

## CHAPTER 94

### STUDIES OF PREDICTION OF RECIRCULATION OF COOLING WATER IN A BAY

By Akira Wada, Dr Eng

Technical Laboratory  
Central Research Institute of Electric Power Industry  
Tokyo, Japan

#### ABSTRACT

In the present report, a few factors of great importance in the determination of the degree of recirculation of cooling water for power station are chosen for theoretical analysis

First, the problem of flow and thermal diffusion in the sea caused by outfall of warmed cooling water is considered. Numerical solution of the equation of thermal diffusion with a complete set of Eulerian equations in which the values of the eddy coefficient of diffusion and viscosity depend upon the relation to the velocity shear and vertical density gradient is presented. The Boussinesq approximation is used throughout. The mechanism of development of upwelling phenomenon is discussed from the numerical experiment taking into account of the interaction between the flow and the thermal distribution. From the result of field survey, the occurrence of the return flow in the bottom layer of the coastal region is confirmed. The numerical solutions give a good representation of all features of flow and thermal profiles.

Next, study on cold water intake from bottom layer is presented. The numerical experiment of the stratified fluid with a continuous density gradient flowing into the submerged intake is conducted to gain additional insight into the mechanism of density flow in the field of the continuously stratified fluid. The results lead to the conclusion that the flow in the field of the continuous distribution of density differs from that for a two-layer system

#### I INTRODUCTION

In a previous paper<sup>1)</sup>, the effect of some dominant factors on the recirculation of cooling water was discussed. A practical theory was also presented from which quantitative prediction of water temperature and velocity fields due to discharge of warmed cooling water may be obtained for a bay characterized by a meteorological condition.

By the development of the technique of this numerical model test, the next problems can be solved. They are the followings, (a) presumption of approaching velocity to intake entrance, (b) discrimination of thermocline stability caused by the appearance of locally high velocity, (c) influence of outfall of warmed cooling water on a vessel navigation and (d) estimation of influenced range of water temperature rise in the sea in relation to compensation for coastal fishery.

On the basis of these studies, the design criteria of intake and outlet structures can be determined. By the application of the developed method of computation to the real planning site, the hydraulic design of the intake and the outlet at Tsuruga Nuclear Power Station, Mizushima Thermal Power Station, Shimane Nuclear Power Station and Oita Thermal Power Station was done

To know the aspect of the thermal diffusion caused by the outfall of warmed cooling water, we can not but rely on the method such as the field survey, the hydraulic model test and the numerical experiment by an electronic computer. The positive field survey is especially important for the research work of recirculation of cooling water. It is because there is a fundamental defect in the method verifying its result however rich the products of theoretical analysis may get. It is necessary to collect the data of field survey as many as possible and to deal with it correctly and then to make efforts approaching the essence of the problem inductively.

There are some methods to solve the problem of recirculation of cooling water. But some questions arise. That is, the method by the hydraulic model test comes into question in the point of similarity of real phenomena. It is because the flow in the sea is very small, as the discharged water from the outlet has a low velocity and heat exchange between the sea surface and the atmosphere must also be considered. In the present study, the method of numerical experiment is used throughout from this point of view.

In the present report, two factors of great importance in the determination of the recirculation degree of cooling water for power stations are chosen for theoretical analysis. In the first place, the problem of flow and thermal diffusion in the sea caused by the outfall of warmed cooling water is considered. Secondly, study on cold water intake from the bottom layer is presented.

In this report, we present some results of a theoretical investigation of turbulent thermal diffusion in a compressible fluid by means of direct numerical solution of a complete set of dynamic equations. The application of numerical experimentation to physical theory is generally justifiable only when more concise analytic methods have been unproductive or have reached apparent limits of usefulness, but these conditions seem to prevail in the field of turbulent fluid mechanics. Although it would be possible to formulate and numerically integrate sets of differential equations, initial and boundary values, appropriate to a broad range of fluid dynamics phenomena, this method would have little merit in the cases where general analytic solutions are available. One should always keep in mind, however, that the results of numerical experiment are purely logical consequences of the various theoretical approximations and simplifications initially assumed, difficult though it may be to trace through the effects of particular assumption.

An accelerated Lieberman method using finite difference in this report is developed for obtaining the distributions of flow and water temperature. In the model to be described, a coupled set of simultaneous nonlinear partial differential equations is transformed into difference equation system, which is solved numerically with the aid of electronic computer IBM SYSTEM 360. The Eulerian grid-point representation at points evenly spaced in a rectangular net was chosen principally because of its relatively straightforward program coding and the relatively large fund of knowledge available, pertaining to its characteristic behavior.

## II. FORMULATION OF THE PROBLEM

In order to obtain the distributions of flow and thermal diffusion in the sea basin off the outlet, it is necessary to consider both dynamic movement of released water and thermal diffusion of water temperature. As shown in Fig. 1, take the Cartesian coordinates in three dimensional space, the origin of which is taken as the center of outlet. The direction of three axis of the coordinates are as seen in the definition sketch (see Fig. 1). Let us assume that the outlet has a rectangular section,  $2B$  in breadth and  $H$  in height, from which the cooling water with an initial constant temperature  $T_0$  is released into the sea in the direction perpendicular to the coast.

In general, a velocity caused by discharged water in the bay has to be kept small. Because the small relative velocity between an upper and a lower layers keep the stability of the stratification in the bay. Therefore, the eddy viscosities are predominant in a field of the flow in the bay, and the field of flow are strongly subject to the influence of the coastal boundaries near the outlet. In this respect, it differs from the general phenomena of jet flow.

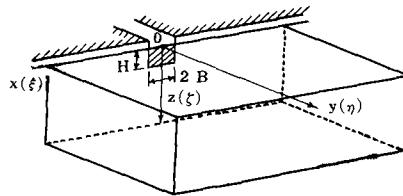


Fig. 1. Definition sketch for thermal spread.

The equations of motion in the  $i$  direction and the equation of continuity can be written as

$$\frac{\partial}{\partial x_i} \left( A_i \frac{\partial u_i}{\partial x_j} \right) = u_i \frac{\partial u_i}{\partial x_i} + \frac{\partial p}{\partial x_i} - \rho g \lambda_k \quad (1)$$

and

$$\frac{\partial(\rho u_i)}{\partial x_i} = 0 \quad . \quad . \quad . \quad (2)$$

where  $j = 1, 2, 3$  corresponds to the  $x, y, z$  direction respectively  $\lambda_k$  is the unit vector along the  $z$  axis  $u_i$  ( $i = 1, 2, 3$ ) are the velocity components along the  $x, y, z$  direction and the eddy viscosities corresponding to those directions are  $A_x, A_y, A_z$  respectively  $p$  the pressure,  $\rho$  the density.

On the other hand, the equation for the thermal diffusion is

$$u_i \frac{\partial T}{\partial x_j} = \frac{\partial}{\partial x_j} \left( \frac{K_i}{\rho} \frac{\partial T}{\partial x_i} \right) + \frac{Q_o}{\rho_u C_w H_u} \quad . \quad . \quad . \quad . \quad (3)$$

where  $K_i$  are eddy thermal diffusivities,  $Q_o$  represents the heat gain or loss for the surface layer of sea basin,  $C_w$  is the specific heat of water and  $H_u$  is the thickness of layer between the sea surface and the atmosphere, across which process of momentum and heat transfer occur.

