

CHAPTER 88

SHEAR VELOCITY IN A TIDAL ESTUARY

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ABSTRACT

A study involving the determination of the bed shear stress and thence shear velocity in the Narrows of the River Mersey (a well mixed tidal estuary) is described. The two dimensional equation of motion is examined and it is concluded that in order to determine the energy slope (S) (and thence the shear stress and shear velocity) it is essential to evaluate density, inertia and kinetic energy terms as well as surface slopes. Although the density term is much smaller than the other terms it is important at periods of low slack-water.

Measurements of the various terms are described and attention is drawn to the difficulties arising in the case of the surface slopes. Comparison is then made between the values of the shear velocity as predicted from the energy slope and those obtained using velocity traverses throughout the depth and with measurements of velocity at three fixed positions near to the bed. Finally consideration is given to the relationship between the cross-sectional mean velocity ($\bar{U}$), the depth-mean velocity ($\bar{U}$) and velocity at 3ft. above the bed (U_3) and the shear velocity obtained from the energy slope. It is concluded that there is good linear correlation of each of these velocities ($\bar{U}$, $\bar{U}$ and U_3) with the shear velocity throughout the tidal cycle except for the period around low slack-water.

INTRODUCTION

The shear velocity (u_*) is one of the basic parameters which govern the vertical distribution of sediment in uni-directional flow. It is to be expected that u_* will be as important in the case of tidal flow. However, in this case the variation of the shear velocity with time can be quite rapid; this added complication implies that the problem is essentially more complicated for tidal flow than for the uni-directional case.

Recent research by the junior author has shown that the distribution of sediment in a tidal flow can be described by equations similar to those of the uni-directional flow, e.g. the equation as used by Rouse, Vanoni, Einstein, (ref. 1, 2 and 3) provided the suspension exponent $z = W/\beta Ku_*$ is adjusted for the temporal variation of u_* (W is the fall velocity of the sediment particles, K is Von-Karman's constant and β is a numerical constant). The determination of u_* for the case of tidal flows is therefore important for the description of sediment distribution which, in turn, is required if a quantitative analysis of the sediment transport within a tidal area is to be made.

ANALYSIS

The longitudinal equation of motion in two dimensions can be written for an estuary (after Agnew⁴) as:

$$\frac{d}{dt}(\rho U) + \frac{\partial p}{\partial x} - \frac{\partial \tau}{\partial y} = 0 \quad (1)$$

where ρ : density of water,
 τ : shear stress,
 y : position on the vertical above the estuary bed,
 U : velocity in the x direction (positive seaward)
 and p : pressure.

If the vertical density gradient and the velocity in the y direction are ignored and atmospheric pressure is assumed constant then the pressure gradient along the estuary can be written:

$$\frac{\partial p}{\partial x} = -\rho g I + g(H-y) \frac{\partial \rho}{\partial x} \quad (2)$$

where I : water surface slope (positive ebb direction)
and H : water depth.

Substituting equation (2) into equation (1) and expanding the first term (ignoring variations of density with time) gives

$$\frac{\partial U}{\partial t} + U \frac{\partial U}{\partial x} + \frac{U^2}{\rho} \frac{\partial \rho}{\partial x} - g I + \frac{g}{\rho} (H-y) \frac{\partial \rho}{\partial x} - \frac{1}{\rho} \frac{\partial \tau}{\partial y} = 0 \quad (3)$$

or

$$\frac{1}{g} \frac{\partial U}{\partial t} + \frac{\partial}{\partial x} \left(\frac{U^2}{2g} \right) + 2D \left[\frac{1-y}{H} + \frac{U^2}{gH} \right] - \frac{1}{\rho g} \frac{\partial \tau}{\partial y} = 1 \quad (4)$$

where D : density slope = $\frac{H}{2\rho} \frac{\partial \rho}{\partial x}$

Substituting $\eta = y/H$ and assuming U^2/gH is very much less than unity, equation (4) can be written:

$$A + F + 2D(1-\eta) - \frac{1}{\rho g H} \frac{\partial \tau}{\partial \eta} = I \quad (5)$$

Integrating equation (5) throughout the depth (i.e. from $\eta = 0$ to $\eta = 1.0$) and assuming the shear stress at the surface is zero gives

$$\frac{\tau_b}{\rho g H} = S = 1 - D + A_1 + F_1 \quad (6)$$

where

$$A_1 = - \int_{\eta=0}^{\eta=1.0} A d\eta = \text{inertia slope,}$$

$$F_1 = - \int_{\eta=0}^{\eta=1.0} F d\eta = \text{kinetic energy slope,}$$

and τ_b = shear stress at the bed.

