CHAPTER 56

A MODEL STUDY OF THE DISTRIBUTION OF RUN-UP OF WIND-GENERATED WAVES ON SLOPING SEA WALLS

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ABSTRACT

The behaviour of wind-generated waves on impermeable slopes of 1:2, 1:4 and 1:10 was investigated in a 40 ft long laboratory wind-wave flume. Apparatus for measuring instantaneous values of the run-up was devised. Wind velocities of up to 44 ft/sec were applied, producing wave heights of up to 2.5 in.

Distributions were obtained for the wave and run-up characteristics. The empirical results were compared with theoretical statistical relationships. A comparison was made with the run-ups in paddle-wave experiments.

The relationship between the waves and their run-ups was further investigated by a comparison of simultaneous recordings, using the methods of spectral analysis.

INTRODUCTION

An understanding of the run-up behaviour of waves on sloping faces is of importance both in relation to sea walls and beaches. The subject has been studied for paddle-generated waves (Granthem 1953, Saville 1956) and for wind waves (Sibul and Tickner 1955, 1956; Paape 1960), principally from the point of view of the overtopping characteristics.

For design purposes, the procedure is often adopted whereby the run-up corresponding to the 'significant' wave height is determined from empirical data, such as that in the Coastal Engineering Research Centre Technical Report No. 4 (1966). It may then be quite reasonably inferred that only a small proportion of the run-ups would exceed this value. Saville (1952) has introduced an interesting refinement by developing an approximate distribution, based on the joint distribution of wave height and period proposed by Bretschneider (1959) and utilising the above run-up data, but still assuming each run-up as being the product of an individual wave.

The aim of the present investigation, undertaken with a wind-wave flume, was to examine the run-up behaviour on impermeable slopes of three different inclinations and to establish the relationship with the incident deep-water waves.

EXPERIMENTAL EQUIPMENT

The wind-wave flume was 40 ft long, 1 ft wide, and 3 ft deep, with a constant speed fan at the downstream end. Air flow was controlled by means of a hand-operated butterfly damper, and the wind velocities associated with various damper settings were measured by a pitot-static tube appropriately positioned. A sheet of perspex, with inclination of 1:10, 1:4, or 1:2 constituted the sloping face. Fig. 1 illustrates the working end of the flume and the assemblage of electronic measuring equipment. A hinged paddle-type generator was located at the windward end, so that an alternative means of generating waves was available.

Initially, the variation of the water surface elevation was measured by a wave-recorder of the twin-wire resistance type, but unsatisfactory calibration drift was experienced owing to contamination of the probes. The capacitance type of recorder proved to be much more stable and two such devices were employed.

Measurement of the run-ups proved difficult, because of the small size of the waves and the relatively high wind velocities. Various methods were investigated, including the suspension of wires of the wave recorder type just above and parallel to the sloping surface, so that the run-up covered a varying proportion of the wire. But this arrangement was found to be depth sensitive as well as being affected by clinging droplets. The type of run-up gauge finally evolved was a bank of 40 pairs of thin wire probes, arranged in parallel lines 0.5 in. apart and suspended with their tips 0.05 in. above the surface. The longitudinal spacing of the probes was made appropriate to the run-up range for each of the slopes. Fig. 2 shows the comb-like measuring head, comprising a perspex frame and wire inserts.

By means of an appropriate circuit, the output of the wave run-up gauge was transmitted in staircase voltage wave form proportional to the number of pairs of probes in contact with the water. Calibration was effected by adjusting its position on the slope so that the points of a known number of probes were immersed. The purpose of the pair of probes was to obviate, so far as possible, the errors caused by foam and droplets coming into contact with probes above the leading edge. A voltage change was only produced if the tips of a pair of probes became immersed. The leading edge during wave run-up was thus defined as the highest point at which the depth of water was sufficient to maintain contact.

The recording equipment included a single-channel high-speed pen recorder and a four-speed four-channel F.M. tape recorder. The former provided a visual record of the waves and run-ups, whilst the latter allowed of simultaneous recordings of waves and run-ups, and facilitated a subsequent analysis by analog or digital means.

EXPERIMENTAL PROCEDURE

The flume was filled to a still-water depth of 10 in. An examination of the wave spectra showed that this depth was sufficient to establish 'deep-water' conditions in the flume, with only a small error of about 8 per cent at the maximum wave lengths.

For a reference location, the wave recorder was sited above the toe of the 1:2 and 1:4 slopes, and 4 ft leeward of the toe of the 1:10 slope, the last position being chosen because of the need for comparable fetches. In order to assess the effect of wave reflections from the sloping faces, wave recordings were taken with and without an absorbent covering on the 1:2 slope. It was found that the average reduction in the standard deviation of the water surface elevation was about 5.3% for various wind speeds. This was considered acceptable, especially in view of the fact that the average error for the 1:4 and 1:10 slopes would have been rather less.

Experiments were undertaken at wind speeds of 26, 30, 35, 40 and 45 ft/sec, the characteristic velocity being that at 10 in. above S.W.L. However, it was found that, if satisfactory conditions were to be maintained on a particular slope, the wind velocity could be varied only within a limited range. Below the lower limit of this range the length of the run-up zone was insufficient for accurate measurements, while above the upper limit the quantity of foam and droplets of water on the slope became excessive.

In order to ensure that conditions within the flume had become stabilised, a steady wind was applied for at least half an hour in each test run before the outputs of the gauges were recorded. Calibrations were made before and after each run. In the case of tape recordings, the wave and run-up processes were recorded simultaneously for a period of 80 minutes at a tape speed of 1⅞ in./sec.

ANALYSIS OF CHART RECORDINGS

Waves:

Forty wave records, each with one hundred consecutive waves, were analysed for apparent wave height $\vec{H}$ and apparent period $\vec{T}$, using a conventional procedure. Values of $\vec{L}$ were defined as $\vec{L} = \vec{T}^2$. The mean water level was also established.

In order that results with different absolute magnitudes could be compared, the set of values obtained for a given wave characteristic in a particular test was normalised by the mean values. Thus

$$H_n = \vec{H}/\bar{H}, \quad T_n = \vec{T}/\bar{T}, \quad L_n = \vec{L}/\bar{L}$$

A typical set of probability distributions of H_n for different wind velocities with a 35 ft fetch is shown in Fig. 3. For the range available, it was found that the fetch had little effect upon the distribution, but an increase in wind velocity caused an increase in standard deviation (S_{H_n}) of H_n.