An approximate relation between density and water temperature is

$$\rho = \rho_0 (1 - \alpha T) \quad . \quad . \quad . \quad . \quad (4)$$

where the density,  $\rho_0$ , is the standard density of the fluid. The main process for the heat balance in any part of the coastal region is shown in the following list (see Fig. 2)

Table - 1

Process of heating the sea basin	Process of cooling the sea basin
1 Absorption of radiation from the sun and the sky, $Q_s = Q' (1 - \bar{r})$	1 Back radiation from the sea surface, $Q_b = \sigma(T+273) \{1 - a - b/e(T_s)\} (1 - K_n)$
2 Convection of sensible heat from atmosphere, $Q_h$	2 Convection of sensible heat to atmosphere, $Q_h = h_a (T_a - T)$
3 Condensation of vapor, $Q_e$	3 Evaporation, $Q_e = k \{e(T_s) - e(T)\}$
4 Addition of waste heat from power plant, $Q_p = (\frac{Q}{A}) \{(1 - r) T_b + T_c - (1 - r) T\}$	

The following symbols are adopted for use in Table-1.

$Q_s$  radiation energy from the sun and the sky.

$\bar{r}$  average reflectance over the integration period

$T$  water temperature in the surface layer.

$\sigma$  Stefan-Boltzman's constant for black-body radiation

$a, b$  constants.

$e(T_s)$  the saturation vapour pressure at the sea surface in mb

- $K$  coefficient depending on the cloud height  
 $n$  cloudiness on the scale 1 to 10.  
 $T_a$ : atmospheric temperature.  
 $e(T)$  saturation pressure for water temperature.  
 $h_a$  heat transfer coefficient ( $= 2.77 \times 10^{-4} (0.48 + 0.272V)$ )  
 $k$  : mass transfer coefficient ( $k \approx 2h$ ).  
 $Q_i$  : intake discharge of cooling water  
 $A$  : surface area of bay.  
 $r$  mixing ratio from the upper layer.  
 $T_c$  rise of water temperature added by condenser of power plant.  
 $T_b$  : water temperature in the bottom layer.

The net-exchange rate across the sea surface to the above processes is then represented by the linear combination,

$$Q_0 = Q_i - Q_t + Q_h + Q_e = Q_i - Q_z T \quad \dots \quad (5)$$

The solution of this problem is divided naturally into two parts, one of which corresponds to the radiation effect and diffusion effect, and the other effect of internal diffusion mechanism which results from nonuniform temperature distribution in the water.

Boundary conditions on the velocity are taken to be on the free surface, flow parallel to the surface; at the fixed boundaries, velocity equal to zero. The thermal flux must be zero in normal direction to the boundaries except the sea surface or at the mouth of bay. The thermal gradient on the sea surface or at the mouth of bay should be remain constant

In the fundamental equations which govern phenomena of flow and thermal diffusion, the density enters explicitly. It might therefore be expected that variations of density in a vertical direction would modify the results, but the variations of the density in the sea are too small to be of importance in this respect. If the variations of the density are related to the gravitational effect, the product term of  $\rho g$ , would play an important role in the interaction between the flow and thermal diffusion. And therefore, it should be taken into account in the equations of motion. From this assumption, the equations of continuity can be replaced by the Boussinesq approximation.\*

\* Most studies of fluids with density gradients use the Boussinesq approximation (Boussinesq, 1903), which neglects density variations in the inertial terms of the equations of motion. The general meaning is that the density difference  $\Delta\rho$  occurs in the acceleration terms as

$$\left(1 - \frac{\Delta\rho}{\bar{\rho}}\right) \frac{dV}{dt}$$

where  $\bar{\rho}$  is the mean density, and therefore, if  $\Delta\rho/\bar{\rho} \ll 1$  the acceleration term may be simplified to  $dV/dt$ . But, the term  $g \Delta\rho/\bar{\rho}$  in the vertical equation of motion should not be neglected because  $g$  is large.

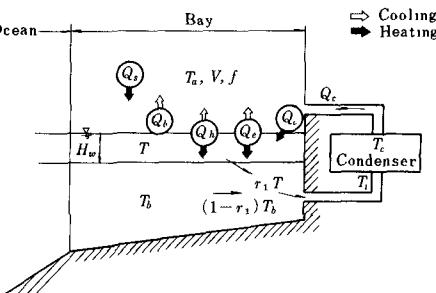


Fig. 2. Schematic diagram of main processes for heat balance

The problem now is to solve this nonlinear set of equations subject to the boundary conditions. Since the set is not tractable by any analytical methods known at present, it is necessary to resort to approximate methods for the finite difference solution.

### III NUMERICAL CALCULATION OF HEAT BUDGET IN A BAY

There would be two major modes of elimination of heat from power plants, one is the dissipation of heat to the atmosphere through radiation and the other is a gradual movement of warm surface water out to the open sea. The relative losses between each other would depend on meteorological and tidal conditions. The determination of the sea surface temperature not only provides a means of predicting the amount of recirculation for intake and outfall conditions, but also plays a definite and specific role in the determination of the effect of intake and outlet design on the amount of recirculation. In this case, it is very important for the effective utilization of the cooling water to investigate the water temperature distribution in the bay and its change as time goes.

The technique with the aid of an analogue computer for estimating the distribution of water temperature in the bay has been developed to provide a reasonably accurate procedure which requires only available meteorological data. The heat balance of the water in the bay is calculated by integrating the equation of heat-conduction over the entire water masses of the bay. The heat transfer in the atmosphere and the bay water are then calculated under the following assumptions:

1) The bay is divided into some blocks for mathematical development. Thermal diffusion and tidal effect would be taken into consideration in executing numerical simulation.

2) During summer, there exists a remarkable layer of thermocline at 3 ~ 4 m below the sea surface in the bay. This interface is stable in spite of tidal changes and wind.

3) The warmed cooling water would spread from the outlet in the form of a thin layer on the surface of the bay, and therefore high temperature content of discharged cooling water is not diffused into the lower layer through the layer of thermocline with large stability.

4) The tidal change is simulated by the sinusoidal curve due to the field survey.

5) The water temperature at the open sea is equal to normal temperature not influenced by the discharged warm water.

In the above case of treating the problem of time history of water temperature in the calculation of heat budget, the meteorological data such as the radiation, atmospheric temperature, need the records with the variation of a day. Such calculations give only a rough estimate these phenomena, but they serve to give an approximately quantitative idea of the interplay between the bay water and the atmosphere. From these calculated values the general features of the daily change of the heat loss or gain from the sea surface can at least be grasped.

Wada, A. and N. Katano<sup>2)</sup> have applied this technique to some bays in which the recirculation of cooling water comes into question, and obtained the balancing water temperature in the surface layer and intake water temperature against parameters such as the dimensions of bay, meteorological condition, the tidal flow, and the cooling water flow.

