Influence of grain size in sediment transport depends on flow conditions. For bed load transport a maximum probably exists for load discharge as a function of grain size. The important parameter seems to be the ratio $\tau_o/\tau$ between the threshold shear stress and the flow shear stress.

1. It has been pointed out** that authors interested in littoral drift sometimes reach very different conclusions as to the influence of grain size.

In fact, prototype observations made in California beaches by Inman have shown that littoral drift was practically independent of grain size.

On the other hand, L. Bajournas concluded from dimensional analysis considerations that littoral drift should increase with the square root of grain size. According to this author, this result agrees with observations made in prototypes and in models for medium and fine sands.

Finally, laboratory tests conducted by Larras and Bonnefille at Chatou Laboratory (France) revealed clearly the existence of a maximum.

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mum for littoral drift as a function of grain size.

In the present paper an attempt is made to show in a more qualitative than quantitative manner that the divergence of results obtained by different researchers is explained by the fact that the influence of grain size should be variable according to circumstances, namely how far one is from the beginning of the sediment movement.

2. It is commonly accepted that littoral drift is a function of wave and sediment characteristics:

\[ Q_1 = f(\text{wave, sediment}) \]

As wave characteristics one should consider \( H \) (wave height), \( T \) (wave period) and \( \alpha \) (wave obliquity). As for sediment, the important parameters are the submerged specific weight \( (\gamma'_s) \) and grain size \( D \).

Some other factors with possible influence on \( Q_1 \), such as beach slope and bed roughness, wind up being functions either of wave or sediment characteristics or both and so we may ultimately write:

\[ Q_1 = f(H, T, \alpha, \gamma'_s, D) \]

3. Now let us consider, according to some authors, the following schematic model for littoral drift: sediments are moved by the longshore current in a direction parallel to the shore line, waves, with their turbulence, merely "prepare" material to be moved.

That being so, to a given wave \((H, T, \alpha)\) on a given beach will correspond a certain longshore current and consequently a certain shear stress \( \tau \), so that we may write:
In the above formula $D$ means the direct influence of grain size, the indirect influence corresponding to beach slope and bed roughness being implicit in $\tau$.

Equation (1) is formally identical to the equations accepted for unidirectional flow, namely Meyer-Peter's and Einstein's formulas if we consider that bed-load transport is predominant relative to the transport in suspension.

The direct influence of grain size in load discharge can be easily understood if in equation (1) we put $\tau = \text{const}$, $\gamma'_s = \text{const}$ and compute $Q_1$ as a function of $D$.

Meyer-Peter's formula is based on the concept of threshold shear stress and can be written

$$q'_s = 8 \left( \frac{\Omega}{\gamma} \right)^{1/2} (\tau - \tau_0)^{3/2} \quad (*)$$

In this formula, $q'_s$ is the load discharge weighed underwater and $\gamma$ is the water specific weight. The threshold shear stress $\tau_0$ may be computed by

$$\tau_0 = 0.05 \gamma'_s D$$

Calculations carried out on equation (2) for natural sand ($\gamma'_s = 1.6$) and for two distinct values of $\tau$ ($\tau = 0.1 \text{ kg/m}^2$ and $\tau = 1 \text{ kg/m}^2$)

* - Rigorously one should write $\tau'$ instead of $\tau$, with $\tau' = \mu \tau$, where $\mu$ is a coefficient which takes into account the relationship between bed form roughness and the roughness due to grain size. For the sake of simplicity we will put $\mu = 1$. 

$$Q_1 = f (\tau, \gamma'_s, D) \quad (1)$$
led to the results presented in Fig. 1, where the weighed out of water load discharge \( q_s \) is plotted against the grain size \( D \).

It can be concluded that after Meyer-Peter's formula load discharge is zero for sufficiently great sizes and grows as size gets smaller.

For sufficiently small sizes, bed-load discharge tends to remain a constant.\( ^* \)

Einstein's formula for bed-load transport is based on the probability of motion of bottom particles and the load discharge \( q_s \) is given by means of two parameters \( \psi \) and \( \phi \):

\[
\psi = \frac{\rho_s - \rho}{\rho} \frac{D}{R l} \quad \phi = \frac{s}{\rho_s g} \left( \frac{\rho_s}{\rho_s - \rho} \right) \left( \frac{1}{g D^3} \right)^{1/2}
\]

\( \psi \) = flow parameter

\( \phi \) = transport parameter

\( \rho_s \) = specific mass of sediment

\( \rho \) = specific mass of water

\( R \) = hydraulic radius\( ^{**} \)

\( l \) = unit head loss

The relationship between \( \psi \) and \( \phi \) is plotted in Fig. 3.

