CHAPTER 29

COMPUTER MODELLING OF DIFFRACTION OF WIND WAVES

by

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ABSTRACT

A digital computer model for diffraction of wind waves behind a breakwater is developed The model combines the hydrodynamic theories and the concept of directional spectra It is designed so that it may be used not only for the study of the wind wave diffraction problem behind breakwaters but also for the investigation of experimental (or field) data analysis procedures of other kinds An extensive study of optimum data length, lag number and gage spacings in wave gage arrays is presented

I INTRODUCTION

Unlike monochromatic waves, wind waves are quite complicated in nature Their heights and lengths are irregular The crest length along each wave crest is relatively short and their forms are not permanent Each portion of the water surface has a different shape The wave speed, frequency, and direction of advance vary from one wave to the next Because of the difficulties of an analytical treatment of such a complicated situation, an approach through simulation techniques offers many advantages[4]

The three types of simulation commonly used are laboratory (physical) electronic analog, and digital computer In the following, only the digital simulation procedures will be outlined

II SIMULATION OF WIND WAVES

Wind waves can be simulated digitally in several ways, such as superposition and digital filter techniques [1] In the following development, only the filtering procedure will be employed

Let y(t) be the wave surface elevations recorded by 4 gages in front of a breakwater Each of the y(t) can be expressed by the summation of the products of d and x (see eq (1)) Here, x is a sequence of normal random deviates For computational convenience the random numbers have been selected so that they have a zero mean and variance of one

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The "d" values are digital filter coefficients and can be represented in terms of the Fourier coefficients, A_n and B_n obtained from the Fourier approximation of the system functions K(f) as shown by eqs (2)-(4)

The values of K(f) are determined from the cross-spectral densities which are Fourier transform of the cross-covariance functions (eqs (5),(6))

If the probability distributions of the spectra are assumed to be circular normal (eq (10)), then the normalized cross-spectral density functions can be expressed as the functions of Bessel functions (eqs (7)-(9))

In short, if the spectral forms of waves are assumed (such as that given by Bretschneider [5] and Pierson and Moskowitz [6] we can simulate the wind waves with digital computation

As already mentioned previously these simulated random waves have very complicated natures and the conventional diffraction theories cannot be applied To deal with this kind of problem, we introduce the concept of directional spectrum

III CONCEPT OF DIRECTIONAL SPECTRUM

Random waves can be thought as composing of infinite number of monochromatic component waves, each of which has a different frequency and phase and propagate along a different direction Since it is known that, to a linear approximation, the conventional diffraction theories are applicable to these monochromatic waves, our problem has then become the question of "how to determine the directional spectrum?"

At the present time, there are several ways available for estimating the directional spectrum But, only one of those methods, [1,2] will be discussed It considers the directional spectral density function as the product of PS(f) and D(θ), where PS(f) is spectral density which varies solely with frequency and D(θ) is a function of direction and possible frequency (see eq (13))

There are several ways to estimate D(θ) One of them is based on the assumption that D(θ) is circular normal In that case, it can be approximated in terms of Bessel functions as shown in eq (14)

The spectral density function can be estimated by the conventional spectral analysis technique using either the covariance function approach or the techniques arising from the fast Fourier Transform algorithm

IV SIMULATION OF DIFFRACTION OF PLANE WAVES [3,4]

Mathematically, the propagation of a plane wave is described by the boundary value problem with a second order partial differential equation of elliptic type (eq (15)) and three boundary conditions (eqs 16,17,18) By applying the method of separation of variables, the boundary value can be solved

Due to the presence of a semi-infinite breakwater, located along the X-axis with one tip at the origin and the other at $x = +\infty$, an additional boundary condition of

$$\frac{\partial \phi}{\partial y} = 0$$

should be introduced For the reason of generality and convenience, a polar coordinate system is adopted here Accordingly, the amplitude for incident wave and diffracted wave can be determined

Since the diffraction coefficient, k' is defined as the ratio of incident wave height over the diffracted wave height, it can be estimated by the modulus of $F(r,\theta)$ for the diffracted waves Their mathematical equations and solutions are given in the Appendix III

V THE APPLICATIONS OF SIMULATION TECHNIQUE

Simulation may be used to explain various features of data sampled from the field or to examine the consequences of selected theories In addition, it can be used to determine

