## **CHAPTER 94**

Breakweter and Quay Well by Horizontel Plates

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### Abstract

Wave action to breakwater and quay wall by fixed horizontal plates as shown in hig 4 and 5 are studied theoretically and experimentally Transmission and reflection coefficients for breakwater are calculated as shown in Fig 6(a)(b), which suggest that only fixing the water surface by rigid plete is effective to reduce the transmitted waves for appropriate wave conditions Pressure distributione to plates ere calculated as shown in Fig 7 and total pressures are in Fig 8 Pressure distributions to norizontal plate and vertical wall of quay wall are shown in Fig 9 and 10, which show thet the plate makes the pressure distribution to vertical wall more uniform than the one without plete and also eubmerged plates of breakwater and quay wall make the pressure distributions smooth

Wave action in case when the region under plates is filled by permeable material are also calculated The result suggest that the void of the material has the effect to elongate the plate and the fluid resistance acte as wave energy absorber

### 1 Introduction

Piling up the franged-column blocks as shown in Fig 1 into three or four layers, permeable breakwater and quay wall with vertical sides ere constructed as shown in Fig 2 and 3 Photo 1 and 2 are actually constructed ones of Fig.2 and 3, respectively, in Japan, as wave absorbing structuree Hydrodynamically, they are regerded es composed of horizontal slabs supported by vertical piles Accordingly, wave actions to these structures are studied by investorating the effect of horizontal plates and then of the permeable material filled uniformely under the plates instead of piles

### 2 Theory of breakwater by horizontal plete (without permeable material)

Suppose thet a rigid horizontal plete of length  $2\ell$  is fixed on water surface with constant water depth h and the water is divided at  $\chi = \pm \ell$  into three regions I, II and III as shown in Fig 4. Assuming small amplitude waves in perfect liquid and velocity potentielin each region to be  $\Phi_1, \Phi_2$  and  $\Phi_3$ , we have the following Laplace's equations

$$\partial^2 \Phi_{1/2} \chi^2 + \partial^2 \Phi_{1/2} \chi^2 = 0$$
 (i = 1 2 3) (1)

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Bottom and surface conditions in region I and III are as follows

$$\frac{\partial \phi_{\iota}}{\partial Z} = 0 \quad \text{at } z = -h, \quad \frac{\partial^2 \phi_{\iota}}{\partial z^2} + \frac{\partial \phi_{\iota}}{\partial Z} = 0 \quad \text{at } z = 0 \quad (\iota = 1, 3) \quad (2)$$

(3)

In region II, we have 
$$\frac{\partial \Phi_2}{\partial Z} = 0$$
 at  $z = 0$  and  $z = -h$ 

$$\frac{\dot{P}_{c}}{f} = -\frac{\partial \phi_{c}}{\partial t} - gZ \left( x = 1, 2 \right) , \quad \zeta_{c} = -\frac{1}{g} \left( \frac{\partial \phi_{c}}{\partial t} \right)_{Z=0} \left( x = 1, 3 \right) \quad (4)$$

where 9 is gravity acceleration and 5 is fluid density

Supposing the incident wave of frequency  $\mathbf{O} = 2\pi/T$ , T is wave period) and reflected wave in region I and transmitted wave in region III, the velocity potential which satisfies Laplace's equation and bottom and surface boundary conditions in each region is given as follows

$$\Phi_{1}(x,z,t) = \tilde{C}^{kat}\left[\left(A\tilde{C}^{kx} + B\tilde{C}^{kx}\right) \frac{\cosh k(z+h)}{\cosh kh} + \sum_{n=1}^{\infty} C_{n}\tilde{C}^{k_{n}x} \frac{\cosh k_{n}(z+h)}{\cosh kh}\right], (m=1,2,3) \quad (5)$$

$$\Phi_{2}(x zt) = e^{i\sigma t} \sum_{T=0}^{\infty} \left( D_{r} \cosh \frac{Y\pi x}{h} + E_{r} \sinh \frac{Y\pi x}{h} \right) \cos \frac{Y\pi Z}{h}, \quad (T = 0, |Z|) \quad (6)$$

$$\Phi_{3}(x,z,t) = e^{i\sigma t} \left[ F e^{i \frac{k z}{\omega \sigma h} \frac{k(z+h)}{\omega \sigma h}} + \sum_{n=1}^{\infty} G_{n} e^{k z} \frac{\omega \kappa (z+h)}{\omega \sigma k n h} \right] \quad (n=1, z 3) \quad (7)$$

where k and kn are eigenvalues determined by the following relations

$$\frac{\sigma^{2}h}{g} = kh tanh kh = -knh tan knh , (n = 1, 2, 3)$$
<sup>(8)</sup>

A, B and F are complex constants which represent the incident, reflected and transmitted waves, respectively, and  $C_{\mathcal{H}}$  and  $G_{\mathcal{H}}$  are standing waves which vanish at  $|\chi| \rightarrow \infty$  $D_{\Upsilon}$  and  $E_{\Upsilon}$  are complex constants to be determined from the following boundary conditions

At the boundaries  $\chi=\pm 1$ , continuities of horizontal fluid velocities and fluctuating wave pressures due to continuities of mass and energy flux through the boundaries require the following conditions

$$\frac{\partial \phi_1}{\partial \chi} = \frac{\partial \phi_2}{\partial \chi}$$
,  $\phi_1 = \phi_2$  at  $\chi = \ell$  (9)

$$\frac{\partial \phi_2}{\partial \chi} = \frac{\partial \phi_3}{\partial \chi}, \qquad \phi_2 = \phi_3 \qquad \text{at } \chi = -k \qquad (10)$$

