CHAPTER 16

# ENERGY LOSSES UNDER WAVE ACTION

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## ABSTRACT

Wave-height attenuation measurements were made in two identical flumes of different widths and the results used to separate bottom energy losses from sidewall energy losses These energy losses, in the form of rates of energy dissipation, were then compared with their theoretical values as calculated by solving the linearized Prandtl boundary layer equations and evaluating the Rayleigh dissipation function Using these results, an adjusted formula for the wave-height attenuation modulus was determined

## INTRODUCTION

Up to the present time no direct measurements of the sidewall and bottom rates of energy dissipation in laminar boundary layers, produced by progressive, oscillatory gravity waves in a wave flume, have been made A comparison between these experimental values and their related theoretical values would be valuable as the results could be used to produce an adjusted formula for the wave-height attenuation modulus $\alpha$, which is defined by the equation

$$\frac{H}{H_o} = e^{-\alpha x} \qquad \text{— — — — — — — — 1}$$

where $H_0$ is the wave-height at position $x = 0$ and H is the subsequent wave-height at position $x$, in the direction of wave propagation Eagleson (1) and Iwagaki and Tsuchiya (2) measured the bottom shear forces produced by a wave by measuring the shear on a plate fitted flush with the flume bottom Their results led to values of $\alpha_b$, (where $\alpha_b$ is the attenuation modulus which results from the bottom boundary layer), which did not agree with the theoretical values of $\alpha_b$, the formula for which is derived by the authors mentioned In model harbour tests it is often necessary to have an accurate estimate of this attenuation modulus in order that the prototype wave-heights may be interpreted from the model measurements This is because harbour models are constructed according to the Froude modelling law whereas the model wave amplitude attenuation is normally a viscous phenomenon

## GENERAL APPROACH

In order to separate bottom and sidewall friction effects it is necessary to have two flumes which are identical except for their widths Since the space rate of change of average wave power per unit plan area, $\partial\overline{P}/\partial x$, is equivalent to the sum of the average time rates of energy dissipation per unit plan area on the bottom and sidewalls, $d\overline{E}_b/dt$ and $d\overline{E}_w/dt$ respectively, the following simultaneous equations can be written

$$\begin{aligned} B_1\ \partial\overline{P}_1/\partial x &= -(B_1\ d\overline{E}_b/dt + 2h\ d\overline{E}_w/dt) \\ B_2\ \partial\overline{P}_2/\partial x &= -(B_2\ d\overline{E}_b/dt + 2h\ d\overline{E}_w/dt) \end{aligned} \qquad \text{— — — 2}$$

where B is flume width, h is still water depth and the subscripts 1 and 2 refer to the two separate flumes If the quantities $\partial\overline{P}_1/\partial x$ and $\partial\overline{P}_2/\partial x$ are known, then the equations can be solved for $d\overline{E}_b/dt$ and $d\overline{E}_w/dt$ These equations must be solved for the case when the wave parameters, wave period T, wave height H and still water depth h, are the same in both flumes because then the values of $d\overline{E}_w/dt$ and $d\overline{E}_b/dt$ are the same in the two flumes Thus it becomes necessary to determine experimental values for $\partial P/\partial x$ in the two flumes Now since $\overline{P}$, from first order wave theory, can be written as

$$\overline{P} = \gamma H^2 C_g/8 \qquad \text{— — — — — — — 3}$$

where $\gamma$ is the specific weight of the fluid and $C_g$ is the wave group velocity, it is possible to write $\partial\overline{P}/\partial x$ as

$$\partial\overline{P}/\partial x = \gamma H/4\ \ dH/dx\ C_g \qquad \text{— — — — — 4}$$

for constant T and h First order wave theory was used because the horizontal water particle velocities are described best, in this case, by this theory, (Le Mehauté et al (3)), and because the first order approximation to the solution of the wave equation satisfies the boundary conditions at the free surface as well as other theories, (Dean (4)) It now becomes obvious that it is necessary to determine wave height versus distance attenuation curves for waves with the same periods and depths in the two flumes Equation 4 can then be evaluated and equations 2 solved

## EXPERIMENTAL TECHNIQUE

The Coastal Engineering Laboratory of Queen's University at Kingston has two similar flumes of width 2 ft and 3 ft and length 150 ft They are constructed from concrete and the inside walls were cement plaster which has been sanded smooth and painted However, in order to obtain sufficient difference between attenuation rates in the two flumes, an aluminum sheet wall was constructed in the 2 ft flume to make a temporary flume 6 inches wide The flumes had identical motors and similar flap-type paddles

