CHAPTER 82

EQUILIBRIUM PROFILES OF MODEL BEACHES

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ABSTRACT

The investigation reported herein covers two aspects of equilibrium beach profiles, namely, (a) the criterion governing their type and (b) their reflection characteristics The problems are first analysed from dimensional considerations and then studied experimentally in laboratory wave flumes using different sizes of quartz, ground walnut shell and ground plastic as movable bed material. Empirical relations between pertinent parameters governing the beach process are given.

Introduction:

The wave and hydrologic climate at a beach location is everchanging and the coastal engineers have always been interested in the deformation of the natural beach caused by wave action over a short or long interval of time. The resulting changes in beach characteristics, such as the type of profile, beach width, berm level, location and size of offshore bars, height of run-up, the rate and mode of sediment transport, areas of deposition and scour and the amount of energy absorbed or reflected from the beach, are often rapid. Coastal management requires a knowledge of the fundamental principles involved in the behaviour of the

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beach material and research attempts have, long since, been directed to obtain analytical and empirical relations between the different pertinent variables involved in the extremely complicated beach process.

The study reported herein covers two aspects of equilibrium beach profiles, namely, criterion governing their type and reflection characteristics. The experiments relating to the criterion for the type of profile have been performed as series A and those of the latter as series B in two separate experimental units in the laboratory.

<u>Part A:</u> Criterion for type of profile.

Almost all the investigations that have been conducted to date have indicated that deep-water wave steepness is an important parameter related to the beach process and a critical value of this parameter characterises various elements of the phenomenon like the type of profile, mode and rate of transport and the type of breakers etc. In this part of the study an attempt is made to relate the variation in the critical value of deep-water wave steepness concerning the type of beach profile, "storm" or "summer", to the characteristic size and specific gravity of the beach material.

The type of equilibrium profile, storm or summer, that results from the action of waves in a two dimensional flume can be assumed to be governed by the following variables.

(i) deep-water wave height, H_o

(ii) wave period, T

(iii) specific gravity of sediment in water, S

(iv) median diameter of sediment, D

(v) depth of water in the channel, d

(vi) acceleration due to gravity, g

(vii) initial slope of beach, i_o

(viii) standard deviation of grain size, σ

i e type of profile = $f_1 (H_o, T, S, D, \sigma, i_o, g, d)$

$$= f_2 (H_o, L_o, S, D, i_o, g, d)$$

The role of viscosity of the fluid is of minor importance if we assume that the Reynolds number is sufficiently high and the entire phenomenon takes place well with in the turbulent range. If the zone of sand movement is limited on the beach slope, the effect of depth of water can be neglected. The gravitational acceleration is constant. The standard deviation of grain size is of minor importance and from earlier studies one can find that the initial beach slope plays no major role in the phenomena Thus considering the equilibrium condition which is independent of duration,

Type of profile = $f_3 (H_o/L_o, H_o/D, S)$

or = $f_4 (H_o/L_o, H_o/SD, S)$

The problem can be also viewed as

Type of profile = $f (H_o, L_o, S, D)$

$$= f (H_o, L_o, V_f)$$

considering that the fall velocity in water, V_f, characterises the sediment,

i.e., type of profile = $f \left(H_o/L_o, \frac{\sqrt{gHo}}{V_f}\right)$

The fall velocity can be taken to be proportional to the quantity $\left(\frac{S\,g\,d}{C_D}\right)^{\frac{1}{2}}$

$$\therefore \text{Type of profile} = f \left(H_o/L_o, \frac{\sqrt{g\,Ho}}{S^{\frac{1}{2}}\, g^{\frac{1}{2}}\, D^{\frac{1}{2}}}\, C_D^{\frac{1}{2}}\right)$$

$= f_5 (H_o/L_o, H_o/SD)$, disregarding the effect of C_D.

The parameter H_o/SD can also be taken as the ratio of a typical unit force exerted by the wave to unit resistance offered by the beach sediment, because the former is proportional to

$$\rho_f \times (\text{a characteristic velocity of flow pattern})^2 \times D^2$$

and the latter to the submerged weight of the grain of bed material, i.e., $g\ (\rho_s - \rho_f)\ D^3$. If the characteristic velocity is taken as $\sqrt{gH_o}$, the ratio becomes

$$\frac{\rho_f \cdot u_c^2 \cdot D^2}{g\ (\rho_s - \rho_f)\ D^3} = \frac{\rho_f\ u_c^2 / g}{(\rho_s - \rho_f)\ D} = \frac{H_o}{SD}$$

The ratio H_o/SD is also comparable to the ratio $1/\psi$ where ψ is the intensity of flow given by the expression $\psi = \frac{\rho_s - \rho_f}{\rho_f} \cdot \frac{Dg}{U^2}$,

an important quantity used in sediment flow problems.