Now, Longuet-Higgins (1952), with some support from observational data, has postulated that a prototype 'sea state' is statistically described by a Rayleigh distribution. On this basis, the probability distribution of H_n is given by

$$P (H_n) = 1 - e^{-\pi H_n^2/4}$$

Theoretical relationships can thus be established between $\bar{H}$ and H_p, where H_p is the average value of the highest p per cent of the waves, also between $\bar{H}$ and the standard deviation σ_H, skewness α_{3H}, and kurtosis α_{4H}.

From a comparison of these theoretical relationships and the experimental data it was evident that the Rayleigh distribution was not a good model for the variability of $\bar{H}$ for laboratory wind waves of the present scale. For example, in Fig. 3, the Rayleigh curve diverges from the plotted points in the upper and lower quartiles. The graphs for H_{max}, H_{10}, H_{33} ('significant' wave), H_{50}, σ_H, α_{3H}, and α_{4H} further confirmed the discrepancy.

As indicated in Fig. 3, a much better degree of conformity resulted from a Gaussian distribution with mean $\mu = 1.0$, and standard deviation $\sigma = 0.35$.

Run-ups:

The apparent wave run-up height $\tilde{R}$ was defined as the difference in elevation between the mean water level above the toe of the slope and a run-up crest. A definition of this type was necessary because it was impossible either to attribute each run-up crest to an individual wave or to measure a run-up height for each wave.

A more suitable datum for a study of the run-up distribution was the elevation of the mean run-up height $\bar{R}$. When this had been standardised $[(R_m)_s = R_m / S_{\tilde{R}}$, where $R_m = \tilde{R} - \bar{R}]$, it was found that a Gaussian distribution with $\mu = 0$, $\sigma = 1.0$ was in good agreement with the experimental results for all three slopes (Fig. 4).

Wave and Run-Up Relationship:

In view of the apparent Gaussian distribution for the run-ups, and the fact that such a distribution is completely defined when the mean and the standard deviation are known, the experimental data for $\bar{R}$ and $\bar{H}$ and for $S_{\tilde{R}}$ and $S_{\tilde{H}}$ were compared. Linear relationships were obtained by the method of least squares, and it is seen (Fig. 5) that they were reasonably appropriate, except possibly for the 1:10 slope, where a very liberal interpretation has been made, the results being of limited range and probably influenced by wind set-up. It was deduced, in general, that the statistical error of the data was unlikely to exceed 10 per cent.

The average values of the ratio mean wave period / mean run-up period were 0.68, 1.00 and 1.05 for the 1:10, 1:4 and 1:2 slopes, respectively.

Although the exact value of this ratio depended upon the definition of $\tilde{R}$, nevertheless it qualitatively indicated the actual run-up behaviour. For example, it was observed, particularly for the 1:10 slope, that waves tended to combine or be absorbed by the backwash of a previous wave. For a very steep slope, the waves quite often created two run-up crests, the first crest being generated by the leading elements of a plunging breaker, whilst the second crest constituted the body of the wave.

The run-up curves of the Coastal Engineering Research Centre (1966), referred to earlier, served as a useful basis for comparison. These curves, for deep-water uniform waves and for the three relevant slopes, are shown in Fig. 6. Corresponding values were deduced from the experimental wind-wave data, although the possible range of comparison was very limited. As will be seen, there was a fair measure of agreement between the plotted points and the curves, although in the case of the 1:10 slope a breaking depth datum (i.e. M.W.L. at location where mean depth was breaking depth, $\frac{4}{3}H$) resulted in a slightly better fit than the normal convention.

The run-up of equivalent paddle-generated waves in the flume was also investigated and the results were found to be below the relevant curves in Fig. 6. The discrepancy could have been due to the effects of reflected waves, wall friction, gauge error, and smallness of scale; indeed a scale effect correction (additive from model to prototype) is recommended in the C.E.R.C. Report. It was also noteworthy that the chart recordings showed the existence of a double run-up crest of the type mentioned earlier.

SPECTRAL ANALYSIS

A greater understanding of the nature of a non-deterministic phenomenon may be gained from an analysis of the process as a whole. This was the purpose of the F.M. tape recordings described previously.

Because equipment suitable for digitising the tape-recorded signals was not available, much of the analysis of the wave elevation $\eta(t)$ and run-up elevation $\lambda(t)$ was carried out in analog form. From preliminary tests it was found that the energy in both processes occurred at frequencies below 4 cycles/sec, with maximum energy as low as 0.25 cycles/sec for the run-up on the 1:10 slope. Therefore, in order to bring these frequencies up to an acceptable level for analog equipment, it was necessary to replay the recorded signals at an increased speed. In this way, a replay time of 19 seconds for 80 minutes recording time applied.

The voltage corresponding to the mean value of $\lambda(t)$ was measured by means of a mirror galvanometer with a suitable time constant. A random noise r.m.s. voltmeter was used to measure the standard deviations of $\eta(t)$ and $\lambda(t)$. For a given slope, approximately linear relationships were found to exist between S_η and $\bar{\lambda}$ and between S_η and S_λ (Fig. 7).

Probability density functions for $\eta(t)$ and $\lambda(t)$ were obtained by measuring the proportion of the sample length during which the voltage of the analogous signal was below a known value. In Fig. 8, showing the results for the standardised functions $p(\eta_s)$ and $p(\lambda_s)$, a Gaussian distribution has been superimposed for purposes of comparison. In the case of $p(\eta_s)$, the divergence may be accounted for by the unsymmetrical nature of the wave profiles - short steep crests and long flat troughs. Although the data for $p(\lambda_s)$ exhibited a considerable degree of scatter, the distributions for the steeper slopes showed a tendency to conform to the pattern of $p(\eta_s)$.