(a) optimal data length (Fig 1) For this case, optional length = 2048

(b) optimal maximum number of lags (Fig 2) For this case, optional lag = 50

(c) effects of smoothing on the spectral density estimates (Fig 3) The results have indicated that

(i) for shorter data length, there is a great difference between the outputs of unsmoothed and smoothed cases but there seems to be no difference between hanning or Hamming smoothing

and that

(ii) for very long record, there is no difference no matter whether they have been smoothed or not

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APPENDIX I

Simulation of Complicated Wind Wave Profiles

Let $y_1(t)$, $y_2(t)$, ,$y_m(t)$ be M - time series

$$y(t) = \sum_{n=N}^{N} d_n \, x(t-n\Delta t) \tag{1}$$

where n = 0 1, 2 ---

Δt = time nterval

x = random deviates

and the digital filter coefficients

$$d_o = A_o \tag{2}$$

$$d_n = A_n + B_n \quad \}$$

$$d_{-n} = A_n - B_n$$

where

$$A_n = \frac{1}{F} \int_o^F R[\hat{K}(f)] \cos\left(\frac{n\pi f}{F}\right) df$$

$$B_n = \frac{1}{F} \int_o^F \imath_m[\hat{K}(f)] \sin\left(\frac{n\pi f}{F}\right) df \tag{3}$$

$$F = \frac{1}{2\Delta t} = \text{Nyquist frequency}$$

and

$$\hat{K}(f) = A_o + \sum_{n=1}^{N} \{A_n \cos\left(\frac{\imath n\pi f}{F}\right) - \imath B_n \sin\left(\frac{\imath n\pi f}{F}\right)\} \tag{4}$$

f = frequency, cps

$\imath = \sqrt{-1}$

The system function can be also written as

$$K_{11}(f_k) = [S_{11}(f_k)]$$

$$K_{m,1}(f_k) = [S_{m,1}(f_k)]^{\frac{1}{2}} \tag{5}$$

$$K_{m,n}(f_k) = \{S_{m,(m-1)}(f_k) - \sum_{j=1}^{n} [K_{mj}(f_k)][K_{jn}(f_k)]\}/K_{nn}(f_k)$$

$$K_{m,m}(f_k) = [S_{mm}(f_k) - \sum_{j=1}^{m-1} |K_{mj}(f_k)|^2]^{\frac{1}{2}}$$

where

$$S_{mj}(f) = CS_{mj}(f) + i\,QS_{mj}(f) \tag{6}$$

$CS_{mj}(f)$ = co-spectral density for gages m and j

$QS_{mj}(f)$ = quad-spectral density for gage m and j

Assuming the circular normal function (eq 10 below) for the angular distribution of energy at a given frequency, the normalized density between gages m and j can be expressed as

$$\frac{CS(f)}{PS(f)} = J_o(A_{mj}) + \frac{2}{I_o(a)} \sum_{n=2,4,6}^{\infty} (i)^n [I_n(a) J_n(A_{mj})] \cos n\gamma_{mj} \tag{7}$$

and the normalized quad-(or quadrature-) spectral density between gages m and j is

$$\frac{QS(f)}{PS(f)} = \frac{2}{I_o(a)} \sum_{n=1,3,5}^{\infty} (i)^n [I_n(a)\, J_n(A_{mj})] \cos(n\gamma_{mj}) \tag{8}$$

where

$$I_n(a) = I_{-n}(a) = (i)^{-n} J_n(ia) \tag{9}$$

= Modified Bessel Function of order n

a = a circular normal parameter = measure of dispersion of the circular normal

α = a circular normal parameter = modal direction of circular normal energy distribution

$$f(\xi,a) = \frac{e^{a\cos(\theta-\alpha)}}{2\pi I_o(a)} \qquad (10)$$

θ = the angle between the positive x-axis and the direction of the wave propagation, measured counterclockwise

ξ = the wave angle departure from the mean = $\theta-\alpha$

$A_{mj} = 2\pi D_{mj}/L$

D_{mj} = distance between wave gages m and j

L = wave length appropriate for the frequency, f

γ_{mj} = the angle between direction of main energy and the line connected gage m and gage j

