CHAPTER 18

VARIATION OF LONGSHORE CURRENT ACROSS THE SURF ZONE

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ABSTRACT

The wave-induced longshore current variation across the surf zone is described for a simplified model The basic assumptions are that the conditions are steady, the bottom contours are straight and parallel but allow for an arbitrary bottom profile, the waves are adequately described by linear theory, and that spilling breakers exist across the surf zone Conservation equations of mass, momentum, and energy, separated into the steady and unsteady components, are used to describe second order-wave-induced phenomena of shoaling waves approaching at an angle to the beach An expression for the longshore current is developed, based on the alongshore component of excess momentum flux due to the presence of unsteady wave motion Wave set-down and set-up have been included in the formulation Emphasis in the analysis is placed on formulating usable predictive equations for engineering practice Comparison with experimental results from the laboratory and field show that if the assumed conditions are approximately fulfilled, the predicted results compare quite favorably

INTRODUCTION

A knowledge of the variation and extent across the surf zone of the longshore current is important in design considerations of structures placed in the littoral area This kind of information is particularly important for groins or similar structures designed to impede sand movement since the longshore current is a primary mechanism for sand transport On the other hand, it is often desirable to have natural bypassing about jetties constructed for navigational purposes at inlets and harbors The distribution of effluent, introduced onto beaches and into the littoral zone, is also influenced by the currents in the surf zone There is a very real need for a more complete understanding of the littoral zone so that further improvements and preservation of our beaches can be based on more rational and concrete approaches

The study of the area in and about the surf zone presents a difficult problem due to its very complex nature A proper treatment of the surf zone must consider a three-dimensional problem of unsteady fluid motion and is further complicated by moving interfaces at the upper and lower boundaries, that

is, at the water surface and sediment bottom The hydrodynamics of the littoral zone can be characterized by two idealized systems of either a longshore current or rip current system which can occur for seemingly similar conditions Thus, it is necessary to make definite and simplifying assumptions in order to make the problem tenable to a theoretical approach

This analysis considers the steady-state distribution of quantities on a line normal to the shoreline A schematic of the surf zone area is shown in Figure 1 The analysis is restricted to the case of an arbitrary bottom profile with straight and parallel contours in the y-direction (parallel to the beach) The x-direction is perpendicular to the beach

CONSERVATION EQUATIONS

A convenient starting point for this analysis is a statement of the general conservation equations of mass, momentum, and energy fluxes applicable to unsteady flow The analysis is not concerned with the internal flow structure of the fluid, hence, the derivation can be simplified by integrating the conservation equations over depth Conservation equations which have already been developed by Phillips [1] are used and are presented below

The conservation equations are applied to wave motion, but they are equally applicable to general turbulent motion The unsteady velocity field of the wave motion can be expressed in the same manner as in the treatment of turbulent motion as the sum of its mean and fluctuating parts

$$\vec{u} = (U_1(x,y,t) + u_1'(x,y,z,t),\ w(x,y,z,t)) \qquad 1 = 1,2 \qquad (1)$$

where (1,2) refer to the horizontal coordinates (x,y), respectively, and z is the vertical coordinate The tensor notation is used only for horizontal components of water particle motion The mean current is assumed uniform over depth for simplicity The pressure term can be stated similarly These expressions can be substituted into the mass, momentum, and energy equations, and the mean and fluctuating contribution identified

The conservation equations are averaged over depth and time (one can consider averaging over a few wave periods) For the case of waves superposed on a mean current, all the wave motion is identified with the fluctuating quantity which, when integrated over the total depth, can contain a mean contribution due to the waves The time averaging of the equations for a general development, being over a short interval compared to the total time, does not preclude long term unsteadiness in the mean motion