Substituting eq 5,6,7 into eq 9,10 and adding and subtracting, we obtain next

relations

$$\left\{Ae^{ikl}(B-F)e^{ikl}\right\}\frac{\cosh k(2+h)}{\cosh k} + \iota \sum_{n=1}^{\infty} (C_n - G_n)e^{ikn}\frac{kn\cosh(2+h)}{k\cos(kn)} = -2\iota \sum_{n=1}^{\infty} \frac{\gamma\pi}{kh} E_r\cosh\frac{\gamma\pi l}{h}\cos\frac{\gamma\pi l}{h}$$
(11)

$$\left\{A\overset{kl}{e} + (B-F)\overset{-kl}{e}\right\} \underbrace{coshk(\overline{z}+h)}_{coshkh} + \sum_{n=1}^{\infty} \left(C_n - G_n\right) \overset{-k_nl}{e} \underbrace{cosk_n(\overline{z}+h)}_{cosk_nh} = 2 \sum_{T=0}^{\infty} E_T \operatorname{such} \frac{TT}{h} \cos \frac{TT}{h}$$
(12)

$$\left\{ \exists \dot{\mathcal{C}}^{kl} = (B+\bar{\mu}) \tilde{\mathcal{C}}^{kl} \right\} \underbrace{\operatorname{cook} k(z+h)}_{\operatorname{cook} kh} + v \sum_{n=1}^{\infty} \left[ (C_n + G_n) \tilde{\mathcal{C}}^{k,l} \frac{k_n \operatorname{cook} k_n(z+h)}{k \operatorname{cook} k_n h} = -2v \sum_{j=0}^{\infty} \frac{Y \pi}{k h} D_r \operatorname{cook} h \frac{T\pi l}{h} \operatorname{cooj} \frac{1}{h} \right]$$
(13)

$$\left\{Ae^{bkl}(B+F)e^{-kl}\frac{\cosh(2th)}{\cosh k} + \sum_{n=1}^{\infty}(C_n+G_n)e^{bnl}\frac{\cosh(2th)}{\cos knh} = 2\sum_{r=0}^{\infty}D_r \cosh\frac{r\pi l}{h} \cos\frac{r\pi 2}{h} \quad (14)$$

Multiplying each terms of above equations by  $\cos(r\pi z/h)$ , integrating from z = -h to z = 0 and using the following relations

$$\int_{-h}^{0} \cosh \frac{2\pi Z}{h} \cos \frac{2\pi Z}{h} dZ = 0 \quad (S+Y) ; = \frac{h}{2} \quad (Y=S+0) , = h \quad (Y=S=0)$$

$$\int_{-h}^{0} \cosh k(Z+h) \cosh \frac{2\pi Z}{h} dZ = \frac{1}{k} \quad \frac{\sinh kh}{1 + (TY/k)^{2}} , \int_{-h}^{0} \cosh k(Z+h) \cosh \frac{2\pi Z}{h} dZ = \frac{1}{kn} \quad \frac{\sinh kh}{1 - (TY/k_{n}h)^{2}}$$

we have following equations

$$\left\{A\underline{e}^{k}\underline{e}^{l}(B-F)\underline{e}^{-k}\underline{e}^{l}\right\} + v\sum_{n=1}^{\infty} (C_{n}-G_{n})\underline{e}^{k}\underline{e}^{l} \frac{tanknh}{1-(Y_{k}h)^{2}} = -vT\pi E_{r}\cosh\frac{Y\pi L}{h}$$
(15)

$$\left\{Ae^{kl}+(B-F)e^{kl}\right\}\frac{\tanh kh}{i+(\gamma K_k)^2}+\sum_{n=1}^{\infty}(C_n-G_n)e^{-k_nl}\frac{k}{k_n}\frac{\tanh k_nh}{i-(\gamma K_kh)^2}=khE_{\gamma}anh\frac{1}{h}\frac{l}{k}$$
(16)

$$\left\{ \mathbf{A} e^{\mathbf{k}\mathbf{l}} - (\mathbf{B} + \mathbf{F}) e^{\mathbf{t}\mathbf{k}\mathbf{l}} \right\} \frac{\mathbf{tank} \mathbf{kh}}{\mathbf{I} + (\mathbf{Y} + \mathbf{K})\mathbf{k})} + \mathbf{v} \sum_{n=1}^{\infty} (C_n + G_n) e^{\mathbf{t}\mathbf{k}\mathbf{n}\mathbf{l}} \frac{\mathbf{tank} \mathbf{kh}}{\mathbf{I} - (\mathbf{T} + \mathbf{K})\mathbf{k})} = -\mathbf{v} \mathbf{T} \mathbf{T} \mathbf{D}_{\mathbf{T}} \mathbf{such} \frac{\mathbf{I} \mathbf{T}\mathbf{l}}{\mathbf{h}}$$
(17)

$$\left\{\underline{Ae}^{kl} + (B+F)\overline{e}^{kl}\right\} \frac{tankkh}{1 + (T^{k}_{kh})^{2}} + \sum_{n=1}^{\infty} (C_{n}+G_{n})\overline{e}^{knl}\frac{k}{k_{n}} \frac{tanknh}{1 - (T^{k}_{kh})^{2}} = kh D_{r} \cosh \frac{Trel}{h}$$
(18)

Eliminating  $\text{D}_{\boldsymbol{Y}}$  and  $\text{E}_{\boldsymbol{Y}}$  in above equations and putting

$$\begin{aligned} &\mathcal{A}e^{kl} = \alpha, \quad \mathcal{B}e^{-kl} = b, \quad \mathcal{F}e^{-ikl} = f, \quad C_{n}e^{knl} = C_{n} \\ &G_{n}e^{knl} = g_{n} \quad \nabla f_{n} = H \quad kh/\pi = \lambda_{0} \quad knh/\pi = \lambda_{n} \\ &\mathcal{F}_{r,n} = \frac{\lambda_{0}^{2} + r^{2}}{\lambda_{n}^{2} - r^{2}} \frac{\lambda_{n} + r \tanh rH}{\lambda_{n}^{2} + (r \tanh rH)^{2}}, \quad \varphi_{rn} = \frac{\lambda_{0}^{2} + r^{2}}{\lambda_{n}^{2} - r^{2}} \frac{\lambda_{n} + r \cosh rH}{\lambda_{n}^{2} + (r \sinh rH)^{2}} \end{aligned}$$

