CHAPTER 27

VELOCITY FIELDS IN THE WAVE BREAKER ZONE

by

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SYNOPSIS

The paper deals with the velocity fields in the neighbourhood of the breakers and the correlation between the wave asymmetry and the velocity asymmetry.

The velocity measurements were made on two beach slopes 1 9 and 1.18. Earlier work (3) showed that the two slopes produced different breaker types; the slope of 1:9 produced plunging breakers and the slope of 1 18 spilling breakers.

The velocities (velocity - time history) of the water particles were measured at a height of 5mm above the bed using the hydrogen bubble method combined with cine photography.

Two types of horizontal velocity asymmetry were defined and investigated, namely

(1) horizontal velocity (Magnitude) asymmetry

and (2) horizontal velocity (time) asymmetry.

It was found that there are both qualitative and quantitative correlation between the asymmetry of the wave, and the asymmetry of the resulting velocity field. As a result of the correlation two alternative expressions are given for the horizontal velocity (magnitude) asymmetry.

INTRODUCTION

In fairly deep water, the water particle horizontal velocities at the bottom are approximately equal for the shoreward and seaward motions, but as the waves moves into progressively shallow water of about $\frac{d}{L} < 0.15$, the asymmetry of the wave gives rise to the velocity differential of the water particles in the beachward and seaward directions, the onshore velocity of the water particles being greater in magnitude and of shorter duration than the offshore velocity. This behaviour thus leads to a definition of two types of horizontal velocity asymmetry, one based on magnitude and the other on time. They are defined as follows.

Horizontal velocity (magnitude) asymmetry =

$$\frac{\text{Maximum horizontal shoreward velocity}}{\text{Maximum horizontal seaward velocity}}$$

$$\text{Horizontal velocity (time) asymmetry} = \frac{\text{Time for the shoreward motion}}{\text{Time for the seaward motion}}$$

The study on wave asymmetry where the effect of beach slope and shoaling on the magnitude of the wave asymmetry was examined had been reported by the author(3). The present study is thus concentrated on the investigation of the velocity fields and the correlation of the velocity asymmetry and the wave asymmetry. The wave conditions used were the same as those used in the study on wave asymmetry(3). The correlation of the wave asymmetry and the asymmetry of the orbital velocity field will be discussed later in the paper. A review of previous work on velocity studies of water particles under the action of waves is given below.

Review of Previous Work

Iversen(1) examined the kinematics of the water movement in breakers. He used particles of a mixture of xylene, carbon tetrachloride and zinc oxide for flow visualization. The movement of the particles was recorded on cine film, from which each particle velocity was obtained by noting the distance moved on the projected frames and the time interval of movement. The velocities obtained were plotted as vectors as shown in Fig. 1. Iversen, in addition, measured the backwash and crest velocities. He obtained the backwash velocities by averaging all particle velocities in the region of minimum depth in the backwash, and the crest velocities were obtained from the gradient of the crest position - time history. Iversen found that the backwash velocity was higher on a steep beach as compared to a flat beach. This was to be expected.

Hamada(5) studied the particle velocity in wave motion in the region of the breakers. The velocity of flow was measured by means of a current meter which had several propellers driven by the action of water flow. A direct electric current in the circuit connected the current meter to an oscillograph, with one element of the oscillograph corresponding to one of the propellers. Hamada made orbital velocity measurements 3cm. from the bed. He noted the differential velocity in the shoreward and seaward directions. He found the ratio of backward velocity to forward velocity to vary between 1 : 1.16 to 1 : 1.29. However, his theoretical approach was to calculate the orbital velocity by the Airy theory and to add an estimated "residual" velocity (equivalent to the mass transport velocity). Since this was always positive, the resulting values gave a differential between the forward and backward velocities. His measurements are of interest, but his theoretical evaluation neglects the existence of orbital velocity asymmetry even in the absence of mass transport.

Inman and Nasu(6) made a study of orbital velocity in shallow water at La Jolla California. The measurements were made near the bottom and just seaward of the breaker zone in water depths ranging from

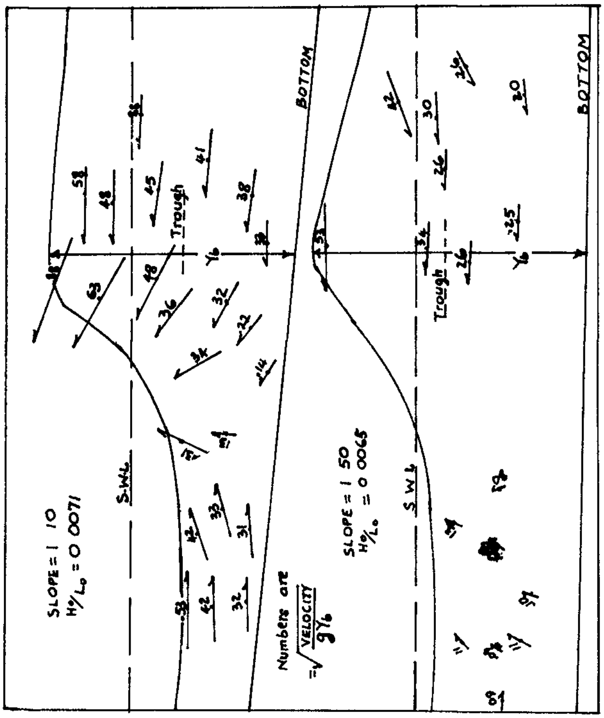


Fig. 1 Kinematics of a Breaking Wave (after Iversen (1)(1952))

about five to fifteen feet and wave heights of up to seven and a half feet. The orbital current meter consisted in the main of a cylindrical rod fixed rigidly at one end like a cantilever, and the system was arranged such that the orbital velocity could be interpreted from the bending of the rod caused by the force exerted by the moving water. A pressure type wave meter was used with the current meter, and both the wave meter and the current meter were mounted on a tripod. The current meter was mounted 0.82 ft. above the bottom. The bed had an average slope of about $\frac{1}{35}$ in the region where the orbital velocities were measured.

Inman and Nasu commented that the graph of the horizontal component of orbital velocity as a function of time resembled that of the wave pressure and they noted that the horizontal velocity seemed to be more dependent on the rate of change in level of the water surface during the passage of the wave than on the actual height of the wave. With reference to Fig. 2 after Inman and Nasu(6) they noted that in the figure, the fourth wave was almost as high as those preceding it, but showed a more gradual rise in level from the preceding trough to the crest. The result was a very low crest velocity. In terms of the work reported in this paper, and in earlier papers by the author(3,4) the statement of Inman and Nasu could be condensed into the statement that for waves of the same height, greater velocities are associated with greater wave slope asymmetry.

Inman and Nasu remarked that the maximum velocity under the wave crest was always onshore and that for almost all the waves analysed the mean onshore maximum velocity exceeded the offshore velocity. They compared the maximum orbital velocities with Solitary and Airy-Stokes waves. They gave the sum of the crest and trough velocities along the bottom aU for both Airy and Stokes waves as:

$$aU = \left| U\text{crest} \right| + \left| U\text{ trough} \right| = \frac{2\pi H}{T} \; \frac{1}{\sinh \frac{2\pi d}{L}} \qquad \text{——} \qquad (1)$$

For the Solitary wave, they gave the maximum orbital velocity at the bottom sU as:

$$sU = \tfrac{1}{2} NC \qquad \text{——} \qquad (2)$$

where $C = \sqrt{g(H + d)}$ is the velocity of propagation of the wave crest. H is wave height and d water depth.