The auto-correlation coefficients $\rho_\eta(\tau)$ and $\rho_\lambda(\tau)$ were determined by means of an analog correlator, developed for high frequency analysis. This equipment accepted samples of 10 seconds duration, equivalent in the present case to approximately 43 minutes of experimental time. Whilst ignoring the mean level of the signal, it gave a digital output from which the value of the auto-correlation coefficient could be calculated for the successive time lags τ, the latter being increased from zero by increments of 0.0256 seconds experimental time. From the relevant graphs it was evident that there was considerable difference between the form of $\rho_\eta(\tau)$ and $\rho_\lambda(\tau)$, particularly in respect of the 1:10 slope.

When the signals for $\eta(t)$ and $\lambda(t)$, recorded in the same test run, were fed into the correlator simultaneously, values for the cross-correlation coefficient $\rho_{\eta\lambda}(\tau)$ were obtained, the relevant curves being shown in Fig. 9. It will be observed that there was very little correlation between the waves and the run-ups for the 1:10 slope, but that correlation improved with increasing steepness. It was interesting to note that for the 1:1 and 1:2 slopes the largest peaks in $\rho_{\eta\lambda}(t)$ were negative. This was thought to be due to the unsymmetrical nature of $\eta(t)$ and $\lambda(t)$.

The energy spectra $[E_\eta(f)$ and $E_\lambda(f)$ versus $f]$ are shown in Figs. 10 and 11 for the three slopes. The maxima of each of the run-up energy spectra obtained for the 1:10 slope (Fig. 10) were found to occur at about 0.35 cycles/sec - a lower frequency than the band containing the wave energy. Also, for this slope, there was little or no response at frequencies corresponding to the peaks of the wave energy spectra. Thus it is not surprising that the mean apparent run-up period was found to be greater than $\bar{T}$. Only in the case of the 1:2 slope was the maximum run-up energy found to be at the same frequency as the maximum wave energy, and, even so, $E_\lambda(f)$ registered half its maximum value in a secondary peak at 0.35 cycles/sec.

CONCLUSIONS

For the waves, a Gaussian distribution ($\mu = 1$, $\sigma = 0.35$) for H_n was found to be reasonably appropriate. For the three slopes that were investigated, a Gaussian distribution ($\mu = 0$, $\sigma = 1$) was a good approximation to the distribution of $(R_M)_s$. Also, linear relationships appeared to exist between $\bar{H}$ and $\bar{R}$, and between $S_{\bar{H}}$ and $S_{\bar{R}}$ for a given slope. If this were found to be true of the prototype then it would be possible to make accurate run-up predictions from a knowledge of the wave conditions.

The visual observations, trace records, and spectral analysis all showed that it was not possible to establish direct correspondence between individual waves and their run-ups. Indeed, in the case of the 1:10 slope, there were over 30 per cent fewer run-up crests than waves approaching the slope. Thus any method which predicts the distribution of apparent run-up heights on the basis of an individual run-up for each wave must be inaccurate unless the previous waves are taken into account.

Unlike the statistical conditions generally assumed to apply in nature, the laboratory wind waves were not in accordance with a Rayleigh distribution. This must obviously suggest caution when considering the practical application of the results. There is clearly a need for further investigation at a larger scale, and preferably in the prototype.

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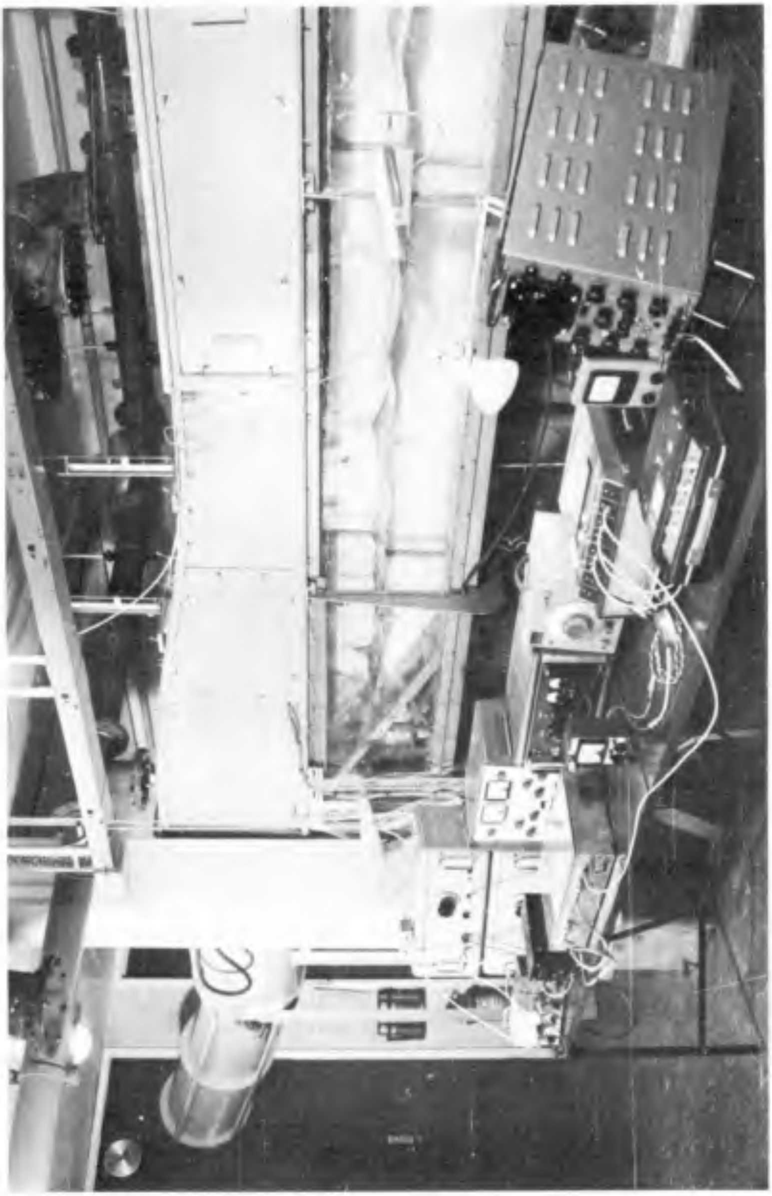

