CHAPTER 23

BREAKING WAVE SETUP AND DECAY ON GENTLE SLOPES

by

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ABSTRACT

Waves of large amplitude on a gentle slope may form spilling breakers which propagate shoreward and are slowly transformed In addition, there occurs a modification of the mean water level termed wave setup An analytical description based upon consideration of momentum flux has been developed which predicts this wave setup and the decay history of breaking wave height The results have been compared with experiments and found quite satisfactory

The effect of wave setup on breaking wave transformation is particularly important near the shoreline, where setup dominates the vanishing mean depth

INTRODUCTION

The transformation of a breaking wave over a slope is a problem of obvious concern in the design and planning of coastal facilities A number of investigators have considered this topic, including Freeman and LeMéhauté (1964), Horikawa and Kuo (1966), LeMéhauté (1962), Divoky, LeMéhauté, and Lin (1970), Nakamura, Shiraishi, and Sasaki (1966), and Street and Camfield (1966) The present contribution considers an essential aspect neglected in previous studies, the phenomenon of "wave set-up "

Set-up, while negligible seaward of the breaking point, becomes dominant with respect to still-water depth as the shore is approached It is apparent, then that any analysis of breaking wave height transformation should account for the set-up Experimental investigations of set-up have been made by Saville (1961) and by Bowen, Inman, and Simmons (1968) with the result that the maximum elevation may be a significant fraction (~ 50%) of the breaking wave height Additionally, measurements obtained during the Mono Lake explosion-wave tests indicated a set-up value equalling the maximum height of the superposed dispersive wave train (Van Dorn, et al, 1968) Hwang (1970) also investigated dispersive wave trains and found a fluctuating set-up of roughly half the peak wave height

Wave set-up has been investigated extensively in a series of papers by Longuet-Higgins and Stewart (1960, 61, 62, 64) from an analytical approach The difficulty in applying their results to the

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surf zone, arises from the problem of finding an adequate description of the waves after breaking Bowen, Inman, and Simmons (1968) assumed that the wave height remains a constant fraction of mean water depth after breaking and found, from a momentum balance, a linear set-up variation

In this paper, a more detailed description of the wave transformation has been used so that height-decay and set-up are calculated simultaneously The approach roughly follows Divoky, LeMéhauté, and Lin (1970) with the wave decay computed from an energy-dissipation model The essential difference is that the set-up is included so that more realistic behavior is found near the shoreline, in particular, a significant surviving wave height is found at the still-water shoreline

MODEL

Two governing equations are adopted in the present model Firstly, a balance of forces across a fluid element (see, for example, Longuet-Higgins and Stewart, 1964), as shown in Figure 1, gives the momentum equation

$$\frac{dM}{dx} + \rho g(h+\xi)\frac{d\xi}{dx} = 0 \tag{1}$$

where M is the momentum flux, h is the still water depth, and ξ is the wave set-up The energy dissipation rate is assumed to be a fixed fraction, B, of that of a bore of the same height Then the standard formula (see, for example, Lamb, 1945) may be adopted--adjusted, of course, to conform to a moving jump, the result is

$$\frac{dE}{dx} = B\left(\frac{dE}{dx}\right)_{BORE} = B\frac{\rho g}{4}\ \frac{H^3 D}{y_t(y_t + H)} \tag{2}$$

In this expression the volume flux Q has been taken to be CD where C is the wave celerity and D is $(h+\xi)$, y_t is the depth below the trough (corresponding to the depth ahead of the jump) and (y_t+H) corresponds to the depth behind the jump The assumption which has been made here is that B, which must account for all forms of energy dissipation, is constant within the breaking region

To implement this simplified model, a suitable wave description must be chosen in order to calculate E and M In this, we have followed the observation of LeMéhauté, Divoky, and Lin (1968), that the cnoidal wave theory of Keulegan and Patterson (1940) appears to give the best description of periodic waves in moderately shallow water and that this description may be adopted for gently spilling breakers with moderate success, of course, any non-breaking wave theory will be inadequate to describe violently plunging or surging conditions