N is defined by the equations:

$$N = \tfrac{2}{3} \sin^2 \left[M\left(1 + \frac{2H}{3d}\right) \right] \qquad \text{——} \qquad (3)$$

and

$$\frac{H}{d} = \frac{N}{M} \tan \tfrac{1}{2} \left[M\left(1 + \frac{H}{d}\right) \right] \qquad \text{——} \qquad (4)$$

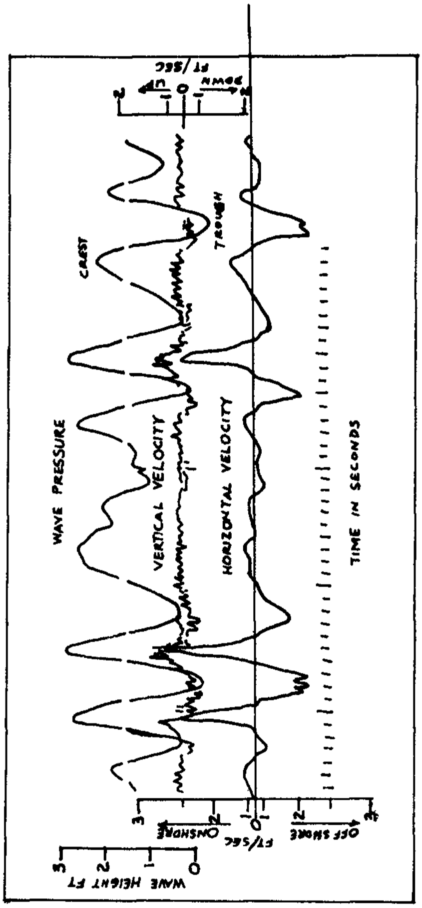


Figure 2 After Inman and Nasu (6) (1956)

Inman and Nasu[6] remarked that the measured velocities were on the average in better agreement with the Solitary wave than with the Airy - Stokes relations.

Ippen and Kulin[7] studied the internal velocities of the Solitary wave at the break-point. They used droplets of a solution of xylene and n-butyl phthalate. The motion picture camera they employed was operated at 20 frames per second and they placed an auxiliary grid in the plane of the particles so that on the photographs the grid lines could be projected down into the wave itself. They conducted their studies on two types of breakers; the plunging breaker on a slope of 0.065 and the spilling breaker on a slope of 0.023. They commented that on the slope of 0.023 with a spilling breaker the maximum velocity was nearly equal to the crest celerity at the wave break point, and they noted that the maximum particle velocity seemed to occur just slightly shoreward of the highest point of the crest.

Some velocity studies were also carried out by Morison and Crooke[11] in a study of deep water, shallow water and breaking waves. They used a mixture of carbon tetrachloride, xylene and zinc oxide having approximately the same specific gravity as water. They found that the greatest horizontal particle velocity occurred when the wave was breaking, and they noted a velocity differential between the shoreward and seaward horizontal velocities. They concluded that the maximum horizontal particle velocity at the crest of the wave might attain the wave celerity as the wave broke, but that the phenomenon was confined to a very narrow region of water at the crest of the wave. This was not borne out by Miller and Zeigler's [8] observations.

APPARATUS AND MEASURING TECHNIQUES

The technique used for the velocity measurements in the studies presented in this paper is a method usually called the "Hydrogen bubble method." Essentially it consists of a fine wire which forms the negative electrode of a D.C. circuit, while two metal plates, positioned at each side of the tank with a reasonable depth of the metal inside the water, serve as the second electrode. Hydrogen bubbles are formed on the negative electrode. The arrangement was such that the support for the wire and metal plates serving as the positive electrode were integral, (see plate 1). Two advantages of such an arrangement were that, as measurements were made at different positions along the beach, the apparatus could be moved intact. Secondly it was found that the quality of the bubbles was affected when the positive electrode was not close enough to the wire, as the circuit then became weak. The wire used in this work was 0.05mm diameter platinum wire, with brass plates as the positive electrodes.

Cine photography was employed and a suitable lighting arrangement was therefore devised. The camera used was a 16mm Reflex Bolex at 64 frames per second. With a wave period of 0.8 seconds, this gave approximately 50 pictures per wave cycle.

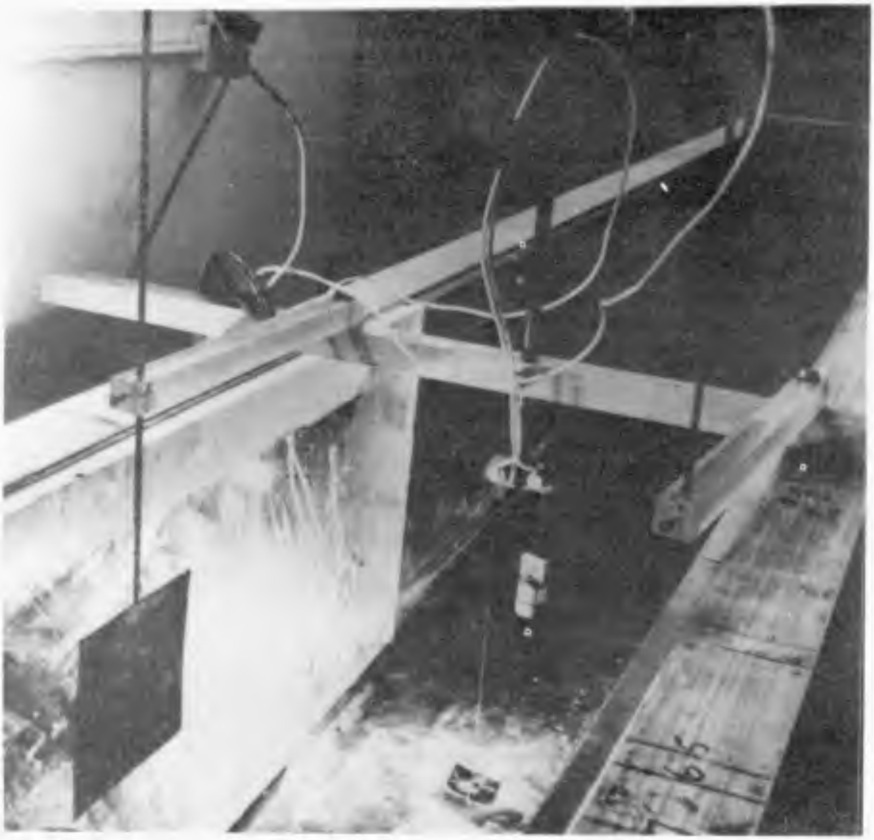


Plate 1

For the lighting arrangement, an Aldis Tutor 1000 Watt slide projector was used. The light source was located on the opposite side of the channel to the camera, and orientated at an oblique angle to the channel. Slits were used to prevent stray lights from scattering in the water.

In operation, one end of the wire was fixed to the beach with a tape, while the other end was passed over a support and was tensioned by a suitable weight. The velocities close to the wire at the different positions were evaluated from the displacement of the bubbles generated at different but known times. The orbital velocity measurements were made at a height of 5mm above the bed, which was outside the wave induced boundary later. These velocity measurements were made at several positions along the beach on two beach slopes 1:9 and 1:18 in the region $d/L \leq 0.15$ including the breakers. The two slopes were chosen on the basis of previous experiments(3, 4) which indicated that they are respectively representative of characteristically steep beaches with plunging breakers and flat beaches with spilling breakers.

The wave profiles and hence the different wave asymmetries for the different positions along the two beach slopes were also obtained using the capacitance-wire probe together with a visicorder.

EXPERIMENTAL RESULTS

Effect of Shoaling on Orbital Velocity Field

The velocity - time history for a full wave cycle at the different positions along the beach were obtained from the analysis of the cine films of hydrogen bubble blocks emitted from a wire positioned at these positions. The resulting curves are shown superimposed in figures 3 and 4. The two types of horizontal velocity asymmetry, one based on the magnitude of the shoreward-seaward velocities and referred to as the horizontal velocity (magnitude) asymmetry and the other based on the duration of the shoreward-seaward motion and referred to as the horizontal velocity (time) asymmetry were computed from the separate curves of the velocity - time variation for a wave cycle.

Figures 3 and 4 show how the orbital velocity-time curve changes in form as the wave progresses into shallower water. It can also be seen from the figures that the horizontal velocities increased in magnitude as the wave advanced into shallower water, and the greatest horizontal velocities occurred at the wave breaker position. It is interesting to note that the curves are closer together on the flatter beach slope of 1:18 than on the slope of 1:9 but that on each beach slope, the velocity seems to have a constant value at all positions along the beach 0.4 secs i.e. half the wave period after the passage of the wave crest, although the constant velocity varies in magnitude from one slope to another. It is evident from figures 3 and 4 that for each position on the beach, the maximum horizontal shoreward velocity did not occur directly under the wave crest but at a time 0.09T seconds after the passage of the crest (T is wave period).