LEEWARD END OF THE WIND-WAVE FLUME WITH THE ELECTRONIC EQUIPMENT

FIGURE 2
MEASURING HEAD OF THE RUN-UP GAUGE

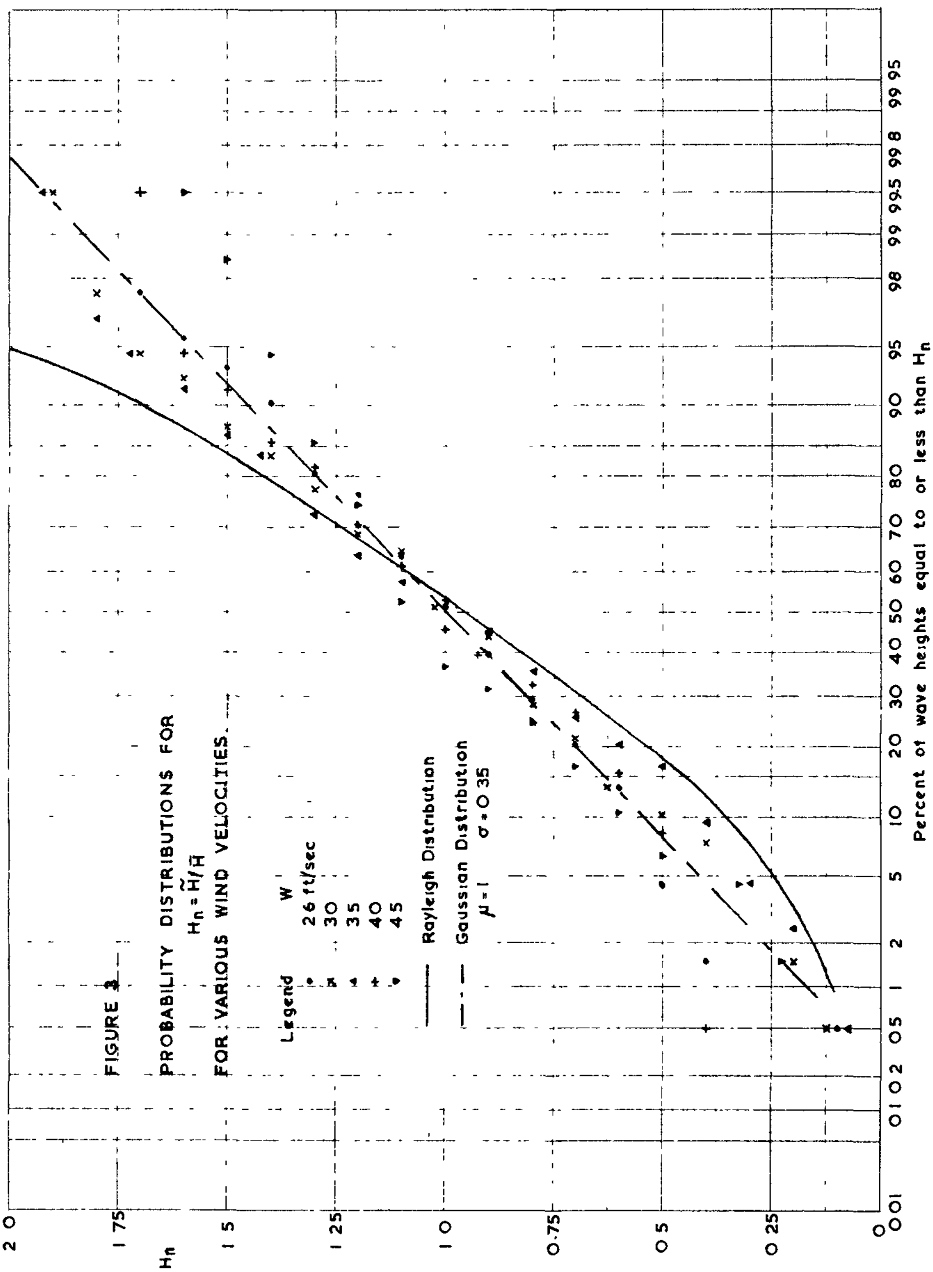
FIGURE 3
PROBABILITY DISTRIBUTIONS FOR
$H_n = \tilde{H}/\bar{H}$
FOR VARIOUS WIND VELOCITIES.
Legend
W
26 ft/sec
30
35
40
45
Rayleigh Distribution
Gaussian Distribution
$\mu = 1$ $\sigma = 0.35$
H_n
2.0
1.75
1.5
1.25
1.0
0.75
0.5
0.25
0.0
0.01
0.1 0.2
0.5
1
2
5
10
20
30
40
50
60
70
80
90
95
98
99
99.5
99.8
99.95
Percent of wave heights equal to or less than H_n