Take, for instance, the case of J. Power Station. This station is planned to be located on the innermost of T. Bay. T. Bay is similar to a rift valley in shape. The width of the bay at the innermost is about 300 m with a depth of about 15 m. The width and depth increase gradually toward the mouth and the widest section is about 700 m at the mouth of the bay, while the depth increases to about 25 m. Its length is about 1,700 m. From the result of the field survey, it is clear that the waters of the inner bay and the outer sea are interchanging, and that the tempera-

ture of the bottom water below the interface between two layers in the bay about  $25^{\circ}\text{C}$

The cooling water intake will be located at the innermost of the bay of 13 m deep, and the warmed water is returned to the same bay through the outlet about 200 m distant from the intake. The water in the bay has a pronounced temperature stratification. One should intend to use the bottom layer water which has a more suitable temperature for the cooling condenser with a view to achieving a higher efficiency and reducing the trouble associated with impurities in the water.

From the above-mentioned assumptions, the rate of temperature rise  $dT/dt$  in each block, taking the effects of tidal current and thermal diffusion into account, is given by

$$\frac{dT_1}{dt} = \frac{1}{C_{w\mu}(H_w + \eta)} \left\{ (Q_{ps} - Q_e - Q_r) + \frac{1}{A_1} (Q_{e_1} - Q_e - Q_{r_1} + Q_{s_1} + Q_{t_1}) \right\} \quad \begin{array}{l} \text{for No 1 Block} \\ \text{for No 2 Block} \\ \text{for No 3 Block} \end{array} \quad (6)$$

$$\frac{dT_2}{dt} = \frac{1}{C_{w\mu}(H_w + \eta)} \left\{ (Q_{ps} - Q_e - Q_r) + \frac{1}{A_2} (Q_{e_2} - Q_e - Q_{r_2} + Q_{s_2} + Q_{t_2}) \right\}$$

$$\frac{dT_3}{dt} = \frac{1}{C_{w\mu}(H_w + \eta)} \left\{ (Q_{ps} - Q_e - Q_r) + \frac{1}{A_3} (Q_{e_3} - Q_e - Q_{r_3} + Q_{s_3} + Q_{t_3}) \right\}$$

in water column of cross section 1  $\text{cm}^2$  and thermocline depth of  $H_w + \eta$  (see Fig. 3). Figure 4 represents schematic diagram of flow and heat advection in T bay. Energy losses from the sea surface to the air by contact with cooler air and that due to evaporation are dealt with as negatives. The sign in the equation (6) represents a positive one for incoming heat energies into blocks. In the above equation (6), the definitions are as follows

$Q_{ps}$  the net radiation ( $= Q_s - Q_b$ )

$Q_s = \alpha T_a + \beta$  ( $\alpha = h(2/m+1)$ ,  $\beta = 2hn(1-f)$ )

$Q_b = rT(r=h(2m+1))$

$\eta(t)$  tidal change,

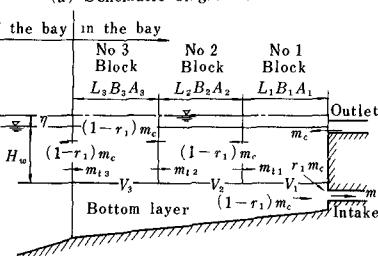
$A_i$  surface area for each block,

$Q_a$  heat energy in each block by the discharge of warm water,

$Q_i$  heat energy transferred by the mixing from upper layer,

$Q_h$  heat energy advected by the tidal current.

(a) Schematic diagram of flow



(b) Schematic diagram of advection of heat energies

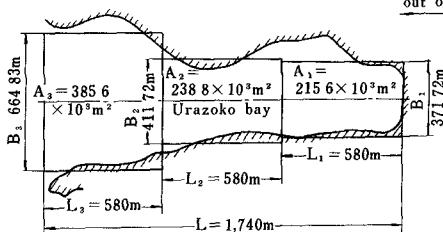
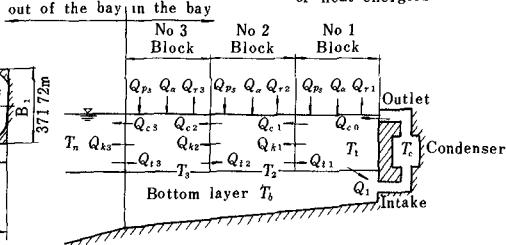


Fig. 3. Choice of block

Fig. 4. Schematic diagram of flow and heat advection in T. Bay

In the following numerical calculations, the undermentioned values were used for the various factors in the heat budget, according to the meteorological data observed in the summertime at T bay over the period of twenty years

Average air temperature  $28.5^{\circ}\text{C}$   
 Average wind velocity  $V = 3.3 \text{ m/sec}$   
 Relative humidity  $\cdot 79\%$   
 Cloud amount  $\cdot n = 6.3$   
 Coefficient depending on cloud height  $K = 0.083$

The above-mentioned records represent the values of average state. However, in the case of treating the problem of time history of water temperature in the calculation of heat budget, the meteorological data such as the radiation, atmospheric temperature, need the records with the variation of a day. These data were chosen from the records taken in site. Figures 5 and 6 represent the daily variations of the radiation and the atmospheric temperature. From the process of convergence of the water temperature in the surface layer at natural condition, the

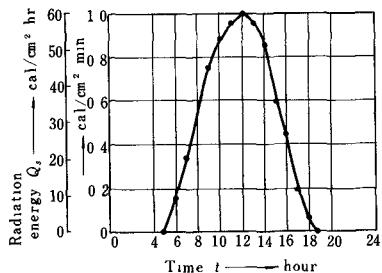


Fig. 5 Daily variation of radiation at the T. site (July-August, 1964)

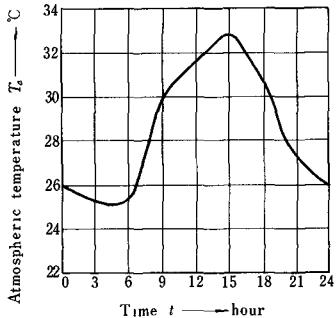


Fig. 6 Daily variation of atmospheric variation at T. site (July - August, 1964)

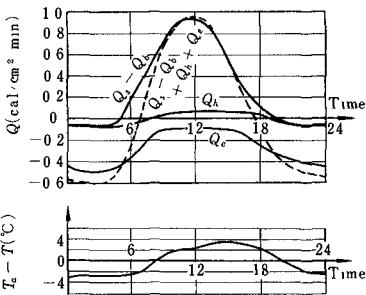


Fig. 7 Heat budget in the surface layer (Natural condition).

