\mathrm{d}t} = u (X,t)$

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A particle released at X_0 at t_0 would be located at X_j+1 at time $t = t_0 + n \cdot \Delta t$. where:

$$X_{j+1} = X_j + u_{x_j} \cdot n \cdot \Delta t$$
, $(n, j = 0, 1, 2, 3, ...)$ -7-

To obtain the flushing pattern of an effluent introduced at any time t at any section X along the reach, the value of u at that section at that time is read and multiplied by the time increment Δt to give the increment $\Delta x'$ which the effluent has advanced. $\Delta x'$ is added algebrically to X to yield the new location X of the effluent at time t + Δt . At this new location X , the value of u is obtained by linear interpolation from the corresponding values of u in the mesh of length Δx to the right and to the left of point X. The same procedure is repeated to obtain the flushing pattern for any length of time.

APPLICATION

Simulation of Spatial and Temporal Velocity Changes. To establish the range of conditions under which the computational scheme would yield satisfactory results, the following factors are considered 1. effect of assumed initial conditions, 2. effect of Manning's coefficient of roughness, 3. stability of the scheme.

Effect of Initial Condition.- The initial conditions employed are the water level elevation E and the flow velocity uat t = 0. Since the water level is known at the two ends of the reach from gaging stations, an elevation of the water level at different sections along the reach by linear interpolation would be satisfactory for obtaining the initial values of E. The corresponding values of u could be computed from Manning's formula, in this case a value of u = 1.45 ft/sec is obtained and is assumed to be the same along the entire reach at t = 0. To establish the effect of u on the convergence of the solution, computations were carried out with u = 1.45, 0.725, and 0.00 ft/sec. It was noted that all values computed converged to a unique value in less than 5 hours (figure 2). Five hours would seem as a short duration in the study of the velocity variation during a whole tidal cycle. However, in the study of the flushing pattern during periods of large velocities, the variation in the velocity obtained from the computational scheme during the first hours could result in an appreciable error in evaluating the flushing pattern. This suggests that a careful estimate of the initial velocity u would be necessary if accurate flushing patterns are to be obtained.

Effect of Manning's Coefficient of Roughness - The effect of the value of n selected for the calculations on the spatial and temporal variation of the velocity was investigated for n values of 0.0, 0.02, 0.03, and 0.10. Other parameters are held constant. The results are shown in figure 3 and indicate that convergence does not occur and a variation in the magnitude of the velocity of as much as 90% occurs by changing the value of n from 0.02 to 0.03. Although the computational scheme is sensetive to variations in the value of n employed in the calculations, this should not be of great concern since the value of n can be estimated with a reasonable degree of accuracy for either an existing canal or for one which is in the planning stage.

Stability.- Questions regarding the stability of the explicit scheme has been raised. For instance (14) showed that the method is unstable when $\Delta x \rightarrow 0$ even though $\Delta t \rightarrow 0$ in such a manner as to satisfy equation 5. It was reported by (19) that satisfying equation 5 does not guarantee that the equation of continuity would be satisfied. According to the same author, over a long period of time the inflow might be greater or less than the outflow plus the accumulated storage. In his study of overland flow (15) noticed a tendency of the hydrograph to overland flow (15) noticed a tendency of the study instability was observed when the rate of change of discharge with respect to time is large. This was corrected for by selecting a value of Y variable in equation 5 to ensure stability in convenient means since a rigorous stability criteria for nonlinear equations cannot be established (16). After exploring the effect of the initial conditions, the coefficient of roughness, and the stability of the method on results obtained by the computational scheme, it would be necessary to examine the effect of the different hydraulic parameters (such as tidal amplitude and inflow hydrograph) on the spatial and temporal variations of the flow velocity. This is summarized in the following paragraphs.

Effect of Tidal Amplitude. The effect of tidal amplitude on the velocity, discharge, and water level for two semiamplitudes of a = 4.00 ft, al = 3.5 ft, and a = 1.50 ft, al = 1.25 ft indicated minor spatial variation in the values of u,Q, and E obtained for the same tidal amplitude. This is believed to be due to the small coefficient of roughness (n=0.002) employed in the numerical model.

Effect of Inflow Hydrograph.- The four inflow hydrographs tested have a sinusoidal form with a peak amplitude (h) of 7.5 ft above existing water level at upstream end of reach. The duration of the inflow hydrograph ($\hat{\tau}$) varied from T/16 to T/4. The results are shown in figure 4 and indicate ar equal peak velocity for different values of τ and a longer duration of high flows for larger values of τ .

Simulation of Flushing Pattern of Index Particles. After examining the effect of the different hydraulic parameters on the spatial and temporal variation of the flow velocity, two typical examples are selected for the study of the flushing pattern. The first situation is representive of conditions during unusual weather activities. The following hydraulic parameters are selected: a=4.0ft, al=3.5ft, q= 0.10 ft /sec, u₀ =1.45 ft/sec, h=4.0ft, τ =T/2, T=24.83hrs., and n=0 02. The flushing pattern of an effluent introduced at time t₀ =T/2 at section 1 through 12 of the reach under study is shown in figure 5. The second situation is representative of conditions that prevail during the dry season with normal tidal activity in the Gulf of Mexico. The following hydraulic parameters are selected: a=0.75ft, al=0.50ft, q=0.0, u₀=0.0, h=0.0, τ =0.0, T=24.834hrs., and n=0.02. The flushing pattern for an effluent introduced at time t₀=T/2 at sections 1 through 12 of the reach under study is shown in figure 6.