Thus the ratio of wave height to the product of specific gravity in water and the median diameter of the sediment can be taken to be a very important parameter in the beach process.

The importance of the size of the beach material has been pointed out by Rector (1954) and the ratio of wave height to median diameter by Iwagaki (1962) (Fig.1). The present experiments were carried out in a wave flume shown in Fig.2. The wave height was noted with the help of a point gauge placed just offshore from the toe of the beach and was taken as twice the distance recorded between the crest of the wave and the still water level. This is correct in deep water. The beach profile was recorded by measuring the horizontal and vertical coordinates on a rectangular grid fixed to the side of the flume. The first four materials whose size distribution is given in Fig.4, were used as beach material in

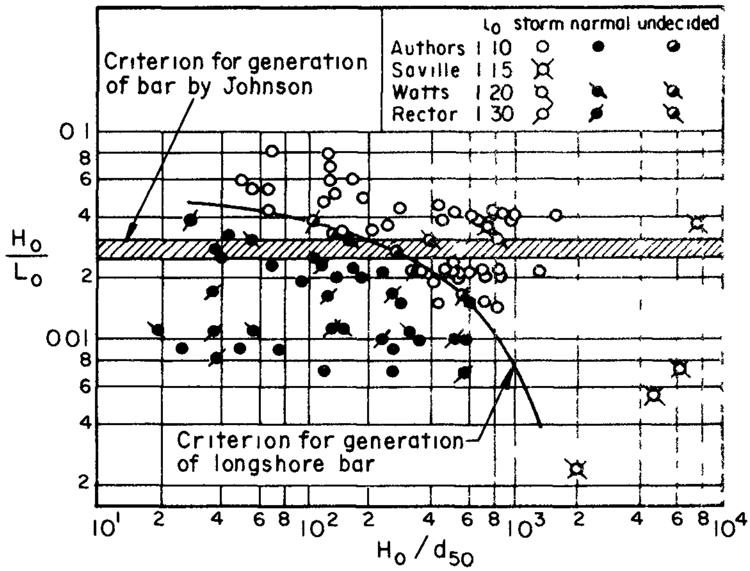

FIG 1 CRITERION OF BAR GENERATION (IWAGAKI)