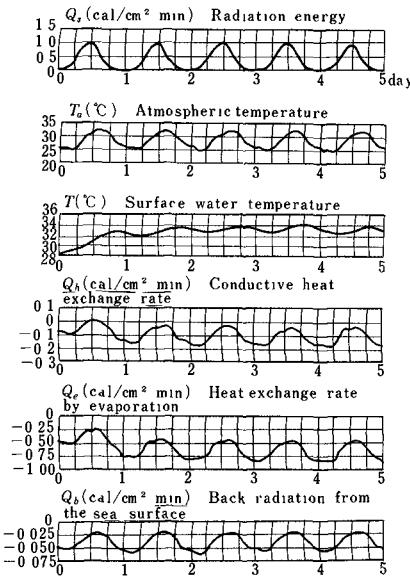


Fig. 8 Process of convergence of water temperature in the surface layer affected by discharged warm water

daily variation of heat budget was obtained as shown in Fig. 7. Figure 8 shows the process of convergence of the water temperature in the surface layer effected by the discharged warm water. Based on heat-budget concepts, graphs have been developed to permit the estimation of the water temperature of sea surface, taking into consideration of the addition of heat by plant to bay (see Fig. 9)

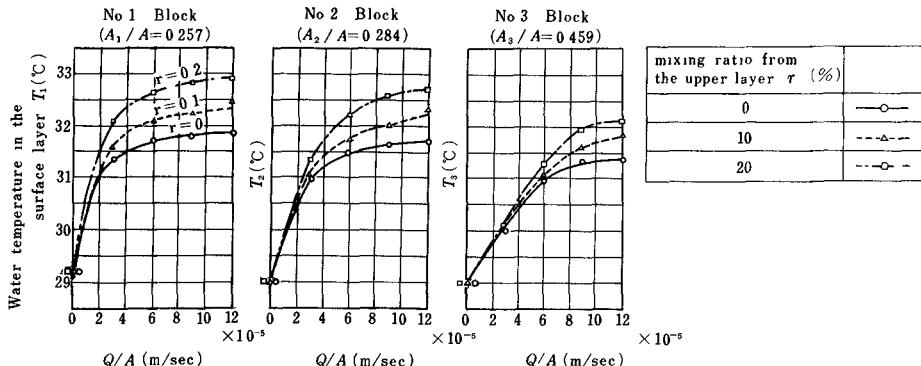


Fig. 9 Relation between water temperature in the surface layer and  $Q/A$  for mixing ratio  $r$

#### IV. INTERACTION BETWEEN FLOW AND THERMAL DISTRIBUTION

##### 4.1 Preface

Generally speaking, phenomena of flow and thermal diffusion caused by the outfall of warmed cooling water seem to be composed of complex processes. The field of flow would change that of water temperature distribution in the vertical section. The diffusion process of warmed water would also change the field of flow. Thus, these processes of two phenomena can not be considered independently each other, but must be taken into account of the interaction between these two phenomena.

In order to confirm the realization of the above mentioned matter, it is necessary to conduct the numerical experiments on the thermal diffusion in the vertical section taking into account of the thermal diffusion-velocity correlation.

##### 4.2 Actual state of thermal diffusion of cooling water and stability of stratified distribution in density

The following points were made clear as the result of the discussion carried out to make clear possibility of intake and outfall of cooling water in the same basin of bay, in confirmation of preparatory investigation data for water temperature of the sea around the Mizushima Bay and other research data

- 1) The thermocline is stable in spite of tidal change. The eddy viscosity near the boundary is very small (about 0.05 c.g.s.) and the stratification of density is not destroyed by mixing caused by the prevailing winds of 5 ~ 8 m/sec.
- 2) Based on the data of the field survey, the thermal diffusion coefficients are calculated. It is found that the horizontal thermal diffusivity is at least 50 times greater than the vertical, the order of which seems to be about 0.01 m<sup>2</sup>/sec. Accordingly, it is concluded that the decrease of the temperature observed in the field tests should be due to the horizontal

mixing with the surrounding waters of lower temperature, with addition to the process of cooling the sea surface and the process of entrainment of the lower colder water to the upper layer current.

Generally, the warmed cooling water discharged from the outlet flows as an upper layer current partly because of its inertial momentum and partly because of its lower density. The upper layer current thus exerts the tangential stress on the lower layer which favours the compensation current along the sea bottom. That is, the water temperature in the lower layer is almost unchanged, but there is a slow movement of lower layer upstream from the open sea to compensate for the water lost by entrainment. It seems that the surface outfall of cooling water gives rise to upwelling motion near the outlet.

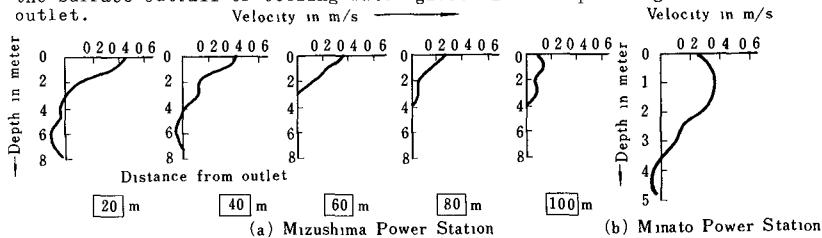


Fig. 10. Vertical variation of velocity off the outlet

Until recently the investigation of upwelling motion was not enough to afford any insight into the inner mechanism of this phenomenon. Detailed systematic field surveys of the upwelling phenomena have been made at power plants located on the Mizushima

Bay and the Miike Port. The results

indicate the occurrence of the return flow in the bottom layer due

to the outfall of cooling water

Figure 10 represents the state of occurrence of the return flow in the bottom layer of the coastal region off the outlet by the outfall of cooling water at the Mizushima and the Miike Thermal Power Plants. The upwelling motion may be also understood by the vertical structure on the thermal diffusion of Fig. 11, in which the uniform rise of the isothermal line towards coast is a particularly marked feature of the thermocline structure of the upwelling region.

This compensation current phenomenon probably occurs at the sea region off shore the outlet. The upwelling brings water of greater density and also leads to change in the distribution of mass, and exercises therefore a widespread influence upon the sea conditions off coasts where the process take place.

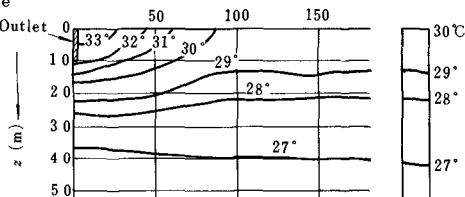


Fig. 11. Vertical distribution of water temperature along the  $y$ -axis perpendicular to the shore in M Bay

## V DIFFUSION IN THE DENSITY STRATIFIED SEA

In the theory of locally isotropic turbulence, it was found originally by Kolmogoroff that the eddy viscosity and eddy diffusivity increase with the scale of the phenomena concerned. The basic idea is that a wide spectrum of turbulent motions, or eddies, exists with scales varying from less than a centimeter up to the dimensions of the body of water itself. There exists an intermediate range of eddies which are in energy equilibrium. It is shown that in this range  $K \propto L^3$ , where  $L$  is a linear dimension representative of the scale of diffusion being

studied. A number of authors have suggested that this law should be applicable to horizontal diffusion in the sea. In a stratified sea the application is complicated by the effect of density currents and the shear effect. It would seem unwise to assume that the 4/3 power law applies in a given stratified condition unless an independent experiment there has indicated that it does so.

The process of turbulent exchange in the stable stratified sea is different from that in the homogeneous sea region. The existence of the density gradient may introduce into the energy spectrum a new equilibrium range—the "buoyancy subrange"—situated in a range of lower wave numbers than the inertial subrange, as suggested by Bolgiano (1959)<sup>3)</sup>. As for the vertical diffusivity  $K_z$ , on the contrary, the 4/3 power law will not be useful as much as for the horizontal diffusivity, not only because of a fact that the integral scale of turbulence is comparable or even smaller than the scale of phenomena in the vertical direction, but also because of a strong dependence of  $K_z$  on the stratification is still lacking.