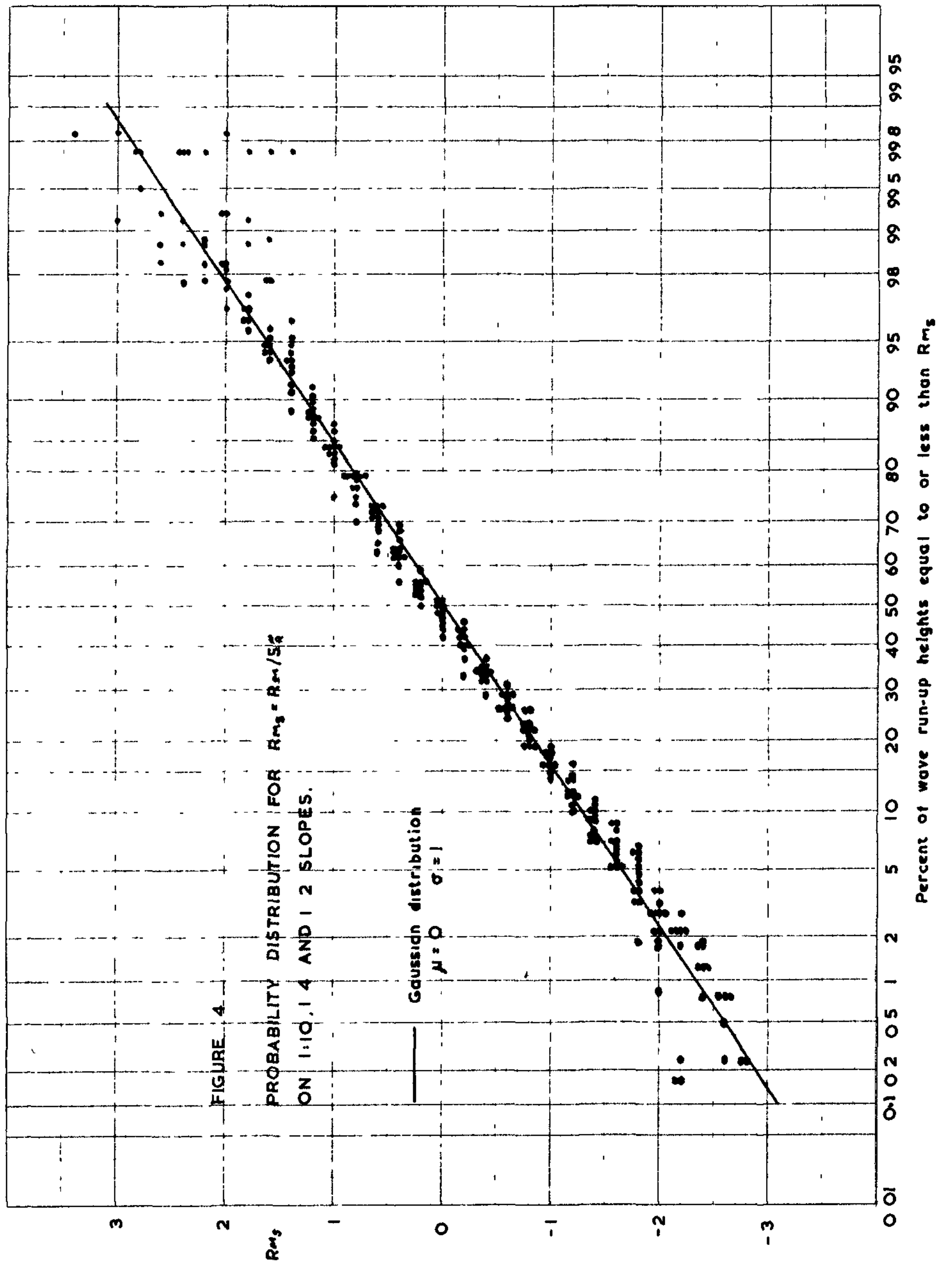

FIGURE 4

PROBABILITY DISTRIBUTION FOR $R_{m_s} = R_m/S_R$

ON 1:10, 1:4 AND 1:2 SLOPES.

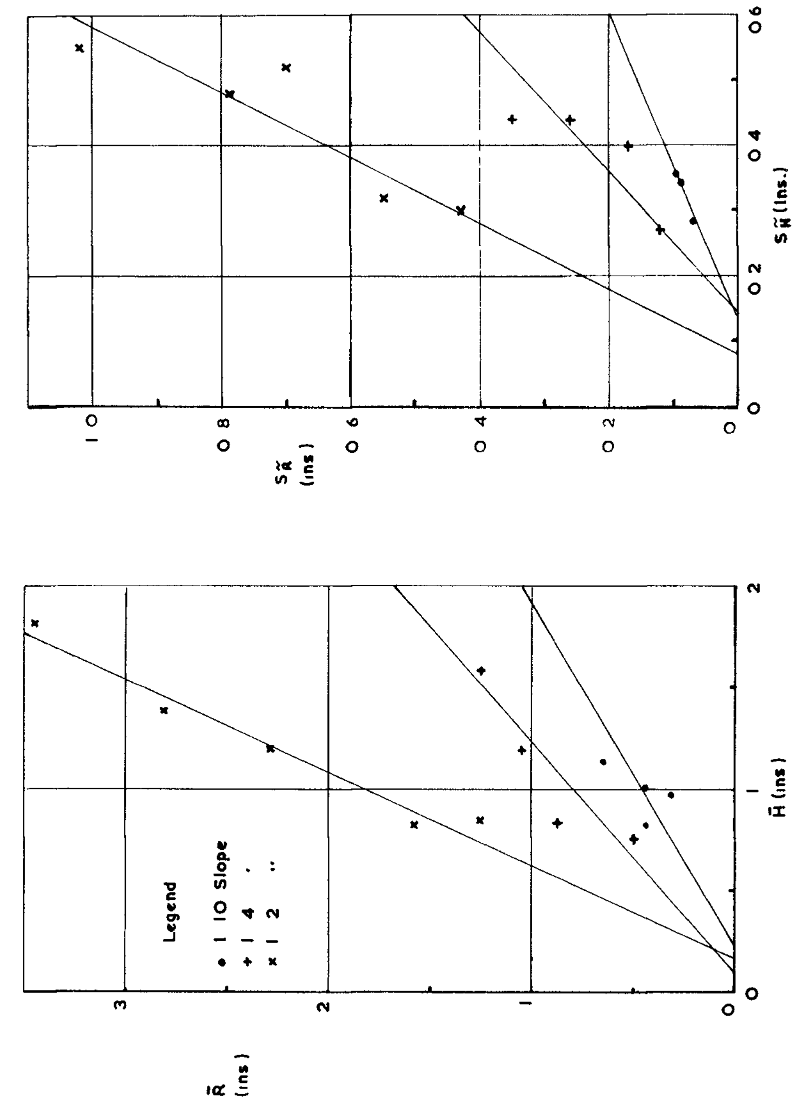

FIGURE 5 VARIATION OF MEAN RUN-UP HEIGHT WITH MEAN WAVE HEIGHT, AND RUN-UP HEIGHT STANDARD DEVIATION WITH WAVE HEIGHT STANDARD DEVIATION

