CHAPTER 52

A Class of Probability Models for Littoral Drift

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Introduction

The major goal in the development of sediment tracer technology is to produce an accurate method for the field measurement of short term volume littoral rate Many of the technical difficulties involved in tagging, injecting, and sensing the movement of radioisotope sand tracers in the littoral zone have been overcome by the RIST project However, quantitative determination of volume drift rate requires more than knowledge of tracer position in time and space A mathematical model is required to relate the flux of tracer material to the sediment flux

A linear (or average) rate of tracer transport along the coastline can be measured to a fair degree of accuracy with tracers These measurements, when determined from tracers injected along a line source which span the transport zone, can be used to provide an estimate of an areal transport rate However, it is not obvious how to measure the third dimension, depth of transport This, of course, is needed to provide the desired estimate of volume transport rate

This problem arises, even if the relation of tracer concentration to burial depth is everywhere known without error Sediment transport does not occur as a sheet of constant thickness moving at a constant rate If this were so tracer concentration would rapidly attain a uniform concentration over a fixed depth and no tracers would appear below that depth In fact no observations of the relation between tracer concentration and burial depth support this model as even a first approximation

Studies such as those of Courtois and Monaco (1969) and Hubble and Sayre (1964) suggest that the concentration of tracers is related to burial depth in a complex fashion The concentration on the surface is finite, but not maximal The concentration increases with depth to some point where a maximum is reached and diminishes in a "long tailed" fashion

If sediment transport occurred as a sheet of uniform thickness, and tracers were thus uniformly distributed over the range of that thickness, one measure of depth of movement would be the mean of the maximum observed tracer depth A more stable estimate, if this condition was assumed only to approximate reality, would be to double the observed mean depth of burial

Yet the observed distribution curves so strongly deny the original hypothesis that neither of these methods can be relied upon What is needed is a model for sediment particle burial which admits the observations as realizations and directs the form of estimation This paper attempts to present a class of such models which lead to a particularly simple form for the calculation of littoral volume drift rate

The Conceptual Model

Assume a two dimensional sediment transport system which includes only depth of burial and transport direction The motion of sand particles is initiated by the passing of a surge As wave crests pass over a point particles are alternately lifted from the sea floor, transported, and redeposited Inasmuch as wave height and period are random variables, the depth to which material is eroded from the sea floor varies with each wave

When the wave train impinges obliquely upon the shoreline a net longshore current is superimposed upon the oscillatory motion so that particles suspended by the passing surge are subjected to a net motion along the shoreline Contrary to the situation in uniform flow, it is the individual energy pulses which control the erosive mechanism and the wave climatology as a whole which controls the transporting mechanism Short period high waves do not effect littoral drift if they are directed onshore The longshore current is the transporting mechanism, but is not the erosive mechanism except in extreme cases

The Probability Model

For the purposes of this development the elevation of a point on the sea floor can then be considered to change instantaneously through a series of erosional and depositional increments Define the thickness of the erosional increment accompanying the passage of the ith surge as ε_i, and the thickness of the following depositional increment as δ_i Both ε_i and δ_i are assumed to be random variables with equal expectations (μ) Particles which are eroded are assumed to mix in transport so that the depth below the sediment-water interface at which they are redeposited is independent of the depth from which they were eroded

At time zero tracer particles are injected on the surface Initially all tracer particles will be eroded, transported and redeposited As successive erosional and depositional episodes occur, some of the tracer particles will become buried below normal erosional depths, thus occupying sites of relative stability Ultimately an equilibrium state will be reached where the concentration of tracer particles with depth of burial will no longer change This state will be that where the probability of erosion of particles buried at a given depth times the proportion of tracer particles buried at that depth precisely equals the probability that a freshly eroded particle will be deposited at that depth times the total probability that a tracer particle is eroded, for all depths

Certain characteristics of the equilibrium distribution can be derived without knowledge of the specific probability density functions describing erosional and depositional increment thicknesses

Define

$f_{i+1}(z)dz$ = the probability that after the ith surge passes, a tracer particle is buried between depths z and z+dz

$f_{new}(z)dz$ = the probability that a particle which is eroded by the ith surge, is redeposited between depths z and z+dz

$P_{move}(z)$ = the probability that a particle buried at depth z is eroded by the ith surge

$P_{stay}(z)$ = the probability that a particle buried at depth z is not eroded by the ith surge

$f_{\varepsilon}(\varepsilon)d\varepsilon$ = the probability that the thickness of an erosional increment is between ε and $\varepsilon+d\varepsilon$

$F_{\varepsilon}(\varepsilon) = \int_0^{\varepsilon} f_{\varepsilon}(x)dx$ = the probability that the thickness of an erosional increment does not exceed ε

Then $P_{stay}(z) = F_{\varepsilon}(z)$, and $P_{move}(z) = 1 - F_{\varepsilon}(z)$