An application of the transfer theory or the mixing length theory has been extensively accepted, as a useful approximation, to practical problems of turbulent diffusion in the sea. Various empirical formulas have been proposed to represent the variation of  $K_z$ , or the mixing length associated with it, with  $R_i$ . Taylor (1931)<sup>4)</sup> examined on turbulent phenomena of very small scale and pointed out that the influence on the vertical exchange and the diffusion of momentum were of many kind and that the process of exchange was isotropic. A semi-empirical study based on the mixing length theory was first developed by Rossby and Montgomery (1935)<sup>5)</sup> by introducing certain stratification parameter—Richardson number—into the expression for the mixing length. Thus, the application of their idea to the coefficient of diffusion results in

$$K_z = \ell_s^2 \left| \frac{du}{dz} \right| = \ell_0^2 \left| \frac{d\bar{u}}{dz} \right| (1 + \beta R_i)^{-1} = K_z^o (1 + \beta R_i)^{-1} \quad \dots \quad (7)$$

where  $\ell_s$  and  $\ell_0$  denote the mixing length for the stable stratified and the nonstratified sea, respectively, and  $R_i$  represents the Richardson number given by

$$R_i = (g \partial \rho / \partial z) / (\rho \partial p / \partial z)^2$$

Different considerations of the exchange mechanism in the stratified sea produce different expressions for the coefficient of diffusion on the degree of stability.

The influence of stability on the vertical mixing process due to turbulence has also been treated by Mamayev (1958)<sup>6)</sup>, who considered that the appropriate forms for  $A_z$  and  $K_z$  were

$$K_z = K_z^o e^{-n R_i}, \quad A_z = A_z^o e^{-m R_i} \quad \dots \quad \dots \quad \dots \quad (8)$$

where  $n - m > 0$ . From Jacobsen's data it was deduced that  $n = 0.8$ ,  $m = 0.4$ .

The effect of the density stratification in the vertical section on the horizontal exchange of turbulence was first investigated by Parr (1936)<sup>7)</sup> and Bowden (1965)<sup>8)</sup>. The results show that the turbulent exchange in the stable stratified sea is very nonisotropic and that the effective coefficient of horizontal diffusion is inversely proportional to the coefficients of the vertical eddy diffusion. The occurrence of a stable gradient of density therefore increases the effective horizontal mixing very considerably. This conclusion is inconsistent with that led by Taylor previously mentioned. But, Taylor's theory concerns with the exchange process of very small scale. The above conclusion is applicable only to phenomena with long duration. R.V. Ozmidov (1965)<sup>9)</sup> proposed a new model of turbulent exchange in the stable stratified sea. From the theory of locally isotropic turbulence, the mean velocity gradient of the turbulent eddies is given by the following equation

$$\frac{du}{d\ell} = c \epsilon^{1/3} \ell^{-2/3} \quad \dots \quad \dots \quad \dots \quad \dots \quad (9)$$

where  $c$  is a universal dimensionless constant,  $\epsilon$  is the energy dissipation rate and  $\ell$  is the distance. The critical scale  $\ell_{cr}$  that the turbulent field loses its non-isotropy may be obtained by taking  $\ell_{cr}$  as  $\ell$  when  $R_i$  approaches a finite value, a

The effect of shear existing in the Richardson number  $R_i$  may be replaced with equation (9). The critical scale  $\ell_{cr}$  depends on energy dissipation rate  $\epsilon$  and on parameter  $\beta = (g/\rho)(\partial\rho/\partial z)$ . In this case  $\ell_{cr}$  is given by

$$\ell_{cr} = \left( \frac{ac^2 \rho^{\frac{2}{3}}}{g(\partial\rho/\partial z)} \right)^{\frac{3}{4}} = \alpha \epsilon^{\frac{1}{3}} \beta^{-\frac{3}{4}} \quad . \quad (10)$$

where  $a$  is a dimensionless constant.

The values of the vertical and horizontal turbulent exchange coefficient  $K_h$  and  $K_z$  also depend on parameters  $\epsilon$  and  $\beta$ . Within scales varying from the critical scale  $\ell_{cr}$ , up to the "Kolmogorov micro scale",  $\ell_0 = (\nu^3/\epsilon)^{\frac{1}{4}}$ , where  $\nu$  is the kinematic viscosity, the turbulent fields have the isotropy even in the stratified sea. For eddies with  $\ell > \ell_{cr}$ , large turbulent eddies have only vertical axis. That is, we have  $K_h > K_z$  for large scale exchange because the diffusion velocity has its upper limit even if the spot of diffusion becomes large.

The maximum value of  $K_z$  can be estimated on the basis of the 4/3 power law from the theory of locally isotropic turbulence. That is

$$K_{z,max} = c_1 \epsilon^{\frac{1}{3}} \ell_{cr}^{\frac{4}{3}} = \frac{a c^2 c_1 \rho \epsilon}{g(\partial\rho/\partial z)} \quad . \quad (11)$$

where  $c_1$  is an universal constant.

The proposed turbulent model in the stable stratified sea is summarized as follows (see Fig. 12). For small scale turbulent exchange there is a three dimensional turbulence, that is  $K_h = K_z$ . But for exchange phenomena larger than  $\ell_0$  we have  $K_h > K_z$ . Then, the value of  $K_h$  is controlled by the 4/3 power law in the horizontal space. As the dimension of the scale becomes large, scale of turbulence amounts to the maximum eddy with  $\ell_1$  in the horizontal direction. This dimension corresponds to several kilometers in the ocean.

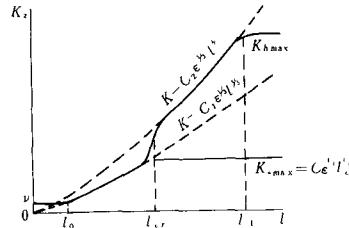


Fig. 12 Relation between scale of phenomena and  $K_h$ ,  $K_z$  in the stable stratified sea

## VI METHODS OF SOLVING FUNDAMENTAL EQUATIONS

The distributions of flow and water temperature in the continuously stratified sea caused by outfall and intake of cooling water must be obtained by taking account of the interaction between flow velocity and thermal distribution. In order to obtain the velocity and the density fields, the numerical experiments is conducted in the vertical section. In the fundamental equations which govern phenomena of flow and thermal diffusion, the density enters explicitly. In this report, the Boussinesq approximation is used throughout, which neglects density variations in the inertial terms of the equations of motion. From this assumption, the equation of continuity can be replaced by this approximation. And therefore we can introduce a stream function  $P$  into the fundamental equations. Replacing the equations (1)~(4) by a Laplacian equation of the stream function, an equation satisfying the vorticity, and an equation for the thermal diffusion, we obtain the following equations (12)~(14). To obtain the equation for the transfer of vorticity, the pressure  $p$  is eliminated between equations in the  $x$ - and  $z$ -directions of equation (1) by cross-differentiation and subtraction. Solution of the equation

of thermal diffusion with a complete set of Eulerian equations may be obtained numerically.

