CHAPTER 89

PRESSURE OF WAVES AGAINST VERTICAL WALLS

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SUMMARY

This paper is concerned with the study of the breaking wave pressure exerted upon the breakwater of the vertical type. The method of calculation of the wave pressure, named the quasistatical method, is based with the theory of the impact of the water jet on the vertical plane and some new results of the experimental data.

The formulas for the calculation of the pressure distribution of the breaking wave and the surf wave on the vertical wall are given.

INTRODUCTION

The following symbols are used in this paper.

\[
\begin{align*}
  h & \quad \text{wave height;} \\
  \lambda & \quad \text{wave length;} \\
  \tau & \quad \text{wave period;} \\
  c & \quad \text{wave celerity;}
\end{align*}
\]

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v  -- orbital velocity,
p  -- pressure intensity,
H  -- depth of water measured from SWL,
$H_{cr}$  -- critical depth of water corresponding to the point of breaking wave,
$H_w$  -- depth of water at the wall,
$\gamma$  -- unit weight of water,
g  -- 9.81 m/sec$^2$  -- gravitation acceleration.

Up to the present time evaluation of the breaking wave force upon a breakwater of the vertical type presents a complex problem. For many decades this problem has requested very much attention from the harbour engineer, who has to deal with design and construction of breakwaters of the vertical type at a shallow sea.

There are several solutions of this problem, one of them makes use of the conception of the theory of the impact of a water jet upon a vertical plane. However, in spite of the fact that these investigations have been carried out during a long time the possibility of the conception of the impact of a water jet upon a vertical plane in full measure is not used.

This expression is given

$$p = k \gamma \frac{u^2}{2g}$$

(1)

where

$u$  -- velocity of the water jet at the crest of a breaking wave,
k  -- experimental coefficient equal to 1.7 (by Gaillard).

In their time B. Gaillard, D. Molitor and V. Tremuhin developed this direction suggesting to take into account $u = c + v$. 
In 1940 N. Djounkovskiy\cite{1} corrected this to $u = 0.75c + v$. In 1958 M. Plakida\cite{2} suggested the new corrections to the distribution of the surf wave pressure exerted upon the vertical wall, based upon the experimental investigation. As a consequence the resultant value of the wave pressure on the vertical wall could be decreased up to 20 per cent in comparison with the value given by N. Djounkovskiy\cite{3}.

It is necessary to note that summing up $c$ and $v$ makes no physical sense, this is explained by the absence of sufficient information about the kinematics of the breaking wave structure.

In this paper we have given as far as it was possible the development of the calculation method of the pressure distribution from the breaking wave and the surf wave on the vertical wall. Remaining at the conception of the impact of the water jet for the calculation of the wave pressure at the still water surface, we have used the standing wave theory of the first approximation and some experimental data for the calculation of the wave pressure at the foot of the vertical wall.

This expression is given

$$P_b = \frac{gH}{ch} \frac{2\pi H_w}{\lambda}$$

where

$P_b$ - wave pressure (above hydrostatic) at the foot of the vertical wall.

We note that it was about twenty years ago, when the method of calculation of the breaking wave pressure, based on the use of the impact impulse of the breaking wave, was suggested. It is the second direction in the solution of this problem. However, the experimental data is insufficient for the full solution at the present time.
KINEMATICS AND DYNAMICS OF BREAKING WAVES

The breaking wave or the surf wave is developed when the deep water wave comes up to a shallow water and reaches the critical depth. The value of the critical depth varies in a very wide range \((1.0 \pm 2.5)h\). The critical depth within the limits \((1.5 \pm 1.8)h\) are accepted for the steepness of the deep water waves within \(1:9 \rightarrow 1:25\).

When the water depth in front of the vertical wall is \(H > \frac{A}{2}\) or at least \(H > 3h\) and the depth on the berme at the foot of the wall is \(H_w \leq H_{cr}\) then the deep water, wave is broken on the berme at the wall surface; in this case the vertical wall is subjected to by the breaking wave action.