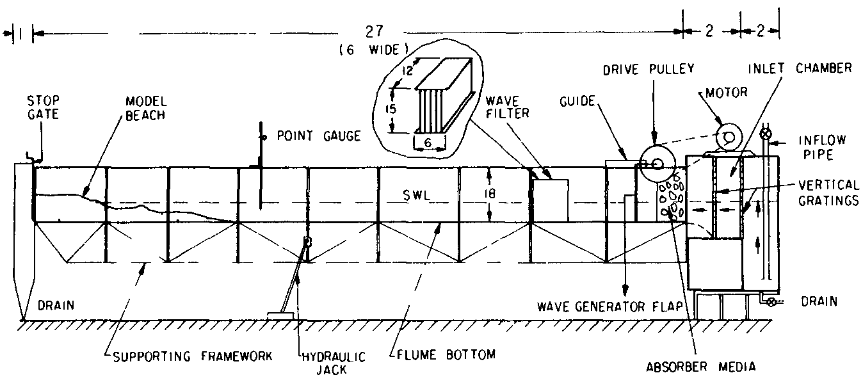

FIG 2 WAVE FLUME USED IN SERIES A

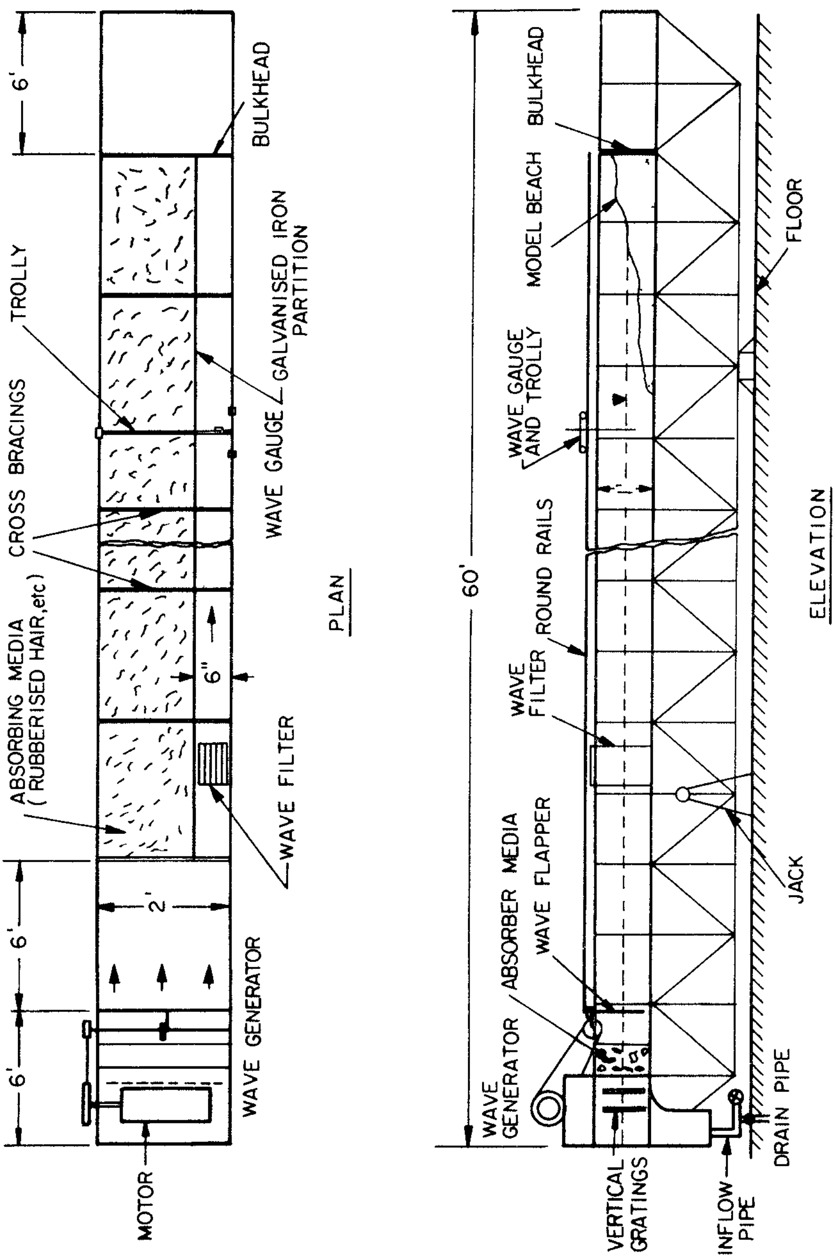

FIG 3 WAVE FLUME USED IN SERIES B

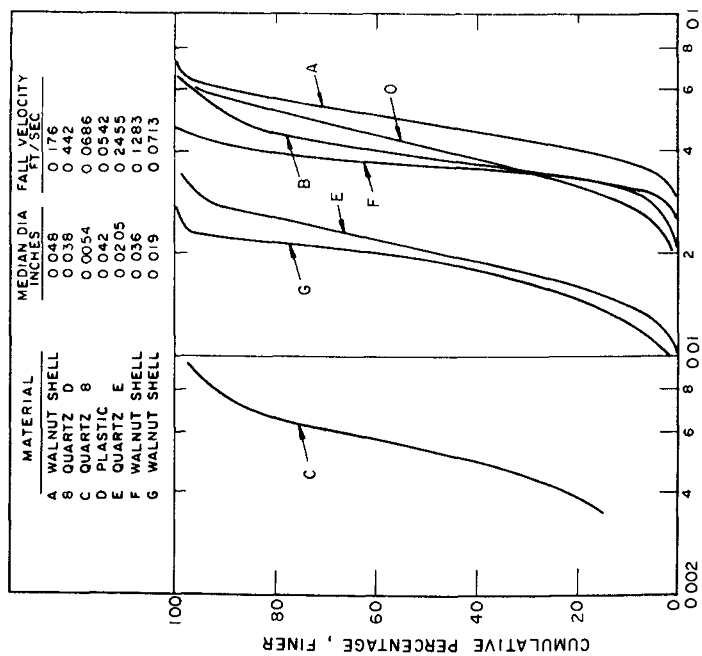

FIG 4 SIZE DISTRIBUTION OF MATERIALS

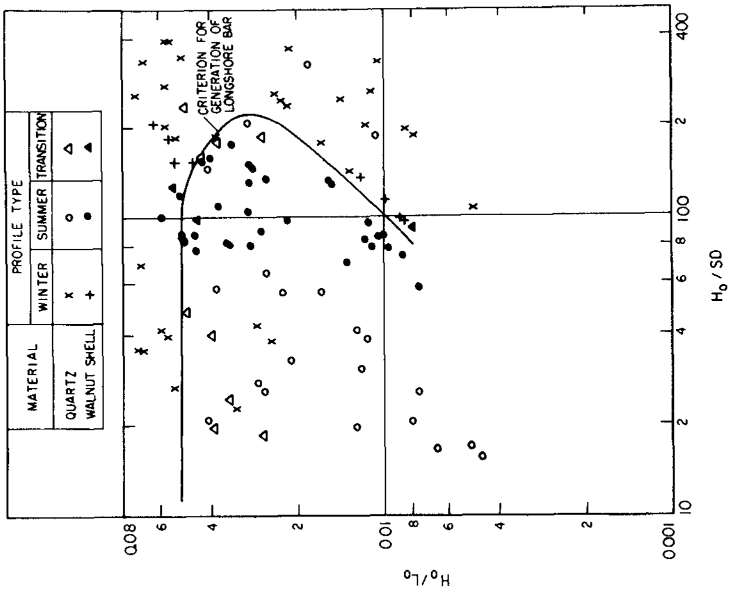

FIG 5 CRITERION FOR GENERATION OF LONGSHORE BAR

series A. In the case of each material, the depths of water used were 0 30, 0 35, 0.40, 0 45 and 0 50 ft. For each depth the following wave periods were used 1.86, 1 60, 1.26, 1 01, 0 845 and 0.72 secs. At the beginning of each group of experiments just enough material was placed in the flume such that the beach crest was generally located 1 to 2 ft. from the end of the flume. No particular initial slope was used. The beach was allowed to shape itself under wave action until equilibrium conditions were reached.

The criterion for the type of profile, summer or winter, based on this study has been shown in Fig.5 in terms of two parameters, namely, deep-water wave steepness, H_o/L_o, and the ratio of deep-water wave height to the product of specific gravity of the beach material in water and its median diameter, H_o/SD. In this plot the results of experiments with the two sizes of quartz sands and the walnut shell have been used.

The behaviour of the ground plastic was quite different In none of the runs with this material were ripples observed or were the waves found to break. The material appeared "soupy" and proved a very efficient absorber of energy Under wave action most of the material went into thick suspension and oscillated to the entire depth of