$$\begin{aligned} &\theta_{r} = tan^{-1}(\frac{r}{\lambda_{0}} tanh rH) \qquad \qquad \mathcal{G}_{r} = tan^{-1}(\frac{r}{\lambda_{0}} corh rH) \end{aligned}$$

$$(19)$$

the following two systems of linear simultaneous equations are provided

$$\frac{l+f}{a} = e^{i\theta r} + i \sum_{n=1}^{\infty} F_{rn} \frac{c_n + g_n}{a} = e^{i\theta r} \qquad (r = 0, 12) \qquad (20)$$

$$\frac{b-f}{a}e^{i\Re r}+i\sum_{n=1}^{\infty}\varphi_{r,n}\frac{Cn-g_n}{a}=e^{i\Re r} \quad (\Upsilon=0\ |\ Z) \quad (21)$$

where

$$\begin{aligned} F_{0,n} &= \frac{\lambda_{0}}{\lambda_{n}}, \quad \Phi_{0,n} &= \lim_{T \to 0} \frac{\lambda_{0}^{2} + Y^{2}}{\lambda_{n}^{2} - Y^{2}} \frac{\lambda_{n} + Y \operatorname{coth} rH}{\sqrt{\lambda_{n}^{2} + (Y \operatorname{coth} rH)^{2}}} &= \frac{\lambda_{0}^{2}}{\lambda_{n}^{2}} \frac{1 + \lambda_{n} H}{\sqrt{1 + (\lambda_{n} H)^{2}}} \\ \Theta_{0} &= 0, \qquad \varphi_{0} &= \lim_{T \to 0} \tan^{-1}(r \operatorname{H} \operatorname{coth} rH/\lambda_{0} H) &= \tan^{-1}(\frac{1}{\lambda_{0}} H) \end{aligned}$$

$$\end{aligned}$$

Eq 20 and 21 are regarded as equations for determining the unknowns (btf)/a,  $(c_{\eta} + g_{\eta})/a$  and (b - f)/a and  $(c_{\eta} - g_{\eta})/a$ , respectively, so that we obtain b/a, f/a,  $c_{\eta}/a$  and  $g_{\eta}/a$  from eq 20 and 21. Thus, from eq 16 and 18,  $D_{\gamma}$  and  $E_{\gamma}$  are determined as follows

$$D_{s} = d_{0}/2 = \frac{\alpha}{2\pi^{2}} \frac{\Omega^{2}h}{g} \left[ \left( 1 + \frac{k+f}{\alpha} \right) \frac{1}{\lambda_{0}^{2}} - \sum_{n=1}^{\infty} \frac{(n+g_{n})}{\alpha} \frac{1}{\lambda_{n}^{2}} \right]$$

$$D_{r} \omega h r H = d_{r} = \frac{\alpha}{\pi^{2}} \frac{\Omega^{2}h}{g} \left[ \left( 1 + \frac{k+f}{\alpha} \right) \frac{1}{\lambda_{0}^{2} + \gamma^{2}} - \sum_{n=1}^{\infty} \frac{(n+g_{n})}{\alpha} \frac{1}{\lambda_{n}^{2} - \gamma^{2}} \right]$$

$$E_{r} \lambda m h r H = e_{r} = \frac{\alpha}{\pi^{2}} \frac{\Omega^{2}h}{g} \left[ \left( 1 + \frac{k-f}{\alpha} \right) \frac{1}{\lambda_{0}^{2} + \gamma^{2}} - \sum_{n=1}^{\infty} \frac{(n-g_{n})}{\alpha} \frac{1}{\lambda_{n}^{2} - \gamma^{2}} \right]$$

$$Letting the incident wave be = \int_{0}^{\infty} = \cos(kx + A_{0}t), we have$$

$$(23)$$

$$\alpha = \iota d g e^{\iota k \ell} / c$$
 (24)

Thus, from eq 5,6 and 7, the velocity potentials are determined as follows

$$\Phi_{\mathbf{r}}(\mathbf{x},\mathbf{z},\mathbf{z}) = a e^{\mathbf{v} \cdot \mathbf{z}} \left[ \left\{ e^{\mathbf{k}(\mathbf{x}-\mathbf{z})} + \frac{\mathbf{f}_{\mathbf{z}}}{a} e^{-\mathbf{k}(\mathbf{x}-\mathbf{z})} \right\} \frac{\cosh \mathbf{k}(\mathbf{z}+\mathbf{h})}{\cosh \mathbf{k}\mathbf{h}} + \sum_{n=1}^{\infty} \frac{C_{n}}{a} e^{-\mathbf{k}_{n}(\mathbf{x}-\mathbf{z})} \frac{C_{n}}{\cos \mathbf{k}\mathbf{h}} \right]$$
(25)

$$\Phi_{2}(\mathbf{x},\mathbf{z},\mathbf{t}) = \Omega e^{i\sigma \mathbf{t}} \left[ \frac{d_{0}}{2\alpha} + \sum_{I=1}^{\infty} \left( \frac{d_{T}}{\alpha} \frac{(\sigma d\mathbf{x}' H X/2}{\sigma d\mathbf{x}' H H} + \frac{e_{T}}{\alpha} \frac{(\sigma d\mathbf{x}' H X/2}{\sigma d\mathbf{x}' H H} \right) \cos \frac{Y T C E}{h} \right]$$
(26)

$$\Phi_{s}(\mathbf{1},\mathbf{1},\mathbf{t}) = \alpha e^{i\sigma \mathbf{t}} \left[ \frac{f}{a} e^{i \mathbf{k} (\mathbf{1}+\mathbf{t})} \frac{\cos \mathbf{k} (\mathbf{1}+\mathbf{t})}{\cosh \mathbf{k} \mathbf{h}} + \sum_{n=1}^{\infty} \frac{g_n}{a} e^{k_n (\mathbf{1}+\mathbf{t})} \frac{g_n}{\cos \mathbf{k} \mathbf{h}} \right]$$
(27)

