CHAPTER 22

PERIODIC WAVES SHOALING IN WATERS OVER STEEPLY SLOPING BOTTOMS

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Abstract

The bottom bed slope plays a vital role in the shoaling and breaking of periodic waves. This study aims at the understanding of the effect of steep slopes (range steeper than 1 10) on the breaking point and the breaker trajectory and the point of impact. This range of slope is often met with in the near-shore structures. Analytical investigations and model studies are outlined. The influence of slope on each of the significant breaker parameters is discussed.

1. INTRODUCTION

A train of periodic waves propogating on a sloping beach presents one of the most interesting and perhaps one of the most exciting phenomena in nature. Similar phenomena occur on the slopes of near-shore structures like sea-walls, breakers, dykes etc. A knowledge of the point of breaking and position of occurrence of maximum impact pressures due to breakers will be useful aids to the designer. It has been observed that beyond the point of breaking the particles move in a confused manner. Wave theories cannot be applied in this zone, at best till the breaking point only.

2. PERIODIC WAVES PROPOGATING IN SHOALING WATERS

It is known from AIRY(1)'s theory of small amplitude waves, that the energy in a wave train is transported in the direction of wave propogation with the group velocity C_G. Considering a train of progressive waves advancing in water of variable depth, with change in depth being uniform and small RAYLEIGH(2) assumed that the mean rate of energy transport past all vertical sections is constant. Further it was assumed that since the change in depth is very gradual, the characteristics of the wave at any section are given by AIRY's equations. This leads to the relation

$$\frac{H}{H_o} = \sqrt{\frac{2\cosh^2 kh}{2kh + \sinh 2kh}} \qquad (1)$$

and
$$\frac{H/L}{H_o/L_o} = \sqrt{\frac{2\cosh^2 kh}{2kh + \sinh 2kh}} \cdot \mathrm{Cotanh}\, kh \qquad (2)$$

where
H_o = Height of the incident wave (deep water)
H = Height of wave at any given location
L_o = Length of incident wave(deep water)
L = Length of wave at any given location
h = Local depth of water below SWL
k = Wave number = $2\pi/L$

This solution, however, does not predict breaking, as it is based on the linear theory of AIRY.

STOKER(3) analysed the breaking problem based on the classical non-linear wave theory and the method of characteristics. The solution is however not explicit and calls for individual treatment. Only solutions for a few special angles of bottom slope were obtained. The solutions of STOKER do not predict the breaking point correctly (LE MEHAUTE(4)) and also predicts a bore even when the wave travels on a horizontal bottom. Moreover the results of STOKER always predict a spilling breaker observed only on mild slopes. On steep slopes plunging breakers are formed invariably.

3. SOLUTION BASED ON CLASSICAL NON-LINEAR SHALLOW WATER WAVE THEORY

The classical non-linear shallow water wave theory equations are

$$\frac{\partial}{\partial x} u(\eta + h) = -\frac{\partial \eta}{\partial t} \qquad (3)$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = -g\frac{\partial \eta}{\partial x} \qquad (4)$$

where
u = particle velocity in the horizontal direction (ie. direction of wave advance)
x = horizontal distance from the origin in the in the direction of wave advance.
η = the vertical elevation of any particle on the free surface above SWL
h = depth of water measured below SWL
g = acceleration due to gravity
t = the time reckoned from the moment the wave enters the sloping bottom.

The origin is taken at the intersection of SWL with the vertical at the foot of the slope (See Fig.1)

The equation can be expressed in terms of dimensionless quantities

$$x' = \frac{x}{l} \quad \text{where} \quad l = nh$$

$$\eta' = \frac{\eta}{h}$$

$$u' = \frac{u}{u_o} \quad \text{where} \quad u_o = \sqrt{gh}$$

$$t' = \frac{t}{t_o} \quad \text{where} \quad t_o = n\sqrt{\frac{h}{g}}$$

The equations (3) and (4) will reduce to

$$\frac{\partial}{\partial x'} u' (1 - x' + \eta') = - \frac{\partial \eta'}{\partial t'} \tag{5}$$

$$\frac{\partial u'}{\partial t'} + u' \frac{\partial u'}{\partial x'} = - \frac{\partial \eta'}{\partial x'} \tag{6}$$