The following equations of cnoidal wave theory have been used

$$\eta = y_s - D = y_t - D + H\,\mathrm{cn}^2\left[2K(k)\frac{x}{L}\right] \tag{3}$$

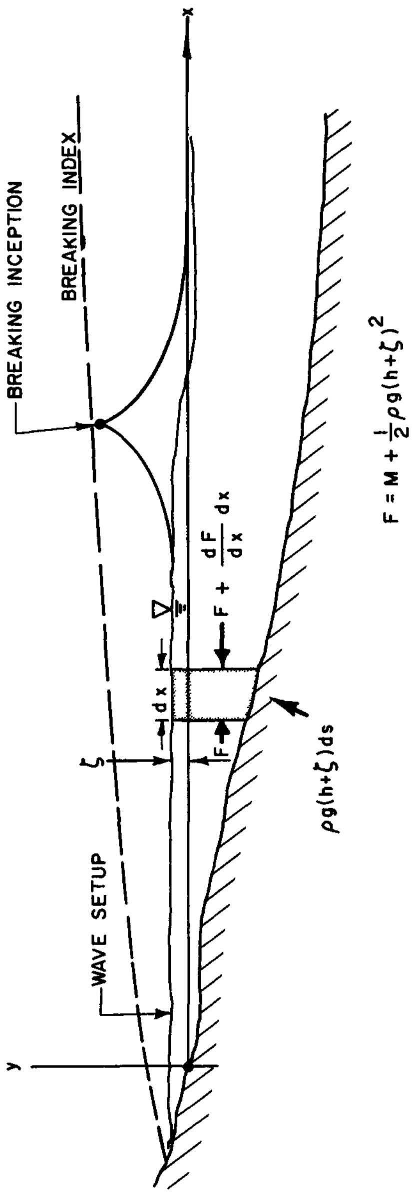


Figure 1 Coordinate System and the Balance of Forces on a Fluid Cross-section

$$L = \sqrt{\frac{16\,D^3}{3H}}\; kK(k) \tag{4}$$

$$T\sqrt{\frac{g}{D}} = \sqrt{\frac{16\,D}{3H}}\left\{\frac{kK(k)}{1+\frac{H}{k^2 D}\left[\frac{1}{2}-\frac{E(k)}{K(k)}\right]}\right\} \tag{5}$$

$$y_t = D - H + \frac{16D^3}{3L^2H}\left\{K(k)\,[K(k)-E(k)]\right\} \tag{6}$$

For given values of H, D, and T, Equation (5) is used in an iteration procedure to find the elliptic parameter k^2, which then allows calculation of all other quantities In particular, we have evaluated the energy as

$$E = 2(\text{POTENTIAL ENERGY}) = \rho g \int_0^L \eta^2\, dx$$

$$= \rho g L \left\{-(y_t - D)^2 + \frac{H^2}{3k^2}\left[1 - k^2 + (2-4k^2)\left(\frac{y_t - D}{H}\right)\right]\right\} \tag{7}$$

Calculation of the momentum flux from defining integrals poses a time-consuming and costly task even on a computer, since it involves repeated evaluation of the cnoidal wave properties We have made the greatly simplifying approximation that

$$M \approx \frac{3}{2}\,\frac{E}{L} \tag{8}$$

a result essentially from Airy theory given by Longuet-Higgins and Stewart (1964) Following them, we consider this to be a not unreasonable assumption within the surf-zone, with the possible improvement that E and L are computed here from a more adequate wave theory

The solution of Equations (1) and (2) begins with specification of a set of initial values for height, depth, set-up, and period Equations (3) - (8) then give the wave properties at that point For an advance by an increment dx toward shore, Equation (2) supplies the corresponding energy loss dE An iteration procedure involving Equation (1) then gives new values of wave height and total mean depth consistent with the available energy