the movable bed. For this reason the results of the ground plastic were not included in the plot of Fig.5 It is also to be noted that no correction has been applied to the observed wave heights to take into account the reflected wave component.

The curve separating the region of summer profiles from that of winter profiles in Fig.5 is a rising straight line in the range of wave steepnesses from 0.008 to 0.02. For the higher values of wave steepness,

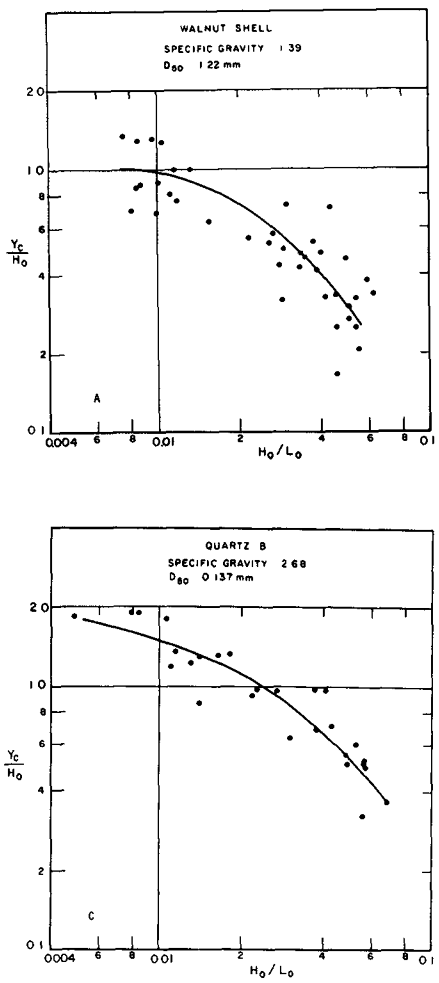

FIG 6 PLOT OF Y_C/H_0 VS H_0/L_0 FOR VARIOUS MATERIALS

the curve is a curved line from $H_o/L_o \approx .05$ to $.03$, the value of the parameter H_o/SD increasing from 20 to about 220, where it turns round to join the straight line from the lower range. The curved line separating the two regions is, at the beginning, nearly horizontal for the range $H_o/SD \approx 20$ to 100. For the range of H_o/SD - 80 to 200 it was possible to get a storm profile for high values of wave steepness as well as low values with an intermediate range in which the profiles were of summer type without any bar When compared with Fig.1, the critical wave steepnesses obtained in this study are slightly higher probably due to the reflected energy not being taken into account. The results indicate that with certain combination of size and specific gravity it is possible to get "winter" profiles at low wave steepnesses as well as high ones with an intermediate range for which the profiles will be of "summer" type. This fact can be taken advantage of in achieving similarity in the type of profile in model studies.