$= \beta_{mj} - \alpha$

β_{mj} = the angle between the x-axis and the line connected gage m and gage j

$i = \sqrt{-1}$

APPENDIX II

DIRECTIONAL SPECTRUM

1 Directional spectral density function, $p(f,\theta)$

(a) It has the property of

$$p(f,\theta)\, df\, d\theta = \Sigma\,[\text{Mean-Square Wave-Surface fluctuations}] \quad (11)$$
$$(df)(d\theta)$$

In other words,

$p(f,\theta)\, df d\theta$ = variance of sea surface fluctuations obtained by adding together only the waves with frequency and direction of travel in the $(df, d\theta)$ rectangle centered at (f,θ) (12)

(b) It gives the allocation of the total variance among the various frequencies and directions

(c) It can also be considered as an allocation of wave energy (since the wave energy per unit sea surface is proportional to the variance)

2 Estimation of directional spectral density

$$p(f,\theta) = PS(f)\, D(\theta) \quad (13)$$

$$D_f(\theta) = \frac{e^{a\text{Cos}\,(\theta - \alpha)}}{2\pi I_0\,(a)} \qquad \text{(Circular Normal Function)}$$

$$= \frac{1}{2\pi} + \sum_{n=1}^{\infty} \frac{I_n\,(a)}{\pi\, I_0\,(a)}\, \text{Cos}\,\left(n\theta - \frac{n\pi}{2}\right) \quad (14)$$

(α and a may be functions of frequency in the general case)

APPENDIX III

Diffraction of Plane Wave

(A) BOUNDARY VALUE PROBLEM FOR PLANE WAVE PROPAGATION

1 Partial diff eq $$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0 \qquad (15)$$

2 Boundary Conditions

(i) At an impervious and rigid bottom

$$\left.\frac{\partial \phi}{\partial z}\right|_{z=d} = 0 \qquad (16)$$

which d = still-water depth

(ii) At the free surface

(a) Kinetic Surface Boundary Condition

$$\frac{\partial \phi}{\partial z} = \frac{d}{dt}[\eta(x,t)] = \frac{\partial \eta}{\partial t} + \frac{\partial \eta}{\partial x} + \frac{\partial \eta}{\partial t} = \frac{\partial \eta}{\partial t} + u \frac{\partial \eta}{\partial t} \text{ at } z = \eta \text{ (non-linear)} \qquad (17)$$

$$\frac{\partial \phi}{\partial z} = \frac{\partial \eta}{\partial t} \text{ at } z = 0 \text{ (linear)}$$

(b) Dynamic Surface Boundary Condition

$$-\frac{\partial \phi}{\partial t} + \frac{1}{2}(u^2 + v^2 + w^2) + g\eta = 0, \text{ at } z = 0 \text{ (non-linear)} \qquad (18)$$

$$\eta = \frac{1}{g}\frac{\partial \phi}{\partial t} \text{ at } z = 0 \text{ (linear)}$$

3 Solutions

$$\phi = F(x,y)\, Z(z)\, T(t) \qquad (19)$$

$$\phi = A\, e^{-ikct} \cosh[k(z+a)]\, F(x,y)$$

For a plane wave travelling in the y-direction

$$F(x,y) = e^{-ıky} \tag{20}$$

Sommerfeld's radiation condition $F = 0\ (\frac{1}{r})$ (21)

$$r = \sqrt{x^2 + y^2}$$

Wave amplitude, $A = \frac{Akc}{g} \cosh (kd)$ (22)

wave period, $T = \frac{2\pi}{kc}$ (23)

wave velocity $c = \sqrt{\frac{g}{k} \tanh (kd)}$ (24)

wave length, $L = \frac{2\pi}{k}$ (25)

wave number, $k = \frac{2\pi}{L}$ (26)

$$= \frac{Akc}{g} \cosh (kd) \sin [k(ct-y)] \tag{27}$$

B THE PROBLEM WITH THE PRESENCE OF A BREAKWATER

(semi-infinite breakwater)

1 an additional boundary condition

$$\frac{\partial \emptyset}{\partial y} = 0 \text{ at } y = 0, x \geq 0 \tag{28}$$