A Conservation of Mass

The general conservation of total mass per unit area can be expressed

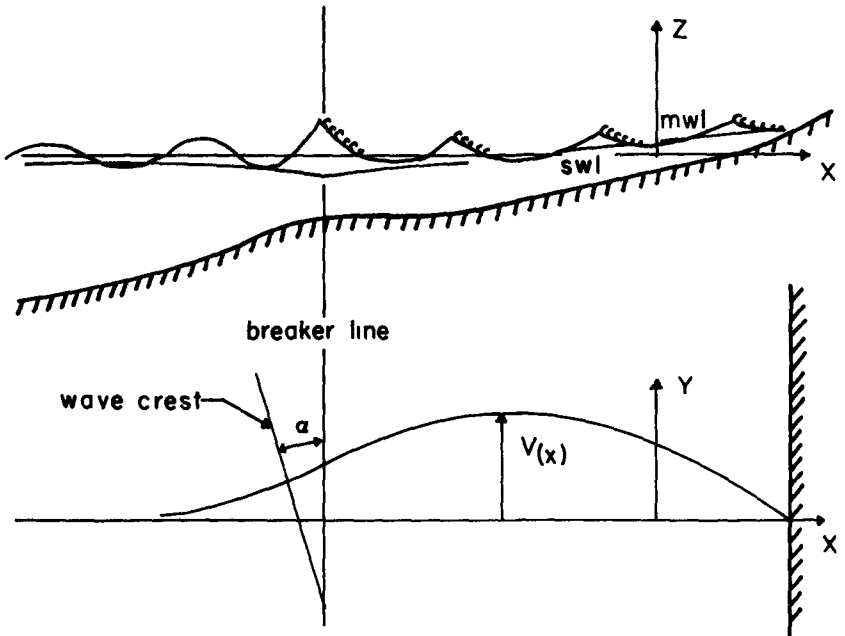

FIGURE 1 SCHEME OF THE SURF ZONE

$$\frac{\partial}{\partial t} \rho D + \frac{\partial}{\partial x_i} \tilde{M}_i = 0 \qquad i = 1,2 \qquad (2)$$

D is the total averaged depth of water which can include a mean elevation $\bar{\eta}$, above (or below) the still water depth h, so that

$$D(x,y,t) = (\bar{\eta} + h) \qquad (3)$$

The overbar shall be used to signify time averages The total mass flux $\tilde{M}_i$ can be partitioned into its mean and fluctuating components

$$\tilde{M}_i = \bar{M}_i + M_i \qquad (4)$$

B Conservation of Momentum

The equation defining the conservation of horizontal momentum is derived by integrating the momentum equation over depth and averaging in time The balance of total momentum per unit area can be expressed

$$\frac{\partial}{\partial t} \tilde{M}_i + \frac{\partial}{\partial x_j} (\tilde{U}_i \tilde{M}_j + S_{ij}) = T_i + R_i \qquad (5)$$

Here $\tilde{M}_i$ denotes the total horizontal momentum per unit area Hence, the first term on the left represents the rate of change of the total mean momentum per unit area which includes both the current momentum and wave momentum

$\tilde{U}_i$ is the total mean transport velocity The second term on the left of Equation expresses the momentum flux of a steady stream together with an excess momentum flux term S_{ij} arising from the superposed unsteady motion, where

$$S_{ij} = \overline{\int_{-h}^{\eta} (\rho u_i' u_j' + p\delta_{ij})\, dz} - \tfrac{1}{2} \rho g D^2 \delta_{ij} - \frac{M_i M_j}{\rho D} \qquad (6)$$

and δ_{ij} is the Kronecker delta The unsteady terms in the integral contain contribution from the mean motion, the last two terms represent the hydrostatic pressure and mean momentum flux contained in the integral term and is subtracted out so that the term represents only the excess momentum flux due to the unsteady motion The last term is generally of higher order, but very near and inside the surf zone the term is of second order and consistent with this analysis, however, its maximum contribution is only four percent and this term will be neglected The term T_i is given by

$$T_i = - \rho g(\bar{\eta} + h) \frac{\partial \bar{\eta}}{\partial x_i} \qquad (7)$$

and represents the net horizontal force per unit area due to the slope of the free water surface

R_i is the time mean averaged shear stress which must be included in any realistic treatment of the surf zone where dissipative effects occur The integrated form of the resistance term is given by

$$R_i = \overline{\int_{-h}^{\eta} \frac{\partial \tau_{ji}}{\partial x_j} dz} + \overline{\tau_{\eta i}} - \overline{\tau_{hi}} \qquad i,j = 1,2 \qquad (8)$$

where τ_{ji} includes the combined lateral shear stresses of waves and currents and the last terms are the surface and bottom shear stress respectively

In the development of the conservation equations, no restrictions are placed on the wave slopes or amplitudes Also, no restrictions are placed on the fluctuating motion so that the equations are equally applicable to wave or turbulent motion

DESCRIPTION OF THE WAVE FIELD

A Waves outside the Surf Zone

It is known that, due to the fluctuating water particle motion of the waves, there is a momentum flux component If the waves have a direction component parallel to shore, a longshore current can be generated due to changes in the longshore momentum flux component of the shoaling waves In waves, the momentum flux is the sum of the pressure and the product of two velocities It can be shown that the average momentum flux is nonlinear in wave height In order to specify the excess momentum flux of the waves, it therefore becomes necessary to consider nonlinear, or higher order, effects of the wave motion