Fig. 6 shows the dimensionless wave run-up observed for different materials plotted against the deep-water wave steepness In each case the wave run-up decreases with increase in wave steepness because the steeper waves break farther offshore in greater depths. Plastic material gave the least uprush and the quartz sands the maximum while for walnut shell the value was intermediate to the other two. Irrespective of the size difference the value of the uprush was almost same for the quartz sands.

Part B Reflection characteristics

In this second part of the article the reflection characteristics of two-dimensional equilibrium profiles are considered It is assumed that the action of a given wave condition on a certain sediment results in a unique equilibrium profile when the waves are allowed to shape the beach for a sufficiently long time. The variables involved in the reflection phenomenon in that case will be

(i) percentage of energy reflected from the profile, E_r
(ii) wave height, H_o
(iii) wave period, T
(iv) median size of grains, D
(v) mass density of sediment, ρ_s
(vi) mass density of fluid, ρ_f
(vii) depth of water, d

One can, then, write

$$E_r = f\,(H_o, T, D, \rho_s, \rho_f, d)$$

the assumptions regarding viscosity of the fluid and standard deviation of the grain size being same as in Part A. If the fall velocity, V_f, of the sediment is taken to characterise the sediment size and its density and the density of the fluid, then one can write

$$E_r = f_1\,(H_o, T, V_f, d)$$

Now

$$E_r \propto K_r^2$$

$$\therefore\ K_r = f_2\,(H_o, T, V_f, d)$$

or

$$f_3\left(K_r, \frac{H_o}{V_f\,T}, \frac{d}{V_f\,T}\right) = 0$$

in which V_f and T are taken as repeating variables.

This can also be written as

$f_4\left(K_r, \frac{H_o}{V_f\,T}, d/H_o\right) = 0$ if V_f and H_o are taken as the repeating variables. The third parameter in these relations is important if the reflection coefficient, K_r, is measured in shallow water. Otherwise, the relation $F\,(K_r, H_o/V_f\,T) = 0$ should apply and it is this relation which is empirically determined.

The experiments related to the second part of the study were conducted in a wave flume (Fig.3) 2' wide, 60' long and 1' deep. With the help of a longitudinal partition, only a narrow 6" wide section of the flume was used in the experiments. This ensured that the results were very little influenced by the secondary reflection taking place from the wave generator. Five different materials, i.e., all excepting the first and the third one in the list given in Fig.4, were used in this series of runs. The following depths of water and wave periods were utilised

d ft.	T secs.	d/L_o
0.50	1.20	0 0678
0.45	1.20	0 0610
0.45	1.00	0 0878
0 40	1.00	0.0543
0.40	1.20	0.0781
0.30	1.00	0.0586

For each depth three different wave heights were applied by changing the stroke of the wave generator. The duration of each run was on an average two hours so that the beach attained substantial equilibrium before observations were made. The coefficient of reflection, K_r, was then found by recording the envelope of the wave system in front of the beach by a movable parallel - wire resistance type wave gauge and noting the wave heights at the node and anti-node of the wave system and using these values in the Keulegan formula. If H_n and H_1 are the wave heights registered at the node and anti node respectively, then according to the linear wave theory

$$K_r = \frac{H_1 - H_n}{H_1 + H_n}$$

The resulting error in using this expression for shallow water waves in the calculation of the reflection coefficient is considered negligible.

The empirical relationship between the reflection coefficient, K_r, and the parameter, H_{o1}/V_fT, as developed from dimensional considerations above is shown in Fig.9. This plot using ordinary scales along the two axes was found to be the best form of representing this relation. The reflection coefficient reduces sharply from a high value of approximately 0.55 to about 0.10 as H_{o1}/V_fT increases from 0.1 to 0.5 and thereafter K_r shows little variation even when H_{o1}/V_fT is increased up to 2.0 The results of the two quartz sands fell in the range of H_{o1}/V_fT - 0.10 to 0.37 and those of walnut shell and plastic in the range - 0 2 to 2.0

Figs.7 and 8 show the same relation for the bed material of grain size 1 mm and 0.5 mm respectively, the results for these diagrams being taken from those of Fig.9. These figures have been drawn to observe the effect of size and specific gravity on the reflection coefficient. Within the same type of sediment, the smaller size showed a higher trend of values of K_r. This is due to the reduced permeability and roughness magnitude. However opposing this tendency is the fact that a larger quantity of the smaller size should be in movement and suspension for the same wave condition and this should lead to a reduction in the value of K_r indicating increased absorption of energy. The resulting trend noticed in the experiments shows, however, that the effect on reflection of the latter factor is not so significant at least in the range involved. A change in