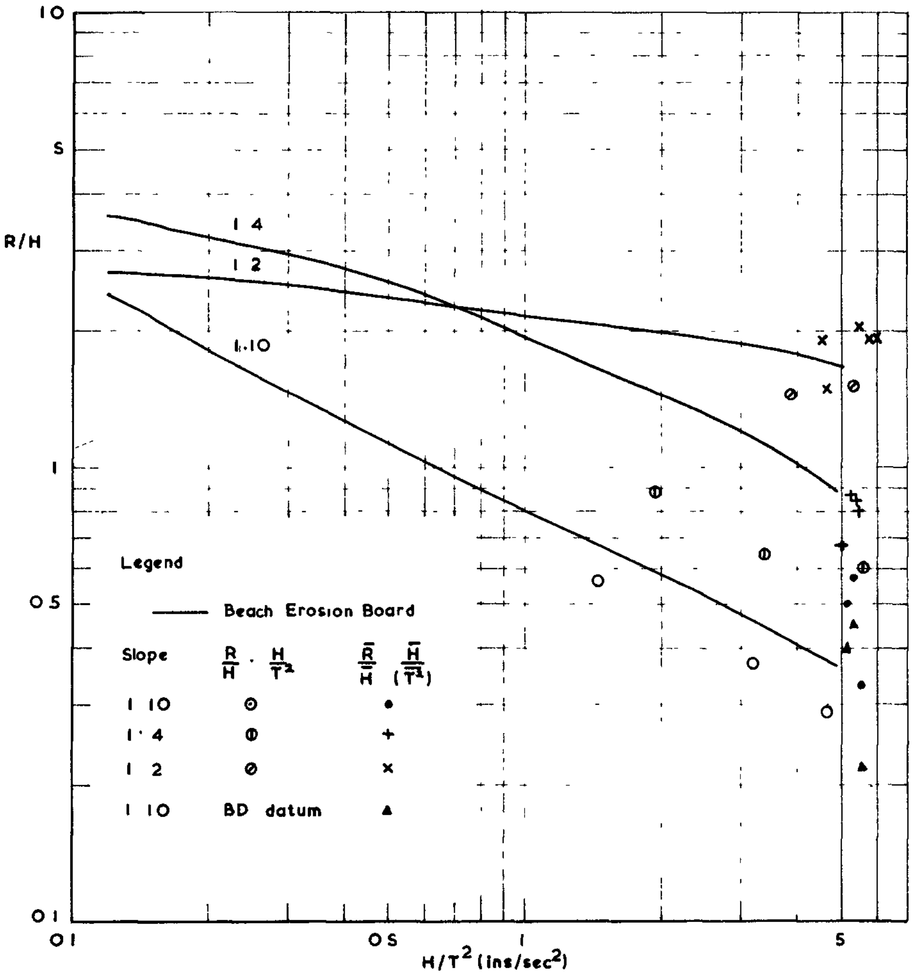

FIGURE 6 THE VARIATION OF RELATIVE RUN-UP WITH WAVE STEEPNESS FOR 1:10, 1 4, AND 1 2 SLOPES

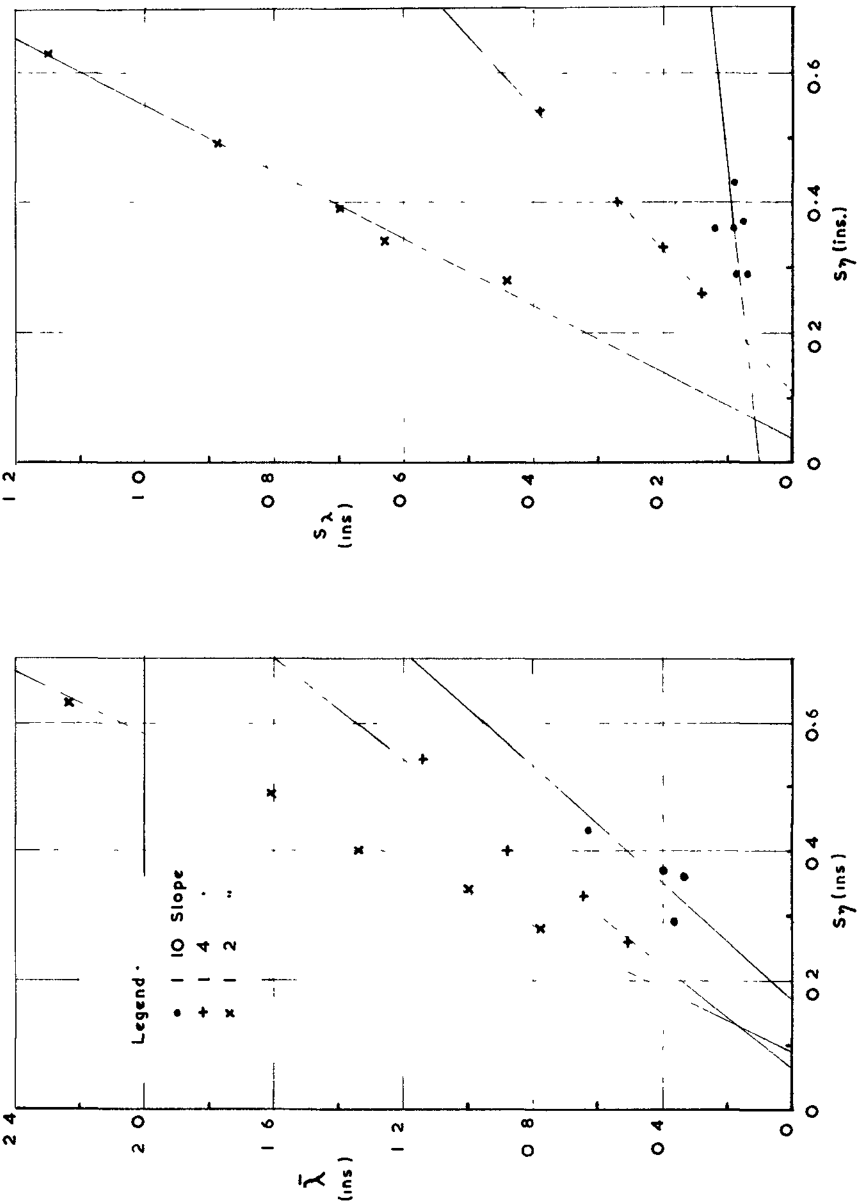

FIGURE 7 VARIATION OF RUN-UP MEAN LEVEL AND STANDARD DEVIATION WITH WAVE STANDARD DEVIATION.