When in front of the vertical wall the sea bed is horizontal or gently sloping at least within \(\frac{A}{2}\) before the wall and the water depth is \(H_1 \leq H_{cr}\) then the deep water wave is broken before the wall, in this case the vertical wall is subjected to the action of the surf wave.

It is seen that from the condition of development of the breaking wave and the surf wave the former exerts a greater wave pressure on the vertical wall than the latter for the same height and length of the deep water wave.

The experimental data show that the velocity of the water particles at the crest of the breaking and surf waves may be nearly equal to the wave celerity and even exceed it.

The value of this velocity is given as

\[ u = \sqrt{gh} \]  
(3)

The formation of the front steep slope of the wave, which is near to breaking, is the result of dragging of the foot of the wave, while the crest of the wave passes ahead because of different wave velocities of the particles in the crest and in the trough. The velocities of the water par-
Particles in the crest of the wave are greater and the velocities of the water particles in the trough of the wave are less than the average velocity of the wave at the still water surface. The kinematics of this phenomenon in detail is described by V. Shuleykin [4] based on the classical hydrodynamics formula

\[ c = \sqrt{\frac{g \lambda}{2 \pi \frac{2 \pi H}{\lambda}}} , \quad (4) \]

taking in (4) \( H = H_1 + 0.5h \) for the wave crest, and \( H = H_1 - 0.5h \) for the wave trough (where \( H_1 \) - water depth from the sea-bed to the still water surface).

From the energetical point of view in the wave near to breaking the concentration of the potential energy occurs, as the deep water wave approaching shallow water decreases in length. In addition the wave energy is carried into shallow water with the full wave velocity as can be seen from the formula (5), when at \( H \ll \lambda \) the second item \( \rightarrow 1 \).

\[ u_o = \frac{c}{2} \left( 1 + \frac{2aH}{sh2aH} \right) \quad (5) \]

where

- \( u_o \) - velocity of the transportation of the wave energy;
- \( a = \frac{2\pi}{\lambda} \).

Phenomenon of the breaking of the wave proceeds very rapidly and during very short time. It is accompanied by the transition of the potential energy of the dragged wave into the kinetic energy of the transitional movement of the water stream. When this stream reaches the vertical wall the phenomenon of the impact of the water jet directed to the wall and the wave reflection from the wall are observed.
The maximum wave pressure on the wall appears earlier than the maximum elevation of the water level at the wall. When the uplift of the water level is maximum the wave pressure decreases. The wave pressure does not exceed the value \((0.6 - 1.0) \gamma h\) at the still water surface.

**EQUIPMENT AND EXPERIMENTS**

The laboratory study of the action of the breaking and the surf waves on the vertical wall has been carried out in a flume of rectangular cross-section 23 m in length, 0.50 m wide. The water depth was 75 cm in the flume and the water depth at the foot of the wall was 15 cm. The vertical wall model was placed on the top of the prism, which was 60 cm above the flume bottom. The prism slope was 1:2.

The positions of the vertical plane of the wall were 25 cm and 82 cm from the prism edge (Fig.1). In the first case the wall is exposed to the action of the breaking wave and in the second case – to the action of the surf wave.

The waves were reproduced by the wave generator of the type of the flat paddle.

The wave pressures were measured by tensemometric gauges. The wave heights were measured by electrical gauges. A sample of the oscillograph record is given on Fig.2.

Phases of breaking, wave deformation details and the moment of small ball indicators (prepared with bitumen and paraffin, their specific weight is equal to that of water) were recorded on 35 mm film at 24 frames per sec. A sample of film record is shown on Fig.3. It is seen that the indicator I between frames 15-17 has moved to the wall with maximum velocity before the impact.

Wave conditions of our experiments are given in table 1.
WAVE PRESSURE

After giving a short description of the kinematics and dynamics of the breaking wave and our experiments, we can begin to evolve the formulas for the calculation of the breaking wave pressure and the surf wave pressure exerted upon the vertical wall.