Along the characteristic directions

$$\frac{dx'}{dt'} = u' + C' \tag{7}$$

$$u' + 2C' + t' = \text{Constant}$$

$$\frac{dx'}{dt'} = u' - C' \tag{8}$$

$$u' - 2C' + t' = \text{Constant}$$

It is shown by GREENSPAN(5) and KISHI(6) that the initial positive characteristic is given by

$$x' = t' - \frac{1}{4} t'^2 \tag{9}$$

in the x-t plane. By changing over to a moving coordinate system, the slope of the wave front is obtained as

$$\frac{\partial \eta'}{\partial x'_m} = (1 - \frac{1}{2} t') \frac{\partial u'}{\partial x'_m} \tag{10}$$

By using the kinematic stability criterion that u = C at the crest at the breaking point and using this relation in the integrated equation (10)

$$\eta' = (1 - \frac{1}{2} t') C' \tag{11}$$

But the property of the initial positive characteristic gives

$$C' = 1 - \frac{1}{2} t'$$

Hence $$\eta' = (1 - \frac{1}{2} t')^2 \tag{12}$$

Suitable parameters are to be chosen to define the kinematics of the breaker. The following are chosen in this analysis.

h_B = Depth at breaking point below SWL
y_B = Height of breaker crest above bed.

From the relations (9) and (12) and the geometry of the breaker, the following final relations will result.

$$\frac{h_B}{h} = \left(\frac{2m}{1+2m}\right)^{4/3} \qquad (13)$$

$$\frac{y_B}{h} = 2\left(\frac{2m}{1+2m}\right)^{4/3} \qquad (14)$$

and

$$\frac{y_B}{h_B} = 2 \qquad (15)$$

where

$$m = \eta \cdot \pi \cdot \frac{H}{L} \qquad (16)$$

When friction and reflection effects are neglected, as was done in this analysis, one gets the relation (15) for the breaker geometry. This also confirms the thumb rule that breaking occurs at about the point where the wave height equals water depth. However the effects of reflection of wave energy are to be accounted for in the case of waves shoaling on steep slopes.

4. LABORATORY STUDIES IN FRANZIUS-INSTITUT AND CONCLUSIONS

As a result of a large number of model tests conducted by the author in the Franzius Institute in the Technical University of Hannover, West Germany the following conclusions were arrived at, by which the determination of the characteristics of breakers is made possible. Various bottom slopes (in the steep slope range) incident wave height and incident wave period were studied in a wave flume 45mX 0.5mX 0.9m with a paddle type wave generator, with provisions for changing the period as well as the height of the wave even during operation.

Instrumentation used consisted of a number of wave gauges of the parallel wire resistance type (gold-plated to prevent any chemical action) of different lengths to suit the local depth of water. The wave profiles were recorded by these wave gauges connectedtto a 3 channel electronic direct recording devices, which recorded the wave profile on a millimeter section paper run at the known speed. To ensure perfect linear relationship between the water level fluctuations and the records on paper special type of variable resistances were introduced in the circuit . This ensured linear relationship over a large range of wave height. The visual determination of the point of breaking was almost impossible. Therefore the wave statistics was collected along the flume and also above the sloping bottom.

A method was developed to determine the breaking point from the wave statistics thus collected, on the assumption that the trajectory of the breaker crest is a second degree curve. This follows from the fact that the particle velocity at the breaker crest at the breaking point is purely horizontal and equal to C, the phase velocity of the wave and in addition the particles are subjected to a gravitational acceleration g. That this is very much so is demonstrated by the shape of the plunging breakers in nature. A computer programme was written using the method of least squares and solving the resulting simultaneous equations by the diagonal matrix method (compact Gauss method). The computations were done in CDC-1604 digital computer. The breaking point was straightaway determined and then the breaker trajectory plotted. A back check was made to verify the degree of the curve (Fig.2). The agreement was quite good.

5. THE RETARDATION OF THE BREAKER

From dimensional analysis of the variables involved, it was found that the factors influencing the breaker characteristics are H/gT^2 and n (T is the period of the wave) which are the incident wave steepness and the bottom slope function respectively. The variation of Y_B/h_B with H/gT^2 was studied for all the slopes tested. The variation showed the same trend for all the slopes. A relation was established as follows

$$\frac{y_B}{h_B} = \log\left\{40.\ n^{0.5}.\ \left(\frac{H}{gT^2}\right)^{0.25}\right\} \qquad (17)$$

(Fig.3) Note that the curves show the variation of y_B/h_B with bed slope as well as the wave steepness (compare theory $y_B/h_B = 2$)

The depth at the breaking point will help us to fix the location of breaking inception (Fig.4). For incident waves of given depth and period, the breaking depth remains almost constant for slopes flatter than 1 5. It was observed that for slopes flatter than 1 5, all the breaking action of the wave takes place shoreward of the breaking point. For slopes steeper than 1 5, the values of h_B increases sharply due to the seaward retardation of the breaker.

