CHAPTER 94

Breakwater and Quay Wall by Horizontal Plates

by

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Abstract

Wave action to breakwater and quay wall by fixed horizontal plates as shown in Fig. 4 and 5 are studied theoretically and experimentally. Transmission and reflection coefficients for breakwater are calculated as shown in Fig. 6(a)(b), which suggest that only fixing the water surface by rigid plate is effective to reduce the transmitted waves for appropriate wave conditions. Pressure distributions to plates are calculated as shown in Fig. 7 and total pressures are in Fig. 8. Pressure distributions to horizontal plate and vertical wall of quay wall are shown in Fig. 9 and 10, which show that the plate makes the pressure distribution to vertical wall more uniform than the one without plate and also submerged plates of breakwater and quay wall make the pressure distributions smooth.

Wave action in case when the region under plates is filled by permeable material are also calculated. The results suggest that the void of the material has the effect to elongate the plate and the fluid resistance acts as wave energy absorber.

1 Introduction

Piling up the franged-column blocks as shown in Fig. 1 into three or four layers, permeable breakwater and quay wall with vertical sides are constructed as shown in Fig. 2 and 3. Photo 1 and 2 are actually constructed ones of Fig. 2 and 3, respectively, in Japan. Hydrodynamically, they are regarded as composed of horizontal slabs supported by vertical piles. Accordingly, wave actions to these structures are studied by investigating the effect of horizontal plates and then of the permeable material filled uniformly under the plates instead of piles.

2 Theory of breakwater by horizontal plate (without permeable material)

Suppose that a rigid horizontal plate of length $2l$ is fixed on water surface with constant water depth $h$ and the water is divided at $x=\pm l$ into three regions I, II and III as shown in Fig. 4. Assuming small amplitude waves in perfect liquid and velocity potential in each region to be $\phi_1$, $\phi_2$, and $\phi_3$, we have the following Laplace's equations:

$$\frac{\partial^2 \phi_i}{\partial x^2} + \frac{\partial^2 \phi_i}{\partial z^2} = 0 \quad (i = 1, 2, 3)$$

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1537
Bottom and surface conditions in region I and III are as follows
\[
\frac{\partial \Phi_i}{\partial z} = 0 \quad \text{at } z=-h, \quad \frac{\partial^2 \Phi_i}{\partial x^2} + g \frac{\partial \Phi_i}{\partial z} = 0 \quad \text{at } z=0 \quad (i=1, 3) \quad (2)
\]

In region II, we have
\[
\frac{\partial \Phi_i}{\partial z} = 0 \quad \text{at } z=0 \quad \text{and } z=-h \quad (3)
\]

Fluid pressure in each region and surface profiles in regions I and III are as follows
\[
\frac{h}{f} = -\frac{\partial \Phi_i}{\partial z} - gh \quad (i=1, 2, 3), \quad \frac{\partial \Phi_i}{\partial z} = -\frac{1}{\delta} \left( \frac{\partial \Phi_i}{\partial z} \right)_{z=0} \quad (i=1, 3) \quad (4)
\]

where \( g \) is gravity acceleration and \( f \) is fluid density.

Supposing the incident wave of frequency \( \sigma \) (\( \sigma = \frac{2\pi}{T}, T \) is wave period) and reflected wave in region I and transmitted wave in region III, the velocity potential which satisfies Laplace's equation and bottom and surface boundary conditions in each region is given as follows
\[
\Phi_i(x, z, t) = e^{i\sigma t} \left( A_i e^{ikx} + B_i e^{-ikx} \right) \frac{\cosh(k(z+h))}{\cosh(kh)} + \sum_{n=1}^{\infty} C_n e^{i\sigma \frac{n\pi x}{2L}} \frac{\sinh(n\pi h)}{\sinh(n\pi h)}, \quad (i=1, 2, 3) \quad (5)
\]
\[
\Phi_2(x, z, t) = e^{i\sigma t} \sum_{n=0}^{\infty} \left( D_n \cosh \frac{n\pi x}{L} + E_n \sinh \frac{n\pi x}{L} \right) \cos \frac{n\pi z}{h}, \quad (i=0, 1, 2) \quad (6)
\]
\[
\Phi_3(x, z, t) = e^{i\sigma t} \left[ F e^{ikx} \cos(k(z+h)) + \sum_{n=1}^{\infty} G_n e^{i\sigma \frac{n\pi x}{2L}} \frac{\sinh(n\pi h)}{\sinh(n\pi h)} \right] \quad (i=1, 2, 3) \quad (7)
\]