Our position is based on the formulas (1) - (3) and our experimental data. The following calculation method and formulas for the determination of the value of the pressure on the vertical wall from the breaking and surf waves is suggested.

Pressure of breaking waves. It is a matter of some difficulty to assume the value of the water depth \( H \) in formula (3). As a solution this complex question after some considerations we suppose possible to take a safe value of

\[
H = H_{cr} = 1.8h
\]
Substituting (6) and (3) in Eq. (1) we arrive at the expression for the maximum pressure of the breaking wave, which occurs at or in the vicinity of still water surface

\[ p_0 = 1.5 \sqrt{h} \]  

(7)

where

\[ p_0 \] — the maximum pressure of the breaking wave at the still water surface.

At the foot of the vertical wall rather calm wave conditions are observed (see the pressure fluctuation on the record of D10 on Fig. 2).

There are traced (see Fig. 4) two curves showing the fluctuation of the relative wave pressure at the foot of the wall. One of them is the experimental curve indicating the pressure of the breaking wave and the second is the theoretical curve showing the pressure of the standing wave. It is seen by the comparison of these two curves that the maximum pressure of the breaking wave at the foot of the wall appears earlier than the maximum pressure of the standing wave, calculated by the wave theory of the first approximation. As a result of this comparison of the two curves, we propose to base the calculation of breaking wave pressure at the foot of the vertical wall upon the formula (2).

The relationship between \( \frac{p_0}{\sqrt{h}} \) and \( \frac{H_w}{h} \) is given on Fig. 5 by the experimental data. This relationship is not linear as it may be expected. The greatest values of the relative wave pressure are found within \( \frac{H_w}{h} = (1.2 - 1.5) \).

The curve accepted by the construction rules acting in the USSR (SN -92-60) is also shown.
The maximum values of the breaking wave pressure measured and calculated are given in Table 2. These data were used for the experimental curve on Fig. 5.

Table 2

Values of wave pressure at the bottom of a vertical wall (gr/cm²)

<table>
<thead>
<tr>
<th>h cm</th>
<th>Hw/h</th>
<th>Calculated</th>
<th>Measured</th>
<th>Pi15</th>
<th>Pi15</th>
<th>Pi15</th>
<th>Pi15</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.8</td>
<td>1.70</td>
<td>7.6</td>
<td>5.7</td>
<td>0.76</td>
<td>0.86</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11.5</td>
<td>1.30</td>
<td>9.9</td>
<td>10.9</td>
<td>1.10</td>
<td>0.94</td>
<td></td>
<td></td>
</tr>
<tr>
<td>15.7</td>
<td>0.96</td>
<td>14.0</td>
<td>12.8</td>
<td>0.92</td>
<td>0.82</td>
<td></td>
<td></td>
</tr>
<tr>
<td>17.7</td>
<td>0.85</td>
<td>14.6</td>
<td>13.0</td>
<td>0.93</td>
<td>0.74</td>
<td></td>
<td></td>
</tr>
<tr>
<td>mean</td>
<td></td>
<td></td>
<td></td>
<td>0.93</td>
<td>0.84</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

where:

\( P_{15} \) — significant wave pressure that for statistical purposes is defined as the average pressure of the highest one-third of all measured breaking wave pressures at the foot of the vertical wall;

\( P_{15}^c \) — wave pressure calculated by formula (2);

\( H_w = 15 \) cm, — water depth at the wall.

The elevation of water surface above the still water level at the vertical wall is observed within \((0.5 - 0.8)h\) when the wave pressure is maximum. Taking into consideration some fluctuation of this value we propose to take

\[ z = h \] (8)
The breaking wave pressure exerted upon the foundation of the vertical wall is given as

\[ W = \frac{1}{2} M_b p_b b \]  

where \( M_b = 0.9 \) - experimental coefficient;
\( b \) - width of the wall.

