CHAPTER 25

PROBABILITIES OF BREAKING WAVE CHARACTERISTICS

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ABSTRACT

Utilizing the hydrodynamic relationships for shoaling and refraction of waves approaching a shoreline over parallel bottom contours a procedure is developed to transform an arbitrary probability density of wave characteristics in deep water into the corresponding breaking characteristics in shallow water A number of probability distributions for breaking wave characteristics are derived in terms of assumed deep water probability densities of wave heights wave lengths and angles of approach Some probability densities for wave heights at specific locations in the surf zone are computed for a Rayleigh distribution in deep water The probability computations are used to derive the expectation of energy flux and its distribution

INTRODUCTION

Many experimental investigations of breaking waves longshore currents and littoral drift have been made Sometimes the results have been expressed in terms of deep water wave characteristics and sometimes in terms of breaking wave characteristics making comparisons of various data a difficult task In some instances the wave characteristics at some intermediate water depth are given The laws of hydrodynamics can be used to relate deep and shallow water characteristics if bottom friction effects are neglected

Earlier theoretical work of LeMéhauté (1961) LeMéhauté and Webb (1964) and LeMehaute and Koh (1967) have proposed methods of computing shallow water wave characteristics in terms of deep water wave characteristics for periodic waves using first third and fifth order Stokian wave theories

It is well known that naturally occurring sea conditions can be characterized only in statistical terms Such terms include the spectrum the average wave height significant wave height mean wave period etc The sea state in deep water can be characterized by a probability distribution of wave heights wave lengths and angles of approach to the shore

The reason for the choice of characteristics in terms of probability distributions rather than the spectrum is purely for convenience in considering the behavior of waves in the surf zone as individual crests Relationships between spectra and probability of wave height and wave period have been demonstrated by Longuet-Higgins (1952 1957) Bretschneider (1959) Collins (1967) and others

The usual method of treating a wave traveling in gradually varying water depths has been to use the wave theories for a horizontal bed and to account for the effects of bottom variation by considering energy conservation Biesel (1951) and many others and more recently Mei Tlapa and Eagleson (1968) have proposed theoretical approaches to wave propagation over variable depth The traditional method of treating waves over gradually varying bottom topography will be adopted It will also be assumed that the linear wave theory (first order) can be applied up to the point of breaking Non-linear effects (peak-up effect) which are important near breaking are, to some extent empirically taken into account in the breaking criteria

In this paper an attempt has been made to apply the hydrodynamic theories of shoaling, refraction and wave breaking to statistical models of the sea Examples of computations are given for a plane beach Generally the wave characteristics in deep water are assumed and the corresponding characteristics in shallow water including the surf zone are computed

One-dimensional (wave height wave length or wave direction) and two-dimensional (wave height and wave length) probability distributions are given The general solution for a three-dimensional probability distribution is derived for two different wave breaking criteria

The results of the computations of wave probabilities in shallow water are used to compute expected values of energy flux in the surf zone as a function of depth of water A similar approach to determine longshore current distribution is proposed

GENERAL DISCUSSION

Governing Equations of Wave Transformation

The dispersion relationship for waves is written

$$L_b/L_o = \tanh 2\pi d_b/L_b \tag{1}$$

where the subscript b refers to conditions at breaking the subscript o refers to conditions in deep water (see figure 1) L is the wave length d is the water depth

Snell's law can be used

$$L_o/L_b = \sin \alpha_o / \sin \alpha_b \tag{2}$$

where α is the angle of the wave crest with the shoreline

The choice of a suitable breaking criterion is required Experimental investigations on this phenomenon in two-dimensional wave tanks have been made notably by Iversen (1952) Hamada (1963) and Suquet (1950) The following criterion fits the data very closely (after LeMehauté and Koh 1967)

$$H_b/H_o = 0\ 76 S^{1/7} (H_o/L_o)^{-\frac{1}{4}}$$

where S is the bottom slope and H is the wave height For waves breaking at an angle the bottom slope is actually S cos α_b and H_o should be replaced by $H_o \cos^{\frac{1}{2}} \alpha_o$ The breaking criterion becomes

$$H_b/H_o = 0\ 76 S^{1/7} \cos^{1/7} \alpha_b (H_o/L_o)^{-\frac{1}{4}} \cos^{3/8} \alpha_o \tag{3}$$