The wave-heights in the 6 inch flume were measured using a carbon-strip probe and those in the 3 ft flume using a capacitance plate probe Both of these instruments have similar accuracy, (± 0 002 ft ) in comparison with a hook gauge Risaltex "horse-hair" mats were used to make beaches which absorbed the energy of the incident waves The reflection coefficient was always less than 5%

The useful measuring length for each flume was about 25 ft out of a total length of some 150 ft This distance was not long enough to produce a sufficient number of wave-height versus distance data points For this reason the flume was "lengthened" using the following known method which is described by Battjes (5) The wave-heights were measured at two stations 25 ft apart, wave-height at x = 0 ft and wave-height at x = 25 ft The wave paddle was then adjusted so that the wave-height measured at x = 25 ft was now produced at x = 0 ft in the flume The wave-height produced at x = 25 ft is now the wave-height which would have been produced at x = 50 ft if the flume had been long enough By repeating this procedure the flumes were "lengthened" considerably In fact the paddle was adjusted so that the wave-height measured at x = 0 ft , for each "lengthening" increment, was within ± 0 3mm of the required wave-height The difference in wave-height decrements for waves of initial height differing by this small quantity would be very small Each measurement was performed three times, the wave-height decrement being taken as the average of these values

The wave-heights at stations were measured by moving the measuring device over a distance of one half wavelength on both sides of the station, thus measuring two maxima and two minima of the incident-reflected wave interference profile Then, assuming linear attenuation over a short distance, and assuming first order wave motion, the wave-height at a station can be shown to be equal to

$$H = 1/8\,(A+3B+3C+D)$$

where, in this case, A, B, C and D are the consecutive values of the maxima and minima taken from the recorder profile

## DIMENSIONAL ANALYSIS OF ENERGY DISSIPATION

The rate of energy dissipation in the laminar bottom boundary layer can be written in the functional form

$$d\bar{E}_b/dt = f(a_\delta, T, \mu, \rho)$$

where $\mu$ is fluid viscosity, $\rho$ is the fluid density and $a_\delta$ is the length of the bottom fluid particle motion Using dimensional analysis this equation becomes

$$d\bar{E}_b/dt = \rho a_\delta^3/T^3 . \phi(a_\delta^2/T\nu) \qquad — \quad — \quad — \quad 5$$

where $\nu$ is kinematic viscosity It is well known that for laminar flow conditions a viscous force is proportional to the first power of a velocity, i e

Force $\propto$ velocity

Now the rate at which work is done, or energy dissipated, is given by the relationship

Rate of work = Force x average velocity

i e Rate of work $\propto$ (velocity)$^2$

or in this case

$$d\bar{E}_b/dt \propto u_o^2 \qquad — \quad — \quad — \quad — \quad 6$$

is taken, where $u_o$ is the horizontal velocity of fluid particles at the upper limit of the boundary layer Thus equation 5 can be written in the form

$$d\bar{E}_b/dt = \rho(a_\delta/T)^3 . \phi((a_\delta/T)^2 . T/\nu)$$

which, upon putting $u_0$ = constant x $a_\delta/T$

$$d\bar{E}_b/dt = \rho u_o^3 \phi_1 (u_o^2 T/\nu) \qquad 7$$

In order that equation 6 can be satisfied, $\phi$ must be of the form

$$\phi = \text{constant} \, (u_o^2 T/\nu)^{-1/2} \qquad 8$$

By suitable arrangement and substitution of equation 8 in equation 7, the following is obtained

$$d\bar{E}_b/dt_{\text{experimental}} = \text{const} \; d\bar{E}_b/dt_{\text{theoretical}}$$

$$= D \; d\bar{E}_b/dt_{\text{theoretical}}$$

and in a similar way the sidewall rate of energy dissipation functional relationship produces

$$d\bar{E}_w/dt_{\text{experimental}} = \text{konst} \; d\bar{E}_w/dt_{\text{theoretical}}$$

$$= F \; d\bar{E}_w/dt_{\text{theoretical}}$$

The subscript "theoretical" indicates the theoretical equation for the rate of energy dissipation D and F are constants