where \( k \) and \( k_n \) are eigenvalues determined by the following relations
\[
\frac{n\pi}{h} = \tan(kh) \tan(kh) = -k_n \tan(kh), \quad (i=1, 2, 3) \quad (8)
\]

\( A_i, B_i, \) and \( F \) are complex constants which represent the incident, reflected and transmitted waves, respectively, and \( C_n \) and \( G_n \) are standing waves which vanish at \( |X| \to \infty \).

\( D_i \) and \( E_i \) are complex constants to be determined from the following boundary conditions.

At the boundaries \( X=\pm L \), continuities of horizontal fluid velocities and fluctuating wave pressures due to continuities of mass and energy flux through the boundaries require the following conditions
\[
\frac{\partial \Phi_1}{\partial x} = \frac{\partial \Phi_2}{\partial x}, \quad \Phi_1 = \Phi_2 \quad \text{at } \quad X=L \quad (9)
\]
\[
\frac{\partial \Phi_2}{\partial x} = \frac{\partial \Phi_3}{\partial x}, \quad \Phi_2 = \Phi_3 \quad \text{at } \quad X=-L \quad (10)
\]

Substituting eqs 5, 6, 7 into eqs 9, 10 and adding and subtracting, we obtain next
Multiplying each terms of above equations by \( \cos(r\pi z/h) \), integrating from \( z = -h \) to \( z = 0 \) and using the following relations

\[
\int_{-h}^{0} \cos(r\pi z/h) \cos(kz) \, dz = 0 \quad (r = S = 0), \quad = h \quad (r = S = 0)
\]

we have following equations

\[
\begin{align*}
\{Ae^{-kl} - (B-F)e^{-kl}\} \frac{\tan k h}{1 + (r\pi h)^2} + \sum_{n=1}^{\infty} (C_n - G_n)e^{-kl} \frac{\tan k h}{1 - (r\pi h)^2} &= -2\pi \sum_{r=0}^{\infty} \frac{E_r \cos r \pi h}{h} \\
\{Ae^{-kl} + (B-F)e^{-kl}\} \frac{\tan k h}{1 + (r\pi h)^2} + \sum_{n=1}^{\infty} (C_n + G_n)e^{-kl} \frac{\tan k h}{1 - (r\pi h)^2} &= 2\pi \sum_{r=0}^{\infty} \frac{D_r \cos r \pi h}{h}
\end{align*}
\]

Eliminating \( D_r \) and \( E_r \) in above equations and putting

\[
\begin{align*}
A e^{-kl} &= a, \quad B e^{-kl} = b, \quad F e^{-kl} = f, \quad C_n e^{k n l} = c_n \\
G_n e^{k n l} = g_n, \quad C_l / k h = \lambda_n, \quad k h / k = \lambda_n, \\
F_{(n)} &= \frac{\lambda_n^2 + r^2}{\lambda_n^2 - r^2} \frac{\lambda_n + r \tanh r h}{\lambda_n^2 + (r \tanh r h)^2}, \quad \phi_{(n)} = \frac{\lambda_n^2 + r^2}{\lambda_n^2 - r^2} \frac{\lambda_n + r \tanh r h}{\lambda_n^2 + (r \tanh r h)^2}, \\
\theta_r &= \tan^{-1} \left( \frac{1}{\lambda_n \tan r h} \right), \quad \zeta_r = \tan^{-1} \left( \frac{1}{\lambda_n \tanh r h} \right)
\end{align*}
\]
the following two systems of linear simultaneous equations are provided

\[ \frac{b+f}{a} e^{-i\theta r} + \sum_{n=1}^{m} \frac{C_n + d_n}{a} = e^{i\theta r} \quad (\gamma = 0, 1, 2) \]  