where S is the slope at breaking An alternate breaking criterion was also used

$$H_b/d_b = 0\ 72 + 5\ 6S \tag{4}$$

if bottom friction effects are neglected then the wave height change up to breaking can be computed from the conservation of energy as $H_b = K_1 K_s H_o$ which over a beach with parallel bottom contours will be used in the form

$$H_b/H_o = \left\{ \tanh k_b d_b \left[1+2k_b d_b/\sinh 2k_b d_b \right] \cos \alpha_b/\cos \alpha_o \right\}^{-\frac{1}{2}} \tag{5}$$

where $k_b = 2\pi/L_b$

Equations 1, 2 5 and 3 or 4 provide a system of equations such that a given deep water wave characterized by H_o L_o α_o yields a unique breaking wave

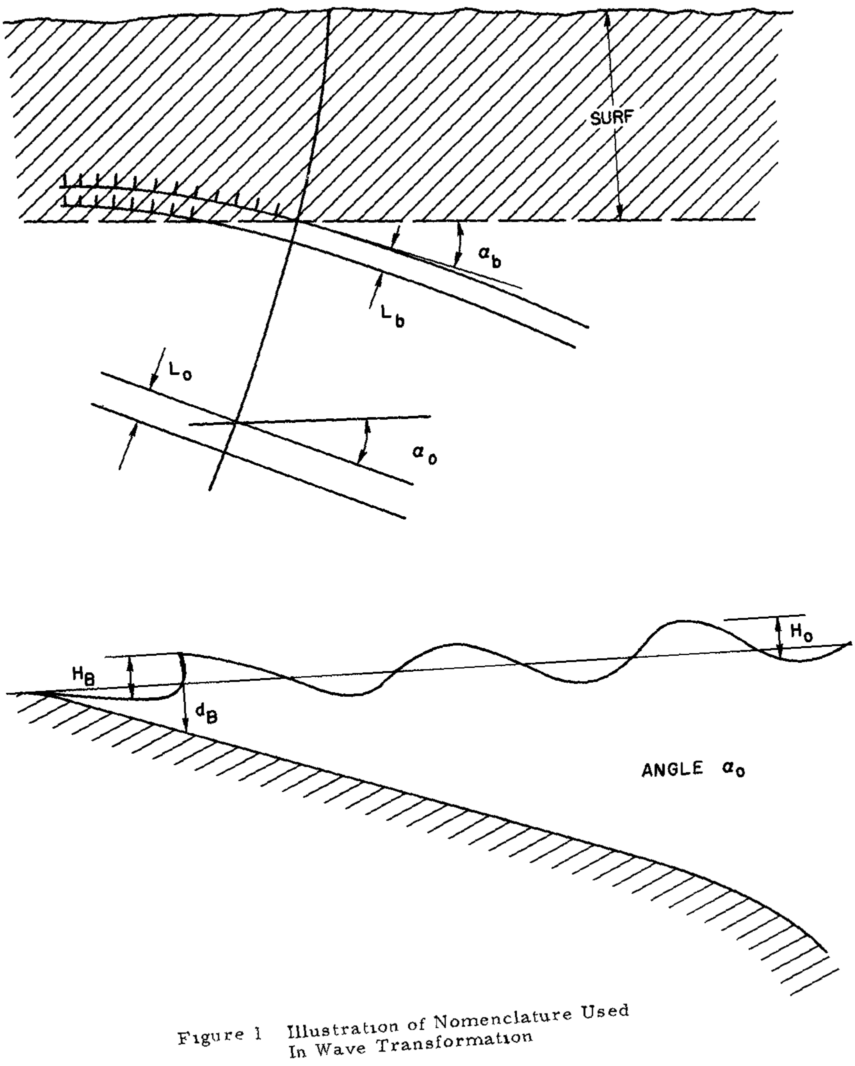

Figure 1 Illustration of Nomenclature Used In Wave Transformation

characterized by H_b L_b α_b In fact the solution of these equations for the breaking characteristics requires a step-by-step or iterative procedure since the dependent variables cannot be separated

Breaking Waves

The methods of the preceding section yield the information H_b L_b α_b d_b for any specified deep water values H_o L_o α_o Another problem of interest can be stated given H_o L_o α_o - what is H L α at a specified location $d = d_c$? Three separate conditions must be recognized