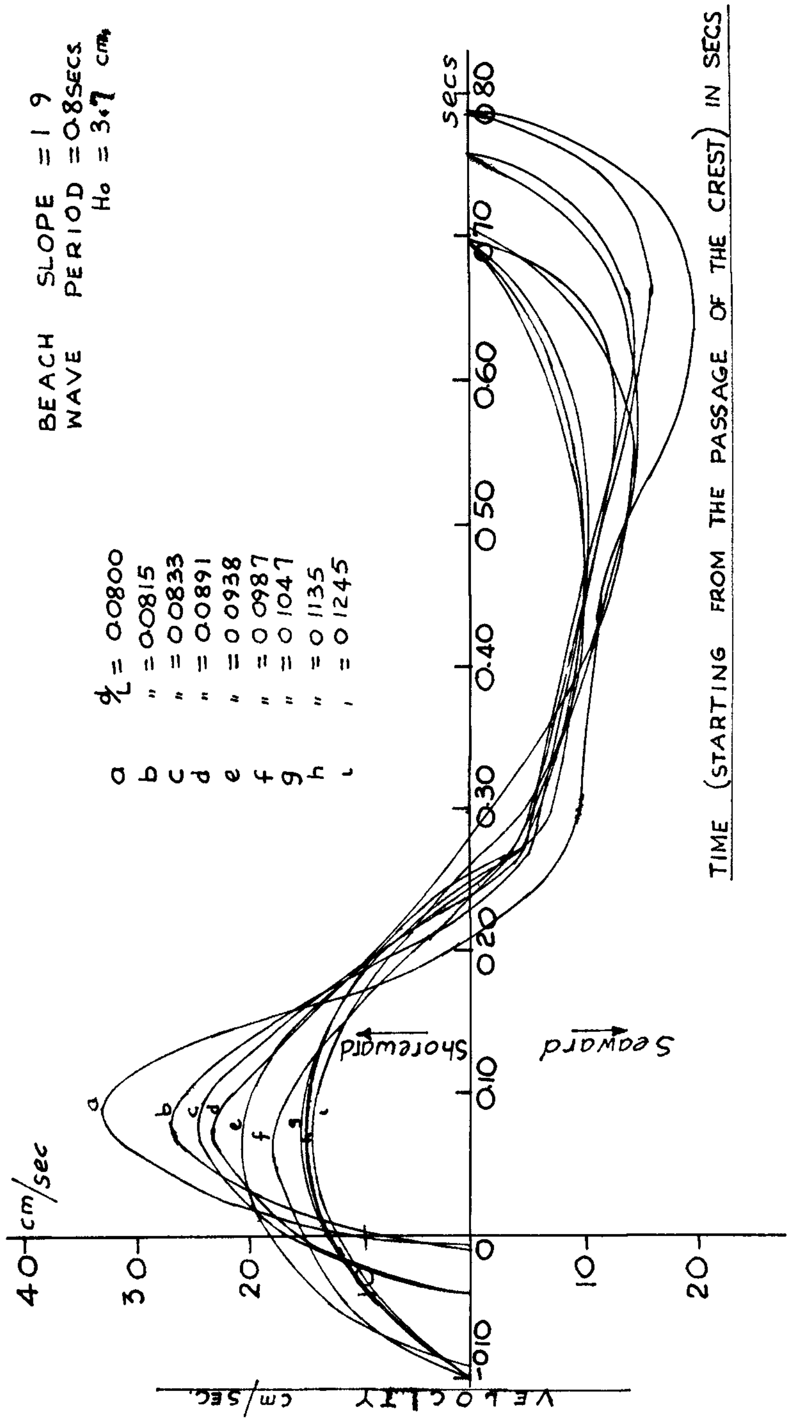


Fig 3 The Superposed Graphs of Horizontal Orbital velocity For a Wave Cycle

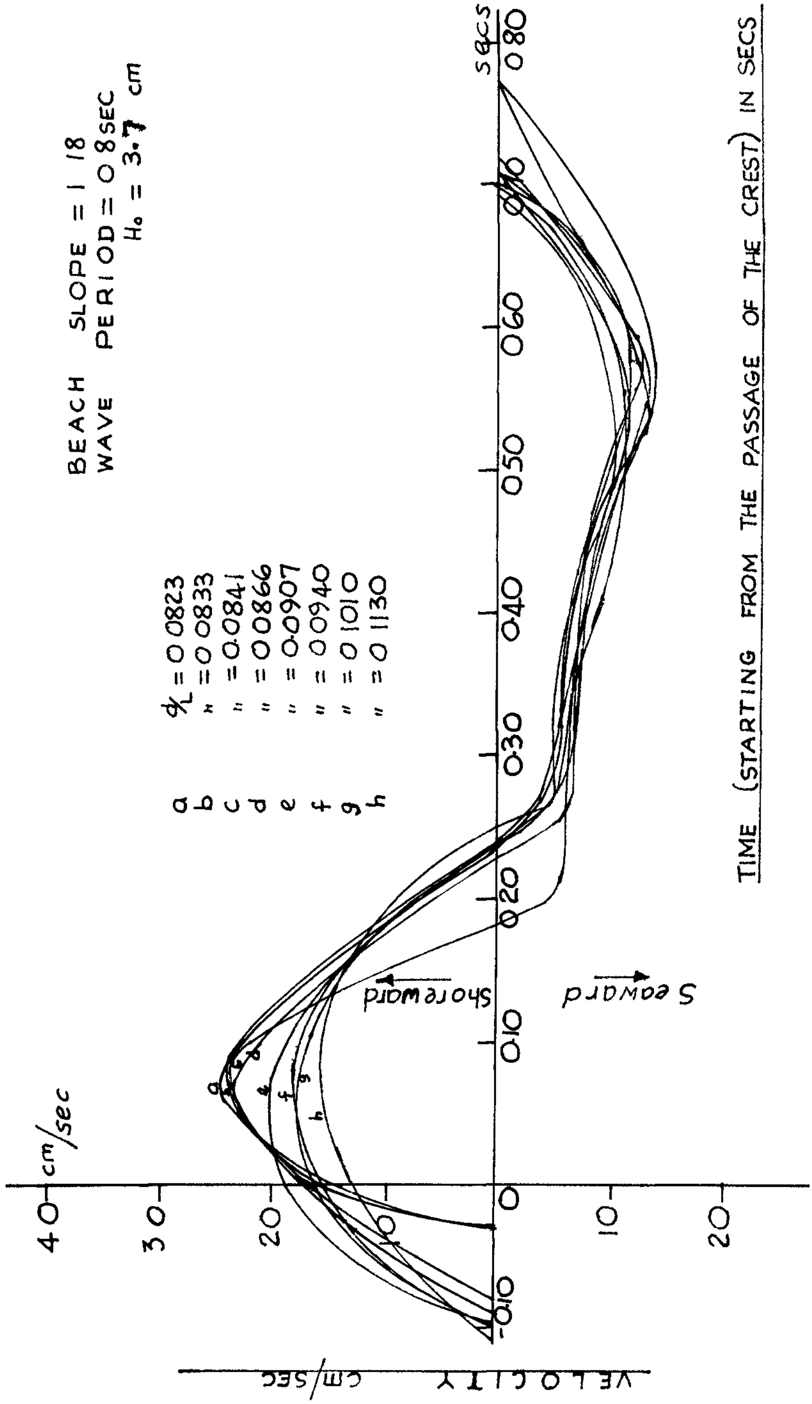


Fig 4 The Superposed Graphs of Horizontal Orbital Velocity For a Wave Cycle.

The graphs of horizontal velocity (magnitude) asymmetry against $\frac{d}{L}$ are shown in figure 5. It was found that for the two slopes of 1:9 and 1:18 the horizontal velocity (magnitude) asymmetry increased as the wave advanced into shoaling water and in each case was highest at the wave breaker position. At the d/L value of 0.10 on the beach slope of 1:9 this asymmetry was found to be 1:37 while on the beach slope of 1:18 it was found to be 1.585. At the wave break-point on the beach slope of 1·9 the value of this asymmetry was found to be 1.63 while the corresponding value on the beach slope of 1:18 was 1.77.

In terms of absolute values of the horizontal velocities, the values on the steeper slope of 1:9 were found to be higher than those of on the flatter slope of 1:18, especially in the neighbourhood of the breakers, but the graph of the horizontal velocity (magnitude) asymmetry against $\frac{d}{L}$ (figure 5) indicated that the asymmetry values were higher on the flatter slope. In other words larger beach material could be set in motion on the steep slope, but a greater shoreward - to - seaward differential would act on material on the flatter slope.