\[ \frac{b-f}{a} e^{-i\theta r} + \sum_{n=1}^{m} \frac{C_n - d_n}{a} = e^{i\theta r} \quad (\gamma = 0, 1, 2) \]

where

\[ F_{0,n} = \frac{\lambda_0}{\lambda_n}, \quad F_{1,n} = \frac{\lambda_0^2 + \gamma^2}{\gamma^2 - \gamma^2} \frac{\lambda_n + \gamma \tan \theta H}{\lambda_n - \gamma^2 (\gamma \tan \theta H)^2} = \frac{\lambda_0^2}{\lambda_n^2} \frac{1 + \lambda_n H}{1 + (\lambda_n H)^2} \]

\[ \theta_0 = 0, \quad \theta = \text{em} \tan^{-1}(\gamma \tan \theta H/\lambda_n H) = \text{tan}^{-1}(\frac{\theta}{\lambda_n H}) \]

Eq 20 and 21 are regarded as equations for determining the unknowns \((b+f)/a, (c_n + d_n)/a\) and \((b-f)/a, (c_n - d_n)/a\), respectively, so that we obtain \(b/a, f/a, c_n/a\) and \(d_n/a\) from eq 20 and 21. Thus, from eq 16 and 18, \(D_T\) and \(E_T\) are determined as follows

\[ D_T = \frac{\omega}{2} \left[ \frac{1}{\lambda_0^2} \left( 1 + \frac{b+f}{a} \right) \frac{1}{\lambda_0^2} - \sum_{n=1}^{m} \frac{C_n + d_n}{a} \frac{1}{\lambda_n^2} \right] \]

\[ E_T \cos \theta H = \frac{\omega}{2} \left[ \left( 1 + \frac{b-f}{a} \right) \frac{1}{\lambda_0^2} + \sum_{n=1}^{m} \frac{C_n - d_n}{a} \frac{1}{\lambda_n^2} \right] \]

Letting the incident wave be \( \psi_i = \cos(kx + \alpha x) \), we have

\[ \alpha = -\frac{\omega}{c_0} e^{i\theta H}/a. \]

Thus, from eq 5, 6 and 7, the velocity potentials are determined as follows

\[ \phi_1(x, z, t) = a e^{i\phi} \left[ \left( e^{i(kx + \alpha x)} + \frac{1}{\alpha} e^{-i(kx + \alpha x)} \right) \cos k^2 H + \sum_{n=1}^{m} \frac{C_n}{a} e^{i(kx + \alpha x) \cos k_1 H + \cos k_2 H} \right] \]

\[ \phi_2(x, z, t) = a e^{i\phi} \left[ \left( \frac{d_x}{2a} \cos \theta H + \frac{e^{i(kx + \alpha x) \cos k_1 H + \cos k_2 H}}{2a} \right) \cos \theta H \right] \]

\[ \phi_3(x, z, t) = a e^{i\phi} \left[ \left( \frac{d_z}{a} e^{i(kx + \alpha x) \cos k_1 H + \cos k_2 H} \right) \cos \theta H \right] \]

Surface profiles in regions I and III are given as follows

\[ S_1(x, t) = a e^{i(kx + \alpha x) \cos k_1 H + \cos k_2 H} \left( \frac{e^{i(kx + \alpha x) \cos k_1 H + \cos k_2 H}}{2a} \right) \]

\[ + \sum_{n=1}^{m} \frac{C_n}{a} e^{i(kx + \alpha x) \cos k_2 H} \]
Reflection and transmission coefficients are as follows:

\[ K_r = \left| \frac{b}{a} \right| , \quad K_t = \left| \frac{f}{a} \right| \]  

Pressure distribution \( p(x=0) \) and the total pressure \( P_n \) to the horizontal plate are as follows:

\[ p(x=0) = e^{ik(x+\phi)} \left[ \frac{b}{a} + \sum_{n=1}^{\infty} \left( \frac{d_r}{a} \coth RH + \frac{c_r}{a} \frac{\sinh RH}{\lambda} \right) \right] \]  

\[ P_n = \left| \frac{\int_{-l/2}^{l/2} e^{ik(x+\phi)} dx}{\int_{-l/2}^{l/2} e^{ik(x+\phi)} dx} \right| = \left| \frac{b}{2a} + \sum_{n=1}^{\infty} \frac{d_r}{a} \frac{\tan \lambda H}{\lambda} \right| \]  