$$\xi = -\frac{1}{2} \left( \frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial z^2} \right) \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (12)$$

$$2 \left( \frac{\partial P}{\partial z} \frac{\partial \xi}{\partial x} - \frac{\partial P}{\partial x} \frac{\partial \xi}{\partial z} \right) = - g \alpha \rho_0 \frac{\partial T}{\partial x} + 2 \left( A_h \frac{\partial^2 \xi}{\partial x^2} + A_i \frac{\partial^2 \xi}{\partial z^2} \right) \\ + \frac{\partial}{\partial x} \left( \frac{\partial A_z}{\partial z} \frac{\partial w}{\partial z} \right) - \frac{\partial}{\partial z} \left( \frac{\partial A_z}{\partial z} \frac{\partial u}{\partial z} \right) + \frac{\partial A_z}{\partial x} \frac{\partial^2 w}{\partial z^2} - \frac{\partial A_z}{\partial z} \frac{\partial^2 u}{\partial x^2} \quad \dots \quad \dots \quad (13)$$

$$\frac{\partial P}{\partial z} \frac{\partial T}{\partial x} - \frac{\partial P}{\partial x} \frac{\partial T}{\partial z} = \frac{\partial}{\partial x} \left( \frac{K_z}{\rho_w} \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial z} \left( \frac{K_z}{\rho_w} \frac{\partial T}{\partial z} \right) \quad \dots \quad \dots \quad \dots \quad (14)$$

The term of thermal horizontal gradient is contained in the first term of the righthand side of equation (13). And this term could probably be interpreted as one having power of binding the flow associated with addition of the warmed cooling water.

The effect of thermal gradient is omitted, in the first order approximation, to simplify the mathematical development. Solutions of the first order approximation are applicable only to bay having negligible stratification or gravitational convection. A more general solution is obtained by the combination of equations (12), (13) and (14), by applying these first order values obtained above. If the processes are repeated, the required solution will be obtained finally. Solutions thus obtained give the values which are influenced, to some extent, by the thermal distribution-velocity interaction.

In the experimental treatments of the previous paper<sup>11</sup>, the coefficients of eddy diffusivity and eddy viscosity were assumed to be constant despite the fact that the magnitude of the coefficient depends strongly on the local turbulence. The existence of thermal activity and buoyancy effects add great complexity to the exchange problem. Generally speaking, the effect of a thermal stratification on the state of turbulence is described by means of the local Richardson number.

It is, in fact, doubtful that a universal equilibrium theory can be established to relate turbulent transfers to mean gradients, regardless of scale and for the entire range of the Richardson number. The assumption we will make essentially imply above mentioned theory. Thus we must apply a considerable amount of empiricism, and can not expect the resulting expressions to be necessarily valid for all physical frameworks.

In this treatment, the eddy coefficients of viscosity and diffusivity are assumed to be of form proposed by Mamayev. This formulation must have a finite value as  $R_i \rightarrow \infty$ . And therefore, the following relations are adopted

$$A_i = (A_i + A_0 e^{-mR_i}) \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (15)$$

$$K_z = (K_1 + K_0 e^{-nR_i}) \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (16)$$

where  $A_i$ ,  $A_0$ ,  $K_1$  and  $K_0$  are constants.

## VII NUMERICAL EXPERIMENTS OF THERMAL DIFFUSION

In this report, the most basic contributions to the velocity fields in the sea are the net outward flow required for disposal of the warmed cooling water, and the gravitational convection due to the density difference between warmed water and lower water. Numerical solutions have been obtained for the coupled system of partial differential equations describing the flow circulation and the water temperature distribution for a given sea condition with meteorological data. These solutions provide new insight into the interaction of the water temperature and velocity fields as a control mechanism in coastal dynamics. The mechanism of development of upwelling motion could also be confirmed by numerical experiment. Figures 13 and 14 represent respectively the vertical profiles of water temperature and velocity along the longitudinal section obtained by the calculation.

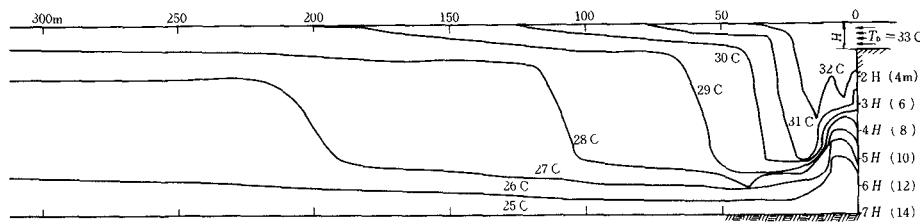


Fig. 13. Water temperature distribution along a longitudinal section.

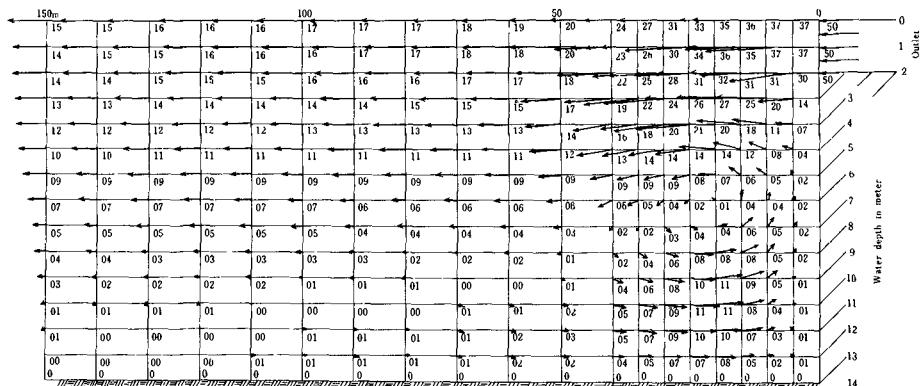


Fig. 14. Velocity distribution along a longitudinal section.

The general feature of the velocity solution is shown in Fig. 15, where the value,  $A_2/A_0 = 0.1$ , has been used. This figure utilizes only the first and the fourth order velocity. These curves show a surface outflow and a deeper inflow of water. Higher order approximation, taking into account of interaction, has a considerable effect on the shape of the vertical profiles of water temperature and velocity.

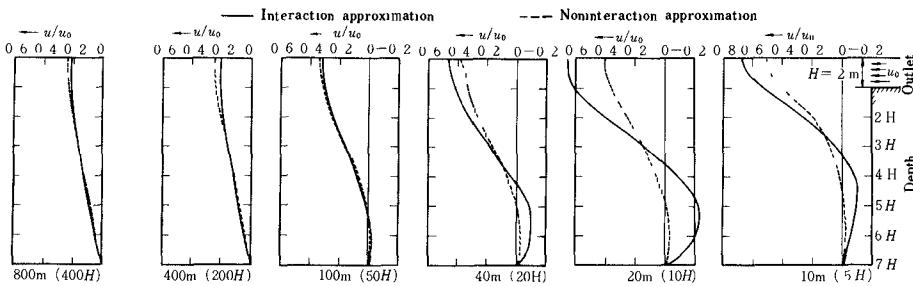
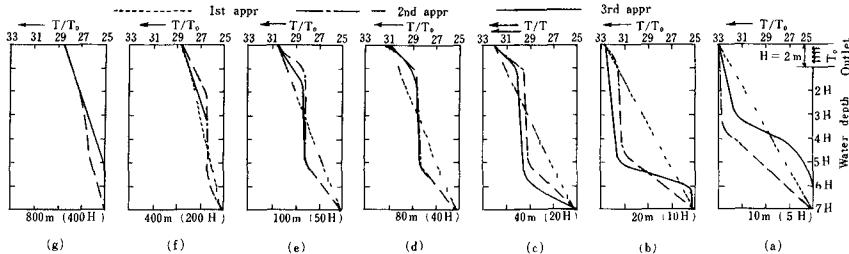


Fig. 15. Variation of velocity profile.