Surface profiles in regions I and III are given as follows

$$S_{1}(\mathbf{I}, t) = \alpha e^{i(\mathbf{k}\mathbf{l} + \mathbf{o} t)} \left[ e^{i(\mathbf{k}(\mathbf{I} - \mathbf{k}))} + \frac{\mathbf{k}}{\alpha} e^{-i\mathbf{k}(\mathbf{I} - \mathbf{k})} + \sum_{n=1}^{\infty} \frac{\mathbf{C}_{n}}{\alpha} e^{i\mathbf{k}_{n}(\mathbf{I} - \mathbf{k})} \right]$$
(28)

$$S_{3}(\mathbf{L},\mathbf{t}) = \alpha e^{i\left(\mathbf{k}^{\mathbf{l}+\alpha\mathbf{t}}\right)} \left[ \frac{f}{a} e^{i\mathbf{k}(\mathbf{L}+\mathbf{k})} + \sum_{n=1}^{\infty} \frac{g_{n}}{a} e^{\mathbf{k}_{n}(\mathbf{L}+\mathbf{k})} \right]$$
(29)

Reflection and transmission coefficients are as follows

$$K_{\mathbf{r}} = |\mathbf{b}/\mathbf{a}| , \qquad K_{\mathbf{b}} = |\mathbf{f}/\mathbf{a}| \tag{30}$$

Pressure distribution  $P_{2(Z=0)}$  and the total pressure  $P_u$  to the horizontal plate are as follows

$$\beta_{2(2=0)}/\text{pgd} = e^{u(kl+ot)} \left[ \frac{d_{\bullet}}{2a} + \sum_{\gamma=1}^{\infty} \left( \frac{d_{\gamma}}{a} \frac{\cosh \tau H \chi/l}{\cosh \tau H} + \frac{e_{\gamma}}{a} \frac{\sinh \tau H \chi/l}{\cosh \tau H} \right) \right]$$
(31)

$$\left| \frac{P_{u}}{2} \frac{P_{u}}{2} \frac{P_{u}}{2} \frac{d_{u}}{d_{u}} d_{u} \right| = \left| \frac{d_{u}}{2a} + \sum_{Y=1}^{\infty} \frac{d_{Y}}{a} \frac{tanhYH}{YH} \right|$$
(32)

3 Theory of quay wall with horizontal plate (without permeable material)

Suppose the vertical wall with horizontal plate of length  $\ell$  as shown in Fig 5 Boundary conditions for fluid motion in region I are just the same as the previous section and velocity potential  $\phi_1$  is given by eq 5 In region II, the condition  $\partial \phi_{3Z} = 0$  at  $\chi = 0$  is to be added, so that the velocity potential  $\phi_2$  becomes, putting  $E_{\gamma} = 0$ , as follows

$$\Phi_{\mathbf{x}}(\mathbf{x},\mathbf{z},\mathbf{t}) = e^{\mathbf{t}\cdot\mathbf{r}\cdot\mathbf{x}} \sum_{\gamma=0}^{\infty} D_{\mathbf{r}} \cosh \frac{\gamma \pi \mathbf{x}}{\mathbf{h}} \cos \frac{\Gamma \pi \mathbf{z}}{\mathbf{h}} , \quad (\gamma=0 \mid \mathbf{z}) \quad (33)$$

Boundary conditions at  $\chi = L$  is the same as eq 9

Following to the same process as previoue section and using the same symbols as eq 19, simultaneous equations for b/a and  $c_{n}/a$  are provided in the same form as eq 20 as follows

$$\frac{\delta}{\alpha} e^{i\theta r} + i \sum_{n=1}^{\infty} F_{rn} \frac{C_n}{\alpha} = e^{i\theta r} \qquad (r = 0 \mid 2) \qquad (34)$$

Thus, the numerical results of eq 20 for breakwater are used as it is and D  $_0$  and D  $_Y$  are given as follows

$$D_{o} = d_{o}/2 = \frac{\alpha}{\pi^{2}} \frac{\rho^{2}h}{g} \left[ \left( 1 + \frac{L}{\alpha} \right) \frac{1}{\lambda^{2}} - \sum_{n=1}^{\infty} \frac{C_{n}}{\alpha} \frac{1}{\lambda^{2}_{n}} \right]$$

$$D_{r} \cosh rH = d_{r} = 2 \frac{\alpha}{\pi^{2}} \frac{\rho^{2}h}{g} \left[ \left( 1 + \frac{L}{\alpha} \right) \frac{1}{\lambda^{2}_{n} + \gamma^{2}} - \sum_{n=1}^{\infty} \frac{C_{n}}{\alpha} \frac{1}{\lambda^{2}_{n} - \gamma^{2}} \right]$$
(35)

For the incident wave  $\int_{a} = d \cos(kx + At)$ , the velocity potential  $\Phi_1$  is the same as eq 25 and  $\Phi_2$  is as follows

$$\Phi_{2}(x,z,t) = \alpha e^{\alpha t} \left[ \frac{d_{0}}{2\alpha} + \sum_{r=1}^{\infty} \frac{d_{r}}{\alpha} \frac{\cosh r H x/\ell}{\cosh r H} \cos \frac{r \pi \ell}{h} \right]$$
(36)

Reflection coefficient  $K_{\gamma} = |b/a|$  is identically equal to unity

Pressure distribution  $p_2(z=0)$  and  $p_2(z=0)$ , and the total pressure  $P_u$  and  $P_H$  to horizontal plate and vertical wall are given as follows

$$p_{2(2r0)}/p_{gd} = e^{i(kl+ot)} \left[ \frac{do}{2a} + \sum_{r=1}^{\infty} \frac{dr}{a} \frac{cosh(rH) t/l}{cosh(rH)} \right]$$
(37)

$$\left| \frac{Pu}{pgdL} \right| = \left| \int_{0}^{L} \frac{p_{2}(z=0)}{p^{2}dL} dz \right| = \left| \frac{d_{0}}{2a} + \sum_{t=1}^{\infty} \frac{d_{t}}{a} \frac{tankYH}{YH} \right|$$
(38)

$$\frac{P_2(\chi=0)}{Pgd} = e^{\iota(k_1^{1}+\sigma_1^{2})} \left[ \frac{d_0}{2\alpha} + \sum_{\gamma=1}^{\infty} \frac{d_\gamma}{\alpha} \frac{\cosh(\pi_1^2/h)}{\cosh(\gamma_1^{1})} \right]$$
(39)

$$\left| \operatorname{Ph}/\operatorname{pgdh} \right| = \left| \int_{-h}^{h} \frac{\beta_{2}(x \cdot o)}{\beta \beta d h} d\xi \right| = \left| \frac{d \cdot a}{a} \right| \tag{40}$$