3 Theory of quay wall with horizontal plate (without permeable material)

Suppose the vertical wall with horizontal plate of length \( l \) as shown in Fig 5. Boundary conditions for fluid motion in region I are the same as the previous section and velocity potential \( \phi_1 \) is given by eq (5). In region II, the condition \( \partial \phi_2 / \partial y = 0 \) at \( y = 0 \) is to be added, so that the velocity potential \( \phi_2 \) becomes:

\[ \phi_2(x, z, t) = e^{it} \sum_{\gamma=0}^{\infty} D_\gamma \coth \frac{Y \lambda H}{h} \cos \frac{T \lambda H}{h} , \quad (Y = 0 \ 1 \ 2 \ 3 \ 4 \ 5) \]  

Boundary conditions at \( Y = \lambda \) is the same as eq (9).

Following the same process as previous section and using the same symbols as eq 19, simultaneous equations for \( b/a \) and \( c_m/a \) are provided in the same form as eq 20 as follows:

\[ \frac{b}{a} e^{-it} \sum_{\gamma=0}^{\infty} F_{\gamma m} \frac{c_m}{a} = e^{it} \text{ (Y = 0 \ 1 \ 2 \ 3 \ 4 \ 5)} \]  

Thus, the numerical results of eq 20 for breakwater are used as it is and \( D_0 \) and \( D_Y \) are given as follows:

\[ D_0 = \frac{d/2}{\text{cos} (\cos \lambda H) - \frac{1}{\lambda H} \sum_{n=1}^{\infty} \frac{c_n}{a} \frac{1}{\lambda^2}} \]  

\[ D_Y \text{cos} \lambda H = \frac{dY}{\text{cos} (\cos \lambda H) - \frac{1}{\lambda H} \sum_{n=1}^{\infty} \frac{c_n}{a} \frac{1}{\lambda^2}} \]  

For the incident wave \( S_0 = \alpha \cos (kx + \phi) \), the velocity potential \( \phi_1 \) is the same as eq (25) and \( \phi_2 \) is as follows:
Reflection coefficient $K_r = \left| b/a \right|$ is identically equal to unity.

Pressure distribution $P_i(x, y)$ and $P_r(x=0)$, and the total pressure $P_u$ and $P_H$ to horizontal plate and vertical wall are given as follows.

$$\Phi_i(z, x, t) = \alpha e^{\omega t} \left[ \frac{d}{2a} + \sum_{n=1}^{\infty} \frac{dr}{\alpha} \frac{\tanh \gamma H}{\gamma H} \sin \frac{\pi n x}{a} \right]$$  \hspace{1cm} (36)

4 Breakwater and quay wall with double plates (without permeable material)

In cases of breakwater and quay wall with double plates at $z = 0$ and $z = -h/2$ as shown by dotted lines in Fig. 4 and 5, velocity potential in region II must satisfy one more condition $\Phi_{/z} = 0$ at $z = -h/2$. This is done by taking even numbers for integers $r$ in eq 6 and 33 (that is $r = 0, 2, 4, 6, \ldots$)

Calculations

For various values of $\alpha h/g = 2 \pi h/\lambda (\lambda$ is deep water wave length for period $T$), $\lambda$, and $\lambda_\infty$ defined by eq 19 are calculated by eq 8 as shown in Table 1. And then, eq 20 and 21 are solved by means of electronic computer. For example, $b/a$, $f/a$, $c_\infty/a$, etc. for the case of $\alpha h/g = 1.0$ and $L/h = 1.0$ are as shown in Table 2.