An interaction solution in the vicinity of the outlet gives the velocity larger or smaller than that given by the first order solution at the surface or at the bottom. However, off the outlet, differential advection between two layers will tend to develop a density instability in these layers, giving a much increased vertical coefficient of diffusivity. The motion of water particles near the outlet can be grasped from velocity distribution along a longitudinal section shown in Fig. 14. From these series of experiments, it can be seen that colder water in the lower layer is sucked into the upper layer along the vertical wall, and that the temperature of released water in the upper layer is decreased, to some extent, by entrainment and its volume also increases as it moves offshore. This upwelling action will also change the vertical profile of the water temperature. This can be understood by the process of formation of the thermocline shown in Fig. 15. The upper movement of isothermal lines near the outlet would be probably determined from the relation between the turbulent intensity of cooling water flow and upwelling motion. In this case, it seems that the upwelling phenomenon is excelled in the fluid motion near the outlet. Figure 13 is similar to the vertical structure on the thermal profile of the field survey shown in Fig. 11, in which the uniform rise of the isothermal lines toward the coast is a particularly marked feature of the thermocline structure of the upwelling region. The numerical solutions give a good representation of all features of flow and thermal profiles. From the velocity profile at the site 80 m (or 40  $H$ , where  $H$  is the height of outlet) shown in Fig. 15, the difference between the first and the higher order approximations can't be almost found.

The process of the formation of water temperature distribution is different from that of velocity fields. As the solution goes to a higher order approximation the heat energy caused by the outfall of warmed cooling water has a tendency to accumulate in the form of wedge shaped in the surface layer (see Fig. 16)

Fig. 16 Profile of water temperature distribution  
 $K_i = K_i + K_{i0} e^{-n_i R_i}$ ,  $A_i = A_i + A_{i0} e^{-m_i R_i}$ .

This phenomena can also be understood by the distribution of water temperature at the site far off the outlet. The results do suggest that the mixing process resulting in the entrainment into the upper layer predominates over that by the eddy diffusion near the outlet. Therefore, refinement of the influenced range of flow and water temperature rise by the surface outfall is obtained by taking the supply of the lower water into account. The subject of future research would be the theoretical description of upwelling in a stratified, two-layer sea. This point is being investigated.

### VIII111 CONSIDERATION OF COLD WATER INTAKE FROM A STRATIFIED FLUID

Finally, problem on the colder water intake from the bottom layer in the sea with relation to design of the intake structure of cooling water is considered.

Up to present, the study on the selective withdrawal intaking colder water of lower layer has been carried out on the assumption that the sea water has a two-layer system with a discrete interface across which the density changes abruptly. In the field, however, such a well-defined interface does not usually occur. The vertical distribution of the density presents continuous profile by development of the intermediate layer when the mixing between two layers is promoted by the wind action. However, an equivalent interface has been assumed to exist at the place in the depth at which the vertical gradient of temperature or density is a maximum.

It appears desirable to extend the theory and experiment to systems with continuous density stratification. Namely, paying attention to the dynamic interactions between the density (or temperature) and the velocity distributions, intake equipments to promote the efficiency of cooling water intake should be designed.

Because of the inherent difficulty in making steady experiments with continuously stratified flows, the results are not very conclusive. For a given intake geometry, the maximum colder water discharge that can be withdrawn without inducing appreciable withdrawal from the upper layer in the sea with two-layers was only determined. It is interesting to investigate the selective withdrawal problem of a fluid with known density from a stably stratified fluid in the sea because little work has been done in the field of continuously stratified flow.

Theoretical studies of incompressible, steady, viscous flow towards a sink in a stably stratified fluid were made by W R Debler (1959)<sup>10</sup>, Yih, O'Dill and Debler (1962)<sup>11</sup>, Robert C.Y. Koh (1966)<sup>12</sup>, and Onishi and Hino (1967)<sup>13</sup>. In these studies, it is assumed that the density increases linearly from the sea surface to the bottom and that the intake opening is taken as a line sink.

In the present report, the numerical experiments of the stratified fluid with a continuous density gradient flowing into the submerged intake with a finite opening is undertaken to gain additional insight into the mechanism of density flow in the field of the continuously stratified fluid. In this section, some calculations are made to obtain the distributions of flow and water temperature at varying velocities.

The intake structure is in the form of submerged curtain wall with a finite opening. The height of opening is 3 m.

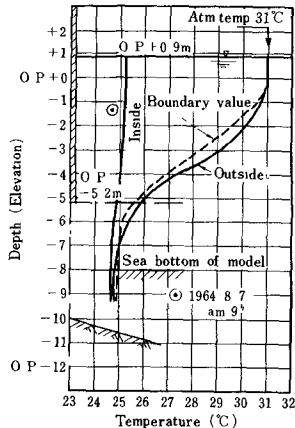


Fig. 17 An example of field survey and setting up of boundary value.

The numerical model contains the dimensions of 9 m in water depth, 390 m far off the intake structure in longitudinal section. This model corresponds to the intake type of curtain-wall structures at Sakai-Port Thermal Power Station. Figure 17 shows the cross section of curtain-wall type for this Power Station. An example of the result of temperature measurements for bottom water intake is shown in this figure. The conditions of calculation is setting up on the basis of the result of the field survey at the Sakai-Port site.

The boundary conditions of numerical calculation are given as follows.

- (i)  $T = 31^{\circ}\text{C}$  at the sea surface,  $T = 25^{\circ}\text{C}$  at the sea bottom,
- (ii) The water temperature for the sea infinitely far off is set up by solving the diffusion equation  $\frac{d}{dz} \left( \frac{K_z}{\rho} \frac{dT}{dz} \right) = 0$  with the model of diffusivity,  $K_z = K_0 + K_1 e^{-mz}$ .