4 Breakwater and quay wall with double platee (without permeable material)

In cases of breakwater and quay wall with double plates at z = 0 and z = -h/2as shown by dotted lines in Fig 4 and 5, velocity potential in region II must satisfy one more condition  $\partial \phi_{32} = 0$  at z = -h/2 This is done by taking even numbers for integers r in eq 6 and 33 (that is r = 0, 2, 4, 6, )

### 5 Calculations

For various values of  $\mathfrak{d}^2 h/g = 2 \pi h/L_0$  (L<sub>0</sub> is deep water wave length for period T),  $\lambda_{\mathfrak{v}}$  and  $\lambda_{\mathfrak{v}}$  defined by eq 19 are calculated by eq 8 as shown in Table 1 And then, eq 20 and 21 are solved by means of electronic computor For example, b/a, f/a,  $c_{\mathfrak{v}}/a$ etc for the case of  $\mathfrak{d}^2 h/g = 10$  and l/h = 10 are as shown in Table 2

### 6 Calculated results for horizontal plate breakwater

Transmission and reflection coefficients  $K_{t}$  and  $K_{r}$  by eq 30 are shown in Fig 6(a) (b) for single and double plates For the case of single plate with l/h=20,  $K_{t}=$ 053, 038 and 028 for  $\sqrt[6]{h/g}=05$ , 10 and 15, respectively, and so only 28%, 14% and 5% of incident wave energy are transmitted through the plate Comparing Fig 6(a) with Fig 6(b), it is seen that if the total length of plates are equal in both cases (that is,  $(l/h)_{1}=20$  corresponds to  $(l/h)_{2}=10$ ),  $(k_{T})_{1}$  is always larger than  $(K_{T})_{2}$ and  $(K_{t})_{1}$  is smaller than  $(K_{t})_{2}$  for all  $\sqrt[6]{h/g}$ , where suffix 1 and 2 show the cases of single plate and double plates, respectively These properties are interpretted ae

## QUAY WALL

the result that when the water surface is fixed by horizontal plate, the fluid under the plate is constrained in motion, increases its inertial resistance to motion and behaves like as semi-rigid breakwater for short waves

Preseure distribution to plate by eq 31 are shown in Fig 7(a)(b), which show that the distribution for double plates is remarkably uniform compared with that of single plate. The total pressure by eq 32 is shown in Fig 8, from which it is seen that the averaged pressure per unit length of plate and per unit amplitude of incident wave for double plates is almost independent of l/h and approaches to the value for l/h=0.5of single plate. These properties of preesure to the plate suggest that the submerged plate has the effect of making the pressure distribution uniform

7 Calculated results for horizontal plate quay wall (without permeable material)

The pressure distribution to horizontal plate by eq 37 is shown in Fig 9 for single plate quay wall As was shown for breakwater, the distribution for double plates is remarkably uniform compared with that of single plate Pressure distribution to the vertical wall is in Fig 10 Compared with the one without plate (that is, f/h=0), the distribution is smooth even for single plate wall and the tendency is much clearer for double plates wall Averaged total pressure to horizontal plate and vertical wall by eq 38 and 40 are in Fig 11 and 12, respectively Comparing both figures, it is seen that for single plate wall the averaged pressure to horizontal direction becomes equal to that of vertical direction for larger f/h, and it is clearer for double plates wall lhis means that the water in region II is constrained in motion by horizontal plate and vertical wall and the wave pressures are equalized to all directions

8 Theory of horizontal plate breakwater and quay wall with permeable material

When permeable material is filled under horizontal plate, it causee resistance to the motion of fluid flowing through the void For simplicity, we assume that the resistance is proportional to fluid velocity and the coefficient of resistance per unit fluid mass is  $\mu$  and the void ratio is V If u, w and p are actual velocities and pressure of fluid, equations of motion and continuity are as follows

$$\frac{\partial u}{\partial t} = -\frac{1}{\beta} \frac{\partial P}{\partial x} - \mu u, \quad \frac{\partial w}{\partial t} = -\frac{1}{\beta} \frac{\partial P}{\partial t} - g - \mu w, \quad \frac{\partial u}{\partial x} + \frac{\partial w}{\partial t} = 0$$
(41)

Letting the average velocities and pressure per unit volume of this region be  $\overline{u}$ ,  $\overline{w}$  and  $\overline{p}$ , respectivel, we have next relations

$$\overline{u} = \nabla u \qquad \overline{w} = \nabla w \qquad \beta = \nabla \beta \qquad (42)$$

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Then, eq 41 are rewritten as follows

$$\frac{\partial \vec{u}}{\partial t} = -\frac{1}{\beta} \frac{\partial \vec{b}}{\partial x} - \mu \vec{u} \quad \frac{\partial \vec{w}}{\partial t} = -\frac{1}{\beta} \frac{\partial \vec{p}}{\partial z} - g \nabla - \mu \vec{w} \quad \frac{\partial \vec{u}}{\partial x} + \frac{\partial \vec{w}}{\partial z} = 0$$
(43)

This average motion has velocity potential  $\overline{oldsymbol{\varphi}}$  which satisfies Laplace's equation

$$\overline{u} = \partial \overline{\Phi} / \partial x$$
,  $\overline{w} = \partial \overline{\Phi} / \partial z$  (44)

fluid pressure  $\overline{p}$  is given by the following equation

$$\frac{\overline{p}}{\overline{y}} = -\frac{\partial \overline{\Phi}}{\partial t} - \mu \overline{\Phi} - g \nabla z$$
<sup>(45)</sup>