VERIFICATION OF NUMERICAL MODEL

For verifying the stages and velocities predicted from the numerical model the stage elevations recorded on three Stevens tide recorders were utilized. Tide charts were obtained for stations No.1 (at Southern Pacific Railroad bridge), No.2 (1 mile downstream from station No.1) and No.13 (0.25 miles below junction with IWW 6 miles downstream from station No.1), figure 1. The tide stages from stations No.1 and 13 were digitized and provided boundary condition input to the model. A comparison between the predicted and measured stage elevations for station No.2 is shown in figure 7 for n=0.045. The agreement between the predicted and measured stage elevations is good and could be improved by optimizing for the value of n. The predicted and meas-ured temporal changes of the average velocity for station No.1 is shown in figure 8 and indicates good agreement. The The velocity measurements were made from the Railroad bridge utilizing a cub current meter. Again it is believed that better agreement between predicted and measured velocities could be obtained by optimizing for the value of n.

CONCLUSIONS

1. The numerical model is sensetive to input values assumed for starting the computations such as the initial velocity, and the Manning's coefficient of roughness. This would call for a careful selection of these values but does not pose a great concern as to the validity of the model. 2. The computational scheme has a tendency to become unstable when the rate of change of discharge with respect to time is large. This difficulty is overcome by taking smaller time increments Δ t, in equation 5.

3. The flushing pattern depends on the prevailing hydraulic conditions and on the time and location of release of effluent. For instance during normal conditions where a tide amplitude at mouth of estuary is about 1.5 ft and about 1.0 ft at a distance 6 miles upstream of the mouth, with no lateral inflow or outflow and no inflow hydrograph, the flushing pattern shown in figure 5 is predicted for effluents released at low water slack ($t_0 = 0.0$). It can be seen from figure 5 that an effluent released at the Southern Pacific Railroad bridge 6 miles upstream of the IWW will be flushed in a downstream direction to a distance of about 14,700 ft after 5 hours. Figure 6 shows typical flushing pattern during unusual weather activities which may prevail during the hurricane season where the variation in water level at the mouth of the estuary may reach as high as 8.0 ft with an inflow hydrograph at the

upstream end of the reach and lateral inflow caused by rainstorms. It can be seen from figure 6 that effluents released at the upstream end of the reach at $t=t_0 + 15.30$ hours (i.e. 15.30 hrs after high water slack) would be flushed in only 1.39 hrs into the mouth of the estuary. The same figure indicates that effluents released during high water slack ($t_0 = T/2$) as far downstream as 0.5 miles from mouth of estuary would reach the upstream end of reach in 3.89 hrs. 4. Computer time is an important factor in the appraisal of simulation techniques. In the scheme represented herein, for a time interval $\Delta t = 1.0$ minute and a section length $\Delta x = 0.5$ miles, a complete run required 2.67 minutes on a IBM 360/65 computer. A complete run involved predicting the spatial and temporal variations in water level elevation, flow velocity, discharge, and flushing pattern for a tidal reach 6 miles long during a tidal period of 24.834 hrs. Time required for plotting or printing out of the results is not included in the 2.67 minutes.

5. Agreement between predicted and measured temporal variations of water stage elevations and velocities is good and could be improved by optimizing for the value of n. 6. Development of two-dimensional stochastic models for predicting the flushing pattern and concentration distribution for non-reactive liquid effluents discharged into large water bodies such as rivers and lakes would seem to be the logical extension of this work.

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APPENDIX II - NOTATION

The following symbols are used in this paper A== flow cross sectional area, (L²), a = semi-amplitude of tide at downstream end of estuary, (L), al= semi-amplitude of tide at upstream end of estuary, (L), B = surface width, (L),c = celerity of gravity wave, (L/T); E = elevation of water surface, (L); g = gravity acceleration, (L/T^2) ; H = average depth of flow in cross section, (L), h = height of inflow hydrograph above pre-existing water level at upstream end of reach, (L), k = a function of Manning's coefficient of resistance, $(T^2/L^{2/3})$; L = length of reach, (L); m = number of time intervals of duration Δt . n = Manning's coefficient of resistance, $(T/L^{1/3})$, N = number of reach sections having a length Δx , q = lateral inflow per unit length of reach, (L²/T), \hat{Q} = discharge, (L³/T), R = hydraulic radius, (L); Se= friction slope, So= bottom slope, T = tidal period, (T); $t = time, (\tilde{T});$ τ = duration of inflow hydrograph, (T), $\Delta t = time interval,(T),$ u = flow velocity in longitudinal direction, (L/T), u_0 = initial velocity in longitudinal direction, (L/T), x = distance measured in longitudinal direction, (L),Y = variable ≥ 1 , Ax= length of equal section of reach,(L), and $\boldsymbol{\varepsilon}$ = phase angle between tides at downstream and upstream ends

of reach.





