6. THE INFLUENCE OF A SLOPE ON BREAKER

Fig.(5) shows the influence of the bottom slope on the breaking of a wave.

The reflection coefficient is negligibly small for a bed slope of 1 50, increases very slowly till a slope of 1 5 is reached and then increases sharply till the bed slope of 1 2 is reached and is then asymptotic till a factor of 1 0 is reached for vertical walls.

The breaking depth h_B is almost constant till a bed slope of 1 5 is reached, since the reflection is still very small of the order of 15%. The small decreasing tendency seen in the breaking depth is indeed negligible. For slopes steeper than 1 5 the breaking depth increases sharply showing that the breaker is retarded seawards. The breaker crest elevation y_B varies in a similar manner.

The variation of y_B/h_B is of interest. The theoretical value of 2 is never reached. The forces of reflection are almost negligible and frictional losses are small compared to the slopes 1 50 etc. For slopes flatter than 1 10 frictional losses increase. The effect of this is to reduce the potential energy growth and hence y_B/h_B decreases. For n=10 to n=5, the breaking action occurs shoreward, the waves still overcome the forces of reflection. A slight decrease is registered. For $n < 5$, h_B increases sharply due to retardation of the breaker. y_B also increases but at a slower rate. y_B/h_B decreases. The lowest value of 1.10 is reached for breaking clapotis.

7. THE POINT OF IMPACT

The point of impact of the breaker on the slope of the bottom boundary can be calculated once y_B and h_B are known. From the geometry of the slope (FUEHRBOETER)(7)

$$\Delta h = - h_B + y_B \left\{ 1 - \frac{1}{2n^2} \left(\sqrt{1 + 2n^2} - 1 \right)^2 \right\} \qquad (18)$$

The determination of the point of impact was checked on the model. With pressure cells connected to electronic direct recording devices (Honeywell ultraviolet light beam oscillograph), the impact pressure variation with respect to time is followed. Impact pressures were recorded at 6 points at 20cm intervals along the slope simultaneously..The maximum pressure corresponds to the direct impact of the breaker on the slope which helps to locate the point of impact. Comparison of the calculated values and measured values showed less than 8% error. The deviation of the curve $\Delta h/h_B$ for the slopes tested, from the curve computed from the linear theory applicable with reasonable accuracy for mild slopes is shown in Fig.(6).

8. ACKNOWLEDGEMENTS

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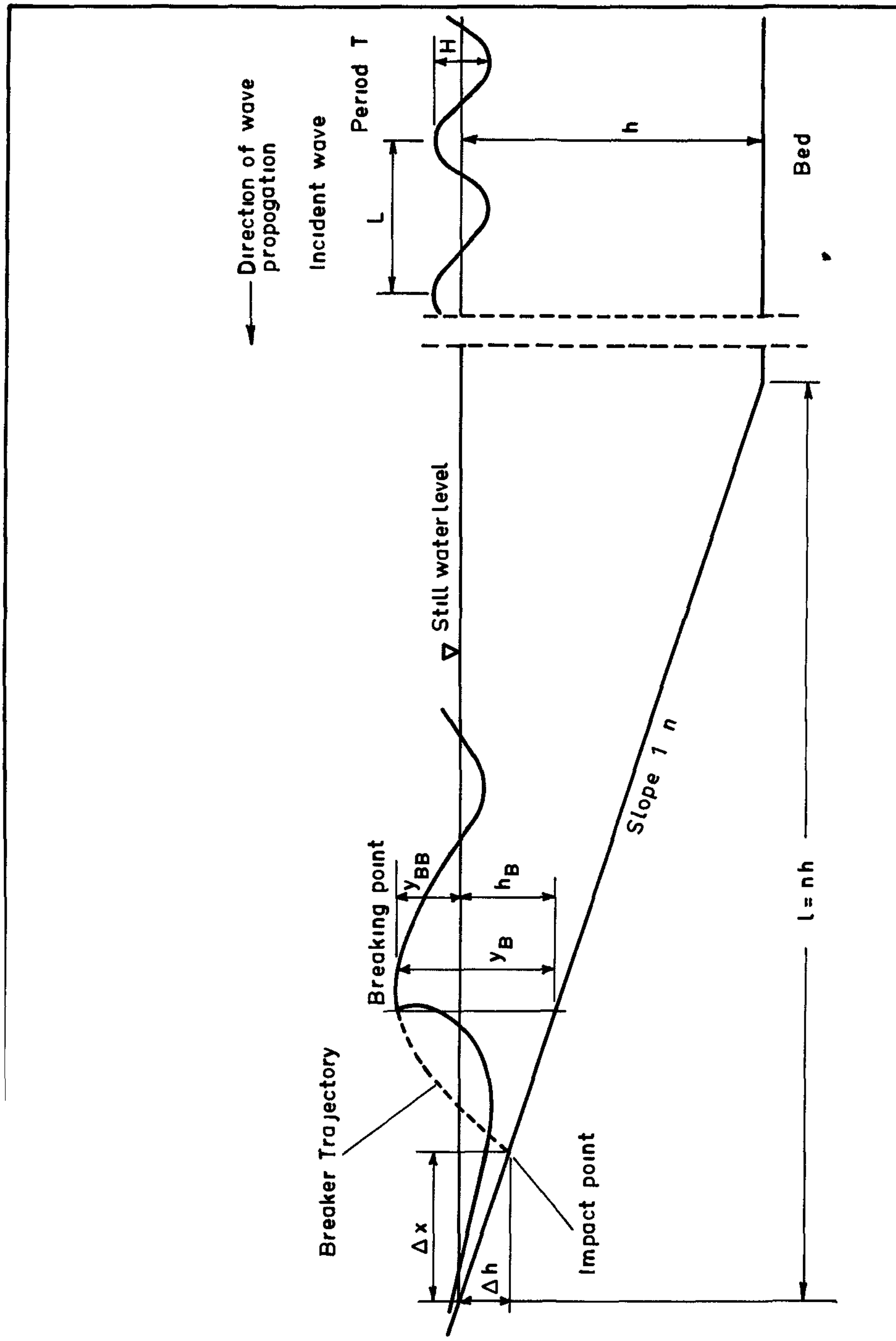