Now the shear velocity

$$u_* = (\tau_b / \rho)^{\frac{1}{2}} \quad (7)$$

so equations (6) and (7) show that the shear stress at the bed and the shear velocity are dependent upon density (D), inertia (A_1) and kinetic energy (F_1) terms as well as surface slope (I). In order to determine the variation of u_* throughout a tidal cycle it is therefore necessary (a) to measure the water surface slope along the estuary, (b) to determine the variations of velocity both along the estuary and with time in order to calculate the inertia and

kinetic energy slopes, and (c) to measure the longitudinal variation of density.

In order to determine the inertia and kinetic energy terms A_1 and F_1 the variation throughout the depth of each of the terms A and F is required. It is convenient to consider A and F to increase linearly from zero at the bed to some values A_b and F_b at some small distance above the bed given by $\eta = \eta_1$, and then to increase to surface values of A_s and F_s respectively according to some chosen mathematical relationship. This arrangement is indicated in Fig. 1.

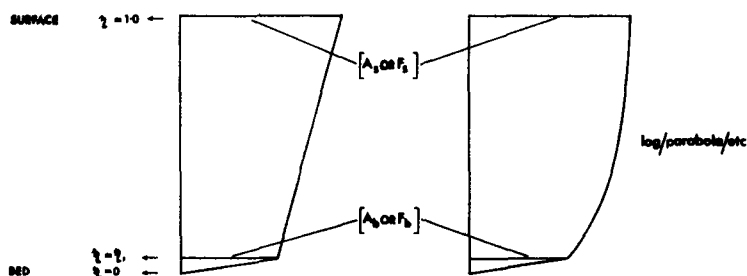


Fig. 1. Assumed variation of inertia and kinetic terms with depth

The simplest relationship to be assumed between $\eta = \eta_1$ and $\eta = 1.0$ is a linear one (Fig. 1a) and the integration of the terms A and F throughout the depth then give:

$$A_1 = -A_b(1 - \frac{1}{2}\eta_1) - \frac{1}{2}(A_s - A_b)(1 - \eta_1) \quad (8)$$

$$\text{and } F_1 = -F_b(1 - \frac{1}{2}\eta_1) - \frac{1}{2}(F_s - F_b)(1 - \eta_1) \quad (9)$$

The special case of uniform velocity distribution from bed to surface is easily obtained from equations (8) and (9) with $\eta_1 = 0$, $A_s = A_b = A_m$ and $F_s = F_b = F_m$ as

$$A_1 = -\frac{1}{2}(A_s + A_b) = -A_m \quad (10)$$

$$\text{and } F_1 = -\frac{1}{2}(F_s + F_b) = -F_m \quad (11)$$

It is of interest to compute the values of inertia and kinetic energy terms when various distributions throughout the depth are assumed and to compare these values with the linear case given by equations (8) and (9). For this purpose a number of cases are presented below and because the same relationships are true for A_1 and for F_1 only the formulae for F_1 are given.

Log curve with 2 points known

If the value of F near the bed and at the surface are known and the distribution of F is assumed to be logarithmic the expression for F_1 is found to be

$$F_1 = -F_b(1 - \frac{1}{2}\eta_1) - \frac{(F_s - F_b)}{\ln(1/\eta_1)} \left[\ln\left(\frac{1}{\eta_1}\right) - 1 + \eta_1 \right] \quad (12)$$

Log curve with 3 points known

In this case it is assumed that a third value of F is known F_1 at η_1 as well as F_b at η_1 and F_s at $\eta = 1.0$. If the distribution is assumed to be linear between the bed and F_b and then two logarithmic curves between F_b and F_1 and F_1 and F_s then the expression for F_1 becomes

$$F_1 = F_b \frac{\eta_1}{2} - F_b \eta_1 - \frac{(F_1 - F_b)}{\ln(\eta_1/\eta_1)} \left[\eta_1 \ln\left(\frac{\eta_1}{\eta_1}\right) - \eta_1 + \eta_1 \right] \\ - F_1(1 - \eta_1) + \frac{(F_1 - F_s)}{\ln(1/\eta_1)} \left[\ln\left(\frac{1}{\eta_1}\right) - 1 + \eta_1 \right] \quad (13)$$