6 Calculated results for horizontal plate breakwater

Transmission and reflection coefficients $K_t$ and $K_r$ by eq 30 are shown in Fig. 6(a) and (b) for single and double plates. For the case of single plate with $L/h = 2.0$, $K_t = 0.53$, 0.32, and 0.28 for $\alpha h/g = 0.5$, 1.0, and 1.5, respectively, and so only 28%, 14% and 8% of incident wave energy are transmitted through the plate. Comparing Fig. 6(a) with Fig. 6(b), it is seen that if the total length of plates are equal in both cases (that is, $L/h = 2.0$ corresponds to $L/h = 1.0$), $K_t$ is always larger than $K_r$ and $K_r$ is always larger than $K_t$ for all $\alpha h/g$, where suffix 1 and 2 show the cases of single plate and double plates, respectively. These properties are interpreted as...
the result that when the water surface is fixed by horizontal plate, the fluid under the plate is constrained in motion, increases its inertial resistance to motion and behaves like a semi-rigid breakwater for short waves.

Pressure distribution to plate by eq 31 are shown in Fig 7(a)(b), which show that the distribution for double plates is remarkably uniform compared with that of single plate. The total pressure by eq 32 is shown in Fig 8, from which it is seen that the averaged pressure per unit length of plate and per unit amplitude of incident wave for double plates is almost independent of \( L/h \) and approaches to the value for \( L/h = 0.5 \) of single plate. These properties of pressure to the plate suggest that the submerged plate has the effect of making the pressure distribution uniform.

7 Calculated results for horizontal plate quay wall (without permeable material)

The pressure distribution to horizontal plate by eq 37 is shown in Fig 9 for single plate quay wall. As was shown for breakwater, the distribution for double plates is remarkably uniform compared with that of single plate. Pressure distribution to the vertical wall is in Fig 10. Compared with the one without plate (that is, \( L/h = 0 \)), the distribution is smooth even for single plate wall and the tendency is much clearer for double plates wall. Averaged total pressure to horizontal plate and vertical wall by eq 38 and 40 are in Fig 11 and 12, respectively. Comparing both figures, it is seen that for single plate wall the averaged pressure to horizontal direction becomes equal to that of vertical direction for larger \( L/h \), and it is clearer for double plates wall. This means that the water in region II is constrained in motion by horizontal plate and vertical wall and the wave pressures are equalized to all directions.

8 Theory of horizontal plate breakwater and quay wall with permeable material

When permeable material is filled under horizontal plate, it causes resistance to the motion of fluid flowing through the void. For simplicity, we assume that the resistance is proportional to fluid velocity and the coefficient of resistance per unit fluid mass is \( R \) and the void ratio is \( V \). If \( u, w \) and \( p \) are actual velocities and pressure of fluid, equations of motion and continuity are as follows:

\[
\frac{\partial u}{\partial t} - \frac{1}{\rho} \frac{\partial p}{\partial x} - \mu u, \quad \frac{\partial w}{\partial t} = -\frac{1}{\rho} \frac{\partial p}{\partial x} - g - \mu w, \quad \frac{\partial u}{\partial x} + \frac{\partial w}{\partial x} = 0
\] (41)

Letting the average velocities and pressure per unit volume of this region be \( \bar{u} \), \( \bar{w} \), and \( \bar{p} \), respectively, we have next relations:

\[
\bar{u} = \nabla \bar{u}, \quad \bar{w} = \nabla \bar{w}, \quad \bar{p} = \nabla \bar{p}
\] (42)
Then, eq. 41 are rewritten as follows:

\[
\frac{2\bar{u}}{\bar{t}} = -\frac{1}{\bar{p}} \frac{\partial \bar{p}}{\partial x} - \mu \bar{u} \quad \frac{\partial \bar{w}}{\partial t} = -\frac{1}{\bar{p}} \frac{\partial \bar{p}}{\partial z} - gV - \mu \bar{w} \quad \frac{2\bar{u}}{\bar{t}} + \frac{\partial \bar{w}}{\partial z} = 0
\]  

(43)

This average motion has velocity potential \( \Phi \) which satisfies Laplace's equation

Putting

\[
\bar{u} = \partial \Phi / \partial x, \quad \bar{w} = \partial \Phi / \partial z
\]

(44)

fluid pressure \( \bar{p} \) is given by the following equation

\[
\frac{\partial \bar{p}}{\partial x} = -\mu \frac{\partial \Phi}{\partial x} - g\bar{V} z
\]

(45)