Fig.1.-Schamatic representation of plunging breaker

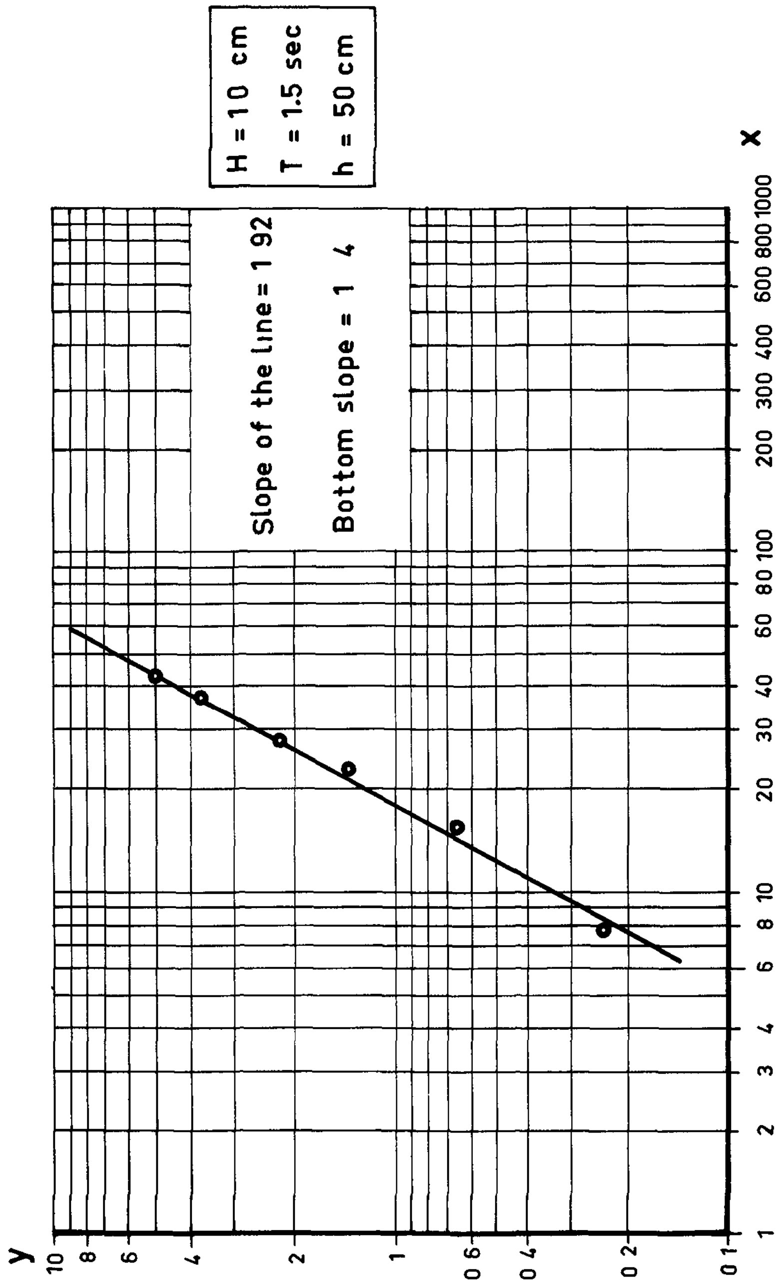

Fig. 2 - Coordinates of the