Introducing in \( \psi \) the concept of shear stress

\( \tau = \gamma R l \) and taking \( \gamma_s = 2600 \text{ kg/m}^3 \) (natural sand) the fundamental parameters can be written, in metric units.

\( ^* \) - This will be true as long as the transport is made predominantly through bed load discharge.

\( ^{**} \) - Again, one should also write \( R^1 = \mu R \) For simplicity we will take \( \mu = 1 \).
\[ \psi = 1600 \frac{D}{\tau} \]
\[ \phi = 10^{-4} \frac{q_s}{d^{3/2}} \]  

Calculations on equations (3) for \( \tau = 0.1 \text{ kg/m}^2 \) and \( \tau = 1 \text{ kg/m}^2 \) led to the results plotted in Fig 1. It can be seen that in this case a maximum of load discharge exists for a size \( D \) varying with \( \tau \).

These results are in qualitative agreement with Larras and Bonefill results.

Also we may say that they agree with Bajournas results. Indeed, if the load discharge increases with the square root of the sediment size for a certain range of diameters, and if, for sufficiently great diameters, the load discharge is zero, then there must exist a maximum of load discharge as a function of grain size.

5 - A more suggestive interpretation of the above results may be achieved by plotting \( \frac{Q_1}{\tau^{3/2}} \) as a function of \( \frac{T_0}{\tau} \) (Fig 2). In this plot the value of \( \tau_0 \) was computed from \( \tau_0 = 0.05 \gamma_s D \) which would be true for unidirectional flow.

It is the author's impression that the curves in Fig 2 may also be assumed valid in case of longshore current movement if for \( \tau_0 \) a smaller value than the above is taken, that is, if \( \tau_0 \) is divided by a parameter \( W > 1 \) which would represent the "help" granted by the wave itself to the longshore current in the "preparation" of the material.

* - The same results would be obtained if a greater value of \( \tau \) were taken.
According to this reasoning, the relative value of littoral drift will depend on the zone of Fig 2 plot where one is working. The influence of diameter will be represented through the parameter $\tau_0$ and the effect of the additional turbulence due to waves will be represented by the parameter $W$. Situations indicated in Fig 2 for waves and currents should be frequent for the usual materials in prototypes and in models. The above reasoning may explain certain observed facts. Let us consider the case of a movable bed model subject to wave and current action in which both waves and current are reproduced in the Froude scale. For a given $\tau$ we may find ourselves in the extreme right of the plot in case of currents, while for waves (lesser $\tau_0$) we may be working in the middle zone. Hence, load discharge by wave action is comparatively greater than the one due to current action. This fact, observed in different laboratories, is one of the great difficulties in the calibration of estuary models, where waves and currents have equal importance. These difficulties have been avoided both by enhancing the effect of currents and by reducing the effect of waves.

* - River or tidal current
6 - CONCLUSIONS

a) Influence of grain size on sediment transport depends on the flow conditions. If the transport is mainly in suspension, then load discharge should decrease when the diameter increases. When the transport is mainly through bed load discharge, then discharge will at the beginning increase with increasing diameter reaching a maximum and decreasing afterwards.

b) The value of the grain size for which the maximum transport is attained depends on the flow shear stress.

c) The influence of grain size is expressed through the parameter $\tau_o/\tau$. It is very different for conditions near the beginning of the sediment movement (great $\tau_o/\tau$) and for conditions of fully developed movement (small $\tau_o/\tau$).

d) In the case of currents (tidal or river currents) $\tau_o$ may be computed from $\tau_o = 0.05 \gamma' \bar{s} D$. In the case of transport due to wave action (longshore current) $\tau_o$ should be divided by a coefficient $W$ which depends on the wave characteristics. (It should be noted that dividing $\tau_o$ by $W$ is the same as multiplying $\tau$ by $W$.)