The boundary condition for potential \( \Phi \) in region II of Fig. 4 are \( \partial \Phi / \partial z = 0 \) at \( z = 0 \) and \( z = -h \), and from the continuity of mass flux at \( x = \pm l \),

\[
\frac{\partial \Phi}{\partial x} = \frac{\partial \Phi}{\partial x} \quad \text{at} \quad x = l, \quad \frac{\partial \Phi}{\partial x} = \frac{\partial \Phi}{\partial x} \quad \text{at} \quad x = -l
\]

(46)

and also from the continuity of energy flux \( \rho = \bar{v} / \bar{V} \) at \( x = l \), \( \rho = \bar{v} / \bar{V} \) at \( x = -l \), so that

\[
\frac{\partial \Phi}{\partial x} = \frac{1}{V} \left( \frac{\partial \Phi}{\partial x} + \mu \frac{\partial \Phi}{\partial x} \right) \quad \text{at} \quad x = l
\]

(47)

\[
\frac{\partial \Phi}{\partial x} = \frac{1}{V} \left( \frac{\partial \Phi}{\partial x} + \mu \frac{\partial \Phi}{\partial x} \right) \quad \text{at} \quad x = -l
\]

(48)

Since the velocity potential in each region is given by eq 5,6 and 7 for breakwater, we obtain next relations corresponding to eq.20 and 21, using above conditions and the same symbols as eq.19

\[
\frac{b + f}{a} + \sum_{n=1}^{\infty} \frac{\lambda_n^2 + \rho^2}{\lambda_n^2 - \rho^2} \frac{C_n + \beta_n}{\lambda_n} = \frac{\lambda^2 + \rho^2}{\lambda^2 - \rho^2} \frac{C_0 + \beta_0}{\lambda^2} \quad (r = 0, 1, 2)
\]

(49)

\[
\frac{b - f}{a} + \sum_{n=1}^{\infty} \frac{\lambda_n^2 + \rho^2}{\lambda_n^2 - \rho^2} \frac{C_n - \beta_n}{\lambda_n} = \frac{\lambda^2 + \rho^2}{\lambda^2 - \rho^2} \frac{C_0 - \beta_0}{\lambda^2} \quad (r = 0, 1, 2)
\]

(50)

where

\[
P_r = \frac{\rho V\tan\beta H}{\rho} \quad Q_r = \frac{\rho V\tan\beta H}{\rho} \quad P_0 = 0
\]

(51)

\[
Q_0 = \frac{\rho V\tan\beta H}{\rho} \quad \beta = \sqrt{1 + (\rho / \rho^2) \quad \nu = \tan^{-1}(\rho / \rho^2)}
\]

Solving eq. 49 and 50, we obtain \( b/a, f/a, c_n/a \) and \( g_n/a \) for breakwater, and then \( D_r \) and \( E_r \) are given as follows

\[
D_r = \frac{d_r}{2} \frac{\rho V\tan\beta H}{\rho} \frac{a}{\alpha} \left[ \left( 1 + \frac{4\rho^2}{a^2} \right) \frac{1}{\lambda^2} - \sum_{n=1}^{\infty} \frac{1}{\lambda_n^2} \frac{C_n^2 + \beta_n^2}{\alpha_n^2} \right]
\]

(52)
\[
D, \coth RH = \frac{\mu}{\beta} e^{\frac{\mu H}{g}} \frac{a}{\beta^2} \left[ \left( 1 - \frac{f^2}{a^2} \right) \frac{1}{\lambda_0^2 + \sum_{n=1}^{\infty} \frac{1}{\lambda_n^2 + \frac{C_n}{a}} \coth^{2} RH \right]
\]

\[
E, \tanh RH = \frac{\nu}{\beta} e^{\frac{\mu H}{g}} \frac{a}{\beta^2} \left[ \left( 1 - \frac{f^2}{a^2} \right) \frac{1}{\lambda_0^2 + \sum_{n=1}^{\infty} \frac{1}{\lambda_n^2 + \frac{C_n}{a}} \tanh RH \right]
\]