The boundary condition for potential  $\overline{\Phi}$  in region II of Fig.4 are  $\partial \Phi / z = 0$  at z = 0 and z = -h, and from the continuity of mass flux at  $\chi = \pm \beta$ ,

$$\frac{\partial \Phi_1}{\partial \chi} = \frac{\partial \overline{\Phi}}{\partial \chi}$$
 at  $\chi = l$ ,  $\frac{\partial \Phi_3}{\partial \chi} = \frac{\partial \overline{\Phi}}{\partial \chi}$  at  $\chi = -l$  (46)

and also from the continuity of energy flux  $p_1 = \overline{p}/V$  at  $\chi = \ell$ ,  $p_3 = \overline{p}/V$  at  $\chi = -\ell$ , so that

$$\frac{\partial \phi_{l}}{\partial t} = \frac{1}{\nabla} \left( \frac{\partial \phi}{\partial t} + \mu \bar{\phi} \right) \qquad \text{at} \quad \mathcal{L} = \mathcal{L} \tag{47}$$

$$\frac{\partial \phi_{3}}{\partial t} = \frac{1}{\nabla} \left( \frac{\partial \bar{\phi}}{\partial t} + \mu \bar{\phi} \right) \qquad \text{at} \quad \chi_{\pm} - \ell \qquad (48)$$

Since the velocity potential in each region is given by eq 5,6 and 7 for breakwater, we obtain next relations corresponding to eq.20 and 21, using above conditions and the same symbols as eq 19

$$\frac{b+f}{\alpha} + i \sum_{n=1}^{\infty} \frac{\lambda_n^* + r^2}{\lambda_n^* - r^2} \frac{\lambda_n + P_r e^{i\nu}}{\lambda_n + P_r e^{i(\nu - \frac{\pi}{2})}} \frac{C_n + g_n}{\alpha} = \frac{\lambda_n + P_r e^{i(\nu - \frac{\pi}{2})}}{\lambda_n + P_r e^{i(\nu - \frac{\pi}{2})}} \quad (\gamma = 0 + 2)^{(49)}$$

$$\frac{b-f}{\alpha} + b \sum_{n=1}^{\infty} \frac{\lambda_{\nu}^{2} + l^{2}}{\lambda_{\nu}^{*} - l^{*}} \frac{\lambda_{n} + Q_{\nu} e^{i(\nu - \frac{1}{2})}}{\lambda_{\nu} + Q_{\nu} e^{i(\nu - \frac{1}{2})}} \frac{C_{n} - g_{n}}{\alpha} = \frac{\lambda_{\nu} + P_{\nu} e^{i(\nu - \frac{1}{2})}}{\lambda_{\nu} + Q_{\nu} e^{i(\nu - \frac{1}{2})}} \quad (\gamma = 0.12) \quad (50)$$

where

$$P_{r} = \frac{r \nabla t_{enth} Y H}{\beta} \qquad Q_{r} = \frac{r \nabla c_{o} th Y H}{\beta} \qquad P_{o} = 0 \qquad (51)$$

$$Q_{o} = \lim_{\gamma \neq 0} r H c_{o} th Y H H = V \beta H, \qquad \beta = \sqrt{1 + (M/o)^{2}}, \quad V = t_{ent}^{-1} (M/o)$$

Solving eq 49 and 50, we obtain b/a, f/a,  $c_{\kappa}/a$  and  $g_{\kappa}/a$  for breakwater, and then  $D_{\gamma}$  and  $E_{\gamma}$  are given as follows

$$D_{0} = \frac{d_{0}}{2} = \frac{\nabla}{2\beta} e^{i\nu} \frac{\sigma^{2}h}{\beta} \frac{\alpha}{\pi^{2}} \left[ \left(1 + \frac{d+f}{\alpha}\right) \frac{1}{\lambda^{2}} - \sum_{n=1}^{\infty} \frac{1}{\lambda^{2}} \frac{(n+g_{n})}{\alpha} \right]$$

Putting

$$D_{r} \cosh tH = dr = \frac{\nabla}{\beta} e^{\nu} \frac{\sigma^{3} h}{g} \frac{Q}{\Pi^{2}} \left[ \left( 1 + \frac{k+j}{\alpha} \right) \frac{1}{\lambda_{b}^{b} + \gamma^{2}} - \sum_{n=1}^{\infty} \frac{1}{\lambda_{n}^{a} - \gamma^{2}} \frac{Cn + \Im n}{\alpha} \right]$$

$$E_{r} \cosh h H = e_{r} = \frac{\nabla}{\beta} e^{\nu} \frac{\sigma^{3} h}{g} \frac{\alpha}{\pi^{2}} \left[ \left( 1 + \frac{k-j}{\alpha} \right) \frac{1}{\lambda_{b}^{2} + \gamma^{2}} - \sum_{n=1}^{\infty} \frac{1}{\lambda_{n}^{a} - \gamma^{2}} \frac{Cn - \Im n}{\alpha} \right]$$
(52)

Surface wave profiles, reflection and transmission coefficients etc. are given in the same form of eq 28, 29 and 30 etc, using b/a, f/a,  $c_{\rm M}/a$  and  $g_{\rm M}/a$  by eq 49 and 50 The fluid pressure in the region II is given by eq 45 for the incident wave  $\zeta_{\rm R} = d \cos(kx + t)$  as follows

$$\frac{b}{f_{T}^{2}d} = \frac{\beta}{V} \frac{c(k+\alpha t-\nu)}{C} \left[ \frac{d_{0}}{2\alpha} + \sum_{T=1}^{\infty} \left( \frac{d_{T}}{\alpha} \frac{(\alpha d_{T}H \lambda/\ell)}{c c h} + \frac{c_{T}}{\alpha} \frac{amLrH\lambda/\ell}{amh rH} \right) cod \frac{r\pi \ell}{h} \right]$$
(53)