Parabolic with 1 point known

The surface value is F_s and the distribution is assumed to be parabolic between the bed and the surface.

$$F_1 = -\frac{2}{3}F_s \quad (14)$$

Parabolic with 2 points known

In this case values of F_s and F_b are assumed to be known at $\eta = 1.0$ and η_1 respectively. The distribution of F is assumed to be linear from the bed to $\eta = \eta_1$ and then parabolic to the surface with the requirement that the parabola is tangential to the linear distribution at $\eta = \eta_1$ i. e. no discontinuity in slope at $\eta = \eta_1$. Then

$$F_1 = -\frac{1}{2}F_b \eta_1 - F_o(1 - \eta_1) - \frac{2}{3} \frac{(F_s - F_o)}{\sqrt{1 - \eta_o}} \left[(1 - \eta_o)^{3/2} - (\eta_1 - \eta_o)^{3/2} \right] \quad (15)$$

where F_o and η_o are given by the equations

$$(F_b - F_o)^2 = \frac{(F_s - F_o)^2}{(1 - \eta_o)} (\eta_1 - \eta_o) \quad (16)$$

$$\text{and } (1 - \eta_o) = \frac{(F_s - F_o)^2}{2F_b(F_b - F_o)} \quad (17)$$

For consideration of the differences between the various relationships considered it was decided to use typical (maximum) values of the velocities in the estuary reach under study and compute the various values of F_1 obtained using equations (9), (11), (12), (13), (14) and (15).

The surface velocity at either end of the reach under consideration was taken to be 3.0 and 5.0 ft./sec. The variation of velocity with depth was taken as logarithmic and the shear velocity was taken as constant along the reach and equal to 0.24 ft./sec. The velocity near the estuary bed (i. e. $\eta = \eta_1 = 0.05$) was then calculated from the logarithmic velocity distributions and yielded values of 1.21 and 3.19 ft./sec. The mean value of the velocity over the vertical at either end of the reach was also calculated at 2.4 and 4.4 ft./sec. From these figures it is seen that F_s is proportional to $(5^2 - 3^2)$ i. e. 16 while F_b is proportional to $(3.19^2 - 1.21^2)$ i. e. 8.7. If F is determined from the depth-mean velocity, then F is proportional to $(4.4^2 - 2.4^2)$ i. e. 13.6.

The differences between the various assumed vertical profiles for F can now be compared by using the values of F_s and F_b quoted above. The linear distribution (equation (9)) gives F_1 proportional to 12.57 while the simple parabolic case (equation (14)) gives F_1 proportional to 10.7. The more complicated parabolic distribution (equation (15)) gives F_1 proportional to 13.06 whilst the single log curve (equation (12)) gives F_1 proportional to 13.48.

If the value of F is calculated for various elevations above the estuary bed from the two velocity profiles as measured throughout the depth, then the depth-mean value of F is found to be proportional to 13.5. Thus, as expected the single log curve agrees most closely with the depth-mean value of F . The simple linear form is, however, only about 7% less than the much more complicated log expression. The one point parabolic curve shows the worst agreement. The use of the depth-mean velocity to calculate F shows excellent agreement with the depth-mean value of F . Thus the use of the depth-mean velocity to calculate the F_1 term would appear justifiable.

If the velocity profile does not increase regularly from the bed to the surface (e.g. in certain flood profiles in stratified estuaries the surface velocity is lower than at mid-depth) then the value of F_1 will be given by equation (13) with $F_1 > F_s$. In order to examine this type of distribution, the surface velocity at either end of the reach was reduced from 3.0 and 5.0 ft./sec. to 1.8 and 3.0 ft./sec. respectively. The velocity profile is assumed to reduce logarithmically from the surface to mid-depth, the latter has the same value as previously i.e. 2.59 and 4.58 ft./sec. and the distribution below mid-depth is as before. Therefore F_s is now proportional to 5.75 while F_1 in equation (13) is proportional to 14.3 with η_1 equal to 0.50 and $\eta_1 = 0.05$. With these values, F_1 is found to be proportional to 10.64, while the value of F computed from the depth-mean velocity is found to be 10.5. The depth-mean value of F computed at various elevations above the bed from the assumed log velocity profiles is found to be 10.6.