(a) $d_c > d_b$ (b) $d_c < d_b$ (c) $d_c = d_b$

When d_c is greater that d_b Eqs 1 2 and 5 can be applied directly to compute H_c L_c α_c The case $d_c = d_b$ can also be included When d_c is less than d_b the wave has broken farther offshore and some of its energy has been dissipated The decay of wave height after breaking has been studied by Horikawa and Kuo (1966) Street and Camfield (1966) Nakamura Shiraishi and Sasaki (1966) and Divoky LeMéhaute and Lin (1969) The experimental results are summarized in Figure 2 (after Divoky et al 1969)

For purposes of the present study, the wave height at $d = d_c$ after breaking was taken as

$$H_c = H_b\, d_c/d_b \tag{6}$$

i e a linear wave height decay from the breaking point to the shore

The angle and wave length of the broken wave at $d = d_c$ were computed from the refraction laws for long waves

$$L_c = L_b\,(d_c/d_b)^{\frac{1}{2}} \tag{7}$$

$$\sin\alpha_c = \sin\alpha_b\,(d_c/d_b)^{\frac{1}{2}} \tag{8}$$

Probability Distributions

Assuming that the statistical properties of the deep water wave characteristics are known the system of equations 2 through 5 will enable the determination of the statistical properties of shallow water waves up to breaking The problem can be stated given $p\,(H_o\ L_o\ \alpha_o)$ determine $p\,(H_b\ L_b\ \alpha_b\ d_b)$ subject to the relationships of equations 1 through 5 At breaking only three conditions need to be specified (The fourth can be found in terms of the other three) The general solution is,

$$p\,(H_b,\ L_b\ \alpha_b) = p\,(H_o\ L_o\ \alpha_o)\,|J|^{-1} \tag{9}$$

where

$$|J| = \begin{vmatrix} \partial H_b/\partial H_o & \partial H_b/\partial L_o, & \partial H_b/\partial \alpha_o \\ \partial L_b/\partial H_o & \partial L_b/\partial L_o & \partial L_b/\partial \alpha_o \\ \partial \alpha_b/\partial H_o & \partial \alpha_b/\partial L_o & \partial \alpha_b/\partial \alpha_o \end{vmatrix} \tag{10}$$

The partial derivities have to be determined from equations 2 through 5 Generally it is seen that if equations 2 through 5 are differentiated by each variable H_o L_o α_o one at a time a system of 12 simultaneous equations are derived which have to be solved for the partial derivatives before substitution into

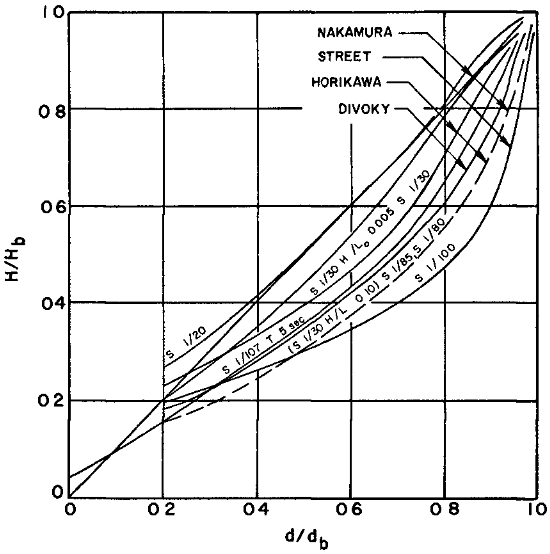

Figure 2 Summary of Experimental Data on Wave Height Decay After Breaking

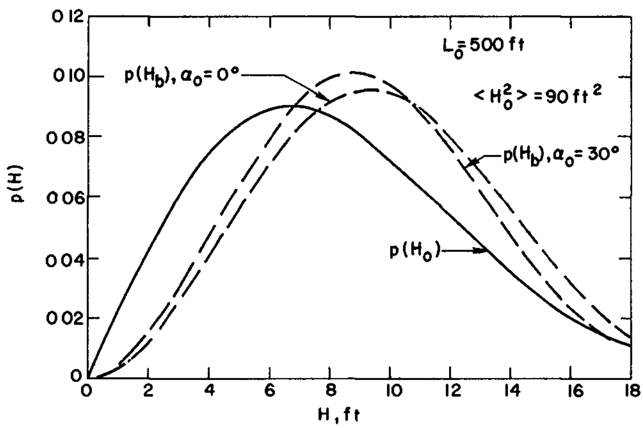

Figure 3 Probability Densities of Breaking Waves Height for Two Angles of Approach in Deep Water

equation 10 The entire procedure is straightforward (but tedious) unless J contains zeros Special consideration must be given to regions where J changes sign