This implies that

$$\frac{\partial F}{\partial y} = 0 \text{ at } y = 0, x \geq 0 \tag{29}$$

2 for incident waves

$$F_1(r,\theta) = e^{-ikr \cos(\theta - \theta_o)} \tag{30}$$

3 the free wave surface can be given as

$$= \frac{Aikc}{g} [\cosh(kd)]\, e^{ikct}\, F(r,\theta) \tag{31}$$

4 for diffracted waves

$$F_d(r,\theta) = f(\sigma)\, e^{-ikr\,[\cos(\theta - \theta_o)]} + f(\sigma)\, e^{-ikr\,[\cos(\theta - \theta_o)]} \tag{32}$$

$$\sigma = 2\left(\sqrt{\frac{kr}{\pi}}\right)\sin\frac{1}{2}(\theta - \theta_o)$$

$$\sigma' = -2\left(\sqrt{\frac{kr}{\pi}}\right)\sin\frac{1}{2}(\theta - \theta_o)$$

$$f(\sigma) = \frac{1}{\sqrt{2}}\, e^{i\pi/4}\int_{-\infty}^{\sigma} e^{(i\pi/2)t^2}\, dt$$

$$f(\sigma') = \frac{-1}{\sqrt{2}}\, e^{i\pi/4}\int_{-\infty}^{\sigma} e^{(i\pi/2)t^2}\, dt$$