Thus again the value of F_1 computed from the assumed log distribution of F gives excellent agreement with the depth-mean value of F from the velocity profiles. This could be expected since a log curve was chosen for the depth variation both of velocity and of F . There is again close agreement between F' computed from the depth-mean velocity and the depth-mean value of F . The depth-mean velocity will therefore be used to compute values of the F_1 term in equation (6).

Consideration of the vertical velocity term in the equation of motion

In deriving equation (6) it was assumed that the vertical velocity could be ignored. For a similar study Sato, Kikkawa and Kishi⁵ included this term and found it to be of importance at certain stations of the River Tone. In the case of the Narrows of the River Mersey it was found that this term is relatively small and unimportant. It is possible that at its maximum value it is of the same order as the value of D , however it is only large when the value of S is large and is in general less than 10% of the value of S . The value of D on the other hand is constant throughout the tidal cycle and is therefore important when the value of S is small, around the periods of slack water.

MEASUREMENTS

The previous analysis indicates that four basic quantities, I , A_1 , F_1 , and D , have to be determined from field observations in order to calculate the bed shear. The necessary field work has formed part of a much larger research programme (which is still in progress) for the investigation of the water and sediment circulation patterns of the Mersey Estuary. This research is sponsored by the Mersey Docks and Harbour Board.

The field observations of velocity, density etc. from which the four basic quantities are determined have been made in an area of the Mersey known as the Narrows. The estuary here runs a straight course for several miles (cross-flow is thus a minimum) and is narrow enough to be unaffected by coriolis forces. It is however, wide enough to be considered two dimensional. The reach is also free of rapid changes in cross-section and the estuary is well mixed so that the vertical salinity distribution in the Narrows is almost uniform (i. e. less than 2% difference between surface and bed). The measurement of I , A_1 , F_1 and D in this research are now considered separately.

(1) Water Surface Slope (I)

In the absence of cross-flow, Coriolis and wind forces, the term I can be taken as constant across the width of the estuary. Its magnitude was found initially by an examination of the automatic tide recordings at Gladstone Dock and Princes Pier. These gauges are both situated on the East side of the river at the ends of the reach under consideration. A preliminary examination of water surface slope from the tide gauge records indicated some very odd results e. g. large flood slopes in the middle of the ebb tide.

Each tidewell was then critically examined. At Princes stage it was found that the tidewell (an 8" O. D. pipe) was situated in a fairly fast flowing stream. Research work by Perry^{6 & 7} indicated that this particular tide gauge was subject to an error which caused the water surface level inside the well to be greater than that outside and this error depended on the magnitude of the flow velocity past the tide gauge orifice. Meanwhile Lennon⁸ had demonstrated that both Princes and Gladstone tide gauges were subject to a salinity stratification i. e. the density of water in the well at high water decreased from the low water salinity value at the surface to the high water salinity value in the region of the orifice. Thus at high water the level inside the tidewell was greater than in the tideway. A further difference of level between the tidewell and tideway can also be caused by the presence of silt which enters the well from the tideway on the flood tide. In the limit the silt completely blocks the tidewell orifice and the water is unable to escape. Further errors can also result from the automatic recording equipment itself e. g. eccentric timing mechanisms and badly aligned pen carriages. Because of all the above mentioned difficulties the water surface slope was determined independently of the automatic tide gauge equipment and of the Princes tidewell.

A self-aligning engineers level was mounted on a floating pontoon attached to Princes Stage (a floating landing stage) at approximately 6-8ft. from a tide staff. The latter could be read to $\pm \frac{1}{4}$ ". Readings on the tide staff were taken continuously for a period of 30 seconds every 15 minutes. The height of collimation of the level was determined at slack water and checked during the course of the experiment. The water level was also read directly from the tide staff as a cross-check.

The level at Gladstone was determined by using an electronic probe device which indicated the water level relative to the top of the well of the tide-gauge. Basically this instrument consists of a pointer attached to a surveyor's tape through which a current is passed, the other electrode usually being the well itself. As the pointer touches the water surface the circuit is completed and this is registered as a frequency-change in an audio-signal transmitted from the instrument. This instrument could be read to $\pm 1/16$ in. The salinity of the water in the well was measured at slack high and low water as also was the salinity in the tide-way at the orifice level. Errors due to silt were eliminated by cleaning the well, the day previous to the tests. There is little velocity past this well except when ships are entering Gladstone Dock, the level in the tideway is therefore assumed to be equal to that in the well, less the salinity error.