The graphs of horizontal velocity (time) asymmetry against $\frac{d}{L}$ are shown in figure 6. These show increasing asymmetry as the wave advances into shallower water. As noted in the introduction above, the horizontal velocity (time) asymmetry was defined as the ratio of the time for the shoreward motion to the time for the seaward motion, thus a small numerical value compared with unity denotes increasing asymmetry.

It was found that at the d/L value of 0.10 the value of this asymmetry on the slope of 1·9 was 0.76 whilst on the beach slope of 1:18 the value was found to be 0.78. At the wave breaker position on the beach slope of 1:9 the asymmetry became 0.37 whilst the corresponding value on the slope of 1:18 was 0.49. Thus the shoreward motion takes a longer time on the flatter slope.

Fig. 7 illustrates the comparison between Stokes theoretical wave profile based on his third approximation, and a typical profile obtained in the present study. It can be seen that the Stokes Wave is less peaked than the experimental wave, and also, by virtue of its mathematical description, does not possess asymmetry about the vertical axis.

Figures 8 and 9 which compare the orbital velocities based on Stokes theory with those obtained experimentally, show that at a $\frac{d}{L}$ value of 0.0833, which is very close to the breakers, the theory clearly does not agree with the measurements (see figure 9). Farther seaward from the break-point, at a $\frac{d}{L}$ value of 0.1135 (figure 8), although the theoretical values were still much higher than the experimental values, for instance, by as much as 73% of the experimental value under the crest, and there was in addition a phase lag in the profile, nevertheless the comparison was better than in figure 9.

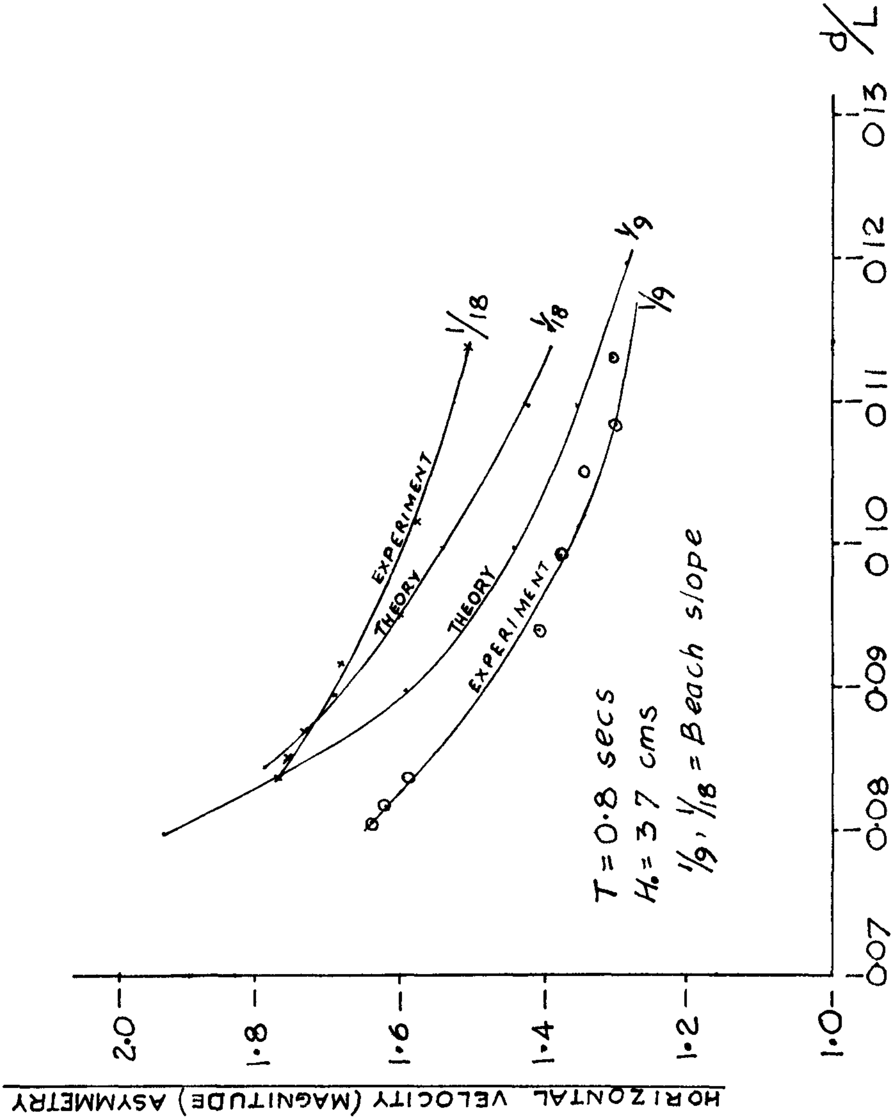


Fig 5 Graph of Horizontal Velocity (Magnitude) Asymmetry against d/L

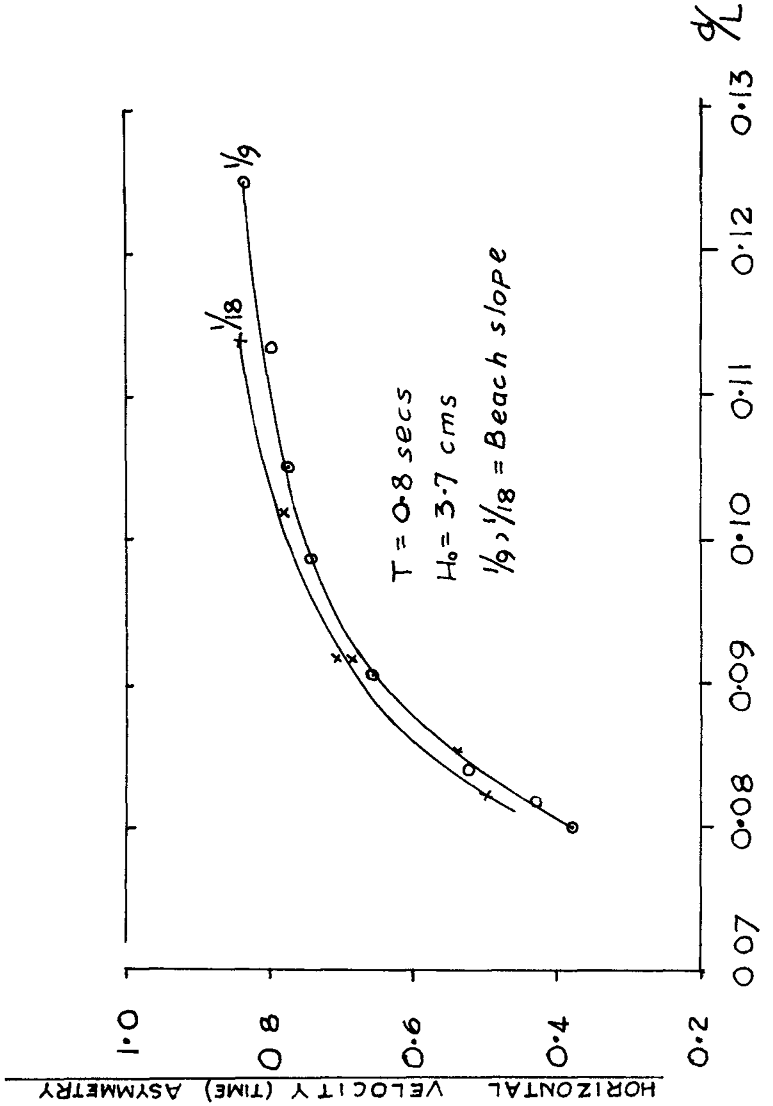
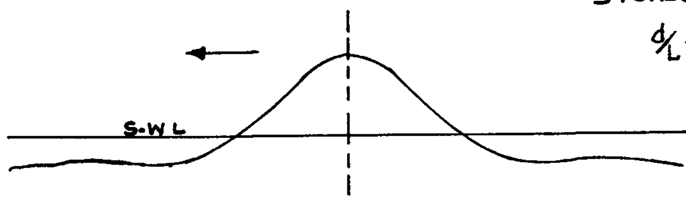


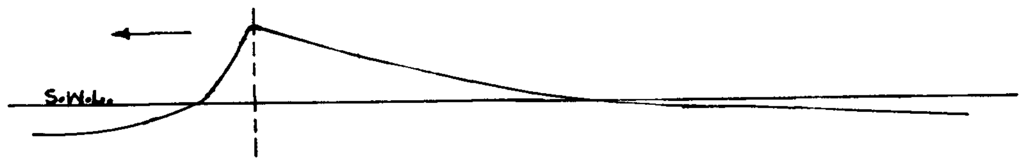
Fig 6 Graph of Horizontal Velocity (time) Asymmetry against d/L

STOKES FINITE AMP WAVE 3rd APPROX.