$$F_d(r,\theta) = \rho(r,\theta)\, e^{i\xi(r,\theta)} \tag{33}$$

$$a_d = \frac{Akc\rho}{g}\cosh(kd) \tag{34}$$

5 Diffraction coefficient

$$k' = \frac{2a_d}{2a_1} = \rho$$

$$k' = |F_d(r,\theta)| \tag{35}$$

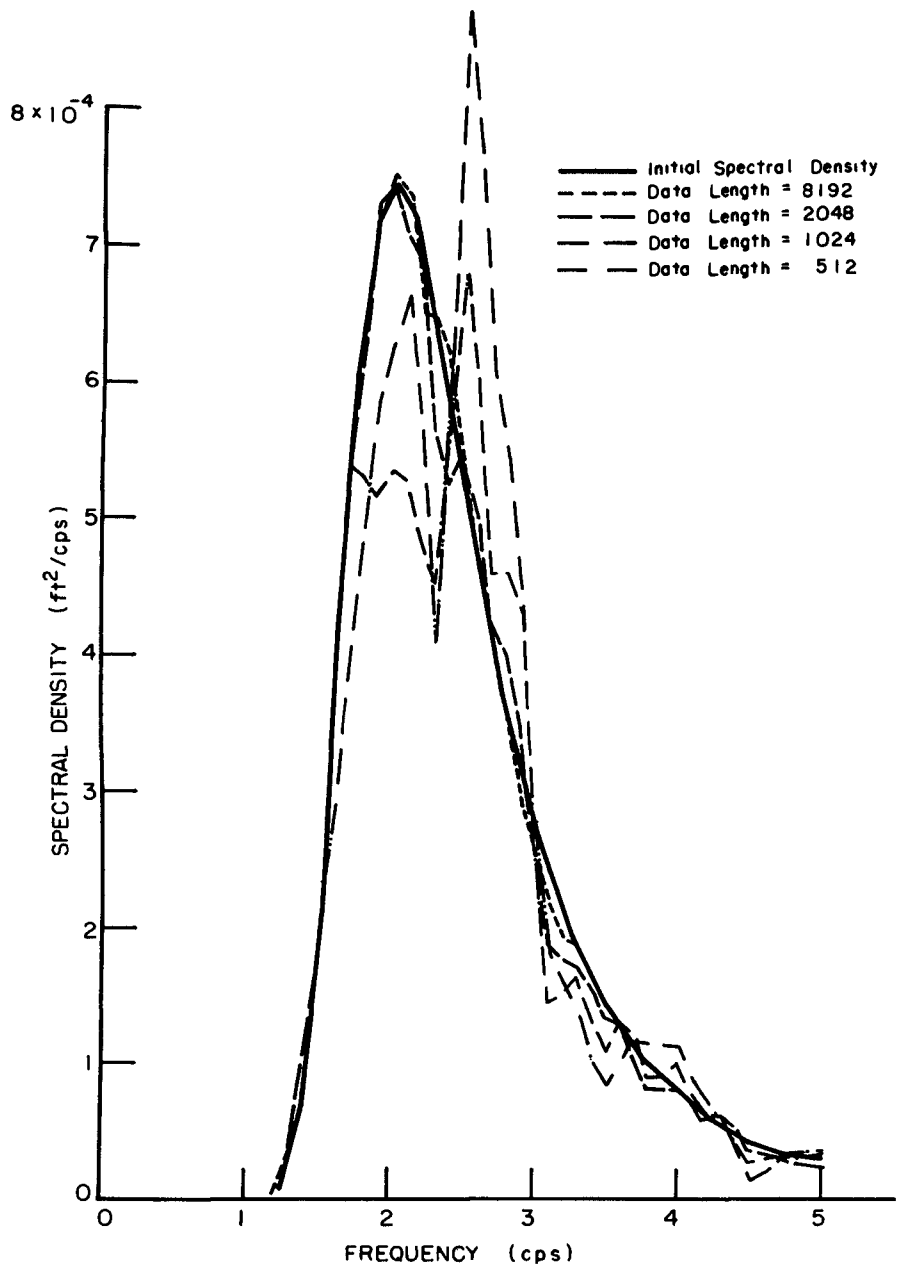