The tide curves at each station were then plotted and smoothed (particularly the Gladstone curve near high water when the passage of ships into the Dock caused the levels to be slightly reduced). The water level at Gladstone was then corrected for the salinity error using the mean value of the salinity (i. e. density) inside and outside the well. The difference at any instant of time between this curve and that observed at Princes is then the water surface slope along the reach (I)

(ii) Inertia Slope (A_1)

In three-dimensional flow this term will not be a constant across the estuary since the lateral distribution of velocity (U) is not constant. In the two-dimensional case the variation of A_1 across the estuary is ignored and the variations of the mean velocity are considered. Thus for the present calculation A_1 will be determined from the cross-sectional mean velocities at the surface and near the bed.

Several days field observations of the water velocity near the estuary bed and at the surface were available for a position in the centre of the river, midway between Princes Pier and Gladstone Dock. These observations were collected as part of the general research work in progress in the Mersey estuary. The velocities were measured using a Kelvin Hughes Direct Reading Current Meter. These field results were plotted out against time and the surface and near-bed velocities taken off the resulting graphs at half-hourly intervals. These velocities were then plotted against high water level (similar to the effect of range in the Mersey) so that a set of velocities could be obtained at half-hourly intervals for a tide of approximately 27.5 ft. H. W. (L. B. D.).

From the work of Burke⁹ it is possible to relate the depth-mean velocity ($\bar{U}$) at the position under consideration to the cross-sectional mean velocity ($\bar{U}$) i. e. $\bar{U} = a\bar{U}$ where a is a constant. Thus it is possible by using this relationship to reduce the half-hourly surface and near bed velocities to the equivalent cross-sectional mean velocity.

Values of A_b and A_s , every half hour were then calculated from the reduced (cross-sectional mean) velocities. A_1 at each time interval then follows from equation (8) with $\eta_1 = 0.05$.

(iii) Kinetic Energy Slope (F_1)

This quantity will also vary across the width of the estuary, depending on the velocity distribution across the width. For the case of two dimensional flow the cross-sectional mean velocity ($\bar{U}$) will again be used to compute F_1 . In the case of a real estuary the value of $F_1 = \frac{\delta}{\delta x} (\bar{U}^2 / 2g)$, will not be equal to the mean value of F_1 , for the cross-section and a factor (α) must be used

to account for this discrepancy. The constant (α) can vary from 1.0-2.0 depending on cross-sectional shape. In the case of the Mersey, in the area of the Narrows, the constant will be of the order of 1.1 and because of this, the correction will not be applied (i.e. α is taken as 1.0). In order to compare the two dimensional equation with river observations, a position was chosen at which the mean depth is close to the cross-sectional mean-depth. Cross-sectional effects should therefore be a minimum.

The calculation of F using F_b and F_s has not been possible since this requires velocity measurements opposite Princes Pier, which is the centre of ship passenger traffic for Liverpool, Birkenhead, North Wales and the Isle of Man. Use has therefore been made of the cross-sectional mean velocity ($\bar{U}$) in order to calculate F . The value of $\bar{U}$ every half hour throughout a tidal cycle at various cross-sections along the river, including Gladstone and Princes, have been computed by Burke⁹ using the continuity equation. The value of F_1 at half-hourly intervals through a tidal cycle can thus be calculated as the difference between $(\bar{U}^2/2g)$ at Princes and Gladstone. Adjustments to $\bar{U}$ for different tidal ranges are also possible since the variation of $\bar{U}$ with high water level (or range) is linear in this reach of the Mersey. In the present computations F_1 has been calculated for a high water level of just over 27ft. i.e. similar to the tides for which the surface slope was measured.

(iv) Density Slope (D)

This term indicates the presence of a longitudinal density gradient in the Mersey which is always present due to the mixing of fresh and saline water. Its order of magnitude in the case of the Mersey is small but it can be significant when the terms I , A and F are small (e.g. slack low water).

In order to indicate the relative magnitude of all the terms (I , A_1 and F_1) these have been expressed in terms of D (Fig. 2). Thus on large Spring Tides the total energy slope can be the order of twenty times the density effect, but on Neap Tides with a large freshwater flow this could be reduced to less than ten times the density effect. For the purposes of the present calculation, the density term is taken as solely due to the variation of salinity along the estuary (i. e. depth variation is ignored) and to be a constant throughout the tide.