breaker trajectory in double log plot

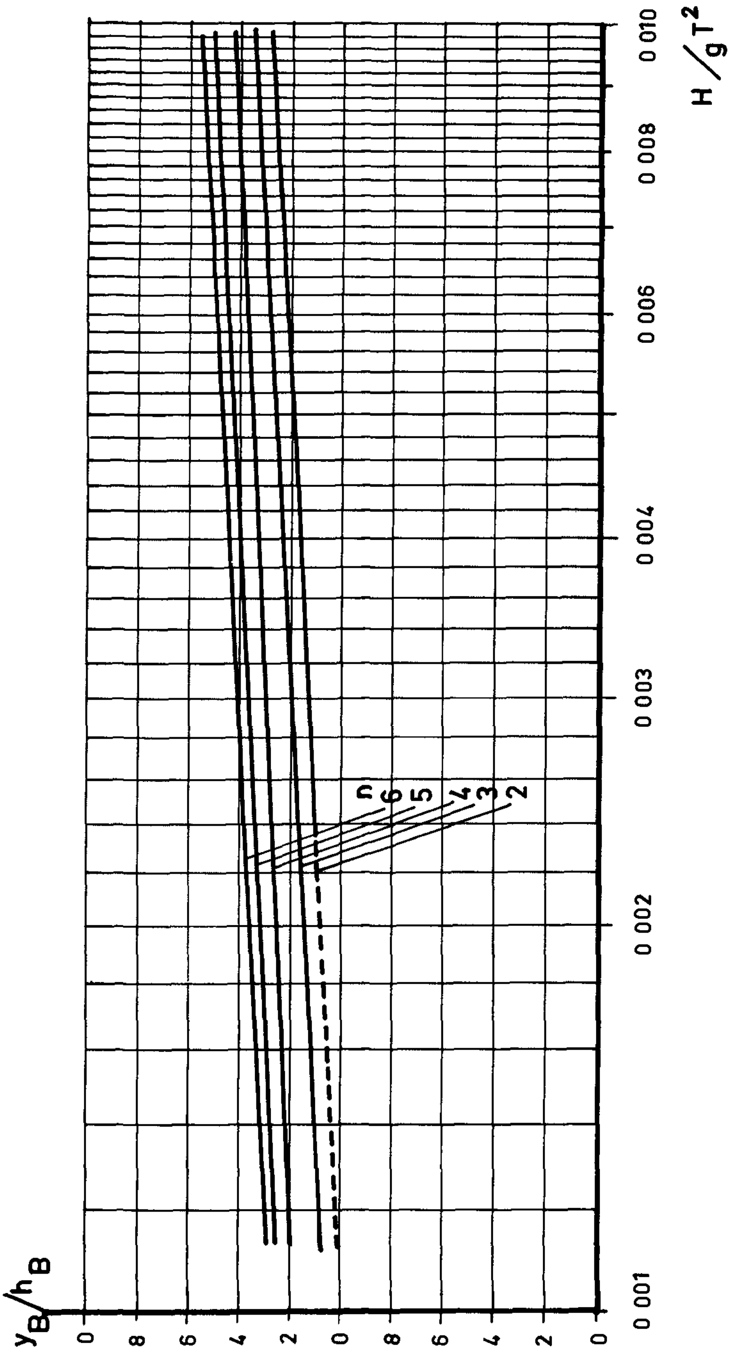

Fig.3 - Variation of y_B/h_B with H/gT^2 and the slope