Surface wave profiles, reflection and transmission coefficients etc. are given in the same form of eq 28, 29 and 30 etc., using \( b/a \), \( f/a \), \( c_w/a \) and \( \gamma/a \) by eq 49 and 50. The fluid pressure in the region II is given by eq 45 for the incident wave \( S_0 = \alpha \cos(kx + \omega t) \) as follows

\[
\frac{P}{\beta g} = \frac{b}{\beta} e^{(kx + \omega t - \nu)} \left[ \left( \frac{d_0}{2a} + \sum_{n=1}^{\infty} \left( \frac{d_n}{a} \coth RH + \frac{c_n}{a} \tanh RH \right) \right) \right]
\]

Pressure distribution and the total pressure to horizontal plate are given as follows

\[
\frac{P}{\beta g} = \frac{b}{\beta} e^{(kx + \omega t - \nu)} \left[ \left( \frac{d_0}{2a} + \sum_{n=1}^{\infty} \left( \frac{d_n}{a} \coth RH + \frac{c_n}{a} \tanh RH \right) \right) \right]
\]

\[
\frac{P}{\beta g} = \frac{b}{\beta} \left( \frac{d_0}{2a} + \sum_{n=1}^{\infty} \frac{d_n}{a} \right)
\]

Above-mentioned results are for single plate breakwater. For quay wall, similar results are obtained by the same process as in section 3.

9 Calculated results for horizontal plate breakwater with permeable material

As an example, \( b/a \), \( f/a \), \( c_w/a \) and \( \gamma/a \) by eq 49 and 50 for \( \epsilon' h/g = 1.0 \), \( \beta h = 1.0 \), \( V = 0.5 \) and \( \mu/\rho = 1.0 \) are shown in Table 3.

Transmission and reflection coefficients for \( \beta h = 1.0 \), \( V = 0.5 \) are shown in Fig 13 for parameter \( \mu/\rho \). When \( \mu/\rho = 0 \), \( K_T \) and \( K_R \) are equal to those for \( \beta h = 2.0 \) in Fig 6(a) for the case without permeable material. When \( \mu/\rho > 0 \), \( K_T \) becomes smaller than that for \( \mu/\rho = 0 \) and then it increases with the increase of \( \mu/\rho \). On the contrary, \( K_R \) decreases steadily with increase of \( \mu/\rho \) This means that the void has the effect to elongate the length \( l \) of the plate to \( \beta h \) and the fluid resistance has the effect to resist to the incident wave and to cause energy dissipation in the breakwater.

The pressure distribution to plate by eq 54 is shown in Fig 14. Compared with Fig 7(a), it is seen that the distribution is steeper than the case without permeable material and the tendency is remarkable for large \( \epsilon' h/g \). This coincides with the fact that the void has the effect to elongate the plate length.

It may be said that the submerged plate has the effect to make the pressure
distribution uniform but the submerged permeable material has the effect to make the
distribution steep  the averaged total pressure by eq 55 is nearly equal to the case
without permeable material

10 Calculated results for horizontal plate quay wall with permeable material

For the same conditions as previous section, reflection coefficient $K_T$ for quay
wall is calculated as shown in Fig 15 $K_T$ decreases steadily with the increase of
$\rho h/g$, but not so with $\rho h/g$ for large values of $\rho h/g$, $K_T$ is minimum for $\rho h/g$
between 1.0 and 2.0 That is, reflection coefficient is the least for a particular
value of $\rho h/g$ depending on $\rho h/g$ and becomes larger for other values of $\rho h/g$. As
seen from Fig 15, $K_T$ is larger than 0.94 for $\rho h/g (\leq 1.5$ and also by calculations
for various values of $\rho h/g$, it is found that $K_T$ is almost constant for $\rho h/g > 1.0$,
that is, $K_T$ is independent on the length of plate for $\rho h/g > 1.0$. The fact that
the reflection coefficient of quay wall does not decrease even for large values of
$\rho h/g$ and $\rho h$ is interpreted to be due to the small fluid velocity and the energy
dissipation inside the permeable material under the horizontal plate.