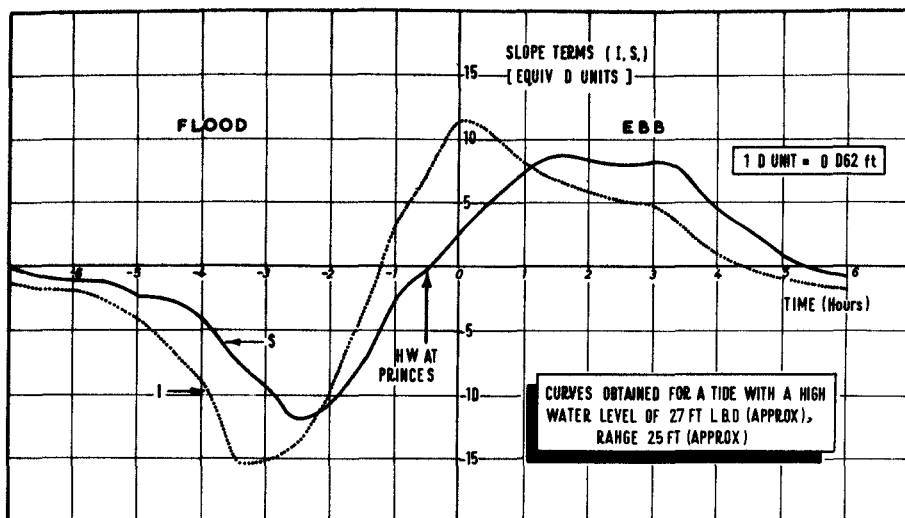


Fig. 2. Variation of Water Surface and Energy Slopes Throughout the Tidal Cycle

Longitudinal salinity data obtained by Wallingford¹⁰ for a model of the Mersey Estuary was examined and a longitudinal profile of mean salinity produced. From this, a mean longitudinal salinity gradient was produced. This was then combined with the centre-line mean-depth of the reach to produce an overall mean figure for the density term D. This amounted to a head difference over the Princes to Gladstone Reach of 0.062 ft. in the horizontal distance of 16,000 ft.

The energy slope (S) can now be calculated from equation (6) using the components I , A_1 , F_1 and D and this is shown in Fig. 2. Once S is known, the shear on the bed (τ_b) can be calculated as also can $u_* = (\tau_b/\rho)^{\frac{1}{2}}$ as shown in Fig. 3.

Shear velocity using velocity profiles

The value of u_* thus obtained can also be compared with measurements taken in the field. A position on the cross-section midway between Gladstone and Princes was selected such that the depth at mean tide level was approximately equal to the mean cross-sectional depth at mean tide level. Measurements of the water velocities were made on a vertical from the surface to approximately 3ft. above the estuary bed. This was done using the direct reading current meter. These velocity transverses were made at approximately half-hourly intervals. The velocities in the region 6in. - 3ft. above the bed were measured using three Ott Mark V Arkansas Current Meters mounted on a vertical freely pivoting rod which was held within a triangular framework. The Ott Meters were connected to a control unit which enabled the three meters to be read simultaneously. The length of time for which the velocities were recorded could be varied to any time interval.

In the case of the present observations a time interval of 10 seconds was used which appears to have been too low since the included results for u_* show a large scatter. Some of this scatter has been reduced by averaging all the results taken within each fifteen minute time interval. The shear velocity is obtained from the results by making a semi-log plot of the velocities against height above the bed, as was done by Lesser¹¹. The same technique was used with the vertical profiles obtained with the D. R. C. M. except that the semi-log plots were confined to below mid-depth so that any wind, ship effects etc. were avoided. The comparison between the river results and those from equation (7) are shown in Fig. 3.

Bowden¹² has suggested that u_* can be related to the depth-mean velocity in a linear fashion i. e. $u_* = k\bar{U}$. In order to examine the relationship between velocity and u_* , the velocity at 3ft. above the estuary bed (U_3), the depth-mean velocity and the cross-sectional mean velocity were plotted against time. Three values of k were determined from these plots such that the sum of the squares of the errors in prediction was a minimum i. e.

$\sum (kU - u_*)^2$ was a minimum (with U equal to U_3 , $\bar{U}$ and $\bar{\bar{U}}$ in turn). The three values found were $k_1 = 0.077$, $k_2 = 0.052$, $k_3 = 0.054$, for the velocity at 3ft. above the bed, the depth mean and the cross-sectional mean respectively. A measure of the best relationship of the three is provided by the term $\sum (kU - u_*)^2$, it was found that the cross-section mean velocity provided the best fit. The comparison of the three values of u_* using the values of U_3 , $\bar{U}$ and $\bar{\bar{U}}$ with that value of u_* calculated from the energy slope method is shown in Fig. 4.