SPECIFIC APPLICATIONS

One-Dimensional Probability Distribution

Assuming that the wave characteristics are specified in deep water such that

L_o = constant α_o = constant

H_o has the probability distribution

$$p\,(H_o) = 2H_o/\langle H_o^2\rangle \exp\left[-H_o^2/\langle H_o^2\rangle\right] \tag{11}$$

$\langle\,\rangle$ denotes mean value

then

$$p\,(H_b) = p\,(H_o)\,\partial H_o/\partial H_b \tag{12}$$

$$p\,(L_b) = p\,(H_o)\,\partial H_o/\partial L_b \tag{13}$$

$$p\,(\alpha_b) = p\,(H_o)\,\partial H_o/\partial \alpha_b \tag{14}$$

The values of the differentials can be determined analytically or they can be calculated numerically by finite difference approximations

Examples of some sample cases computed for one-dimensional deep water probabilities are given as Figures 3 through 5 The probabilities shown are computed for breaking characteristics at whatever location they occur Figure 3 illustrates the effect on the probability of breaking wave heights for different angles of approach in deep water This probability is not too sensitive to the angle of approach

Figure 4 indicates the effect of various deep water wave lengths on the probability distribution of breaking wave heights for a deep water angle of approach of 30° As expected it is seen that increased deep water wave lengths lead to larger breaking wave heights The probability distributions are shifted toward higher values of wave height Figure 5 shows the effect of increased wave length in deep water on the angles of breaking waves for a Rayleigh-type deep water wave height distribution Once more as anticipated larger wave lengths tend to refract more and break more nearly parallel to the shore

Examples of computations for the probability densities of wave characteristics at a specific location are given as Figures 6 and 7 These two figures differ from figures 3 through 5 in that they present wave height probabilities at a fixed location Both broken and non-breaking waves are present All of the figures show a sharp rise in the probability density of wave heights at the breaking limit

Two-Dimensional Probability Distributions

The two-dimensional probability of breaking characteristics indicate the joint probability of breaking wave height and wave length Complete numerical computations have not been made since in this case the roots of the Jacobian must first be investigated One simplification which permitted numerical results to be obtained follows from the assumption that wave breaking occurs

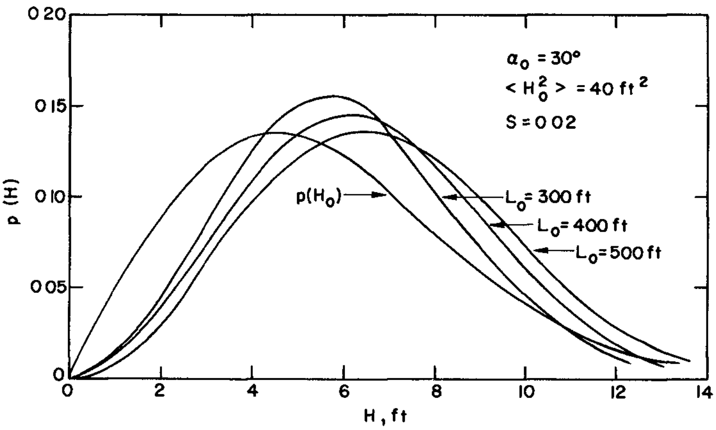

Figure 4 Probability Densities of Breaking Wave Heights for Various Deep Water Wave Lengths

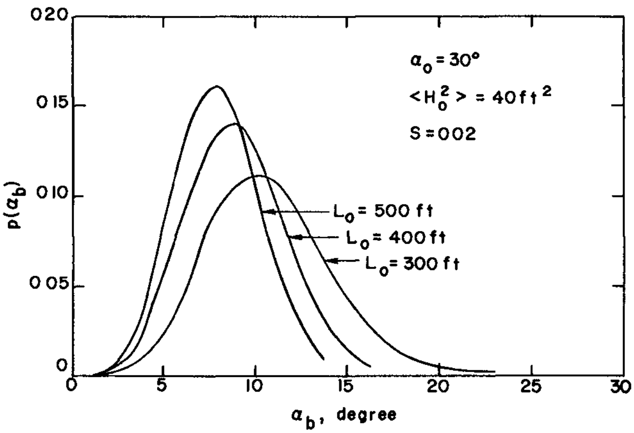

Figure 5 Probability Densities of Breaking Wave Angles for Various Deep Water Wave Lengths