FIG 1 ESTIMATES OF SPECTRAL DENSITIES FOR VARIOUS LENGTH OF DATA

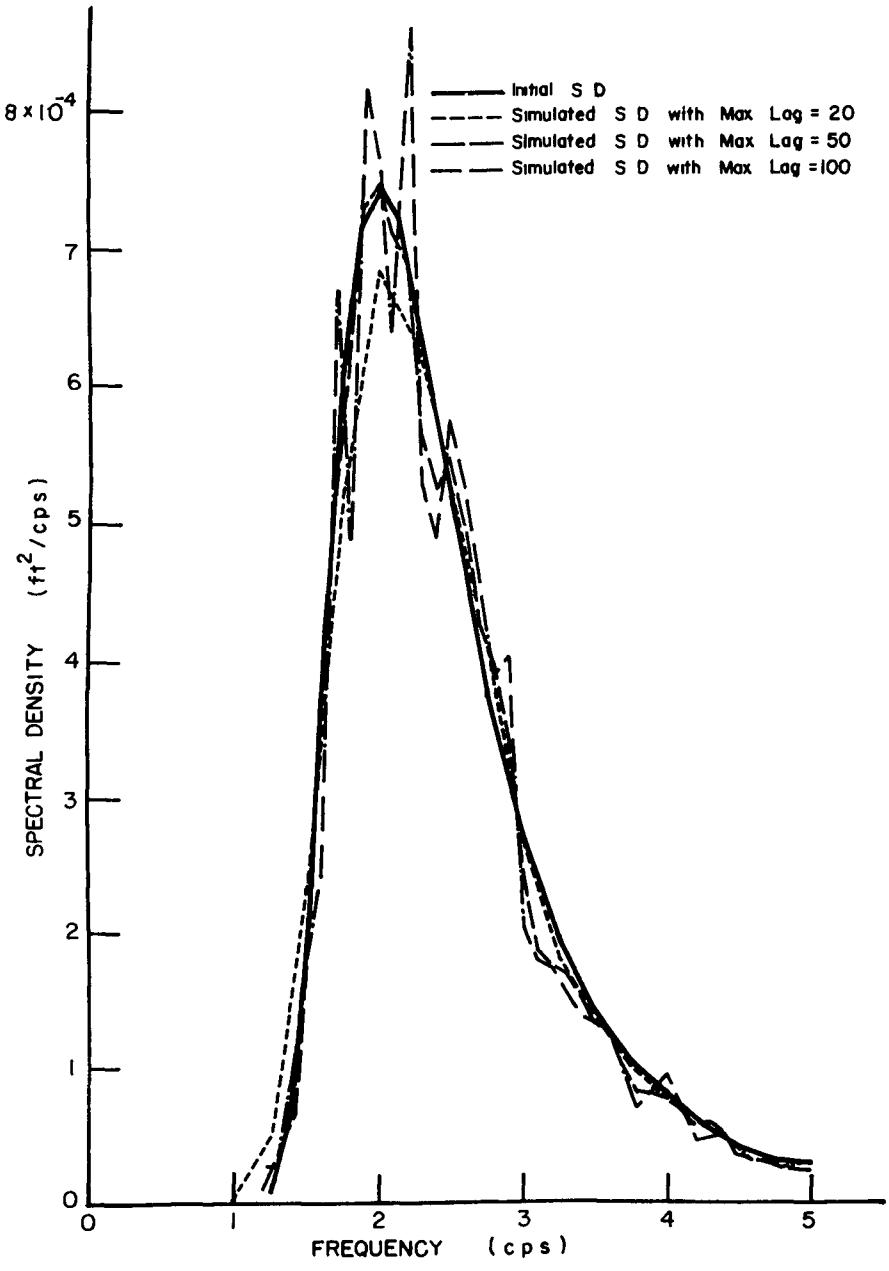

FIG 2 ESTIMATES OF SPECTRAL DENSITIES (2048 DATA LENGTH) FOR VARIOUS MAXIMUM LAGS ON THE COVARIANCE FUNCTIONS

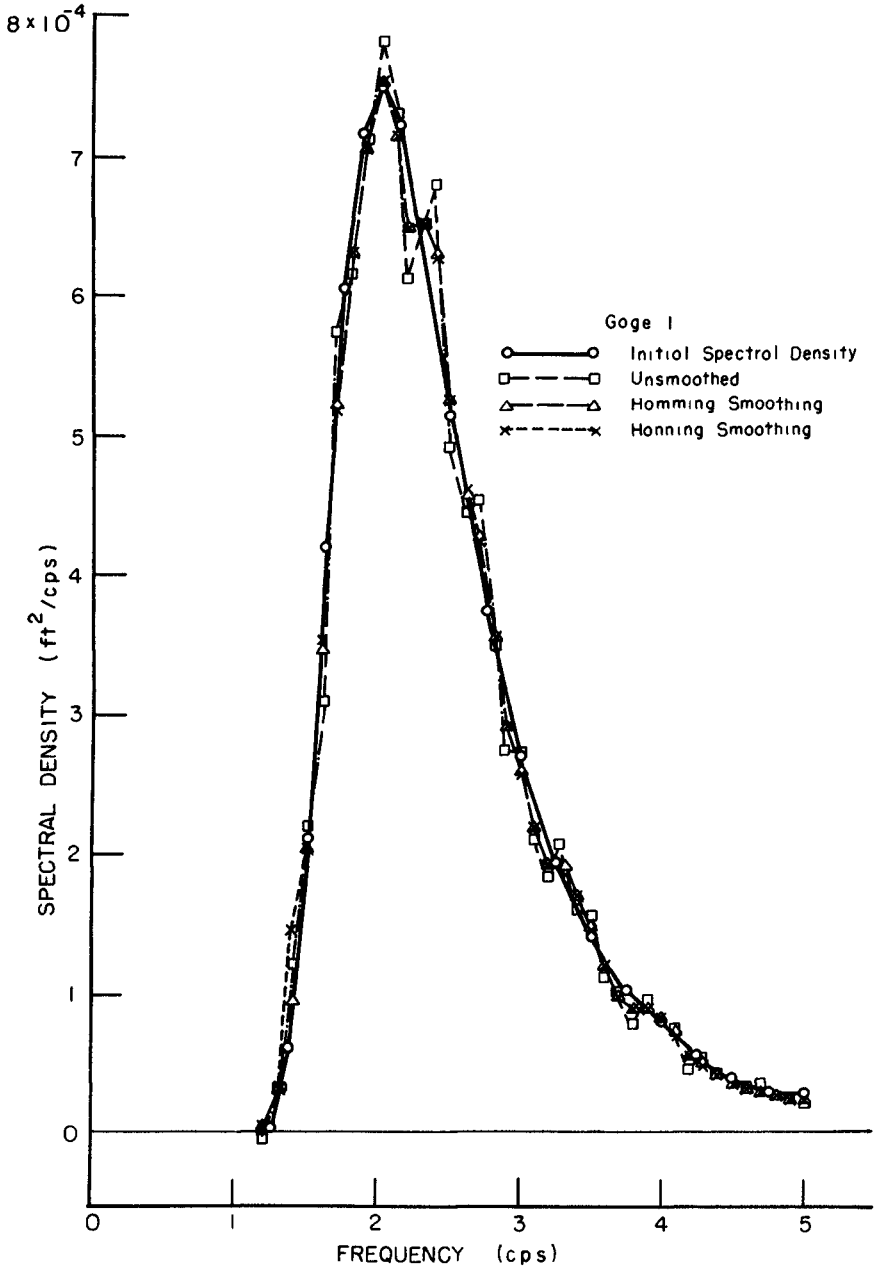