$d/L = 0.07$ H = 91 cm T = 8 secs



Theoretical Near-Breaking Wave. Stokes Finite Amplitude Wave, Third Approximation (after Miller & Zeigler[10])



Typical Experimental Profile Near the Break-point

Figure 7

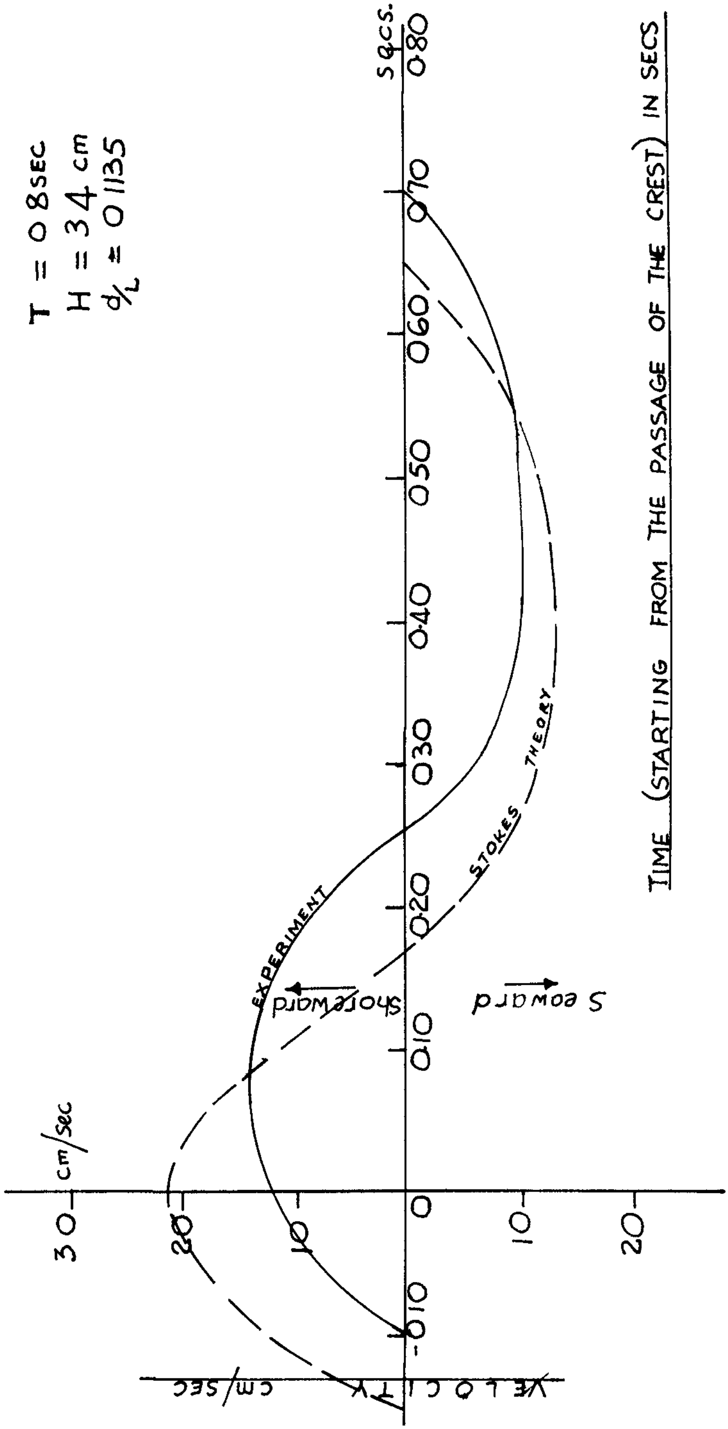


Fig 3 Comparison of Stokes Theory with Experimental Results.

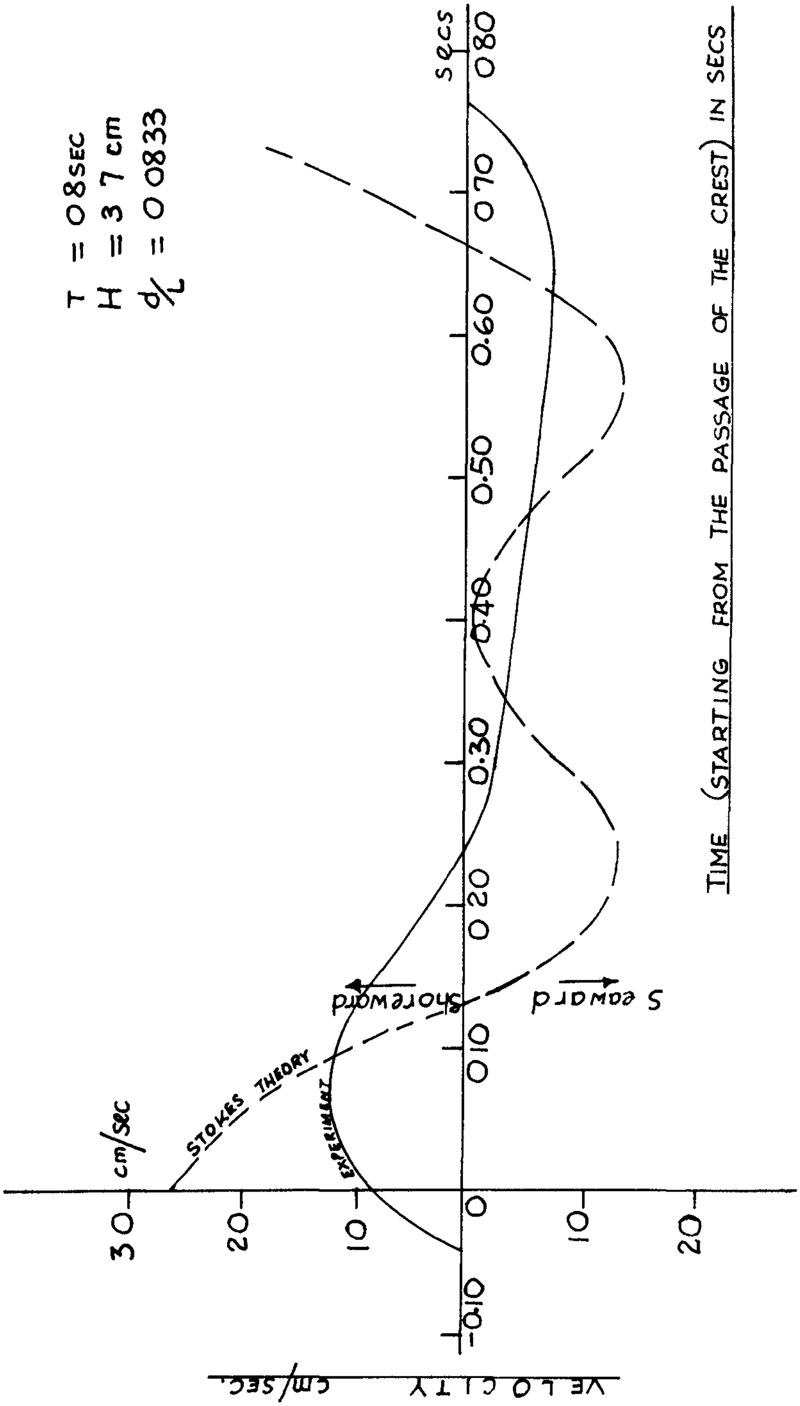


Fig 9 Comparison of Stokes Theory with Experimental Results.