The development presented here retains terms to the second order in amplitude (first order in energy and momentum) and neglect all higher order terms The wave solution is substituted directly into the conservation equations providing a means for describing the wave-induced mean motions In making this substitution and dropping all terms of orders higher than the second, only knowledge of the first order (linear) wave water particle velocities and surface elevation is necessary This is because, in expanding and then averaging over the period, the terms involving higher order quantities in velocity and surface elevation go to zero The pressure must be known to the second order in wave height, however, the average second order pressure component can be determined from the first order water particle velocities and surface elevation terms Thus, only the linear wave solution is required

B Waves inside the Surf Zone

Inside the surf zone, energy is dissipated due to the generation of turbulence in wave breaking, bottom friction,

percolation, and viscosity The waves in the surf zone constitute a non-conservative system in which the use of potential flow theory is no longer valid In fact, there is no analytical description available for the waves in the surf zone Hence, one is required to make rather gross assumptions and then to test these assumptions experimentally The linear wave theory will be retained as the input to the conservation equations, but with modification to the wave amplitude and speed The wave height inside the surf zone is controlled by the depth and is of the same order of magnitude Thus, even the second order theory in wave height is a rather poor assumption, but seems to agree surprisingly well with measurements of some phenomena

Spilling breakers lend themselves to a physical treatment since the potential energy and momentum flux of the waves inside the surf zone can be expressed approximately in analytical form If the beach slope is very gentle, the spilling breakers lose energy gradually, and the height of the breaking waves approximately follows the breaking index curve The height of the wave, H, is then a function of the total depth, D, as given by

$$H = \kappa D \tag{9}$$

where the breaking index, $\kappa = 0\ 78$, as predicted by the solitary wave theory

It is further assumed that the kinetic and potential energy are equally partitioned so that the total wave energy can be described in terms of the wave height which is a function of the depth

$$E = \frac{1}{8} \rho g H^2 = \frac{1}{8} \rho g \kappa^2 D^2 \tag{10}$$

This is a non-conservative statement of the energy distribution within the surf zone

The waves inside the surf zone are assumed to retain their simple harmonic character so that the wave profile and water particle velocity are described by

$$\eta = \frac{H}{2} \cos(k_1 x_1 - \sigma t) \tag{11}$$

$$\vec{u} = \frac{H}{2} \frac{g}{c} \frac{k_1}{k} \cos(k_1 x_i - \sigma t) \tag{12}$$

where k is the wave number, σ the wave frequency and c the wave speed The expression for the horizontal water particle velocity is based on the Airy wave theory and has been simplified for shallow water

In very shallow water, the waves are non-dispersive with the wave speed being only a function of the depth It has been found experimentally that a reasonable approximation to the wave speed in the surf zone is that predicted by the solitary wave theory

$$c = \sqrt{g(H + D)} = \sqrt{g(1 + \kappa) D} \tag{13}$$

Since the bottom contours are parallel, Snell's law can be used to account for the changes in wave direction due to refraction The wave angles are referred to the breaker wave angle which is the commonly measured angle in the study of the surf zone Hence,

$$\sin\alpha = \frac{c}{c_b} \sin\alpha_b \tag{14}$$

where the subscript "b" refers to the breaker line Refraction due to shear flow is neglected and can be shown to be of minor importance

The excess momentum flux tensor can be determined by substituting the wave expressions into Equation (6) In general terms of energy, group velocity, c_g, and wave speed, c, an expression applicable to both inside and outside the surf zone is given by

$$S_{ij} = \begin{vmatrix} E \frac{c_g}{c} \cos^2\alpha + \frac{E}{2} \left(\frac{2c_g}{c} - 1\right) & \frac{E}{2} \frac{c_g}{c} \sin 2\alpha \\ \frac{E}{2} \frac{c_g}{c} \sin 2\alpha & E \frac{c_g}{c} \sin^2\alpha + \frac{E}{2} \left(2 \frac{c_g}{c} - 1\right) \end{vmatrix} \tag{15}$$

The effects of turbulence and surface tension have not been included

LONGSHORE CURRENT FORMULATION

The wave field has been completely specified These results may now be substituted into the general conservation equations to describe wave-induced phenomena inside and outside the surf zone

Recalling the analysis is restricted to the case of an arbitrary bottom profile with straight and parallel contours in the y-direction The water depth is then a function of the x-direction only Since the distribution of mean properties of the wave field is a function of the depth, this eliminates any y-dependence An exception to this was found by Bowen [2] Using the fact that incident waves can excite transversal waves, commonly called edge waves, he showed that if these waves are standing waves, or only slowly progressive, gradients in the mean water surface can be developed in the longshore direction which in turn can result in

circulation cells Thus, a more exact formulation has to assume suitable spatial averaging in the longshore direction so as to preclude the effects of any transversal waves

It is assumed that wave reflection is negligible This assumption is justifiable outside the surf zone for gently sloping bottoms The present analysis is most valid for spilling breakers which implies a gently sloping bottom The wave reflection is least for this type of breaker condition and is assumed negligible inside the surf zone as well