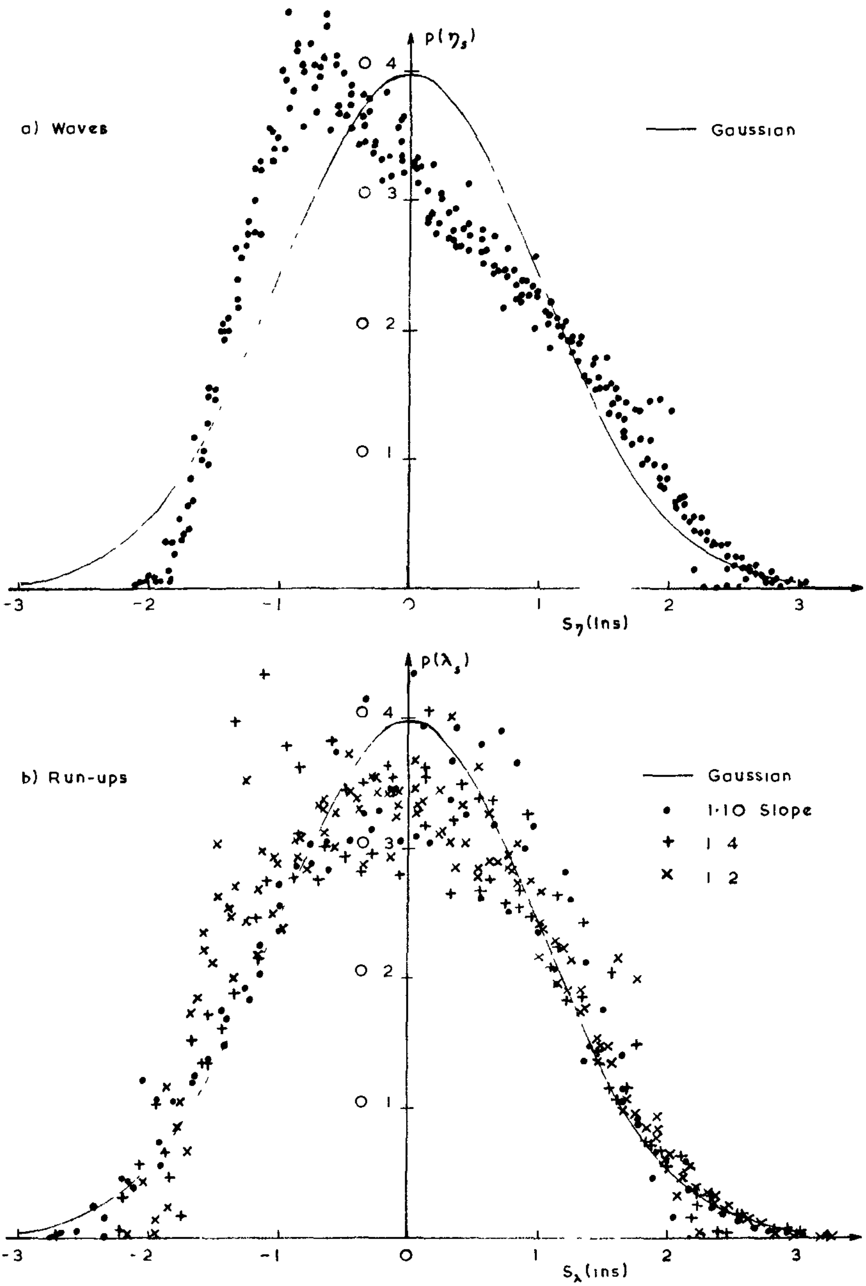

FIGURE 8 PROBABILITY DENSITY FUNCTIONS FOR WATER SURFACE ELEVATIONS IN WAVES AND RUN-UPS

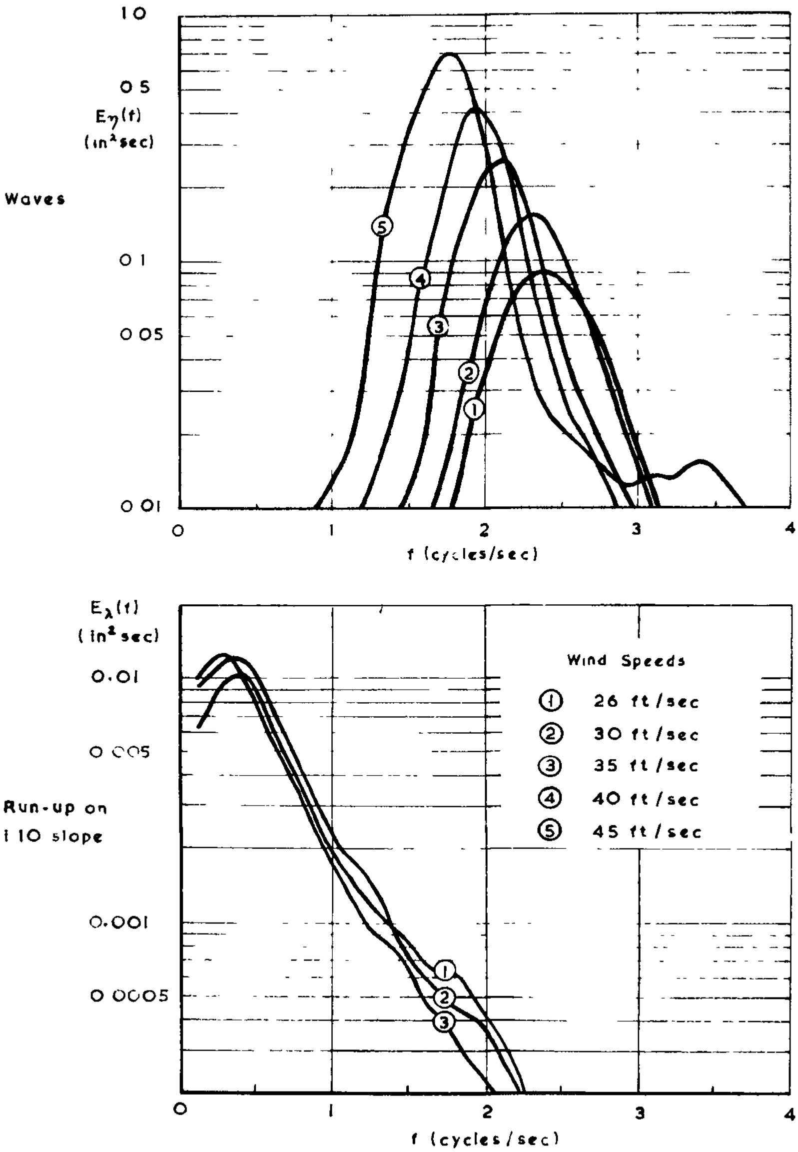

FIGURE 9 ENERGY SPECTRA FOR WAVES AND THEIR RUN-UPS ON A 1:10 SLOPE