Pressure distribution and the total pressure to horizontal plate are given as follows

$$\frac{\beta_2(z=0)}{\beta_{\text{gd}}} = \frac{\beta}{V} e^{\frac{(kl+0L-V)}{2a}} \left[ \frac{d_0}{2a} + \sum_{Y=1}^{2a} \left( \frac{d_Y}{a} \frac{\cosh H L/l}{\cosh YH} + \frac{e_Y}{a} \frac{\sinh H L/l}{\sinh YH} \right)$$
(54)

$$\left| \frac{Pu/2fgdl}{V} \right| = \frac{\beta}{V} \left| \left( \frac{do}{2a} + \sum_{\gamma=1}^{\infty} \frac{d\gamma}{a} \frac{tankrH}{\gamma H} \right) \right|$$
(55)

Above-mentioned results are for single plate breakwater For quay wall, similar results are obtained by the same process as in section j

9 (alculated results for horizontal plate breakwater with permeable material

As an example, b/a, f/a,  $c_n/a$ ,  $g_n/a$  and  $d_r/a$ ,  $e_r/a$  by eq 49 and 50 for  $f^h/g = 1.0$ , f/h = 1.0, V = 0.5 and f/h = 1.0 are shown in Table 3

Transmission and reflection coefficients for k/h = 10, V = 05 are shown in Fig 13 for parameter k/c. When k/c = 0,  $K_Y$  and  $K_t$  are equal to those for k/h = 20in Fig 6(a) for the case without permeable material When k/c = 0,  $K_Y$  becomes smaller than that for k/c = 0 and then it increases with the increase of k/c. On the contrary,  $K_t$  decreases steadily with increase of k/c. This means that the void has the effect to elongate the length k of the plate to k/V and the fluid resistance has the effect to resist to the incident wave and to cause energy dissipation in breakwater

The pressure distribution to plate by eq 54 is shown in Fig 14 Compared with Fig 7(a), it is seen that the distribution is steeper than the case without permeable material and the tendency is remarkable for large  $\int_{-\infty}^{\infty} h/g$  This coincides with the fact that the void has the effect to elongate the plate length

It may be said that the submerged plate has the effect to make the pressure

## COASTAL ENGINEERING

distribution uniform but the submerged permeable material has the effect to make the distribution steep — lhe averaged total pressure by eq 55 is nearly equal to the case without permeable material

### 10 Calculated results for horizontal plate quay wall with permeable material

For the same conditions as previous section, reflection coefficient  $K_{\Upsilon}$  for quay wall is calculated as shown in Fig 15  $K_{\Upsilon}$  decreases steadily with the increase of  $\delta^2h/g$ , but not so with  $M_{\Lambda^*}$  for large values of  $\delta^2h/g$ ,  $K_{\Upsilon}$  is minimum for  $M_{\Lambda^*}$ between 1 0 and 2 0 Fhat is, reflection coefficient is the least for a particular value of  $M_{\Lambda^*}$  depending on  $\delta^2h/p$  and becomes larter for other values of  $M_{\Lambda^*}$  As seen from Fig 15,  $K_{\Upsilon}$  is larger than 0 94 for  $\delta^2h/g \leq 15$  and also by calculations for various values of L/h, it is found that  $K_{\Upsilon}$  is almost constant for  $L/h \geq 10$ , that is,  $K_{\Upsilon}$  is independent on the length of plate for  $L/h \geq 10$ . The fact that the reflection coefficient of quay wall does not decrease even for large values of  $M_{\Lambda^*}$  and L/h is interpreted to be due to the small fluid velocity and the energy dissipation inside the permeable material under the horizontal plate

### 11 Comparisons with the experiment

Using the wave channel of length 22 meters, width 1 0 meters and depth 0 6 meters with flap type wave generater, experimental measurement was carried out for single plate breakwater of h = 40cm and l = 40cm ( $l_h = 1.0$ ) without permeable material, for incident wave amplitude of nearly constant of 3cm fransmission and reflection coefficients are shown in Fig 16, in which each plotted datum is the mean value of 10 Heasured trnsmission coefficients agree well with theoretical times measurements values but reflection coefficients are lower than theory The measured pressure distribution to plate is shown in Fig 17, which shows that the measured distributions are remarkably steeper than those of theoretical ones These discrepancies might be due to wave overtopping and eddies generated at the front edge of plate Measured average total pressure to the plate are shown in Fig 18, which shows good agreement with theory For the case with permeable material, experiments are now under taken

#### 12 Conclusions and remarks

Theory of single plate breakwaver for long waves are introduced by J J "toker (1957) and the mathematical study of dock problems are presented by K O Friedrichs and H Levy(1948) and others In this paper we have tried the extension of Stoker's result to the case of general wave conditions by different method from these authors

# QUAY WALL

dain results of our study are summerized as follows

(1) For the case of breakwater without permeable material, a horizontal plate fixed rigidly it water surface behaves like as a semi-rigid breakwater for short surface waves and more incident wave energy is reflected for shorter waves and longer plate The longer the plate is, the more steeply distributes the pressure along the plate In case of double plates, the pressure distribution becomes remarkably smooth (ii) For the case of quay wall, the pressure distribution to vertical wall is remarkably uniform by the effect of horizontal plate, compared to the case without plate The plate is, the more equally becomes the averaged pressures to vertical wall and to horizontal plate. In other words, the horizontal vlate at water surface of quay wall plays the role of pressure distributions to both directions becomes more smooth than the case of single plate

(111) When the region under the horizontal plate is filled by permeable material, the void has the effect to elongate the plate, so that for breakwater the transmission coefficient decreases and the reflection coefficient increases and for both of breakwater and quay wall the distribution of pressure along the plate becomes steeper than the case without permeable material the effect of fluid resistance is partly to reflection coefficient increases and quay wall the distribution coefficient increases and the transmission coefficient decreases and quay wall the decrease of reflection coefficient is not so remarkable. The pressure distribution along the plate and the vertical wall is not changed remarkably by the effect of fluid resistance

#### References

Stoker,J J (1957) "Water Waves " pp 450 436, Interscience Publishers Friedrichs, 6 0 and 1 Levy(1948) "The Dock <sup>C</sup>roblem " Communications on Pure and Applied Mathematics, Vol 1, pp 135 148