A word concerning calculation of the elliptic parameter is in order The cnoidal wave theory is of greatest interest when k^2 is extremely near unity A direct expansion of E and K in terms of k^2 is then limited by problems of round-off in k^2 This problem has been obviated by defining

$$P = -\log_{10}(1-k^2)$$

E and K are written as power series in P rather than k^2, and all calculations involved in Eqs (3) - (8) similarly suppress k^2 P is an easily manageable number and is roughly the length of the string of nines in the decimal form of k^2 In the present calculations P has varied from less than unity to above fifty

RESULTS

A number of observations on wave transformation are summarized in Figure 2 The data of Horikawa and Kuo (1966) for a slope of 1/65 is shown in Figure 3 with the solid lines representing results of the present model with B taken arbitrarily as 0 8 The general trend appears quite good, and a somewhat better fit might have been obtained with other values of B The comparison indicates that the data scatter is accounted for by the variation in deep water steepness, so that the transformation is not solely a function of bottom slope

The dependence upon slope is indicated in Figure 4 Here, only slope is varied and the theoretical height transformation is shown It is noted that the present model is probably better for gentle slopes than steep slopes, since all calculations of wave properties are based upon the assumption that the water depth is constant and equal to the local value

Measurements of set-up are relatively few A comparison of the present model with observations of Saville (1961) is shown in Figures 5 and 6

The curves labelled "Calculated" represent the contributions to set-up in the breaking and non-breaking zones The latter has been computed from (Longuet-Higgins and Stewart, 1964)

$$\xi = -\frac{1}{8}\frac{H^2 k}{\sinh 2kD}$$

where k is the wave number The contribution due to breaking has been calculated by using observed values at the breaking point as initial values with ξ taken as zero (the portion due to breaking) The figures show that the general trends and magnitudes are correct and that the observed set-up "patches together" the portions contributed before and after the breaking point

In all of the calculations shown so far B has been taken arbitrarily as 0 8, representing an energy dissipation rate 80% of that for a hydraulic jump of equal height The dependence of results on this parameter is indicated in Figure 7, showing set-up for the conditions of Figure 6 It is seen that B = 0 6 would have shown better agreement with observation, but that the differences are relatively small

DISCUSSION

The model presented here appears, from limited comparison with data, to be capable of adequate prediction of set-up and height decay in the breaking zone Considerable improvement is possible, however