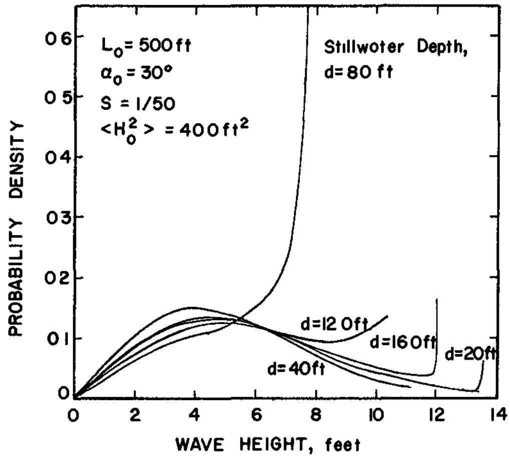

Figure 6 Probability Densities of Wave Heights at Specific Locations for L_o = 500 ft α_o = 30° S = 1/50

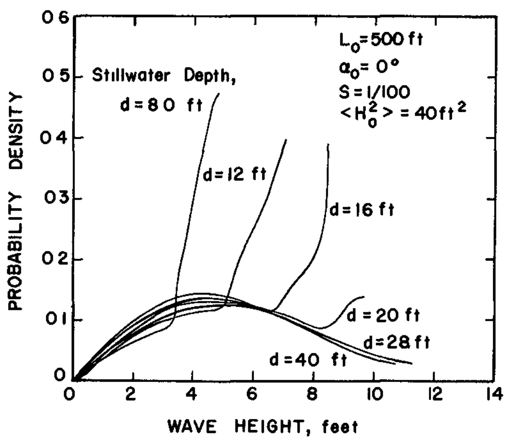

Figure 7 Probability Densities of Wave Heights at Specific Locations for L_o = 500 ft α_o = 0° S = 1/100

in shallow water only, i e d/L at breaking is small Then it follows that equation 5 becomes

$$H_b/H_o \simeq \left[\cos\alpha_o/\cos\alpha_b \quad 1/\left(2k_d\, d_b\right)\right]^{\frac{1}{2}} \tag{15}$$

equation 1 becomes

$$L_b/L_o \simeq k_b\, d_b \tag{16}$$

A comparison between equation 15 and equation 5 when $\alpha = 0$ is shown as Figure 8 It is seen that the differences are very small when d/L is less than 0 05

Substitution of equations 16 and 2 into equation 15 yields the interesting result

$$H_b/H_o = 0\ 5\ [\sin 2\alpha_o/\ \sin 2\alpha_b]^{\frac{1}{2}} \tag{17}$$

which has the limit $H_b/H_o = \left[L_o/L_b\right]^{\frac{1}{2}}$ when $\alpha_o = 0$ as might be expected

The corresponding Jacobian is

$$J = \frac{\partial(H_o\ \ L_o)}{\partial(H_b\ \ L_b)} = \left\{\frac{L_b}{L_o} - \frac{\cos\alpha_b}{\sin\alpha_o}\ \frac{\sin 2\alpha_o}{\cos 2\alpha_b} \bullet \right.$$

$$\left. \bullet\ \frac{1}{6\,A^2}\left(\frac{H_o}{L_o}\right)^{\frac{1}{2}}\right\}\ \frac{3A}{4}\left(\frac{H_o}{L_o}\right)^{-\frac{1}{4}}\ \cos^{3/8}\alpha_o \tag{18}$$

where $A = 0\ 76 S^{1/7}$

Figure 9 shows an assumed deep water probability distribution $p\,(H_o\ \ L_o)$ corresponding to a joint Rayleigh distribution with zero correlation between wave height and wave period (Bretschneider 1959) The corresponding breaking wave characteristics $p\,(H_b\ \ L_b) = p\,(H_o\ \ L_o)\ |J|^{-1}$ are shown for $\alpha_o = 0$ and $\alpha_o = 30^o$ as Figures 10 and 11 for the breaking condition of equation 3 Figure 12 presents $p\,(H_b\ \ L_b)$ for the depth limited breaking criterion

It is seen from Figures 10 and 11 that the effect of the deep water angle of approach on breaking wave heights and lengths is relatively minor A comparison of Figure 10 with 12 shows that the breaking wave steepness is very sensitive to the choice of breaking criteria