DISCUSSION OF RESULTS

The comparison between u_* computed from the energy slope method and from the tripod results (Fig. 3) is seen to be reasonable. It is felt that much of the scatter on the tripod results is due to two main causes. Firstly, the timing interval for the Ott current meters was too low. This enabled large velocity fluctuations, and thus fluctuations in shear to be recorded. Secondly the two calculations of u_* from the velocities at 6" and 12" and the velocities at 12" and 27" indicate that the top Ott meter (27") may have

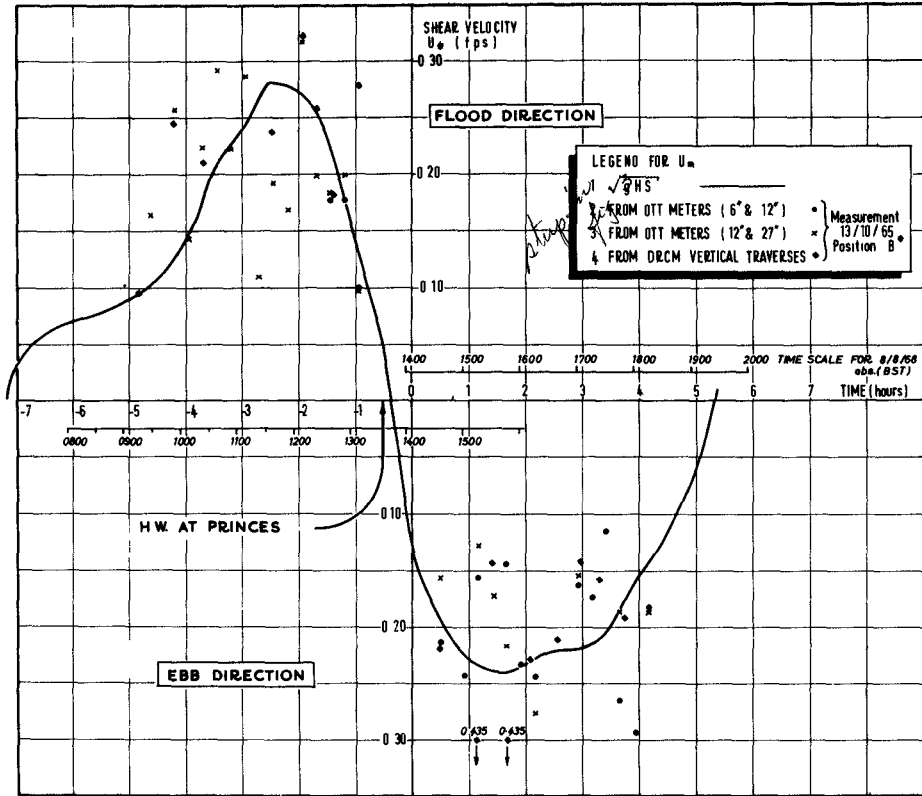


Fig. 3. Comparison between measured values of shear velocity with those obtained from the energy slope

been affected by the supporting framework of the tripod rig. If this is so some re-design of the framework will be necessary. The question of velocity fluctuation can be overcome by a longer timing interval. Another possible reason for the values of u_* obtained from the meters at 12" and 27" generally being too small may be that the meter at 27" had a different blade pitch than the other two meters and this may cause it to react differently to any turbulence.

The shape of the u_* curve (from the energy slope) is seen to closely follow the shape of the velocity distribution near the bed. However, the water surface slope is seen to be out of phase with both the u_* and the velocity curves, particularly on the ebb tide.

If the velocity is used to determine u_* by assuming a linear relationship ($u_* = kU$) i. e. bed roughness is assumed constant, then the cross-sectional mean velocity, rather than the velocity near the bed, is found to be a better fit for u_* as calculated from the energy slope. The reason for this is probably the uncertainty of the position of the D. R. C.M. near the estuary bed; the averaging over the vertical etc. for the mean velocities tending to reduce these sort of effects. The reasonable fit of the cross-sectional mean velocity to the u_* curve (except at low water) does however mean that the mean velocity could be used to determine the shear velocity in those areas remote from land (e. g. Liverpool Bay) where it is not possible to measure the water surface slope etc. and where density effects are small. These velocities could be obtained by field measurements or even by computation if an accurate mathematical model is available.