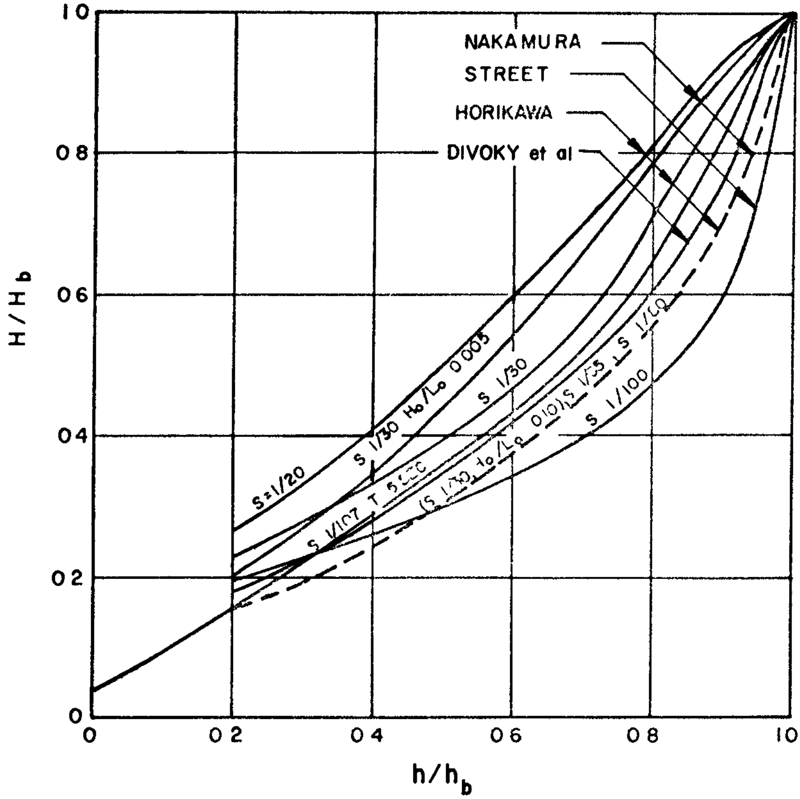


Figure 2 A Summary of Wave Transformation Data Shoreward of the Breaking Point

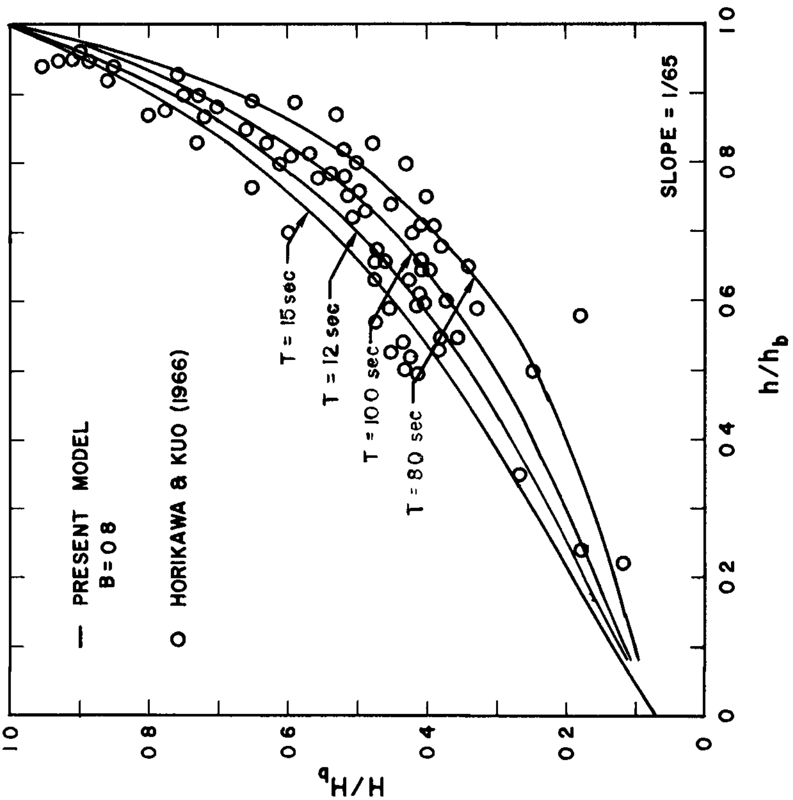


Figure 3 A Comparison of the Present Model with Data of Horikawa and Kuo (1966) for Wave Height Decay in the Breaking Zone