Shear stresses at the surface due to the wind are neglected

The problem can be conveniently discussed by considering separately the areas outside and inside the surf zone A determination of the distribution of mass transport and energy of the waves is first necessary in order to solve for the wave-induced currents

A Mass Transport Velocity

Due to the absence of any y-dependence, the mass conservation Equation (2) reduces to

$$\frac{\partial \tilde{M}_x}{\partial x} = 0 \tag{16}$$

Integration gives

$$\tilde{M}_x = \text{constant} = 0 \tag{17}$$

which must be equal to zero since the beach forms a boundary in the x-direction This then says

$$U_x = - \frac{M}{\rho D} \cos\alpha \tag{18}$$

which states that there is a mean reverse current balancing the mass transport onshore due to the wave motion This must be true everywhere, both inside and outside the surf zone, to ensure that there is no accumulation of mass or growth of currents in the y-direction in order to maintain steady-state conditions in accordance with the original assumptions

B Changes in the Mean Water Level

The changes in the mean water level must be determined inside the surf zone in order to specify the variation of wave energy since the waves are assumed proportional to the total depth of water The changes in the mean water level can be determined from the x-momentum equation given by

$$\frac{\partial}{\partial x} S_{xx} = T_x = - \rho g D \frac{\partial \bar{\eta}}{\partial x} \tag{19}$$

where shear stresses are neglected and time dependence, y-gradients and net mass flux in the x-direction are all zero This equation states that there is a change in the mean water level to balance the excess momentum flux of the waves Longuet-Higgins and Steward [3] have solved Equation (19) for the case of wave approaching to an arbitrary plane bottom using linear wave theory Outside the surf zone they found

$$\bar{\eta} = -\frac{H^2}{8}\frac{k}{\sinh 2kh} \tag{20}$$

where the negative sign indicates a set-down which is a function of the local conditions only

Inside the surf zone the changes in mean water level are again determined from Equation (19) It is assumed that the excess momentum flux tensor inside the surf zone can be expressed in terms of the energy and wave speed in the same form as in shallow water This assumption implies that even under the breaking waves, water particle motion retains much of its organized character as described by linear wave theory

The excess momentum flux decreases inside the surf zone due to the decreasing wave height as energy is dissipated This results in a wave set-up inside the surf zone given by

$$\bar{\eta} = K(h_b - h) + \bar{\eta}_b \tag{21}$$

where the set-down at the breaker line $\bar{\eta}_b$ can be determined from Equation (20) and

$$K = \frac{1}{1 + \frac{8}{3\kappa^2}} \tag{22}$$

Bowen et al [4] conducted laboratory studies verifying the theory predicting the changes in mean sea level for waves normally incident to the shoreline The effect of waves approaching at an angle is neglected in the above formula but was shown by Thornton [5] to result in a maximum change in the total depth of less than 2 per cent

C Distribution of Currents Outside the Surf Zone

There is a component of excess momentum flux directed parallel to the shore due to the oblique wave approach The question of whether a current can be generated is investigated by considering the general y-momentum equation Applying the previous assumptions, Equation (5) can be written

$$\frac{\partial S_{xy}}{\partial x} = R_y \tag{23}$$

where the time dependent term is zero, the gradients in the y-direction are zero and the conservation of mass equation showed that $\tilde{M}_x = 0$ If it is assumed that outside the surf zone that energy is conserved, that is, no dissipation of energy, then the stress term will be zero Therefore, the change of momentum flux due to the waves and mean motion in the y-direction is zero--there is no driving force for generating a current outside the surf zone The only wave-induced current far outside the surf zone is then due to the mass transport velocity which is weak

With the mean water profile and energy distribution specified, the variation of the longshore current across the surf zone can be determined The y-momentum equation inside the surf zone can be written the same as that outside the surf zone Inside the surf zone energy is dissipated due to turbulence and bottom friction and the stress term is important In order to solve for the longshore current inside the surf zone, an appropriate description of the resistance term composed of both bottom and internal shear stresses is required

D Bottom Shear Stress

It is desired to determine the combined bottom shear stress due to waves and currents It is assumed that the total instantaneous bed shear stress for combined waves and currents is related to the velocity by

$$\vec{\tau}_h = \rho \frac{f}{2} \vec{v}|\vec{v}| \tag{24}$$

where $\vec{v}$ is the resultant instantaneous velocity vector of the combined wave and current motion, and f is the friction factor Since the problem has been formulated as a combination of wave and current motion, it proves convenient to resolve the shear stresses into components in the direction of the wave and current components Resolving the component shear stresses in the direction of the velocity vectors results in the shear stress and velocity vectors being conformal Hence, the shear stress component for the wave motion can be written

$$\vec{\tau}_{hw} = \vec{\tau}_h \frac{\vec{u}_w}{\vec{v}} = \rho \frac{f}{2} |\vec{v}|\vec{u}_w \tag{25}$$

where $\vec{u}_w$ is the instantaneous velocity of the wave motion measured just above the frictional boundary layer near the bottom and V is the mean motion which was assumed uniform over the depth and in the longshore direction only