The horizontal component of water particle velocity at a point (x, y) in water depth d is given to the second order of Stokes theory by the expression:

$$U = \frac{\pi H}{T} \frac{\cosh 2\pi\left(\frac{y+d}{L}\right)}{\sinh \frac{2\pi d}{L}} \cos 2\pi\left(\frac{x}{L}-\frac{t}{T}\right) +$$

$$\frac{3}{4}\left(\frac{\pi H}{T}\right)\left(\frac{\pi H}{L}\right) \frac{\cosh 4\pi\left(\frac{y+d}{L}\right)}{\sinh^4 \frac{2\pi d}{L}} \cos 4\pi\left(\frac{x}{L}-\frac{t}{T}\right)$$

or in simplified form

$$U = A \cos 2\pi X + B \cos 4\pi X \qquad \text{———} \qquad (5)$$

An inspection of equation (5) shows that the velocity components due to the first term on the right hand side are positive under the wave crest and negative under the trough, whilst the second term is positive under both crest and trough, and gives maximum negative components at $\frac{L}{4}$ and $\frac{3L}{4}$ from the crest. The net effect is to increase the value of the magnitude under the crest, and decrease it under the trough, thus producing asymmetry in the magnitudes of the orbital velocity. Fig. 10 compares the values of the horizontal velocity (magnitude) asymmetry given by the Stokes theory-equation(5) with the experimental results, and the theoretical values from equation (10). It can be seen that the divergence between the Stokes prediction and the experimental results was about 32% at a $\frac{d}{L}$ value of 0.11, with the divergence increasing shoreward but improving seaward.

For these reasons, one can conclude that Stokes theory is not directly applicable to the study of velocities in the near breaker zone.

Correlation Between Wave Asymmetry and Velocity Asymmetry

An important aspect of the work carried out by the author is the study of the correlation between the different types of wave asymmetry and the correlation between these and the resulting velocity asymmetry as the wave progressed into shallow water. The first aspect has been reported[3].

Figure 11 shows the graphs of both the horizontal velocity (magnitude) asymmetry and the horizontal velocity (time) asymmetry plotted against the wave horizontal asymmetry H_A and $H_A{}^1$ for the beach slope of 1:18. The graphs show that a correlation exists between the velocity asymmetry and the wave asymmetry. The graph of horizontal velocity (magnitude) asymmetry against wave horizontal asymmetry H_A indicated that as the wave progressed into shallower water and the value of H_A decreased, indicating increased asymmetry,

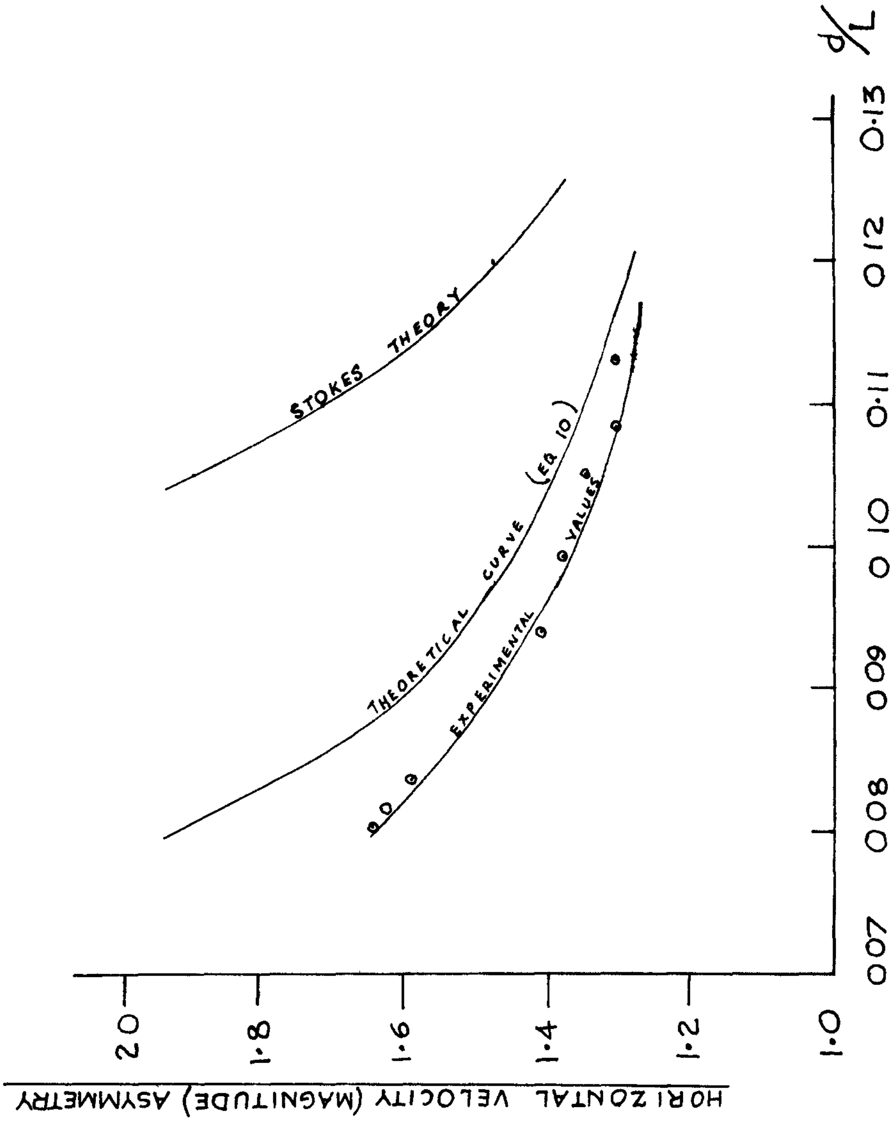


Fig 10 Comparison of Stokes theory with Experimental Results.

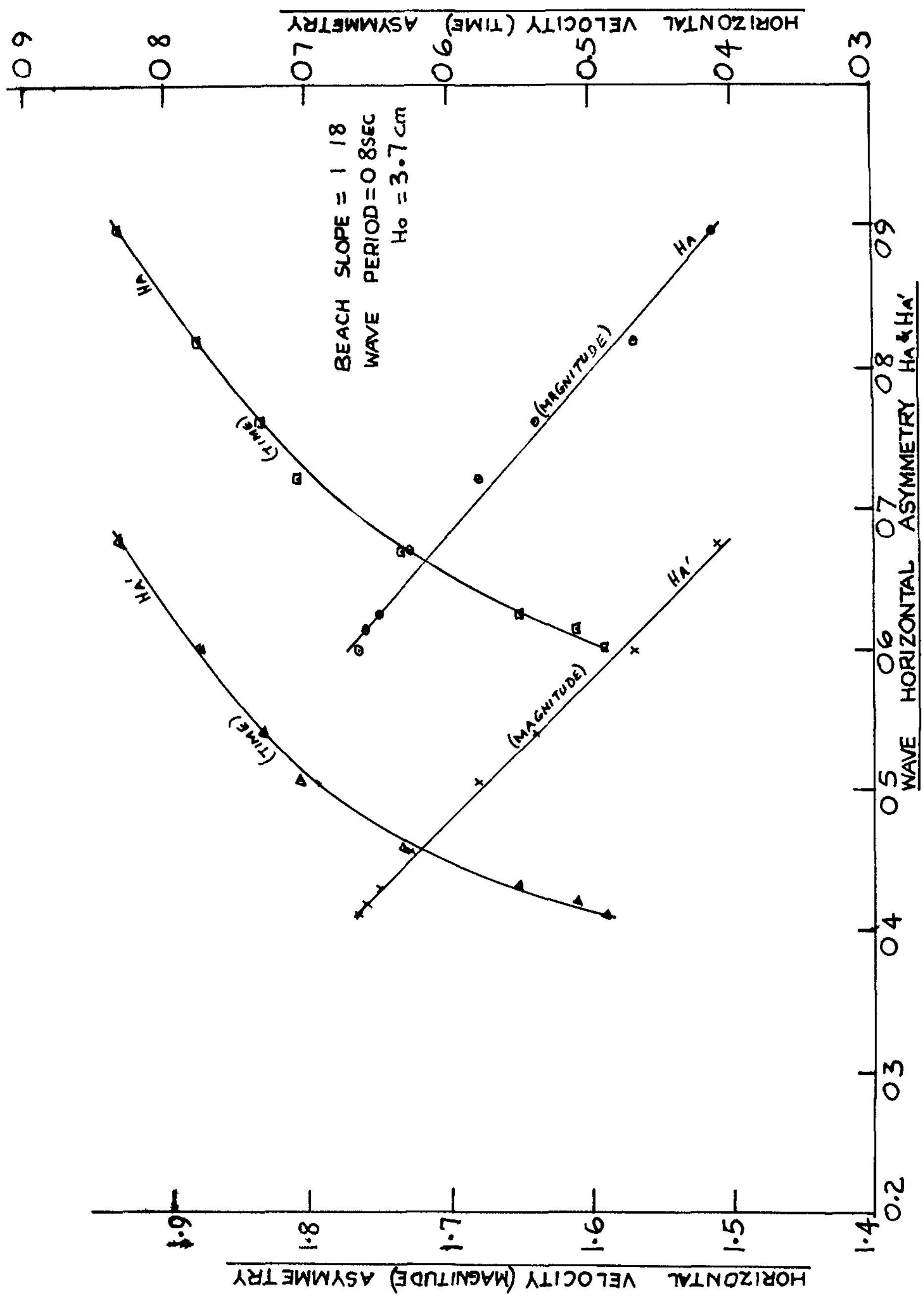


Figure 11 Graph of Velocity Asymmetry Against Wave Asymmetry.