) ash/g	01	05	10	15	20
入。	<ul> <li>0 10237</li> <li>0 98977</li> <li>1 99492</li> <li>2 99662</li> <li>3 99747</li> <li>4 99794</li> <li>5 99831</li> </ul>	0 24564	0 38187	0 51624	0 65742
入1		0 94700	0 89075	0 83459	0 78263
入2		1 97440	1 94845	1 92251	1 89699
入3		2 98303	2 96597	2 98489	2 93194
入4		3 98730	3 97456	3 96182	3 94913
入5		4 98985	4 97968	4 <b>9</b> 6951	4 95953
入6		5 99155	5 98308	5 <b>9</b> 7462	5 96622

Table 1  $\lambda_0$  and  $\lambda_n$  for  $\sigma^2 h/g$ 

Table 2 (a)  $\sigma^2 h/g = 1.0$ , l/h = 1.0 (single plate) b/a = 0.5621 + 0.5434 1 f/a = 0.4334 - 0.4482 1

n	c <sub>n</sub> /a	g <sub>n</sub> /a
1	- 0 1340 - 0 03753 1	- 0 05896 + 0 02837 1
2	- 0 03114 - 0 01006 1	- 0 01050 + 0 008075 1
3	- 0 01392 - 0 004658 1	- 0 004308 + 0 003788 1
4	-0 007891 -0 002683 1	- 0 002340 + 0 002195 1
5	- 0 005077 - 0 001742 1	– 0 001467 + 0 001430 ı
6	- 0 003532 - 0 001220 1	- 0 001003 + 0 001003 1
7	- 0 002582 - 0 0008751 1	- 0 0007238 + 0 0007374 1
r	dr/a	e <sub>t</sub> /a
<b>r</b> 0	dr/a 0 7063 + 0 03396 1	er/a
<b>r</b> 0 1	d <b>r</b> ∕a 0 7063 + 0 03396 1 0 08366 + 0 003990 1	e <sub>f</sub> /a 0 06392 + 0 05616 1
<b>r</b> 0 1 2	$\frac{d_{r}}{a}$ 0 7063 + 0 03396 1 0 08366 + 0 003990 1 0 02246 + 0 001071 1	e <sub>f</sub> /a 0 06392 + 0 05616 1 0 01522 + 0 01337 1
<b>r</b> 0 1 2 3	$\frac{d_{\Upsilon}/a}{0.7063} + 0.03396 1$ 0.08366 + 0.003990 1 0.02246 + 0.001071 1 0.01004 + 0.0004788 1	er/a 0 06392 + 0 05616 1 0 01522 + 0 01337 1 0 006499 + 0 005710 1
<b>r</b> 0 1 2 3 4	$\frac{d_{\Upsilon}/a}{0\ 7063\ +\ 0\ 03396\ 1}$ $0\ 08366\ +\ 0\ 003990\ 1$ $0\ 02246\ +\ 0\ 001071\ 1$ $0\ 01004\ +\ 0\ 0004788\ 1$ $0\ 005633\ +\ 0\ 0002687\ 1$	er/a 0 06392 + 0 05616 1 0 01522 + 0 01337 1 0 006499 + 0 005710 1 0 003563 + 0 003131 1
r 0 1 2 3 4 5	$\frac{d_{\Upsilon}/a}{0\ 7063\ +\ 0\ 03396\ 1}$ $0\ 08366\ +\ 0\ 003990\ 1$ $0\ 02246\ +\ 0\ 001071\ 1$ $0\ 01004\ +\ 0\ 0004788\ 1$ $0\ 005535\ +\ 0\ 0002687\ 1$ $0\ 003594\ +\ 0\ 0001714\ 1$	$e_{\gamma}/a$ 0 06392 + 0 05616 1 0 01522 + 0 01337 1 0 006499 + 0 005710 1 0 003563 + 0 003131 1 0 002242 + 0 001969 1
<b>r</b> 0 1 2 3 4 5 6	$\frac{d_{\mathbf{r}}/a}{0.7063} + 0.03396 1$ $0.08366 + 0.003990 1$ $0.02246 + 0.001071 1$ $0.01004 + 0.0004788 1$ $0.005633 + 0.0002687 1$ $0.003594 + 0.0001714 1$ $0.002488 + 0.0001187 1$	er/a 0 06392 + 0 05616 1 0 01522 + 0 01337 1 0 006499 + 0 005710 1 0 003563 + 0 003131 1 0 002242 + 0 001969 1 0 001537 + 0 001350 1

	b/ a = 0 0014/2 4 0 0002 a	
n	cn/a	gn/a
1 2 3 4 5 6 7	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$
r	dr/a	er /a
0 2 4 6 8 10 12 14	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	0 002614 + 0 004496 1 0 0003059 + 0 0005262 1 0 00008513 + 0 0001464 1 0 00003464 + 0 00005959 1 0 00001745 + 0 00003002 1 0 00001007 + 0 00001732 1 0 00001007 + 0 00001732 1

Table 2 (b)  $\delta^{h}/g = 10$ ,  $\delta^{h}/h = 10$  (double plate) b/a = 0.001452 + 0.86821, f/a = 0.4963 - 0.00083011

h/g = 10, $h/h = 10$ ,	V=05, M/~=10
b/a=0 7769+0 1685 1	f/a = 0 1930 - 0 1239 1

		1/4 = 0 1990 = 0 1299 1	
n	c <sub>n</sub> /a	gn/a	
1 2 3 4 5 6	$\begin{array}{rrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrr$	- 0 02255 - 0 009455 1 - 0 003733 - 0 0008074 1 - 0 001490 - 0 0001913 1 - 0 0007981 - 0 00006291 1 - 0 0004964 - 0 00002332 1 - 0 0003377 - 0 00008349 1	
7	- 0 001384 - 0 001046 1	- 0 0002429 - 0 000002015 1	
r	d <b><sub>1</sub> /a</b>	er/a	
0 1 2 3 4 5 6 7	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{rrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrr$	





















Fig.- 5 Quay Wall with Horizontal Plate



Photo. Z











Fig 13 Kr and Kt of Breakwater





Fig 15 Reflection Coefficient of Quay Wall



Fig 16 Kr and Kt Single Plate Breakwater