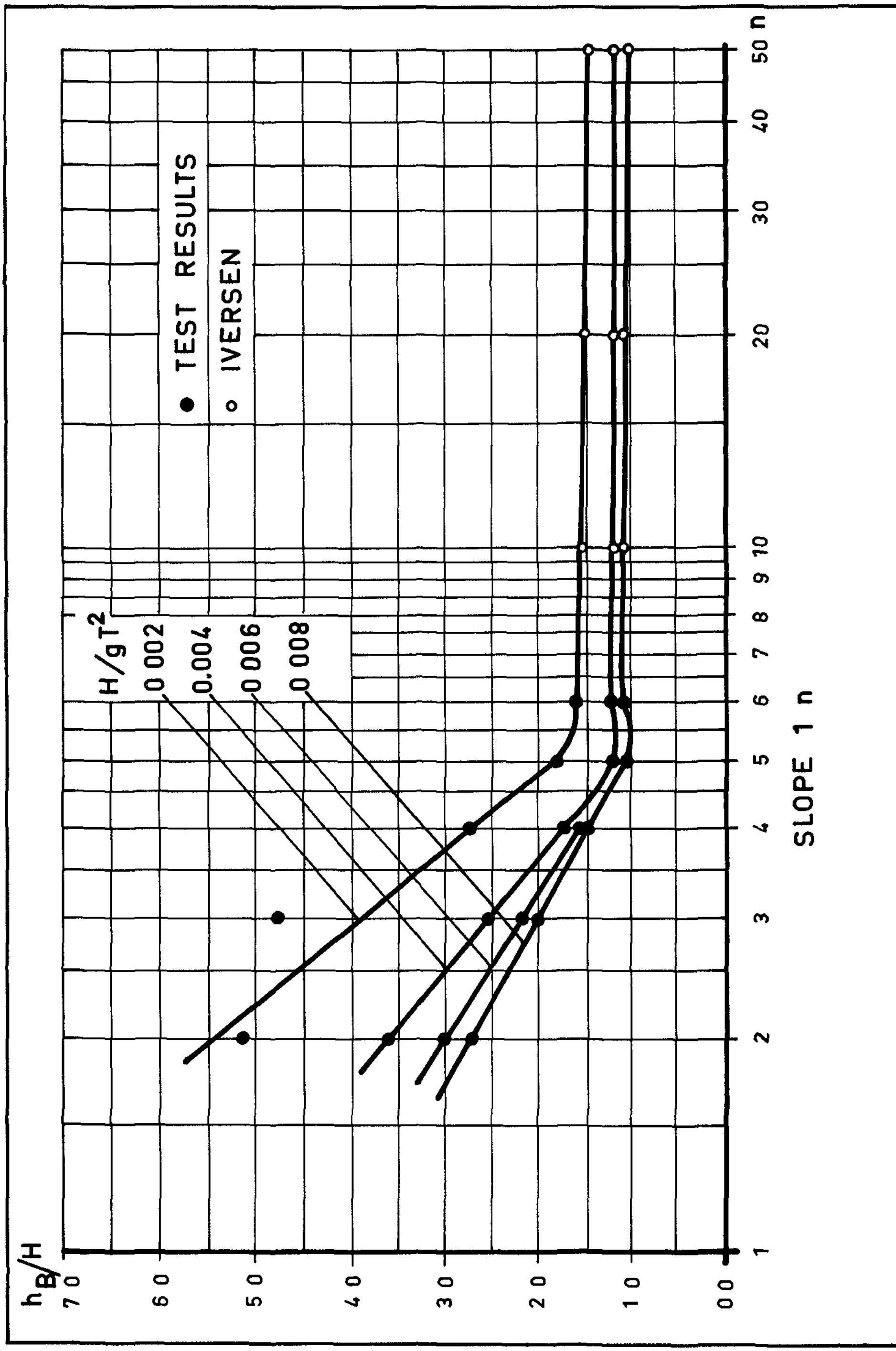

Fig. 4 - Variation of h_B/H with the slope