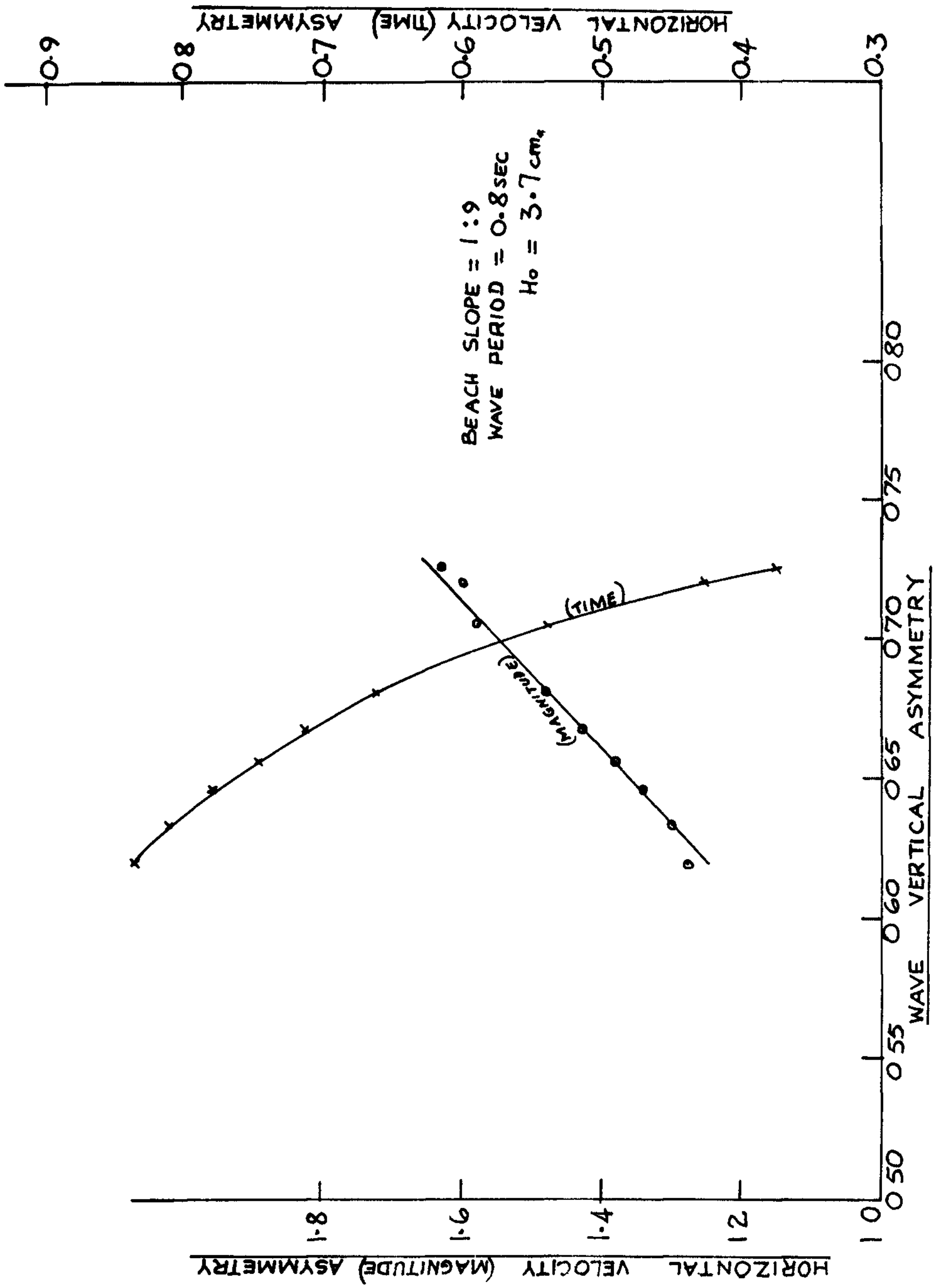


Fig 12 Graph of velocity Asymmetry Against Wave Asymmetry.

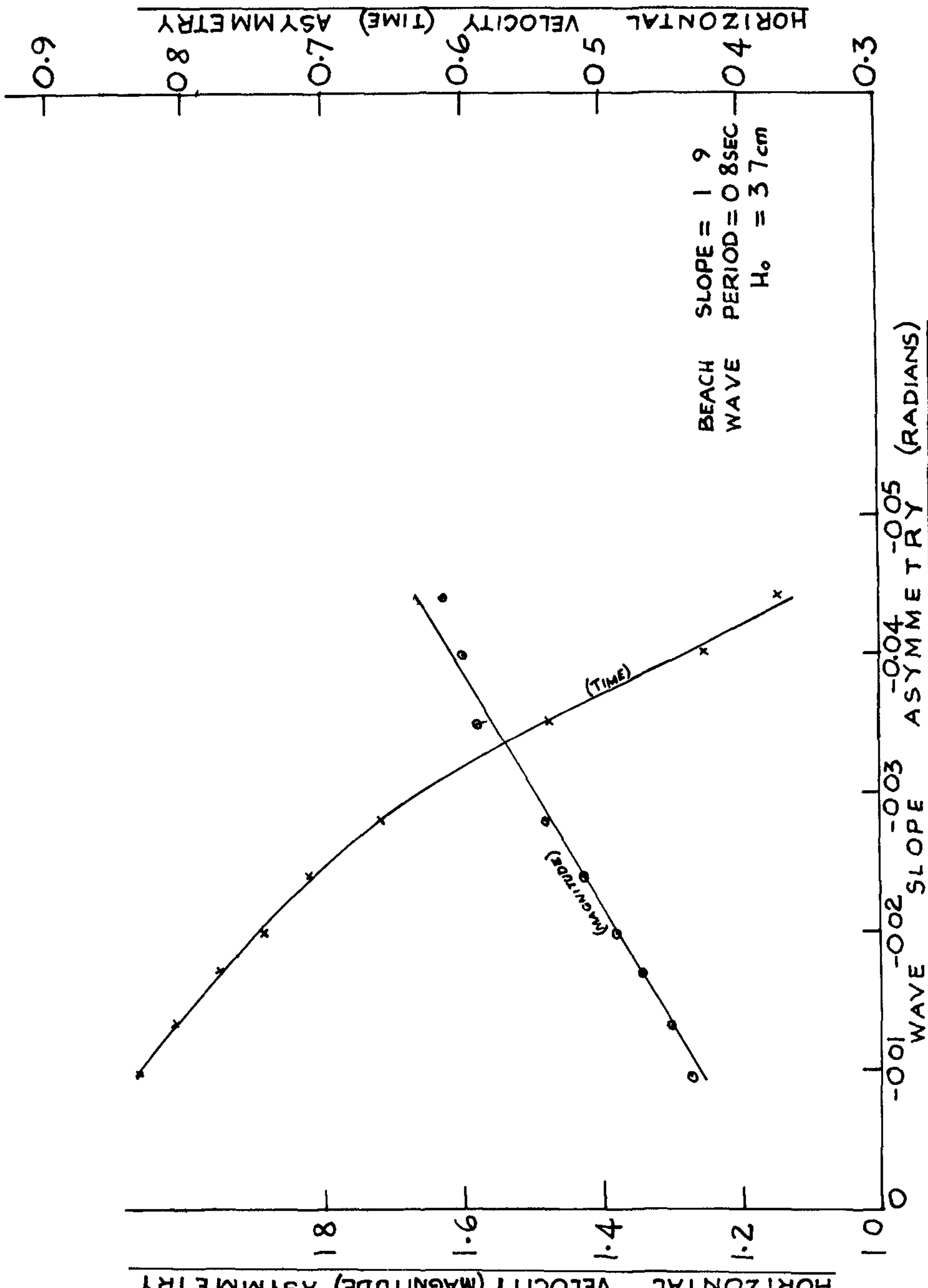


Fig 13 Graph of velocity Asymmetry Against Wave Asymmetry.