Similarly, for the shear stress in the direction of the mean current

$$\vec{\tau}_{hy} = \vec{\tau}_h \frac{V}{\vec{v}} = \rho \frac{f}{2} |\vec{v}|V \tag{26}$$

If friction factors for the wave and current motion are now defined,

$$f_y = f \left| \frac{\vec{v}}{V} \right| \qquad f_w = f \frac{|\vec{v}|}{|\vec{u}_w|} \tag{27}$$

This results in the shear stress components given in a form consistent with experimental results

$$\vec{\tau}_{hw} = \rho \frac{f_w}{2} \vec{u}_w |\vec{u}_w| \tag{28}$$

$$\tau_{hy} = \rho \frac{f_y}{2} V^2 \tag{29}$$

The shear stress for waves, as defined in Equation (28) is in the same form as given by Jonsson [6] for which information of the friction factor was found experimentally He found, further, that the friction factor was practically constant over an oscillation period The constancy of the friction factor for particular flow conditions is an important result which allows for a better analytical determination of the combined shear stress due to waves and currents Using the available data from several sources, he found that the friction factor for wave motion alone for rough turbulent boundary layers (as usually found in nature) could tentatively be represented by

$$\frac{1}{4\sqrt{f_w}} + \log \frac{1}{4\sqrt{f_w}} = - 0\ 08 + \log \frac{\xi_h}{r} \tag{30}$$

where r is a measure of roughness, and ξ_h is the maximum water particle excursion amplitude of the fluid motion at the bottom as predicted by linear wave theory

$$\xi_h = \frac{H}{2} \frac{1}{\sinh kh} \tag{30a}$$

Equation (30) is based on the roughness parameter r being a measure of the ripple height The wave friction factor is seen to be a function of the wave characteristics This is because, for granular beds consisting of a particular grain size, the ripples adjust their dimensions according to the wave motion, and it is the ripple geometry that determines the effective roughness The difficulty in using the quadratic shear stress formula is in stipulating the friction factors The wave friction factor is seen to be a function of the wave properties for a deformable bed, that is, the fluid motion, whereas, the friction factor for steady currents for rough turbulent boundary layers is only a function of the system geometry It would seem reasonable to expect that for weak currents, as compared to the water particle motion of the waves, that the wave dynamics would dominate the hydrodynamical system For this reason, it is desirable to use the combined bottom shear stress in terms of the wave

friction factor alone, even though it is less well defined than the friction factor for steady currents

For the problem at hand, the bottom shear stress directed parallel to shore can be written in terms of the wave friction factor by combining Equations (27) and (28) which results in the form

$$\tau_{hy} = \frac{\rho f_y}{2} V^2 = \rho \frac{f_w}{2} \left|\overline{u_{wh}}\right| V = \rho f_w \frac{H}{2\pi} \frac{g}{c} V \qquad (31)$$

where shallow water waves are assumed

E Lateral Shear Stress

It is necessary to include the lateral shear stress in the formulation If the lateral shear stress is not included, a velocity discontinuity at the breaker line is predicted due to the abrupt change in momentum flux here The lateral shear stress effectively couples the adjacent elemental water columns together resulting in a diffusion of momentum in a direction perpendicular to shore The lateral diffusion of momentum flux seaward across the breaker line results in the momentum flux inside the surf zone driving the currents outside, the lateral diffusion of momentum shoreward results in the maximum velocity displaced shoreward

Considerable success has been achieved using Prandtl's mixing length hypothesis for specific problems This concept will be utilized to relate the internal shear stresses to the mean flow The expression for the internal shear stress can be given in terms of the mean turbulent Reynolds stress which is compared to a "Boussinesq" approach

$$\tau_{xy} = - \rho \overline{u'v'} = \rho \varepsilon_v \frac{\partial V}{\partial x} \qquad (32)$$

where ε_v is the kinematic eddy viscosity

The variation of the turbulent velocity component in the y-direction is given in accordance with Prandtl's hypothesis by

$$v' = \ell' \frac{\partial V}{\partial x} \qquad (33)$$

where ℓ' denotes a mixing length which can fluctuate with time From Equation (32), the kinematic eddy viscosity can then be written

$$\varepsilon_v = - \left|\overline{u'\ell'}\right| \qquad (34)$$

For the case of superposed waves and currents, it is natural to consider the length over which momentum is transferred as equivalent to the water particle excursion due to the wave motion and the velocity fluctuation u' equal to that of the water particles in the waves