Pressure of surf waves. The breaking wave and the surf wave possess the same physics of the breaking phenomenon. This allows us to use, in the case of the surf wave, the same formulas, that were mentioned above in the case of the breaking wave with following replacements:

1. \( h \) - the height of the deep water wave (which is equal to the breaking wave) is replaced by the height of the surf wave \( h_1 \), calculated by the formula

\[ h_1 = 0.65H \]  

where

\( H = H_w \) - in the case when the sea-bed is horizontal in front of the wall, and
\( H = H_o = H_w + 0.5 \lambda_1 i \) - in the case, when the sea-bed is gently sloped;

where \( i \) - sea-bed slope.

2. \( \lambda \) - the length of the deep water wave is replaced by the length of the surf wave \( \lambda_1 \), calculated by the formula

\[ \lambda_1 = \lambda \text{ th} \frac{2 \pi H}{\lambda_1} \]
The maximum pressure of the surf wave exerted upon a vertical wall occurs at $1/3 \, h_1$ above still water level according to our experimental data.

The elevation of water surface above the still water level in the case of the surf wave may be also taken as in the case of the breaking wave by the formula (8).

The surf wave pressure exerted upon the foundation of the vertical wall is given as

$$W = 1/2 \, M_s p_b b$$  

(12)

where:

$$M_s = 0.7$$ - experimental coefficient.

The distribution of the pressure of the breaking and surf waves is shown on Fig. 6.

CONCLUSION

1. The action of the breaking and surf waves upon the vertical wall was studied in the wave laboratory from the point of view of the kinematics and the dynamics. It was found possible to advance somewhat a quasistatical method of the calculation of the wave pressure at the depth conditions

$$h \leq H_w \leq H_{cr} = 1.8h$$

This suggestion is based on the conception of the impact of a water jet upon a vertical plane.

2. Our experimental data lies within of the following values:
   - the wave steepness $h/\lambda$ from 0.05 to 0.10; and
   - the relative water depth at the wall $H_w/h$ from 0.85 to 1.71.
3. The list of the formulas is given in the table 3.

**Table 3**

<table>
<thead>
<tr>
<th>Denomination</th>
<th>Breaking wave</th>
<th>Surf wave</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Wave height and wave length</td>
<td>$h, \lambda$</td>
<td>$h_1, \lambda_1$</td>
</tr>
<tr>
<td>2. Maximum wave pressure $p_0$</td>
<td>$1.5 \gamma h$</td>
<td>$1.5 \gamma h_1$</td>
</tr>
<tr>
<td>3. Maximum wave pressure is found</td>
<td>At the S.W.L.</td>
<td>At $\frac{1}{6}h_1$ the S.W.L. above</td>
</tr>
<tr>
<td>4. Maximum wave pressure at the foot of the wall $p_b$</td>
<td>$\frac{\gamma h}{\text{ch} \frac{2\pi H_w}{\lambda}}$</td>
<td>$\frac{\gamma h_1}{\text{ch} \frac{2\pi H_w}{\lambda_1}}$</td>
</tr>
<tr>
<td>5. Elevation of the water surface above the still water level $s$</td>
<td>$h$</td>
<td>$h_1$</td>
</tr>
<tr>
<td>6. Wave pressure exerted upon the foundation of the wall $W$</td>
<td>$0.9 \frac{p_b b}{2}$</td>
<td>$0.7 \frac{p_b b}{2}$</td>
</tr>
</tbody>
</table>
REFERENCES

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Fig. 1 Model scheme

Fig. 2 Pattern of the oscillograph record
Fig. 3 The breaking wave surface and the trajectories of the movement of water particles (Film 24 cm/sec).
Fig. 4. Variation of the relative pressure at the bottom of the vertical wall
Fig 5 Relation between $P_b^8$ and $\frac{H_w}{h}$
Fig 6. Vertical distributions of the pressure of the breaking wave and the surf wave against the vertical wall.