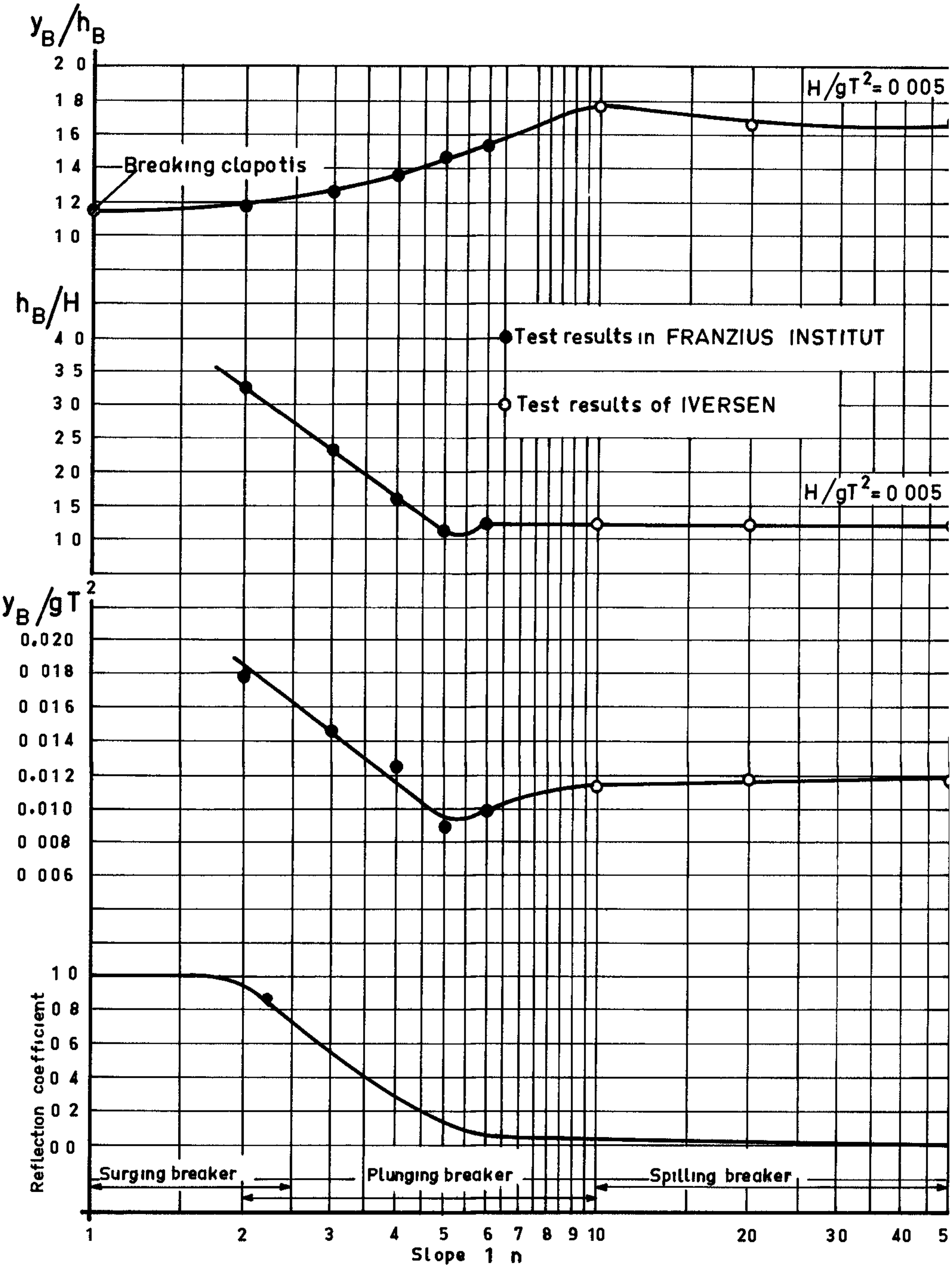

FIG. 5 - Influence of slope on breaking of waves

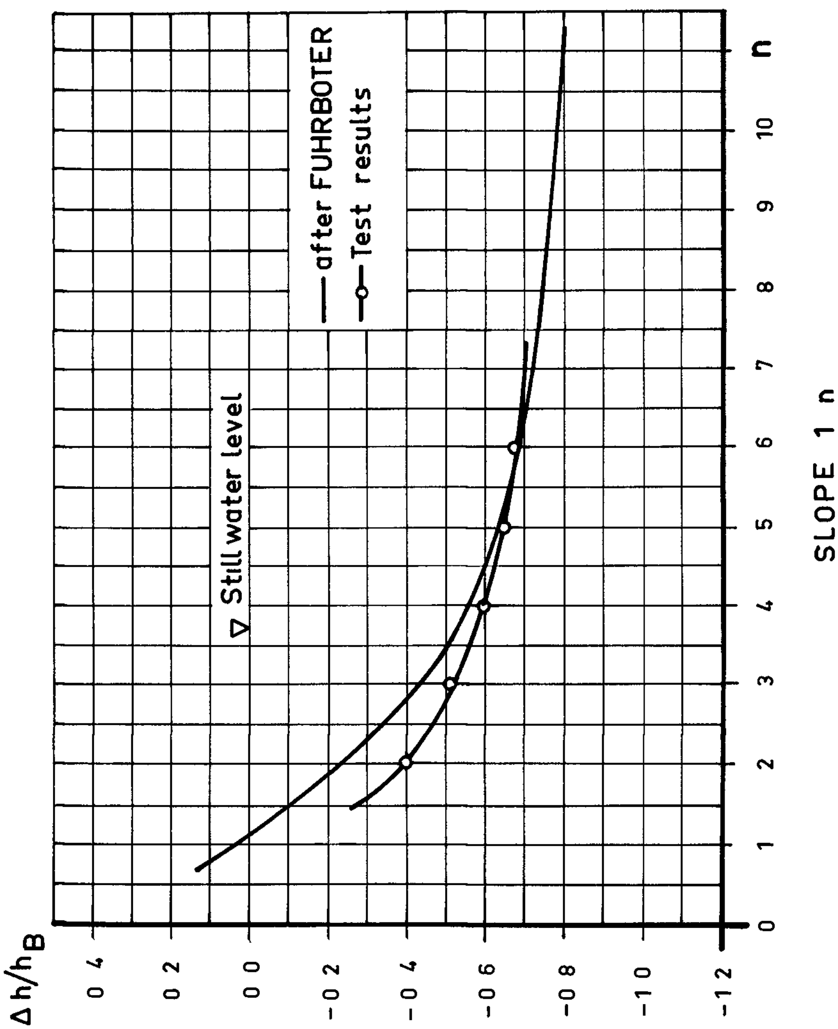