Because u' and ℓ' are in quadrature, the absolute value of the product is necessary in order to obtain a non-zero value The mixing length ℓ' can be interpreted as a measure of the turbulent scale and u' as a measure of turbulent intensity This interpretation is not unreasonable physically Examination of actual energy spectra of turbulence occurring in the surf zone shows that most of the fluctuating energy is associated with the waves

The kinematic eddy viscosity can be evaluated by recalling Equations (12) and (30a) It will be assumed for simplicity that shallow water wave conditions apply, so that

$$\varepsilon_v = \frac{H^2}{8\pi^2} \frac{gT}{h} \cos^2\alpha \tag{35}$$

Evaluating the internal shear stress term by substituting Equation (32) gives

$$\overline{\int_{-h}^{\eta} \frac{\partial \tau_{xy}}{\partial x} dz} = \int_{-h}^{\eta} \rho \frac{\partial}{\partial x} \left(\varepsilon_v \frac{\partial V}{\partial x}\right) dz = \rho D \frac{\partial}{\partial x} \left(\varepsilon_v \frac{\partial V}{\partial x}\right) \tag{36}$$

where ε_v and V are independent of z

The total resistance term, including internal and bottom shear stresses, is then given by

$$R_y = \rho D \frac{\partial}{\partial x} \left(\varepsilon_v \frac{dV}{dx}\right) - \rho \frac{f_w}{2\pi} \frac{g}{c} H V \tag{37}$$

F Distribution of Currents Inside the Surf Zone

The variation of the longshore current inside the surf zone can be solved by equating the changes in excess wave momentum to the resistance forces Substituting for S_{xy} from Equation (15) gives

$$\frac{\partial}{\partial x} [E \sin\alpha \cos\alpha] = R_y = \rho D \frac{\partial}{\partial x} \left(\varepsilon_v \frac{dV}{dx}\right) - \rho \frac{f_w}{2\pi} \frac{g}{c} H V \tag{38}$$

The height, celerity, refracted angle of the waves, and, hence, changes in momentum flux inside the surf zone can be expressed in terms of the local depth of water Substituting Equations (10), (14), and (35) into Equation (38) gives

$$AD^{3/2}\left(1 - 0.7 \frac{D}{D_b} \sin^2\alpha_b\right) \frac{\partial D}{\partial x} = \rho D \frac{\partial}{\partial x} \left(\varepsilon_v \frac{\partial V}{\partial x}\right) - \frac{\rho\kappa}{2\pi} f_w \sqrt{\frac{gD}{(1+\kappa)}}\, V \tag{39}$$

where

$$A = \frac{5}{16} \rho g \kappa^2 \frac{\sin\alpha_b}{\sqrt{D_b}} \tag{40}$$

This is a general equation expressing the changes in momentum flux across the surf zone in terms of the total local depth of water Included in the formulation is the set-up of water and the effects of wave refraction inside the surf zone This equation is subject to the restriction that the waves be described as spilling breakers, and, hence, the depth continuously decreases shoreward from the breaker line Thus, the momentum flux decreases monitonically inside the surf zone since both the energy and wave angle decrease with decreasing depth

This development is similar to that by Bowen [7] in an investigation limited to a plane beach in which he assumes linear bottom friction and constant kinematic eddy viscosity in order to obtain an analytical solution

The inclusion of the bottom friction requires Equation (39) to be solved numerically Also a general method of solution is sought for comparison to the arbitrary field conditions Boundary conditions imposed on the problem inside the surf zone are for $D = 0$, $V = 0$ corresponding to conditions at the intersection of the water line and the beach, and for $D = D_b$, $V = V_b$ corresponding to conditions at the breaker line

A similar solution can be sought outside the surf zone where it is now assumed that the driving force for the currents outside the surf zone is zero--the changes in the momentum flux directed parallel to shore are zero The y-momentum Equation (38) reduces to

$$D \frac{\partial}{\partial x} \left(\varepsilon_v \frac{\partial V}{\partial x}\right) - \frac{f_w}{2\pi} \frac{g}{c} H V = 0 \qquad (41)$$

where now the force driving the currents is due to lateral momentum flux resulting from a coupling of the adjacent vertical faces of the differential water column across the breaker line This is to say that currents outside the surf zone are being driven by the longshore currents inside the surf zone due to coupling across the breaker line

The boundary conditions imposed on the formulation outside the surf zone are that the velocity approaches zero far away from the breaker line ($D \to -\infty$) and that the velocities and velocity gradients inside and outside the surf zone match at the breaker line