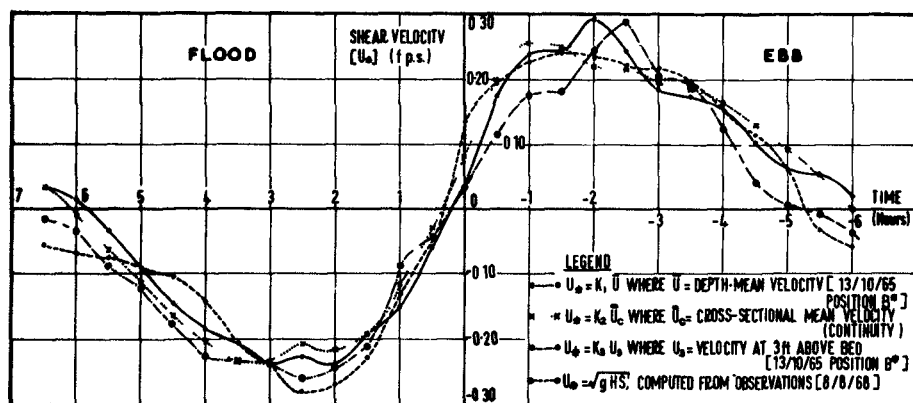


Fig. 4. Comparison between shear velocity obtained from velocity near the bed, depth-mean velocity and cross-sectional mean velocity

The results also indicate that u_* can be found for an estuary without taking many field observations. The water surface slope and density terms can be measured at the shore, while the kinetic and inertia effects could be obtained using velocities computed by applying the continuity equation to the estuary. This would produce the cross-sectional mean velocity ($\bar{U}$) from which the A and F terms could be found.

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NOTATION

A	:	inertia term
D	:	density slope
F	:	kinetic energy term
H	:	water depth
I	:	surface slope (positive when sloping downwards from land to sea)
U	:	velocity in the x direction (seawards)
u_*	=	$(\tau_b/\rho)^{\frac{1}{2}}$: shear velocity
x	:	distance measured along the estuary, seawards
y	:	distance vertically above the bed
α	:	coefficient
$\eta = y/H$	:	percentage depth
ρ	:	density
τ	:	shear stress

Subscripts -

b	:	near the bed
s	:	surface
i	:	intermediate position

- m : mean value
- l : when used with A and F implies integrated values throughout the depth.

REFERENCES

1. Rouse, H. "Modern conceptions of the mechanics of fluid turbulence" Trans. A.S.C.E. Vol. 102, 1937, Fig. 30 p.536.
2. Vanoni, V. A. "Transportation of suspended sediment by water" Trans. A.S.C.E. Vol. 1946, pp.67-102.
3. Einstein, H. A. "The bed load function for sediment transportation in open channel flows". United States Dept. of Agric. Tech. Bull. No. 1026, Sept. 1950.
4. Agnew, R. "Estuarine currents and tidal streams" Proc. 8th Coastal Eng. Conf. 1960, Chapter 28, pp.510-535.
5. Sato, S., Kikkawa, H. and Kishi, T. "On the hydraulic characteristics of the tidal reach of the Tone River" Paper A6, Proc. I.A.H.R., The Hague, 1955.
6. Perry, J. G. "Hydrodynamic Studies on tide gauges" University of Liverpool, M. Eng. Thesis 1967.
7. Halliwell, A. R., and Perry, J. G. "Errors in tide gauges" Dock and Harbour Authority, Vol. XLVIII, February 1968, pp.305-308.
8. Lennon, G. W. "The behaviour of a stilling well in the presence of periodic density variations" University of Liverpool, Tidal Institute, Internal Report No. 7.
9. Burke, C. "Velocities in tidal flows" M. Eng. Thesis, Liverpool University, August 1966.
10. D. S. I. R. Hydraulic Research Station, 2nd Internal Report on the Mersey Investigation, June 1959.
11. Lesser, R. M. "Some observations of the velocity profile near the sea floor". Trans. A.G.U. Vol. 32, No. 2, April 1951, pp.207-211.
12. Bowden, K. F. "The mixing processes in a tidal estuary". Int. J. Air Wat. Poll. Pergamon Press 1963, Vol. 7, pp.343-356.