Figure 13 compares the probability density of wave height determined from the one-dimensional computations and the marginal distribution for the two-dimensional distributions using two breaking criteria

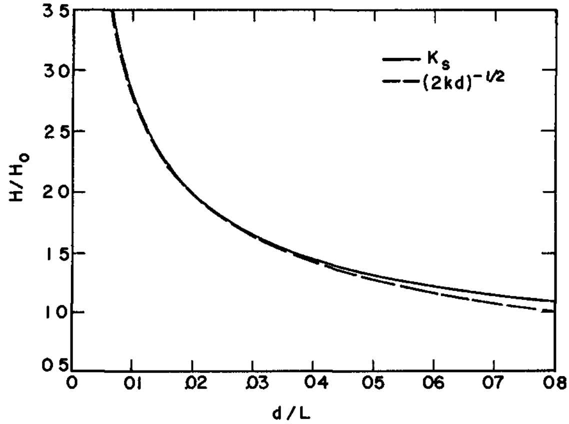

Figure 8 Comparison of Shoaling Coefficients Given by Equation 5 and Equation 17

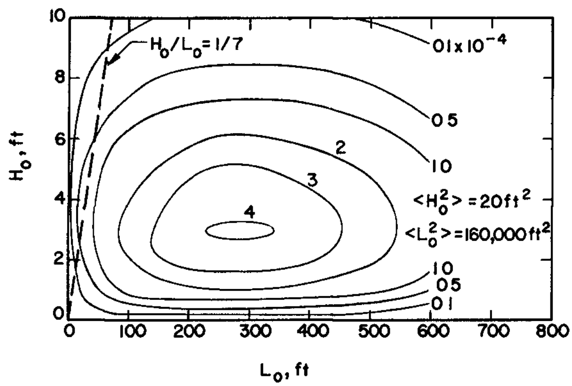

Figure 9 Two Dimension Probability Density of Deep Water Length and Wave Height for Joint Uncorrelated Raleigh Distribution

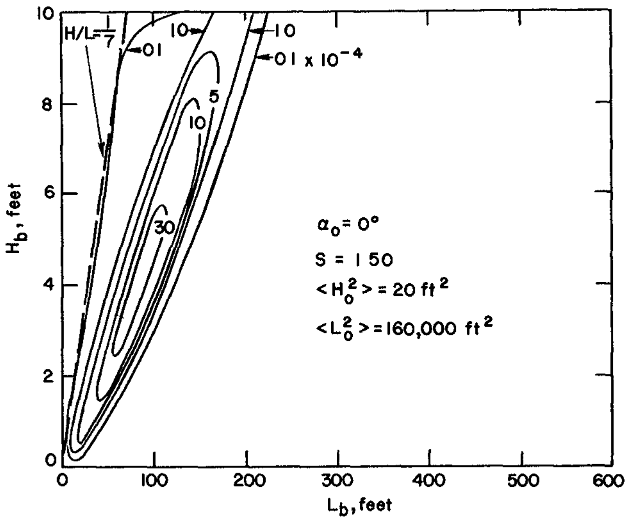

Figure 10 Two Dimensional Probability Density of Breaking Wave Characteristics for $\alpha_0 = 0$

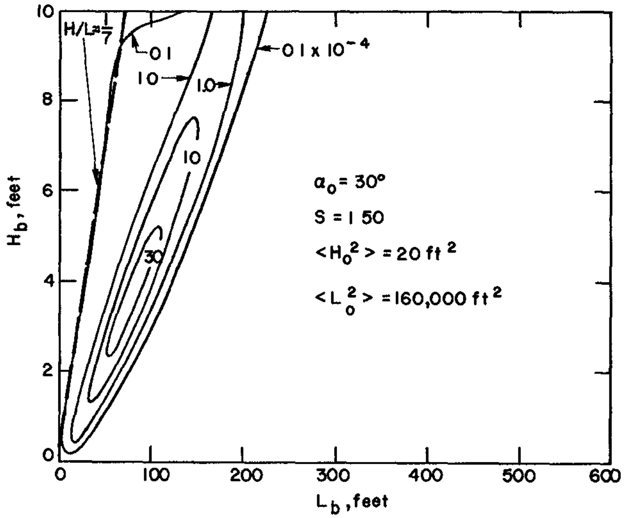

Figure 11 Two Dimensional Probability Density of Breaking Wave Characteristics for $\alpha_0 - 30^{\circ}$