COMPARISON OF THEORY AND EXPERIMENT

The laboratory results of Galvin and Eagleson [8] are used to test the predictive equations The only parameter that is necessary to be chosen is the roughness in order to utilize the predictive equations A value of $r = 0.0033$ feet (1mm) was chosen for the concrete beach which is a reasonable value The kinematic eddy viscosity is completely specified by the kinematics of the flow field The resulting

velocity distribution is shown in Figure 2 The distributions of the kinematic eddy viscosity and friction factor are also shown in Figure 2 The kinematic eddy viscosity is a maximum at the point of breaking, where the maximum momentum exchange would be expected to take place, and decreases to zero at the shoreline The friction factor varies slowly except near the beach where f_w increases very rapidly and approaches infinity at the intersection of the beach and the still water line

Other cases and a more complete text is given by Thornton [5] In general, surprisingly good correlation is found considering that the only parameter chosen is the roughness in order to completely specify the longshore velocity distribution

The same equations are applied to the field data taken by Ingle [9] The velocity distribution is shown in Figure 3 The bottom profile is shown in the same figure Information concerning the bathymetry outside the surf zone is lacking so a bottom slope of 0 01 is assumed The kinematic eddy viscosity and friction factor distributions are also shown and are similar to those found for the laboratory beaches The friction factors for the laboratory and field are of the same order of magnitude, owing to the fact that the friction factor is not only dependent on the roughness but also the wave characteristics The kinematic eddy viscosity for field conditions is several orders of magnitude greater than the value found for laboratory conditions owing to the greater turbulent scales

CONCLUSIONS

It was shown that the component of excess momentum flux due to the presence of the unsteady wave motion (sometimes called a "radiation stress") directed parallel to shore can generate longshore currents Changes in the excess momentum flux, as the waves shoal, must be balanced by a resistance force in order to maintain the assumed steady-state conditions The component of excess momentum flux perpendicular to the beach is responsible for wave set-down outside the surf zone and wave set-up inside the surf zone These changes in the mean water level were included in the longshore current formulations

Logical means of introducing the friction factor associated with the bottom shear stress term was presented--the friction factor being related to the wave and bottom roughness characteristics A mixing length hypothesis and the kinematics of the wave motion were combined in order to define the internal shear stresses Comparison of experimental results from the laboratory and field with the derived theory shows that the predicted results compare favorably if the assumed conditions are approximately fulfilled