the horizontal velocity (magnitude) asymmetry also increased. For instance, when the value of the wave horizontal asymmetry H_A is 0.80 the value of the horizontal velocity (magnitude) asymmetry is 1.60, whereas when the value of H_A becomes 0.70 the value of the horizontal velocity (magnitude) asymmetry increased to 1.685 very near the wave break-point, the value of H_A becomes 0.60 the value of the horizontal velocity (magnitude) asymmetry is found to be 1.77. The graph of the horizontal velocity (magnitude) asymmetry against H_A1 followed a trend fairly similar to the graph of the horizontal velocity (magnitude) asymmetry against H_A. The graph of the horizontal velocity (time) asymmetry against H_A and H_A1 also show good correlations. The remaining graphs of the velocity asymmetry against the different types of wave asymmetry all show reasonably good correlation between the velocity asymmetry and the wave asymmetry. (see figures 12 and 13).

As a result of the studies the following relationships were obtained:

Let Av = wave vertical asymmetry = $\dfrac{\text{Vertical distance from crest to S.W.L.}}{\text{Total wave height.}}$

S = wave slope asymmetry = $\frac{1}{2}$(Front face slope at S.W.L. + Back Face slope at S.W.L.) radians

$|S|$ = modulus of s

γ = beach slope.

V_{HMA} = Horizontal velocity (magnitude) asymmetry

H_A & H_A1 = wave horizontal asymmetry

$H_A = \dfrac{\text{Horizontal distance from crest to front face at S.W.L.}}{\text{Horizontal distance from crest to back face at S.W.L.}}$

$H_A1 = \dfrac{\text{Horizontal distance from crest to preceding wave trough}}{\text{Horizontal distance from crest to following wave trough}}$

The empirical relationships between V_{HMA}, S, γ, Av and H_A are:

$$H_A = \frac{1.52}{e^{\gamma}}(1.18 - \text{Sinh Av}) \qquad (6)$$

$$V_{HMA} = 3.4\,\text{Av} - 0.78 \qquad (7)$$

$$\text{Av} = \frac{8.8}{e^{\gamma}} \tanh |S| + 0.5 \qquad (8)$$

From the Cnoidal wave theory as developed by Korteweg and De Vries[8], the wave vertical asymmetry Av is given by

$$Av = \frac{y_o - d}{H} = \frac{16d^3}{3L^2H} \left\{ K_{(k)} \left[K_{(k)} - E_{(k)} \right] \right\} \qquad (9)$$

where y_c = distance from the ocean bottom to the wave crest

d = still water depth

H = wave height (trough to crest)

L = wave length

$K_{(k)}$ = complete elliptic integral of the first kind.

$E_{(k)}$ = complete elliptic integral of the second kind.

k = modulus of the elliptic integral

From equations (9) and (7) we have

$$V_{HMA} = \left[\frac{18.1\ d^3}{L^2H} \left\{ K_{(k)} \left[K_{(k)} - E_{(k)} \right] \right\} - 0.78 \right] \qquad (10)$$

substituting for Av from eq. (8) into eq. (7) gives

$$V_{HMA} = 3.4 \left\{ \frac{8.8}{e^{\gamma}} \tanh |S| + 0.271 \right\} \qquad (11)$$

Also from the work of Biesel(2), the wave slope asymmetry S is given by

$$s = m^2 \left(\frac{H_o}{2}\right)^2 \gamma \left\{ \frac{3 + \frac{md}{\tanh md} - 3\ md \tanh md}{D^2 (\sinh md)^2 \tanh md} \right\} \qquad (12)$$

where $D = 1 + \dfrac{md}{\sinh md \cosh md}$

and $m = \dfrac{2\pi}{L}$

substituting s from eq. (12) into eq. (11) gives

$$V_{HMA} = 3.4 \left\{ \frac{8.8}{e^{\gamma}} \tanh \left[m^2 \left(\frac{H_o}{2}\right)^2 \left\{ \frac{3 + \left(\frac{md}{\tanh md}\right) - 3\ md \tanh md}{D^2 (\sinh md)^2 \tanh md} \right\} \right] + 0.271 \right\} \qquad (13)$$

Thus equations (10) and (13) give expressions for V_{HMA}. However, of the two, equation (10) gives closer values to the experimental results in this work, and is the theoretical curve shown on figure 5. Values of $K(k)$ and $E(k)$ are tabulated in Masch and Wiegel(9) (1961).

Comment

Figure 5 gives the impression that the theoretical predictions for horizontal velocity asymmetry are closer to the experimental values, for the steeper beach slope category. The correspondence is to within 5% in the region of $d/L = 0.10$ to 0.13, and shows a difference of about 8% very close to the breakers where d/L is approximately 0.08. In fact, similar percentages apply in the case of the flatter beach slope, but the difference in the slope of the curves is more apparent and has the effect of offsetting the correlation.

Conclusions

The results show that the horizontal velocities increase in shoaling water, and for both slopes of 1:9 and 1:18 the greatest horizontal velocities occurred at the wave breaker position. The curves of the velocity-time history figures 3 and 4 show that the curves were closer together on the slope of 1:18 than on the slope of 1:9. It was however found that the velocity seems to have a constant value on each beach slope at all positions along the beach half the wave period after the passage of the wave crest; although the constant varies from slope to slope.

It was found that the maximum horizontal shoreward velocity did not occur directly under the wave crest but at a time 0.09T seconds after the passage of the crest. In terms of the values of the horizontal velocities, the values on the steeper slope of 1:9 were higher than those on the flatter slope of 1:18 especially in the neighbourhood of the breakers, but it was found that the values of the horizontal velocity (magnitude) asymmetry were higher on the flatter slope. Also the graph of the horizontal velocity (time) asymmetry against $\frac{d}{L}$ showed that the shoreward motion takes a longer time on the flatter slope.

The author is of the view that there must be a correlation between the wave asymmetry and the asymmetry of the velocity field resulting from the wave. The graphs of the different types of wave asymmetry defined by the author(3) against the velocity asymmetry confirm that there are both qualitative and quantitative correlation between the wave asymmetry and the velocity asymmetry. As a result of the correlation two alternative expressions are given for the horizontal velocity (magnitude) asymmetry, one based on the Cnoidal wave theory and the other based on the work of Biesel(2).

The paper also shows that Stokes theory is not directly applicable to the study of the velocity patterns in the near breaker zone.

In drawing conclusions from the study of the velocity fields associated with asymmetrical waves on varying beach slopes it is important to note that Miller and Zeigler[10] observed three broad classes of wave asymmetry on a beach of only one slope. It is clear, as they suggest, that in nature in addition to the importance of beach slope, breaker shapes are affected by the interaction of different wave trains, and by the timing and magnitude of the backwash. It seems probable that the observations of Inman and Nasu[6] that higher orbital velocities are associated with wave shape asymmetry must be considered in relation to the fact that their measurements were also made in nature, and that more than one wave train is likely to be present.

The author considers that investigations of the velocity fields very close to the bed should eventually provide a quantitative basis for evaluating the accretion and depletion of sediment under wave action. The magnitude and duration of the shoreward and seaward orbital velocities are important in sediment movement under wave actions. The magnitude of the maximum velocity is associated with initiating the motion and the asymmetry of the velocity fields determines the direction and extent of the net movement. The author thus considers that the quantitative correlation of the wave characteristics with the asymmetry of the orbital velocity fields would most probably lead to the correlation of wave characteristics with sediment movement.

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