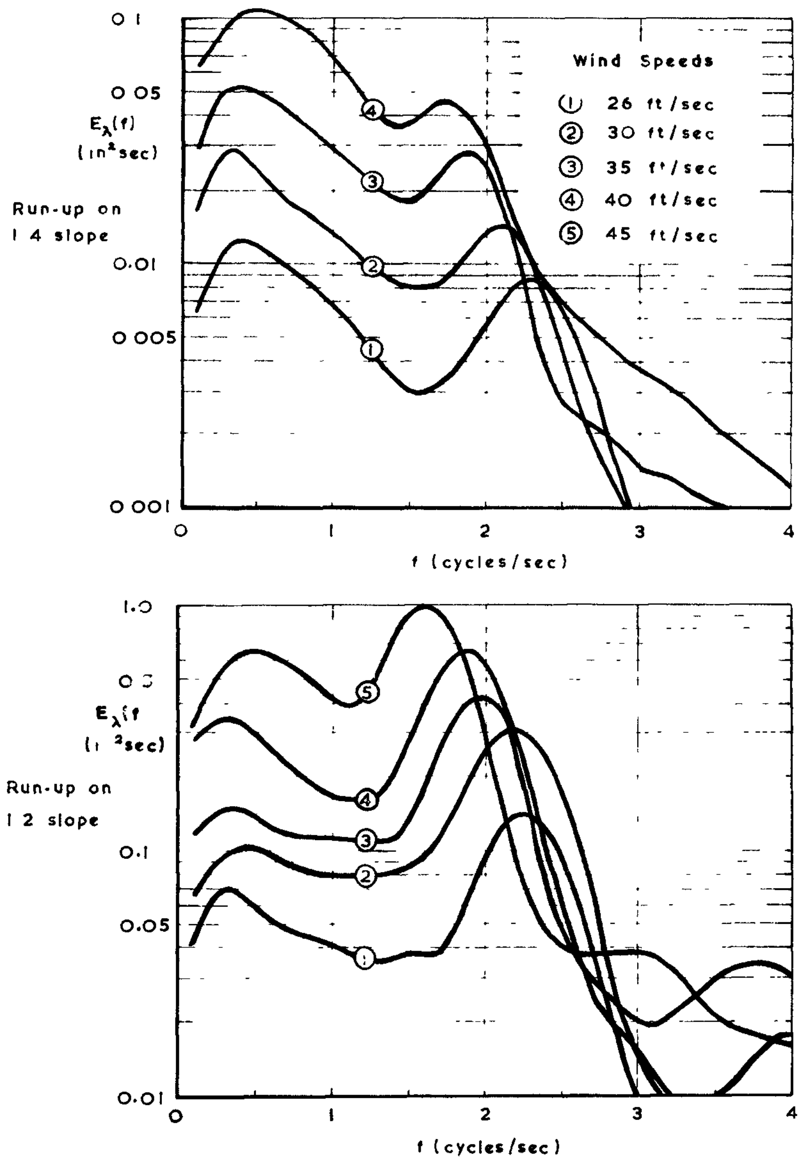

FIGURE 10 ENERGY SPECTRA FOR WAVE RUN-UPS ON 1 4 AND 1 2 SLOPES

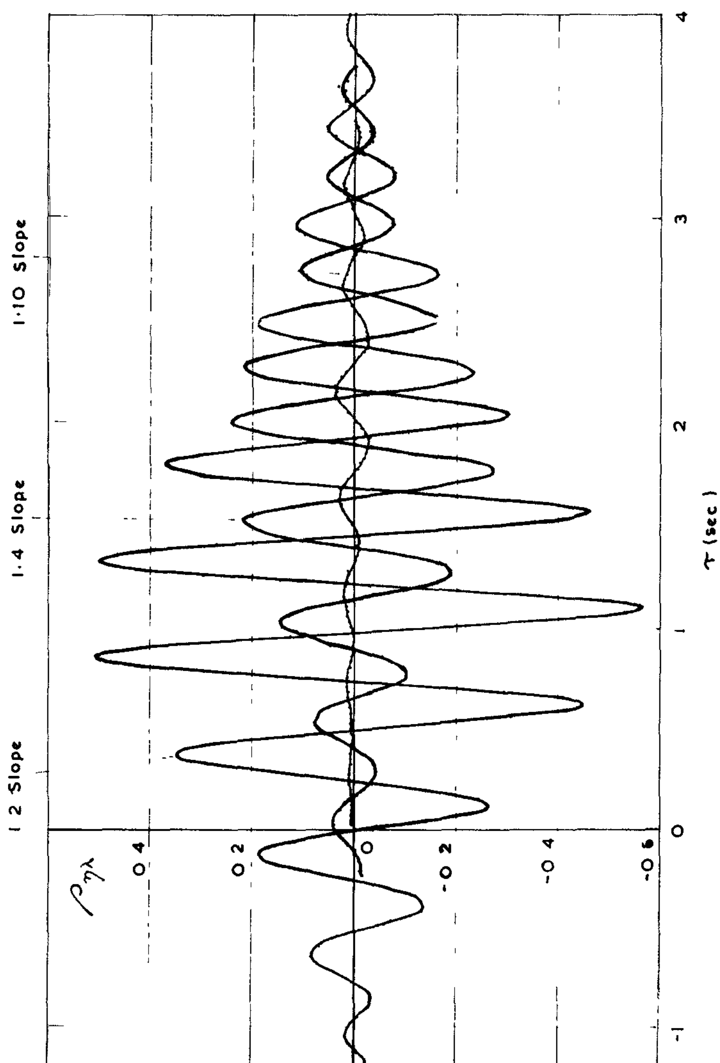

FIGURE 11 CROSS-CORRELATION COEFFICIENT FOR WAVES AND THEIR RUN-UP ON VARIOUS SLOPES AT A WIND VELOCITY OF 35 FT/SEC