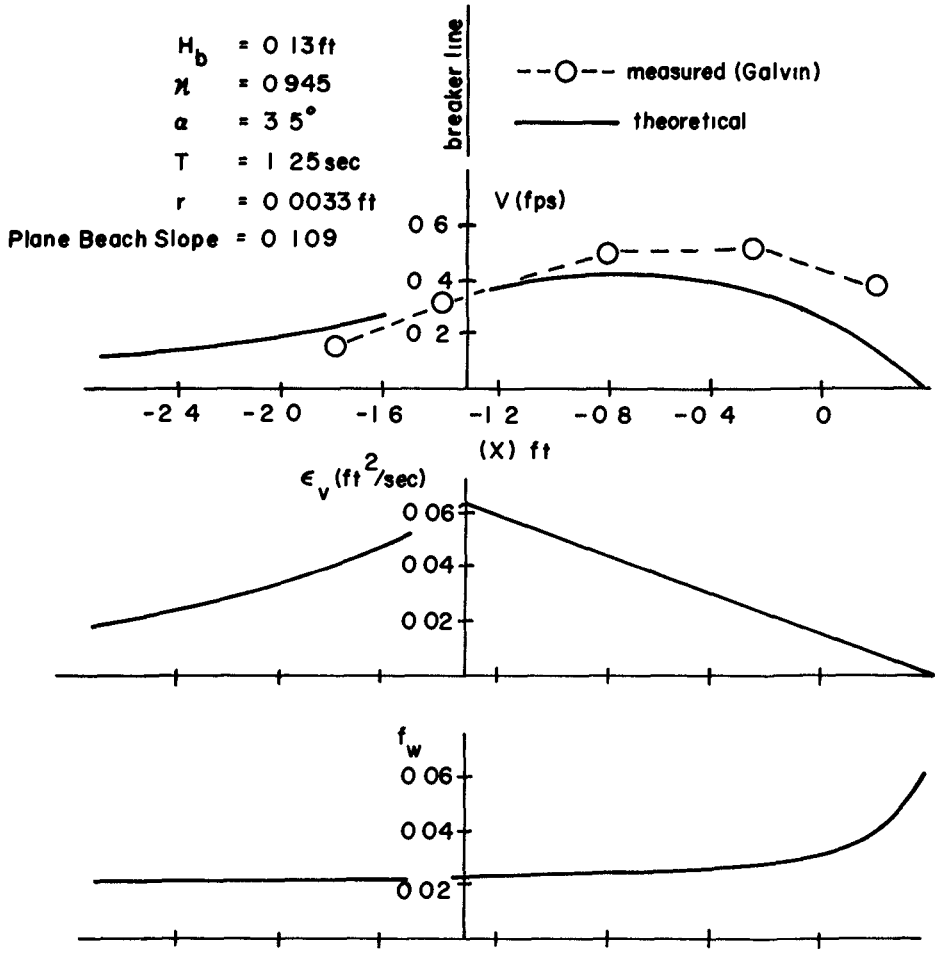

FIGURE 2 VELOCITY DISTRIBUTION ACROSS THE SURF ZONE FOR LABORATORY BEACH

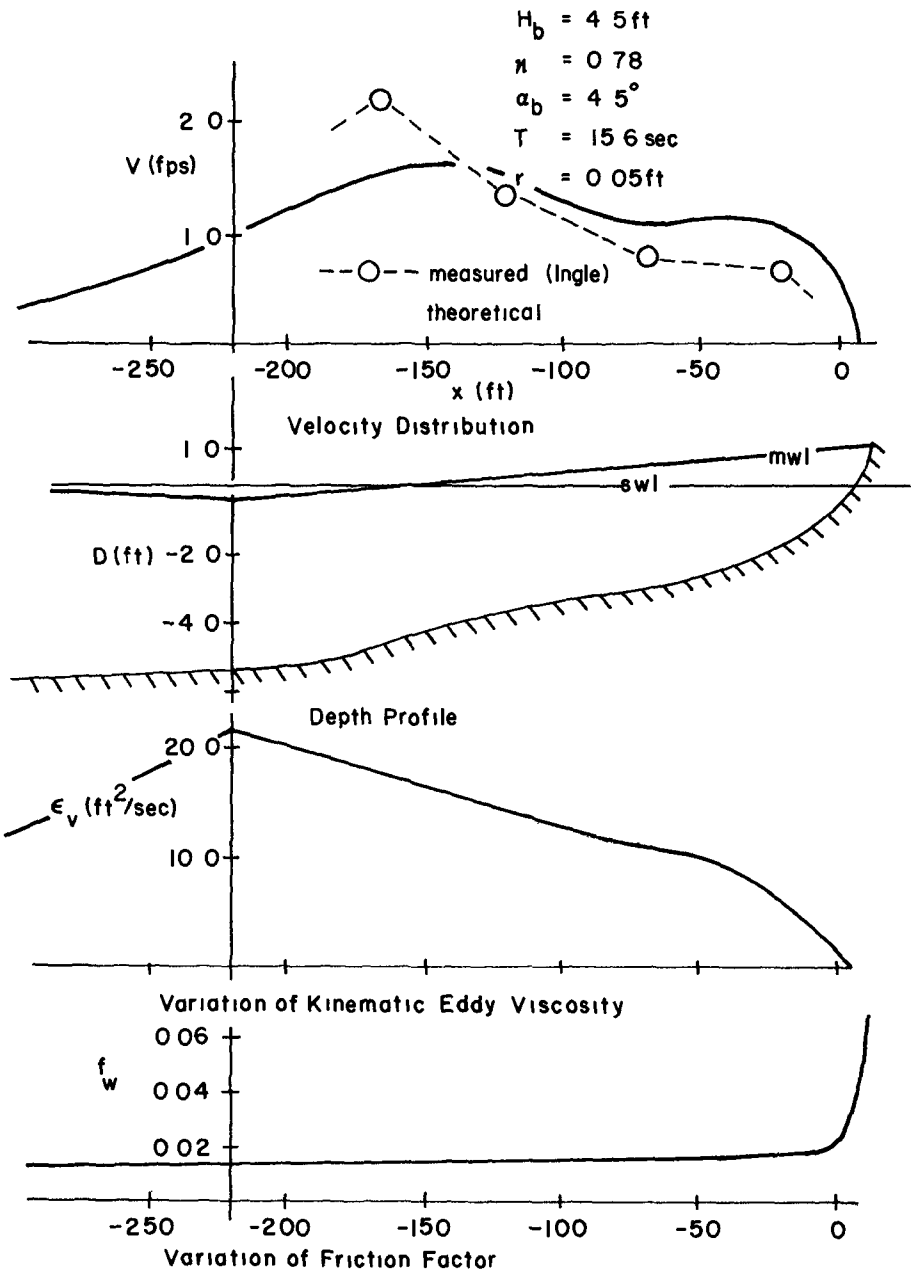

FIGURE 3 VELOCITY DISTRIBUTION ACROSS SURF ZONE, TRANCAS BEACH, CALIFORNIA

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