11 Comparisons with the experiment

Using the wave channel of length 22 meters, width 1.0 meters and depth 0.6 meters
with flap type wave generator, experimental measurement was carried out for single
plate breakwater of $h = 40$ cm and $l = 40$ cm ($l/h = 1.0$) without permeable material, for
incident wave amplitude $\alpha$ nearly constant of 3 cm  Transmission and reflection co-
efficients are shown in Fig 16, in which each plotted datum is the mean value of 10
times measurements  Measured transmission coefficients agree well with theoretical
values but reflection coefficients are lower than theory  The measured pressure dis-
tribution to plate is shown in Fig 17, which shows that the measured distributions
are remarkably steeper than those of theoretical ones  These discrepancies might be
due to wave overtopping and eddies generated at the front edge of plate  Measured
average total pressure to the plate are shown in Fig 18, which shows good agreement
with theory  For the case with permeable material, experiments are now under taken

12 Conclusions and remarks

Theory of single plate breakwater for long waves are introduced by J J Stoker
(1957) and the mathematical study of dock problems are presented by K O Friedrichs
and H Levy(1948) and others  In this paper we have tried the extension of Stoker's
result to the case of general wave conditions by different method from these authors
Main results of our study are summarized as follows

(i) For the case of breakwater without permeable material, a horizontal plate fixed rigidly at water surface behaves like a semi-rigid breakwater for short surface waves and more incident wave energy is reflected for shorter waves and longer plate. The longer the plate is, the more steeply distributes the pressure along the plate. In case of double plates, the pressure distribution becomes remarkably smooth.

(ii) For the case of quay wall, the pressure distribution to vertical wall is remarkably uniform by the effect of horizontal plate, compared to the case without plate. The pressure distribution to plate is similar to that of breakwater, and the longer the plate is, the more equally becomes the averaged pressures to vertical wall and to horizontal plate. In other words, the horizontal plate at water surface of quay wall plays the role of pressure equalizer to horizontal and vertical directions. In case of double plates, pressure distributions to both directions becomes more smooth than the case of single plate.

(iii) When the region under the horizontal plate is filled by permeable material, the void has the effect to elongate the plate, so that for breakwater the transmission coefficient decreases and the reflection coefficient increases and for both of breakwater and quay wall the distribution of pressure along the plate becomes steeper than the case without permeable material. The effect of fluid resistance is partly to reflect the incident wave and partly to absorb wave energy, so that for breakwater the reflection coefficient increases and the transmission coefficient decreases and quay wall the decrease of reflection coefficient is not so remarkable. The pressure distribution along the plate and the vertical wall is not changed remarkably by the effect of fluid resistance.

References
### Table 1

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### Table 2 (a)

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<td>$f/a = 0.4334 - 0.4462 i$</td>
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### Table 3

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| $b/a = 0.0$ | $t/a = 0.0$ |

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### QUAY WALL

1549
Fig. 1 Franged-Column Block

Dimension in Meters

Fig. 2 Breakwater by Franged-Column Blocks (Port of Hakata)
QUAY WALL

Fig. 3 Quay Wall by Franged-Column Block (Tabira Harbour)

Fig. 4 Breakwater by Horizontal Plate

Fig. 5 Quay Wall with Horizontal Plate
Fig 6 (a) Single Plate

Fig 7 (a) Pressure Distribution To Plate (Single Plate Breakwater)

Fig 6 (b) Double Plates

Fig 7 (b) Pressure Distribution To Plate (Double Plates Breakwater)
QUAY WALL

Fig 8: Averaged Uplift to Plate

Fig 9: Pressure Distribution to Plate (Single Plate Quay Wall)

Fig 10: Vertical Distribution of Pressure at Wall (Single Plate)
Fig 11 AVERAGED UPLIFT TO PLATE

Fig 12 AVERAGED HORIZONTAL PRESSURE AT WALL

Fig 13 \( Kr \) and \( Kt \) of Breakwater
Fig 14 Pressure Distribution to Plate (Breakwater)

Fig 15 Reflection Coefficient of Quay Wall

Fig 16 Kr and Kt Single Plate Breakwater
Fig 17 Pressure Distribution to Plate Theory and Experiment (Single Plate Breakwater)

Fig 18 Total Pressure to Plate Theory and Experiment (Single Plate Breakwater)