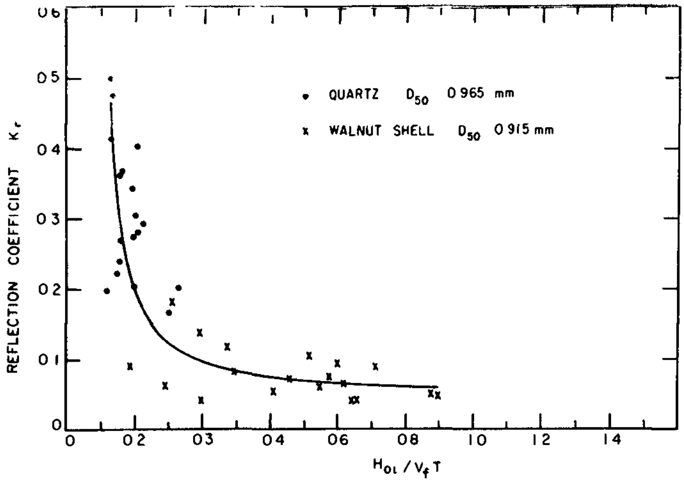

FIG 7 PLOT OF K_r VS H_{oi}/V_fT (D ≈ 1 mm)

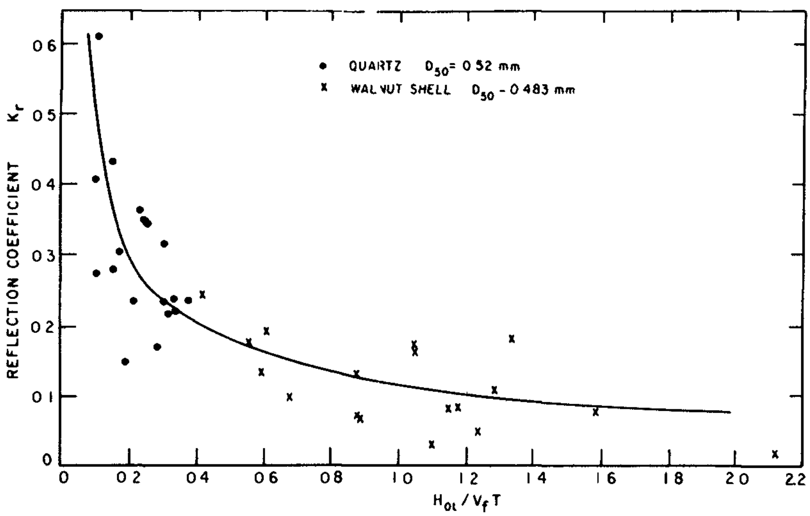

FIG 8 PLOT OF K_r VS H_{oi}/V_fT (D ≈ 0 5 mm)