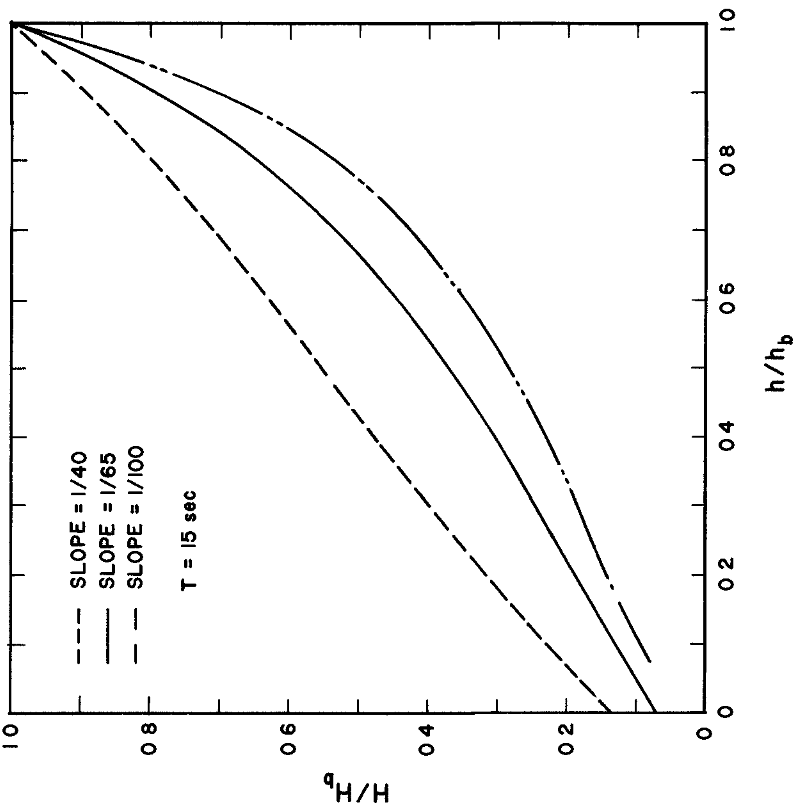


Figure 4 Results of the Present Model Showing Dependence of Height Decay in the Breaking Zone on Beach Slope