FIG 3 EFFECTS OF SMOOTHING ON THE SPECTRAL DENSITY ESTIMATES FOR DATA LENGTH 8192 AND MAXIMUM LAG 50

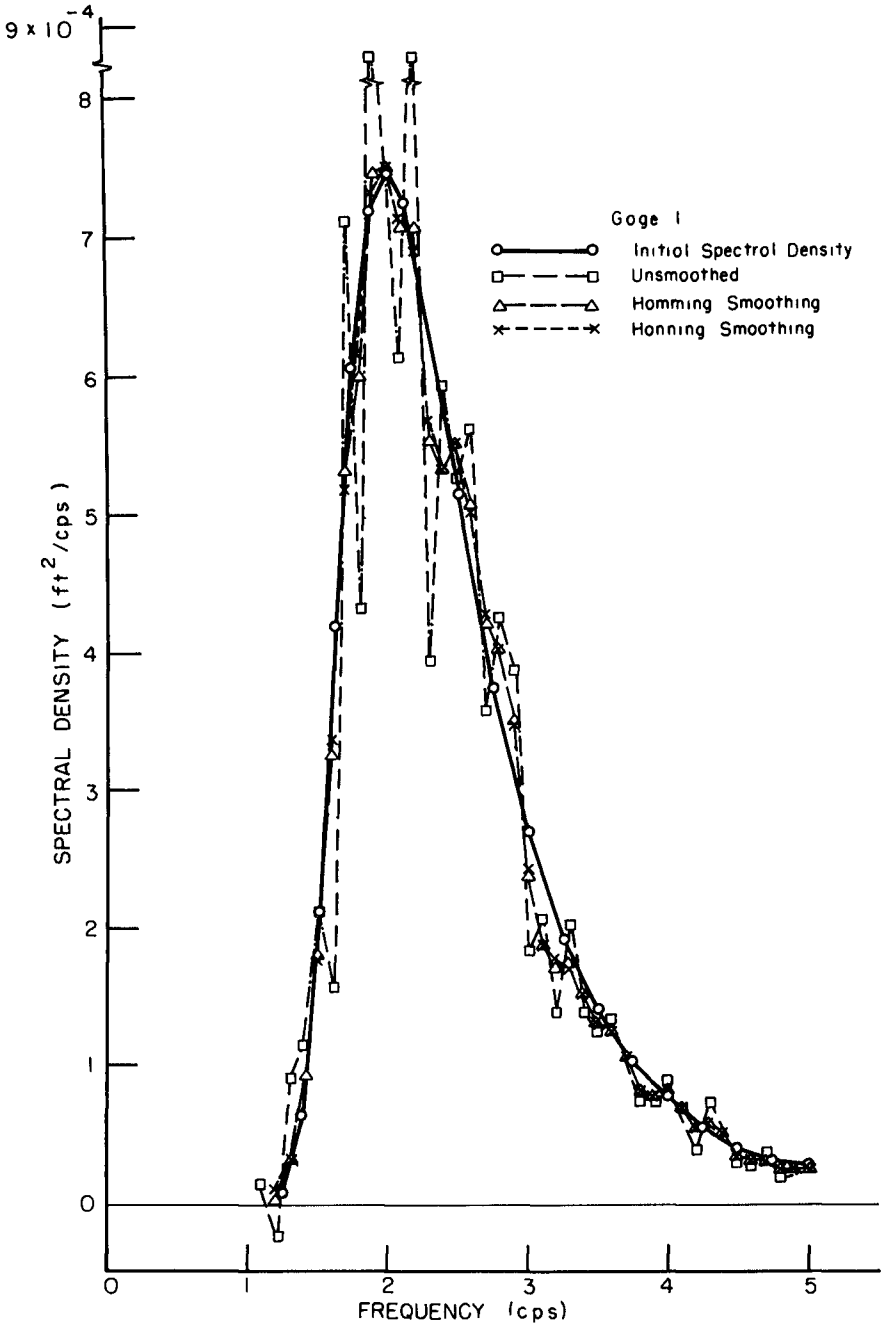

FIG 4 EFFECTS OF SMOOTHING ON THE SPECTRAL DENSITY ESTIMATES FOR DATA LENGTH 2048 AND MAXIMUM LAG 50