Define Pm_i = the total probability of a tracer particle being eroded by the ith surge, independent of burial depth

$$Pm_i = \int_0^{\infty} [1 - F_{\varepsilon}(z)]f_i(z)dz$$

Then

$$f_{i+1}(z) = F_{\varepsilon}(z)f_i(z) + Pm_i f_{new}(z)$$

An equilibrium probability distribution of tracer burial depth will be obtained when

$$f_{i+1}(z) = f_i(z)$$

Define this distribution as $f_{eq}(z)$

Then

$$f_{eq}(z) = F_{\varepsilon}(z)f_{eq}(z) + Pm_{eq}f_{new}(z)$$

or,

$$f_{eq}(z) = \frac{Pm_{eq} f_{new}(z)}{[1 - F_{\varepsilon}(z)]}$$

It now remains to relate the fresh deposition burial law ($f_{new}(z)$) to the probability density for depositional increment thicknesses

Define

$f_{\delta}(\delta)d\delta$ = the probability that a random sample from the probability distribution of depositional increments has thickness between δ and $\delta+d\delta$

$g_{\delta}(\delta)d\delta$ = the probability that the depositional increment within which a freshly deposited particle falls has thickness between δ and $\delta+d\delta$

Then in order to maintain a volume balance

$$g_{\delta}(\delta)d\delta \propto \delta f_{\delta}(\delta)d\delta$$

As, $\int_0^{\infty} g_{\delta}(\delta)d\delta = 1$

and, $\int_0^{\infty} f_{\delta}(\delta)\, d\delta = \mu$

then, $g_{\delta}(\delta)d\delta = \dfrac{\delta f_{\delta}(\delta)d\delta}{\mu}$

From the mixing in transport assumption, a particle which falls within a depositional increment of size δ is equally likely to fall at any depth between the surface and the base of the increment

Thus

$$f_{new}(z|\delta) = \frac{1}{\delta} \qquad 0 < z < \delta$$

$$= 0 \qquad z > \delta$$

Then the joint probability density of depth of burial and size of depositional increment is given by

$$f_{new}(z,\delta) = f_{new}(z|\delta)\, g_{\delta}(\delta)$$

$$= \frac{(1/\delta)\,\delta f_{\delta}(\delta)}{\mu}$$

or,

$$f_{new}(z,\delta) = \frac{f_\delta(\delta)}{\mu} \qquad 0 < z < \delta < \infty$$

$$= 0 \qquad z > \delta$$

Then the probability distribution describing burial depths of freshly deposited particles is given by

$$f_{new}(z) = \int_z^\infty f_{new}(z,\delta)\,d\delta$$

$$= \int_z^\infty (1/\mu) f_\delta(\delta)\,d\delta$$

or,

$$f_{new}(z) = \frac{1 - F_\delta(z)}{\mu}$$

where $F_\delta(z)$ = the probability that a random sample from the probability distribution of depositional increments has thickness less than z

The resulting equilibrium distribution of burial depths is thus

$$f_{eq}(z) = \frac{Pm_{eq}}{\mu} \frac{[1 - F_\delta(z)]}{[1 - F_\varepsilon(z)]}$$

A significant special case being

$$f_{eq}(0) = (Pm_{eq}/\mu)$$

For this class of probability models, the total surface concentration of tracer material is independent of the specific probability laws governing erosional and depositional increment thicknesses

Volume Littoral Drift Rate

Even more important is the implied relation between the rate of tracer transport and the volume rate of littoral drift

At equilibrium

1 The expected proportion of tracer particles moving with each passing surge is Pm_{eq}

2 The average distance of movement is defined as $E(\Delta X)$ Thus the average longshore tracer velocity is

$$v_x = Pm_{eq} E(\Delta X)/\Delta t$$

$$Q \simeq \mu E(\Delta x)/\Delta t$$

$$\mu = Pm_{eq}/f_{eq}(0)$$

$$E(\Delta x)/\Delta t = V_x/Pm_{eq}$$

Thus $$Q = \frac{Pm_{eq}}{f_{eq}(0)} \quad \frac{V_x}{Pm_{eq}}$$

Or $$Q \simeq V_x/f_{eq}(0)$$

Volume drift rate is simply the tracer centroid velocity divided by the total surface concentration of tracer material, both of which are measurable quantities

These results can be applied to the three dimensional case if one assumes that diffusion of material in the shore-normal direction is negligibly small In this case, for a line injection the littoral drift rate is given by

$$Q = \int \frac{V_x(y)}{f_{eq}(0/y)} dy$$

where the integral is taken across the entire zone of transport

Concluding Remarks

Before the above equations can be relied upon, it is necessary to test some of the underlying assumptions both in the laboratory and in the field Experiments are presently being designed at CERC and elsewhere for this purpose

References

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Hubbell, D W and Sayre, W W (1964), "Sand Transport Studies with Radioactive Tracers", ASCE, vol 90, HY 3, pp 39-68