Fig. 6 - Variation of $\Delta h/h_B$ with slope

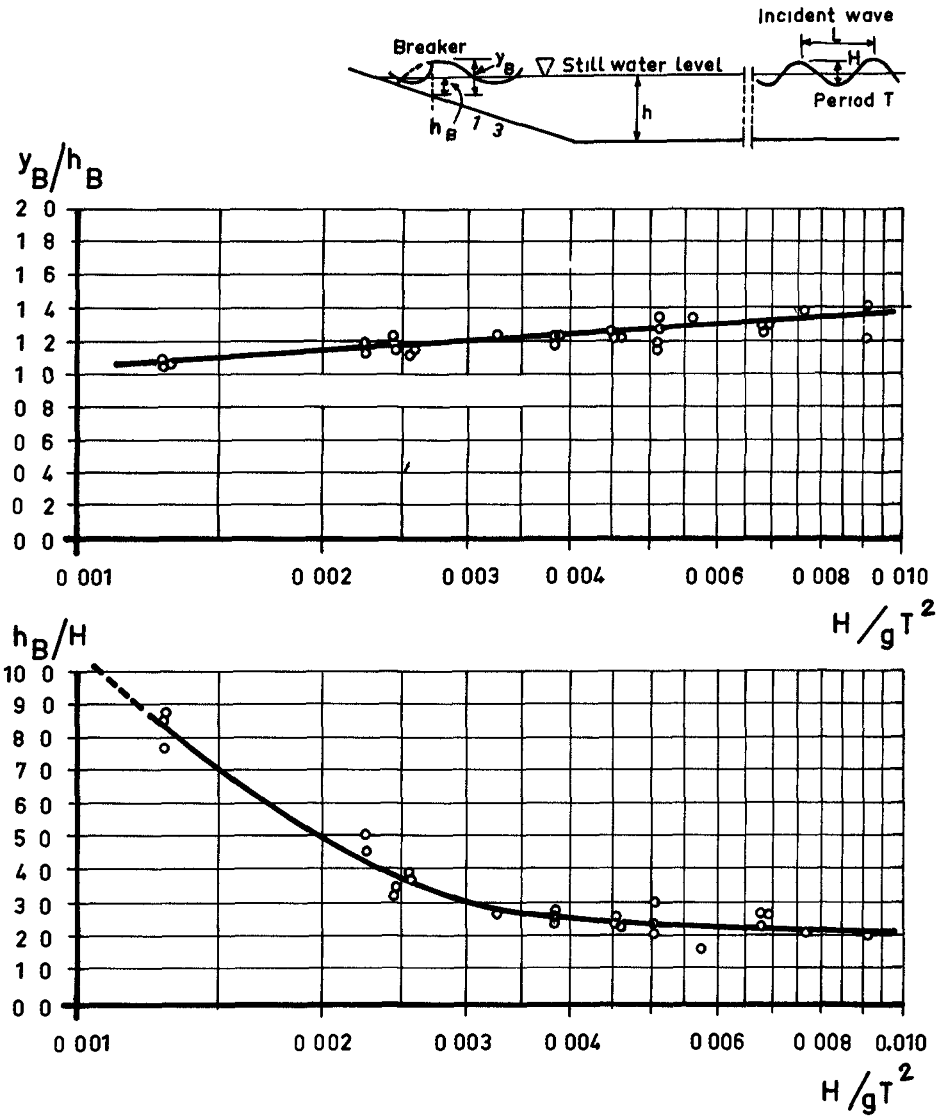

Fig.7 - Test results for the slope 1:3