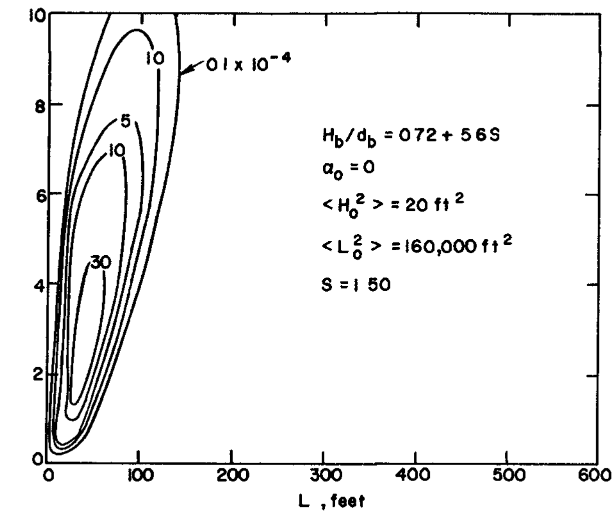

Figure 12 Two Dimensional Probability Density of Breaking Wave Characteristics for $\alpha_o = 0$ and Depth Limited Breaking Criterion

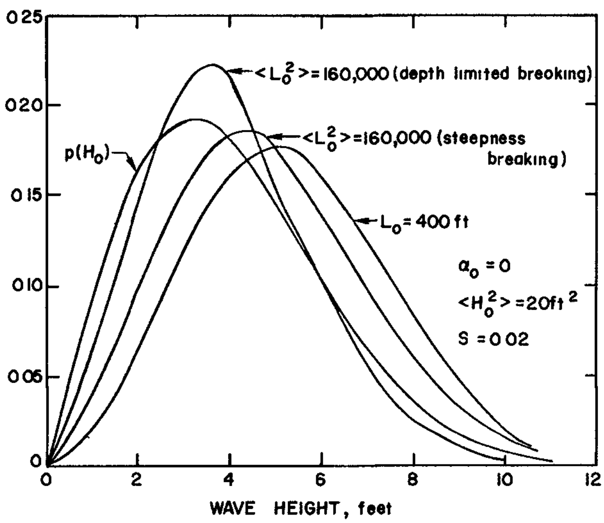

Figure 13 A Comparison of Marginal Distributions of Breaking Wave Heights from Two Dimensional Probability Densities with the Results for a One Dimensional Computation

LONGSHORE CURRENT AND ENERGY FLUX FOR RANDOM WAVES IN THE SURF ZONE

The mean value of a function of H L and α in a random sea can be derived from the equation for uniform periodic waves following a well known theorem of probability given a function of f (x y) in which the probability distributions of x and y are known, the mean value of the function f (x y) is given by

$$< f(x\ y) > = \int\int f(x\ y)\ p(x, y)\ dx\ dy$$

where p (x y) is the joint probability density of x and y Hence a longshore current equation derived for uniform periodic waves i e $V_L = V_L (H, L\ \alpha)$ can be applied to random waves if the random values of H L and α are weighted correctly

The mean value of V_L in a random sea is given by

$$<V_L> = \int\int\int V_L (H,\ L\ \ \alpha)\ p\ (H\ \ L\ \ \alpha)\ dH\ dL\ d\alpha$$

which can be considered as the result of superimposing all of the wave heights wave lengths and angles which are present and weighting the effect of each one by its probability of occurrence

A set of sample calculations for longshore and onshore energy flux has been worked out The mean longshore energy flux is given by

$$\frac{1}{\rho g} < F >_L$$

$$= \int\int\int \frac{1}{8} \sqrt{\frac{g}{2\pi}}\ \frac{H^2\ L\ n}{\sqrt{L_o}}\ \cos\alpha\ p\ (H\ \ L\ \alpha)\ d\,H\ d\,L\ d\,\alpha \tag{19}$$

non-breaking

$$+ \int\int\int \frac{8}{3\sqrt{3}} \sqrt{\frac{g}{2\pi}} \left[\frac{(H\ d)^3}{L_o}\right]^{\frac{1}{2}} \cos\alpha\ \ p\ (H\ \ L, \alpha)\ dH\ dL\ d\alpha$$