Its result is shown in Fig. 17 with the dotted line. The general features of the velocity solution by numerical experiments is shown by Fig. 18. In this case, the values,  $A_z/A_x = 0.1$ ,  $K_z/K_x = 0.1$ ,  $K_1 = A_1 = 0.01 \text{ m}^2/\text{sec}$  and  $K_0 = A_0 = 0.1 \text{ m}^2/\text{sec}$  have been used. The values of intake velocity as a parameter are  $u_s = 0.2 \text{ m/sec}$ ,  $0.5 \text{ m/sec}$ . This figure gives the first and the fifth order velocity. Figure 19 represents a result of the vertical profiles of water temperature accompanied by bottom water intake ( $u_s = 0.5 \text{ m/sec}$ ). Higher order approximation has a considerable influence on the vertical profiles of the velocity and the water temperature in the sea region. The interaction solutions in the vicinity

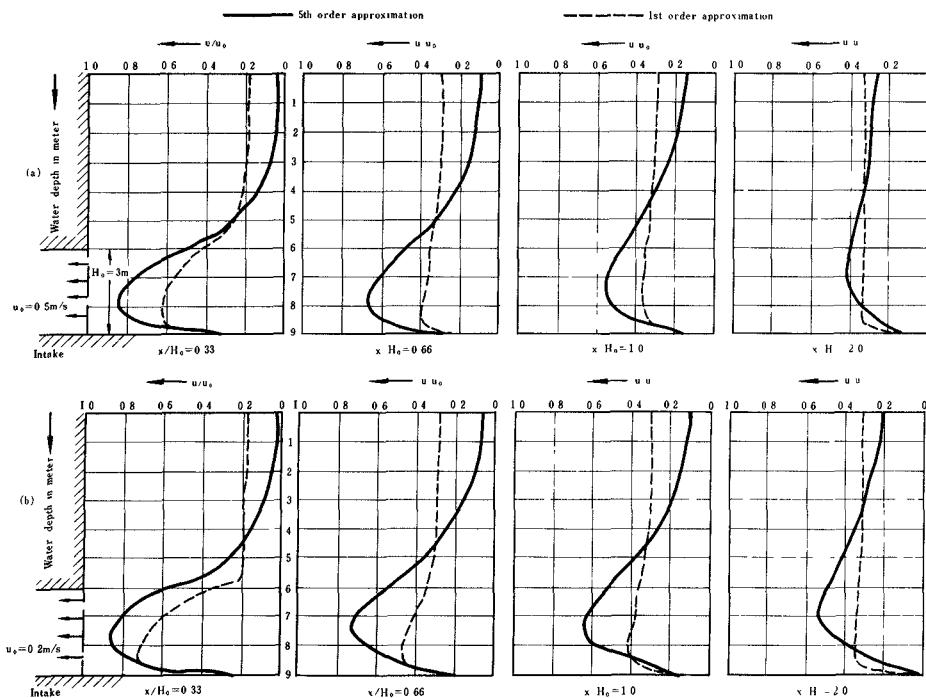


Fig. 18. Variation of velocity profile.

of the inlet give larger than that give by the first order solution especially at the bottom layer. From Figs. 18 (a) and 18 (b), the characteristics of the intake for the different values of intake velocity may be understood. If one makes the intake velocity small, it can be seen from Fig. 18 that we can expect effectiveness of intake.

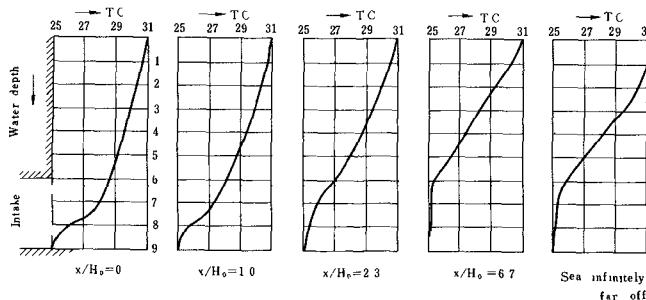


Fig. 19 Profile of water temperature

Figure 20 shows the distribution of the water temperature near the inlet for the two values of intake velocity. The full lines means the case of  $u_0 = 0.2$  m/sec. From Figs. 19 and 20, it can be seen that the vertical profile of water temperature in the sea is not subject to the influence of the intake for the section far off  $x/H_0 = 5 \sim 6$  ( $H_0$  is the intake opening,  $x$  is the distance from the inlet) from the intake structure.

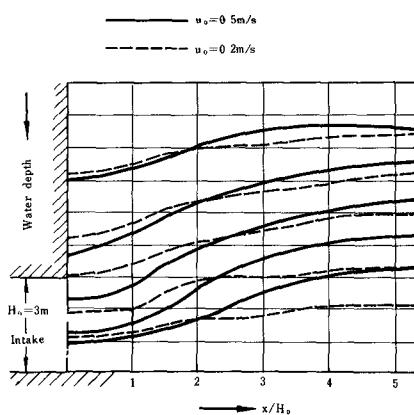


Fig. 20 Water temperature near outlet.

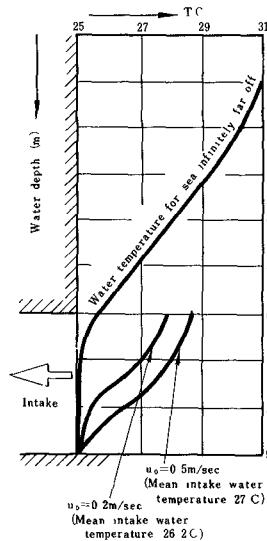


Fig. 21 Intake water temperature.

The result of calculation of the intake water temperature from the continuously stratified field is shown in Fig. 21 for the two values of intake velocity. The water temperature of intake,  $T_i$ , is 27 °C at the intake velocity of  $u_0 = 0.5$  m/sec,  $T = 26.4$  °C at  $u_0 = 0.2$  m/sec. There is less change in the value of the water temperature of intake than it was expected. In the continuously stratified field, there exist a flow which has only a small velocity in the surface layer. This flow toward the coast in the surface layer changes into the flow towards vertical down in the neighborhood of vertical wall upper the intake opening.

The results lead to the conclusion that the flow in the field of the continuous distribution of density differs slightly from that for a two layer system having a discrete interface. However, these results seem support the view that the application of a two-layer system to the design of the practical intake structure is possible. Further studies in this respect must be the subject of future research.

#### CONCLUSIONS

The following points were made clear as the results of the field surveys and the numerical experiments. Those were carried out to make clear the possibility of the intake and the outfall of cooling water in the same basin.

- 1) Numerical solution of the equation of thermal diffusion with a complete set of Eulerian equations in which the values of the eddy coefficients of diffusion and viscosity depend upon the relation to the velocity shear and vertical density gradient was presented. Our efforts to develop a numerical model capable of simulating density flow problems have evidently been partially successful. Results of the computation are in reasonable agreement with the observed features. It also seems that the eddy exchange terms in the vertical section partially satisfy a real physical requirement.
- 2) The mechanism of development of upwelling phenomenon was made clear from the numerical experiment, taking into account of the interaction between the flow and the thermal distribution. It was made clear that the colder water in the lower layer was sucked into the upper layer along the vertical wall of the coast. And therefore, the mixing process resulting in the entrainment into the upper layer predominates over that by the eddy diffusion near the outlet. These phenomena were also confirmed by the field surveys with respect to the velocity and the water temperature off the outlet.
- 3) The numerical experiments of the stratified fluid with a continuous density gradient flowing into the submerged intake opening were conducted. The results lead to the conclusion that the flow in the field of the continuous distribution of density differs slightly from that for a two layer-system having a discrete interface. However, these results seem support the view that the application of a two-layer system to the design of the practical intake structure is possible.

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