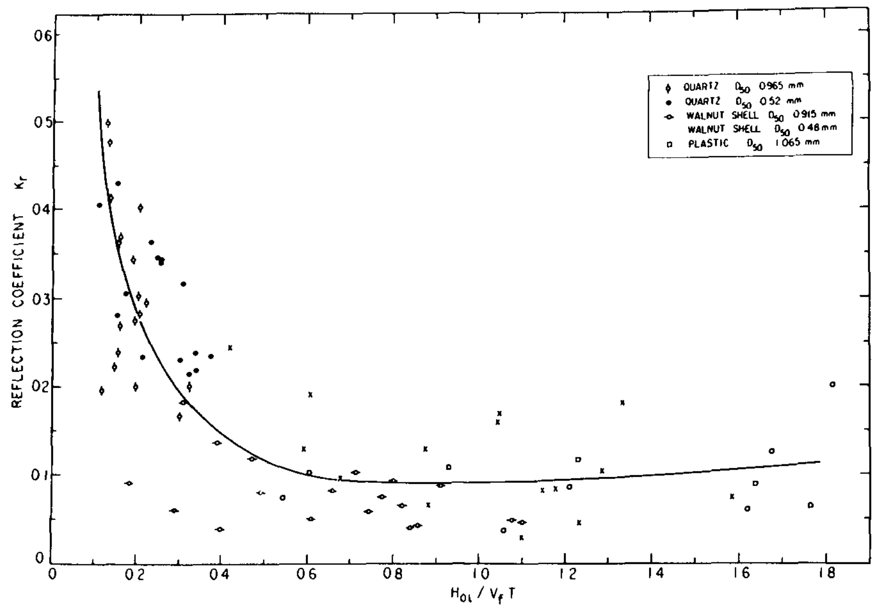

FIG 9 REFLECTION COEFFICIENT K_r VS H_{oi}/V_fT

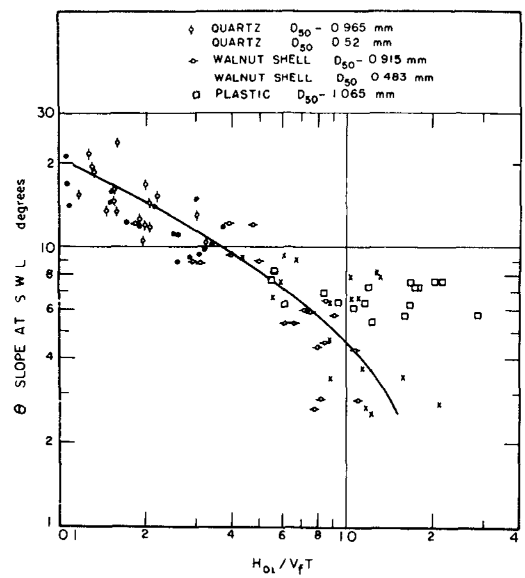

FIG 10 PLOT OF θ VS H_{oi}/V_fT

specific gravity, however, markedly effects the value of K_r. If one assumes that the median diameter characterises the roughness magnitude as well as permeability, the effect of absorption of energy by the particles in motion at the bottom and those in suspension can be estimated by looking at the variation of K_r with H_{01}/V_fT for nearly the same size but different type of material. This can be done with the help of Figs. 7 and 8.

Figure 10 shows the variation of the slope of the profile at the still water level, θ, with the dimensionless parameter H_{01}/V_fT and Fig. 11, the variation of K_r with θ. The use of a less dense artificial sand leads to a lowering of the slope, θ. If the slope at still water level is taken to represent the slope of the profile, the effect of greater quantity of material being in motion and suspension at higher values of H_{01}/V_fT is enhanced by that due to the reduction in slope. The slope decreases regularly from about 22° to 2.5° as H_{01}/V_fT is increased from 0.1 to about 2.0. For the plastic material, however, the slope remains nearly the same with an average value of about 6.5°. The behaviour of this material was characteristically different from the rest as in series 7.

Fig. 12 is a logarithmic plot of slope at still water level versus incident wave steepness for all the materials. In all cases except the plastic the slope is found to increase as the steepness is decreased.

Fig. 13 is a comparison plot of the results of this study using a movable bed with Miche's theoretical values as well as those of Greslou and Mahe, empirically obtained, for smooth impermeable slopes at constant values of deep-water wave steepness. The range of values for wave steepness in all cases is .005 to .03 Although