From the above analysis it is obvious that, once the rates of attenuation curves for wave-height have been determined in the two flumes for waves with the same period and water depth, the constants D and F can be determined One experiment only is required However nine experiments were performed in order to reduce the effect of experimental error The periods used were 0 91, 1 08 and 1 21 seconds The water depths were 4 1, 5 6, 7 9 and 10 0 inches The selection of these values ensured that the wavelength to depth ratios would be in the range 0 $\lessgtr$ L/h $\lessgtr$ 10, the lower limit being for deep water waves and the upper limit being the commonly accepted limit for cnoidal waves Maximum wave steepness was about 0 05 To ensure that the boundary layers remained laminar, the criterion for laminar oscillatory boundary layers under waves derived by Collins (6) was used, this being that

$$R_E = \frac{u_o \delta}{\nu} < 160$$

where $\delta$ is a boundary layer thickness parameter defined as

$$\delta = (\nu T/\pi)^{1/2}$$

In addition, the water surface was kept clean and the condition of an immobile surface never occurred Energy dissipation as a result of surface films was not considered to be of importance

## RESULTS AND CONCLUSIONS

The experiments were performed as described and the results tabulated The next step was to fit a suitable equation to the experimental results so that equation 4 could be evaluated. This was done by assuming exponential attenuation and linearizing equation 1

$$\text{i e} \quad \ln H = \ln H_o - \alpha x$$

Using the least squares technique and a weighting function of $H^2$, the values of $\alpha$ and $H_o$ were determined Table 1 shows the experimental values of $\alpha$ together with their related theoretical values The data was found to fit this form very well The fit was tested using the parameter

$$R^2 = 1 - \frac{\Sigma (H - H_i)^2}{\Sigma (H_i - \bar{H})^2}$$

where

$$\bar{H} = \frac{1}{n} \sum_{i=1}^{n} H_i$$

n is the number of data points obtained for a particular attenuation curve and $H_1$ is the wave-height at position $x_1$ along a flume The values of $R^2$ were better than 0 995 in all cases

Grosch and Lukasik (7) determined an attenuation equation for finite amplitude waves This equation was also tried and the results of aggression analysis showed that their equation fitted the data almost as well as the exponential equation However, the latter, because of its easier mathematical form, was used for evaluating equation 4

With experimental values of $\alpha$ and $H_0$ determined, the values of rates of energy dissipation on the bottom and sidewalls can be separated For constant values of T and h, the constants D and F will not vary with change in H as both the theoretical and experimental rates of energy dissipation are then functions of $H^2$ only Tables 2 and 3 show theoretical and experimental rates of energy dissipation, calculated for an H value of 20 mm , for the bottom and sidewalls respectively, together with the values of D and F The values of D and F are

$$D = 1\ 48 \pm 0\ 15$$

$$F = 0\ 94 \pm 0\ 09$$

at the 95% confidence limits These values show that theory considerably underestimates the rate of energy dissipation on the flume bottom, whereas for the sidewalls, theory is very close to experiment The fact that first order theory and experiments agree quite well for energy dissipation on the side walls, whereas this is not the case for bottom losses, cannot be satisfactorily explained It is not thought that energy dissipation at the fluid surface as a result of surface films is the cause for the approx 50% difference since the experimental surface was certainly not immobile by any stretch of the imagination

Using a method similar to that demonstrated by Eagleson (1), an equation for the adjusted attenuation modulus for bottom and sidewalls was determined

$$\alpha_{b+w} = \frac{k}{B}\left(\frac{T\nu}{\pi}\right)^{1/2}\left(\frac{1\ 48Bk + 0\ 94 \sinh 2kh}{2kh + \sinh 2kh}\right) \qquad - - - - 9$$

where $k = 2\pi/L$ and L is wavelength This adjusted modulus would, for the case of a wide, shallow flume, be considerably larger than the well-known theoretical value