breaking

since

$$\Gamma = \sqrt{\frac{2\pi}{g}}\ \sqrt{L_o} \tag{20}$$

Also by definition $p\ (H\ \ L\ \ \alpha)\ d\,H\ d\,L\ d\,\alpha \equiv p\ (H_o\ ,\ L_o\ \ \alpha_o)\ d\,H\ d\,L_o\ d\,\alpha_o$

and the integrals can be approximated by summation

$$\frac{1}{\rho g} < F >_L$$

$$= \frac{1}{8} \sqrt{\frac{g}{2\pi}} \sum \left\{ \frac{H^2 L}{\sqrt{L_o}} \cos\alpha\ \ p(H_o, L_o, \alpha_o)\ \Delta H_o\ \Delta L_o\ \Delta\,\alpha_o \right\}$$

non-breaking (21)

$$+ \frac{8}{3} \sqrt{\frac{g}{6\pi}} \sum \left\{ \left[\frac{(H\ D)^3}{L_o}\right]^{\frac{1}{2}} \cos\alpha\ p\ (H_o\ \ L_o,\ \alpha_o)\ \Delta H_o\ \Delta L_o\ \Delta\alpha_o \right\}$$

breaking

The mean onshore energy flux is defined with sin α replacing cos α in equation 21 An example of the computations using equation 21 is given as Figure 14 Figure 14 illustrates the variation of onshore energy flux in the surf zone The variation of longshore energy flux is similar but absolute values of flux are smaller In this figure the deep water wave length and angle of approach were taken as constant and the deep water waves were characterized by a probability distribution of wave heights

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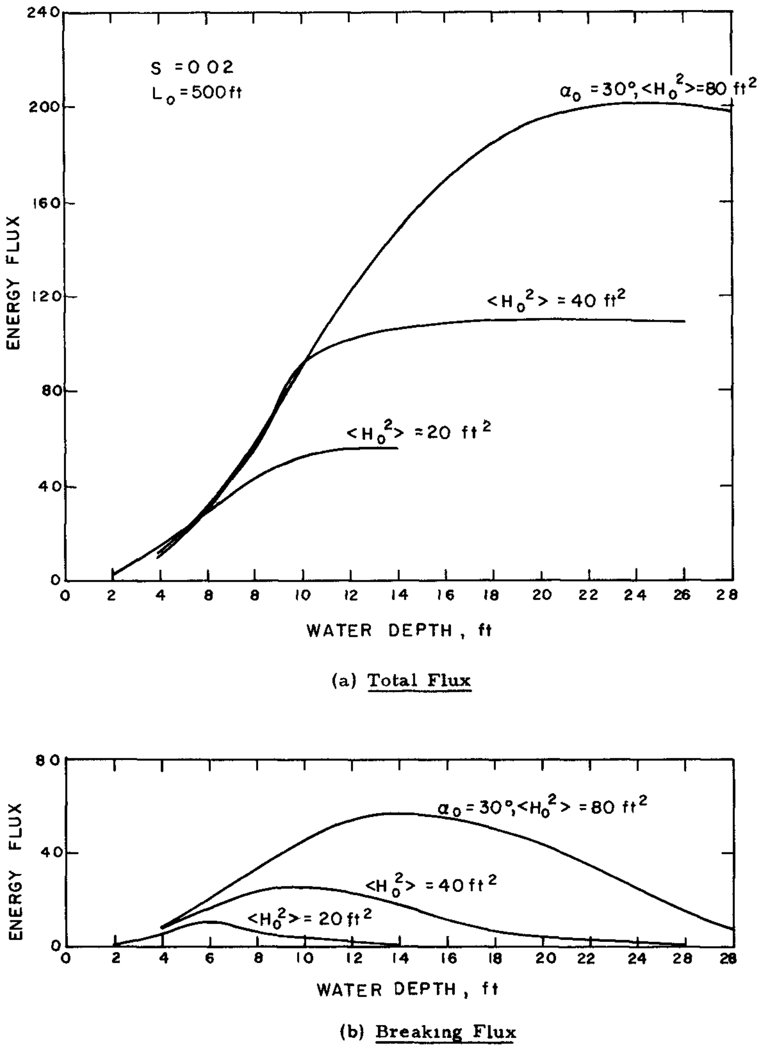

(a) Total Flux

(b) Breaking Flux

Figure 14 Onshore Energy Flux as a Function of Water Depth for Various Mean Square Wave Heights

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