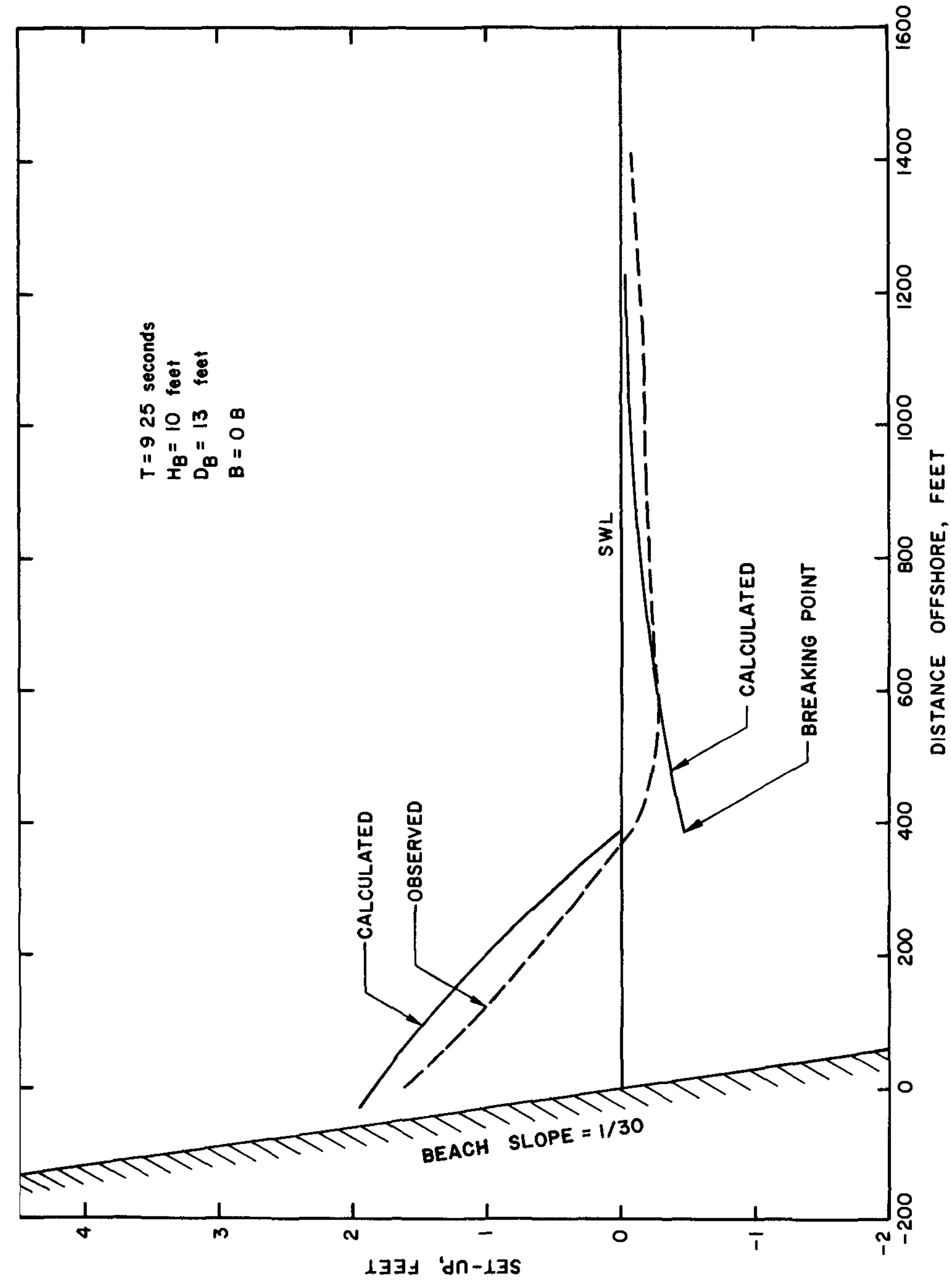
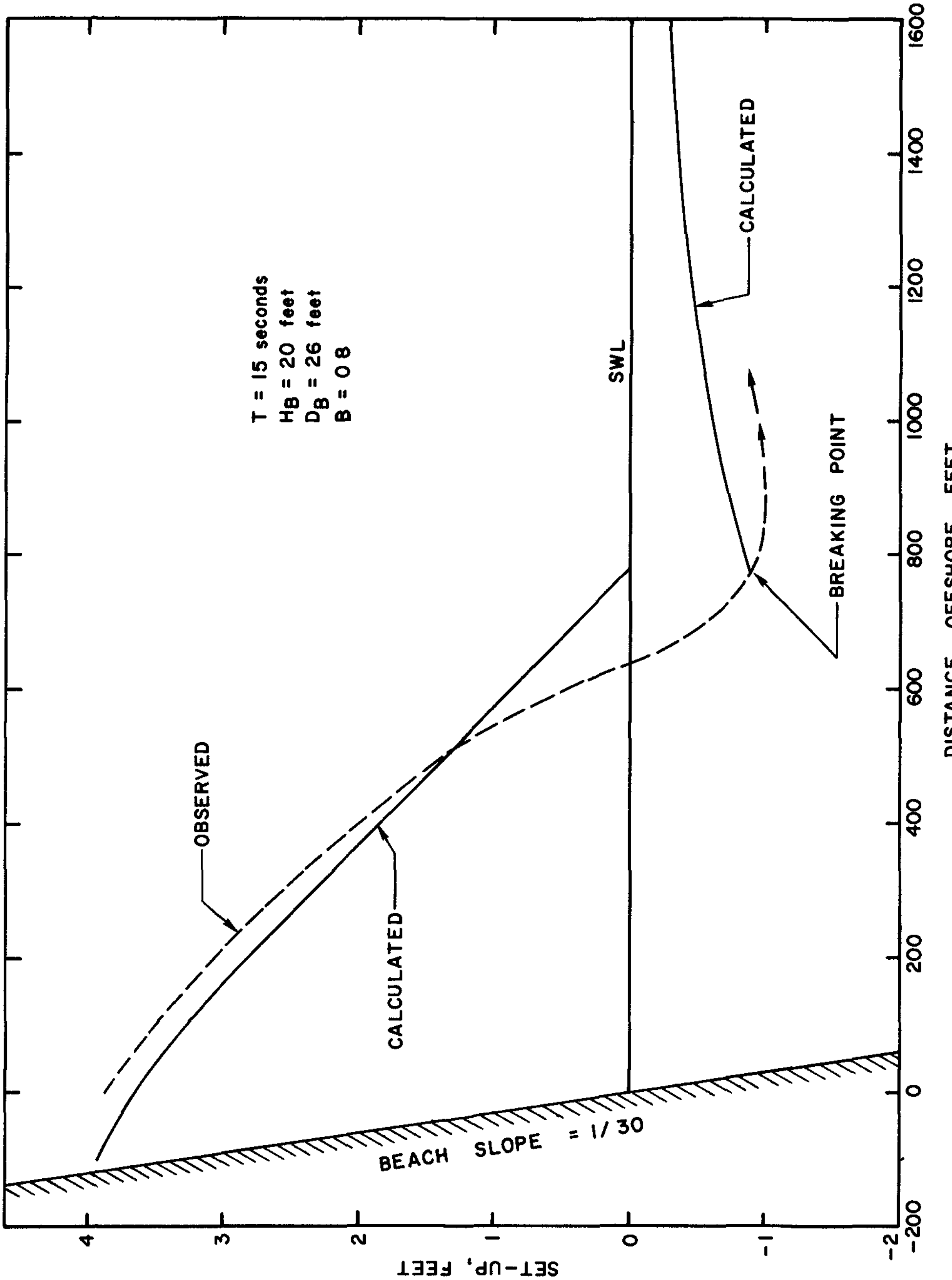