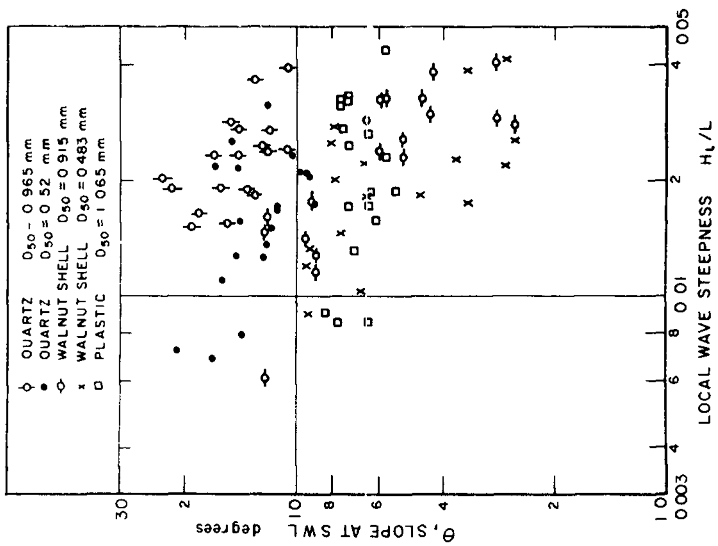

FIG 12 PLOT OF θ VS H_1/L

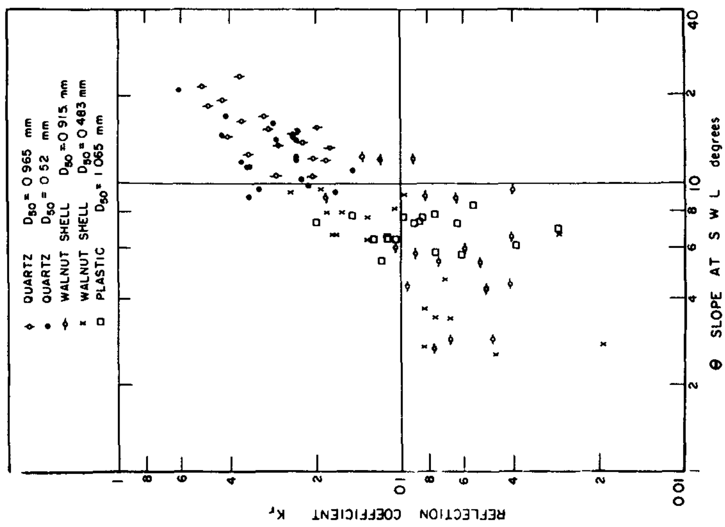

FIG 11 PLOT OF K_r VS θ

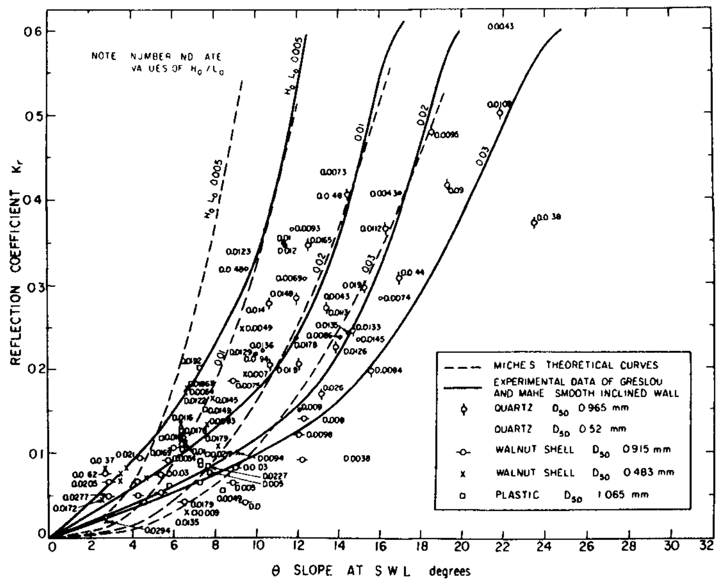

FIG 13 COMPARISON PLOT OF K_r VS θ

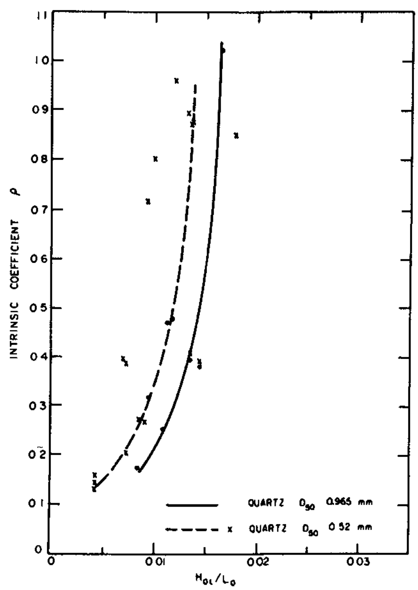

FIG 14 PLOT OF INTRINSIC COEFFICIENT, ρ VS H_{01}/L_0

the total spread of the results agreed very closely with Miche's and Greslou's curves, no clear pattern of distribution of wave steepness as a third parameter could be noticed in the case of the movable bed which includes the effects of roughness, permeability and suspension.

Fig. 14 gives the values of Miche's intrinsic coefficient, ρ, plotted as a function of the deep-water wave steepness for the two sizes of quartz sands. The average value of ρ was found to be 0.54 and 0.49 for the larger and smaller size of sand respectively. The values of the intrinsic coefficient for walnut shell and plastic were found to be higher than 1.00 on an average as the slope, θ, produced by these materials were much lower than 10° and Miche's theory was not applicable The variation of ρ with H_{01}/L_0 follows the same pattern as that found in the studies at Minnesota for artificial wave absorbers.

Conclusions

1. The criterion for generation of a longshore bar can be described in terms of deep-water wave steepness and the term H_0/SD Certain combination of size and specific gravity may produce storm-type profiles at high wave steepnesses as well as low ones with an intermediate range where the profiles are of summer type.

2 The parameter H_{01}/V_fT is very significant in determining the reflection coefficient of model beaches formed of noncohesive, movable bed material of fairly uniform given size and density

3. The reflection coefficient rapidly decreases as the parameter H_{01}/V_fT increases and appears to attain a constant value of about 0.07 over the range of the present experiments It does not appear to have any

distinct relationship with deep-water wave steepness.

4. The slope at still-water level is related to the term $H_{o1}/V_f T$ in a well defined fashion. The slope is affected to a greater extent by specific gravity than grain size of sediment. It is also influenced by the incident wave steepness.

5. The value of Miche's intrinsic coefficient for beaches composed of natural quartz is found to be 0.5

ACKNOWLEDGEMENTS

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