TABLE 1

TABLE OF THEORETICAL AND EXPERIMENTAL ATTENUATION MODULI

| Depth | 3 ft flume Theoretical | 3 ft flume Experimental | 6 inch flume Theoretical | 6 inch flume Experimental |
|---|---|---|---|---|
| T = 1 21 seconds | | | | |
| 10 0 inches | 0 00107 | 0 00108 | 0 0037 | 0 00306 |
| T = 1 08 seconds | | | | |
| 4 1 inches | 0 00315 | 0 00464 | 0 00675 | 0 00784 |
| 5 6 inches | 0 00213 | 0 00246 | 0 00543 | 0 00570 |
| 7 9 inches | 0 00143 | 0 00170 | 0 00451 | 0 00446 |
| 10 0 inches | 0 00115 | 0 00116 | 0 00412 | 0 00356 |
| T = 0 91 seconds | | | | |
| 4 1 inches | 0 00337 | 0 00453 | 0 00758 | 0 00933 |
| 5 6 inches | 0 00227 | 0 00300 | 0 00620 | 0 00714 |
| 7 9 inches | 0 00156 | 0 00214 | 0 00543 | 0 00550 |
| 10 0 inches | 0 00125 | 0 00159 | 0 00515 | 0 00521 |

TABLE 2

TABLE OF THEORETICAL AND EXPERIMENTAL VALUES

OF $d\overline{E}_b/dt$

| Depth | $d\overline{E}_b/dt$ has units ft lb /ft $^2$/sec | | F |
|---|---|---|---|
| | $\frac{d\overline{E}_b}{dt}$ exp x $10^3$ | $\frac{d\overline{E}_b}{dt}$ theor x $10^3$ | |
| T = 1 21 seconds | | | |
| 10 0 inches | 0 166 | 0 131 | 1 27 |
| T = 1 08 seconds | | | |
| 4 1 inches | 0 742 | 0 450 | 1 65 |
| 5 6 inches | 0 361 | 0 290 | 1 24 |
| 7 9 inches | 0 249 | 0 178 | 1 40 |
| 10 0 inches | 0 151 | 0 118 | 1 28 |
| T = 0 91 seconds | | | |
| 4 1 inches | 0 615 | 0 437 | 1 41 |
| 5 6 inches | 0 390 | 0 270 | 1 45 |
| 7 9 inches | 0 275 | 0 148 | 1 86 |
| 10 0 inches | 0 162 | 0 090 | 1 79 |

TABLE 3

TABLE OF THEORETICAL AND EXPERIMENTAL VALUES OF $d\overline{E}_w/dt$

| Depth | $d\overline{E}_w/dt$ has units ft lb /ft$^2$ /sec | | D |
|---|---|---|---|
| | $\frac{d\overline{E}_w}{dt}$ exp $\times 10^3$ | $\frac{d\overline{E}_w}{dt}$ theor $\times 10^3$ | |
| T = 1 21 seconds | | | |
| 10 0 inches | 0 173 | 0 225 | 0 77 |
| T = 1 08 seconds | | | |
| 4 1 inches | 0 521 | 0 583 | 0.89 |
| 5 6 inches | 0 422 | 0 420 | 1 01 |
| 7 9 inches | 0 273 | 0 303 | 0 90 |
| 10 0 inches | 0 191 | 0 239 | 0 80 |
| T = 0 91 seconds | | | |
| 4 1 inches | 0 726 | 0 636 | 1 14 |
| 5 6 inches | 0 485 | 0 462 | 1 05 |
| 7 9 inches | 0 287 | 0 329 | 0 99 |
| 10 0 inches | 0 244 | 0 262 | 0 93 |

## REFERENCES


1 Eagleson, P S (1962), "Laminar Damping of Oscillatory Waves", Proc A S C E, Vol 88, pp 155-181, Hy 3

2 Yuichi Iwagaki and Yoshito Tsuchiya, (1966), "Laminar Damping of Oscillatory Waves Due to Bottom Friction", Coastal Engineering Conference, Tokyo, 1966, pp 149-174

3 Le Mehauté, B, Divorky, D and Lin, A, (1968), "Shallow Water Waves A comparison of Theories and Experiments", Coastal Engineering Conference, London, 1968

4 Dean, R G (1965), "Stream Function Wave Theory, Validity and Application", Coastal Engineering Specialty Conference, Santa Barbara, 1965

5 Battjes, J S (1965), "Wave Attenuation In A Channel With Roughened Sides", Coastal Engineering Specialty Conference, Santa Barbara, 1965

6 Collins, J I (1963), "Inception of Turbulence At the Bed Under Periodic Gravity Waves", Jour Geophs Res, Vol 68, pp 6007-6014

7 Grosch, C E and Lukasik, S J (1963), "Discussion of Laminar Damping Of Oscillatory Waves", Proc A S C E, Vol 89, No Hy1, pp 232-239