Figure 5 A Comparison of the Present Model of Breaking Wave Set-up with Data of Saville (1961)


T = 15 seconds
HB = 20 feet
DB = 26 feet
B = 0 8
SWL
CALCULATED
OBSERVED
CALCULATED
BREAKING POINT
BEACH SLOPE = 1/30
DISTANCE OFFSHORE, FEET
SET-UP, FEET
-200
0
200
400
600
800
1000
1200
1400
1600
4
3
2
1
0
-1
-2

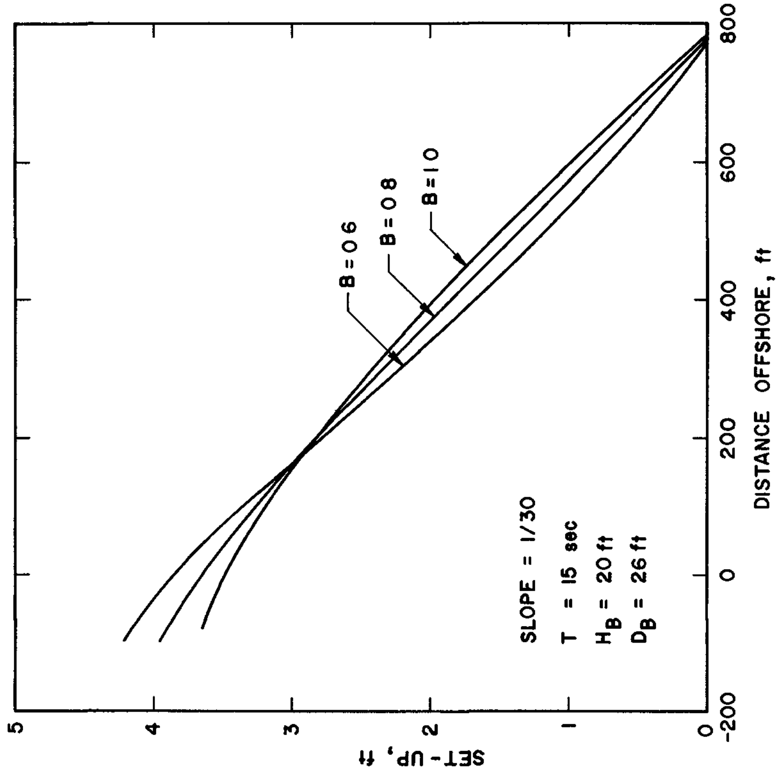


Figure 7 The Dependence of Breaking Wave Set-up (for the conditions of Figure 6) on the Breaking Parameter, B

In particular, the approximation to the momentum flux given in Equation (8) is especially primitive as noted, M can be computed directly from defining integrals, although this is difficult and inherently approximate for breaking waves if a non-breaking theory is adopted

The magnitude of the set-up at the shoreline is found, in accordance with the data of Saville (1961), to be quite large It is pointed out that the Saville data, given in field magnitudes, was actually obtained in very small scale tests Similarly, the data of Bowen, Inman, and Simmons (1968) was also taken at a small scale with periods on the order of one second and maximum set-up heights of a very few centimeters For this reason, comparison with available data, including that of Saville, is somewhat problematic owing to the possibility of scale effects

An interesting phenomenon observed by Bowen, Inman, and Simmons was a strong tendency for the set-up profile to become tangent to the beach slope, their analysis, which assumed a breaking wave height equal to a constant fraction of the total water depth, also showed this feature The present model was unable to duplicate this, always showing a set-up profile convex-upward intersecting the beach, the data of Saville appears generally similar

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