CHAPTER 29

LABORATORY STUDY ON OSCILLATORY BOUNDARY LAYER FLOW

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ABSTRACT

The characteristics of oscillatory boundary layer flow have been treated with keen interest during the last decade from the various aspects. Among the previous results the theoretical treatment by K. Kajiura must be the most important and fruitful one to advance in our knowledge on the present phenomena. On the other hand the senior author has had a real interest in the behaviour of sediment particles due to oscillatory fluid motion, and has conducted his systematic investigations on the sediment movement in nearshore area.

The aim of this paper is to introduce some results of the recent investigations conducted at the Coastal Engineering Laboratory, University of Tokyo, with the intention of investigating the applicability of Kajiura's theory to the oscillatory flow in the vicinity of bottom with sand ripples.

In order to accomplish the above purpose, the authors applied the hydrogen bubble technique to measure accurately as much as possible the time history of velocity distribution especially in the very thin layer above the bottom boundary associated with the progressive waves. The above measuring technique was verified to be enormously powerful even in the case of unsteady flow. The bottom conditions tested in the present investigation were of 1) hydrodynamically smooth bottom, and 2) rough bottom with artificial ripples. The pictures of hydrogen bubble lines released successively from a fine platinum wire were taken by using a 16mm cinecamera specially equiped, and were analyzed flame by flame to take the digitalized data of horizontal velocity of fluid. The data were analyzed in Fourier series and the first mode of the series was only taken to compare with the numerical curves obtained through the Kajiura's theory. The horizontal velocity amplitude and its phase, the eddy viscosity

and the shear stress at each elevation above bottom were calculated by using not only the experimental data of present investigation, but also the laboratory data obtained by I. G. Jonsson.

The agreement between the analysed data and the theoretical curves is basically consistant; this fact verifies the usefulness of the Kajiura's theory. Even though, there are some questions in the theoretical treatment which are arising out of the present investigations. The accumulation of more accurate data is basically requested to clarify these problems. Lastly the comparative study on eddy viscosity averaged over wave cycles in the neighbourhood of sand ripples was established by using the laboratory data of suspended sediment concentration.

INTRODUCTION

The characteristics of oscillatory boundary layer flow induced by surface waves seem to take a primarily important role on the generation, growth and decay of shallow water waves and on the behaviour of coastal sediment in nearshore area. Up to now some prominent treatments on this subject have been made mainly from the theoretical point of view by M. S. Longuet-Higgins,[1] K. Kajiura[2] and others, while from the experimental point of view by I. G. Jonsson,[3] getting a great deal of advanced insight into the characteristics of oscillatory boundary layer flow. But still numerous questions have been remained unsolved due to mainly the profound difficulty in observation and measurement of actual state as well as to the complexity of questioned phenomena.

The aim of this paper is to introduce the main results of the laboratory studies on the above subject carried out at the Coastal Engineering Laboratory, University of Tokyo, during the last few years. The authors have been applying the hydrogen bubble techniques to measure the time history of vertical distribution of horizontal velocity component, especially in the vicinity of bottom, associated with the propagation of surface waves. The laboratory measurements have been repeated under the various conditions of wave and bottom surface. After the laborious improvement of measuring procedures, it was finally successful to visualize the instantaneous velocity distribution and/or the intensity of turbulent fluctuation and to measure the velocity within a really thin boundary layer developed along the flume bottom. In this paper will be presented the data of horizontal velocity distribution in the oscillatory boundary layer which were measured under the conditions of 1) hydrodynamically smooth bottom and 2) roughened bottom with artificial ripples. In the following the comparison of the Kajiura's theory for the bottom boundary layer in water waves with the laboratory data will be conducted by use of the present data mentioned above and the one reported by I. G. Jonsson in 1963. Finally the values of eddy viscosity evaluated by using the measuring data of vertical distribution of suspended sediment concentration will be given and compared with the calculated ones on the basis of the Kajiura's assumption.

BASIC THEORIES ON OSCILLATORY BOUNDARY LAYER FLOW

Equation (1) was introduced by M. S. Longuet-Higgins[1] as an expression for the horizontal velocity component u in a laminar boundary layer induced by progressive waves:

$$u = \hat{U}[\cos(\sigma t - kx) - e^{-z/\delta_L}\cos(\sigma t - kx - z/\delta_L)] \tag{1}$$

where $\hat{U} = \pi H/(T \sinh kh)$, $k = 2\pi/L$, $\sigma = 2\pi/T$, $\delta_L = (2\nu/\sigma)^{1/2}$, ν kinematic viscosity of fluid, H:wave height, T:wave period, L:wave length, h:water depth, t:time, x:horizontal axis, and z:vertical axis taking upward positive from channel bottom.

On the other hand, K. Kajiura[2] presented a model of the bottom boundary layer in water waves and obtained quite important results. He introduced and/or defined such relationships as shown in the following:

$$\frac{\tau}{\rho} = \hat{u}_B^* u^* = K_z \frac{\partial u}{\partial z} \tag{2}$$

$$\delta^* = \mathrm{Amp}\int_0^\infty (U-u)\,dz \,/\hat{U} \tag{3}$$

where τ:friction stress, ρ:density of fluid, u^*:friction velocity, u_B^*:friction velocity at bottom, u:horizontal velocity component in a boundary layer, U:horizontal velocity component at the outeredge of frictional layer, K_z:vertical eddy viscosity, δ^*:wave displacement thickness, and $\hat{}$ denotes the amplitude of physical property.

Following the hypothetical concept of the wall and the defect layers established for the case of a steady turbulent boundary layer, K. Kajiura assumed the frictional layer of the oscillatory flow to consist of three layers: the inner layer, the overlap layer, and the outer layer. He also introduced three numbers such as $N(=12)$, $\kappa(=0.4)$ and $K(=0.02)$ corresponding to the above three layers, and defined the value of K_z as follows:

For the case of hydrodynamically smooth bottom:

$$K_z = \begin{cases} \nu & 0 \leqq z \leqq D_L \quad \text{(inner layer)} & (4a) \\ \kappa \hat{u}_B^* z & D_L < z \leqq d \quad \text{(overlap layer)} & (4b) \\ \kappa \hat{u}_B^* d & d < z \quad \text{(outer layer)} & (4c) \end{cases}$$

where D_L, the thickness of laminar sublayer (or inner layer), is determined by $\hat{u}_B^* D_L/\nu = N$, while d, the upper limit height of the overlap layer above bottom, is by $\kappa u_B^* d = K\hat{U}\delta^*$.

For the case of hydrodynamically rough bottom:

$$K_z = \begin{cases} \alpha\kappa \hat{u}_B^* D_R & 0 \leqq z \leqq D_R \quad \text{(inner layer)} & (5a) \\ \kappa \hat{u}_B^* z & D_R < z \leqq d \quad \text{(overlap layer)} & (5b) \\ \kappa \hat{u}_B^* d & d < z \quad \text{(outer layer)} & (5c) \end{cases}$$

where z_0:roughness length, D:Nikuradse's equivalent roughness, and D_R:thickness of inner layer. The following relations are also assumed:

$$\left.\begin{aligned} D&=30\ z_o \\ D_R&=(1/2)D=15z_o \\ \alpha&=\ln(D_R/z_o)=2\ 708 \end{aligned}\right\} \tag{6}$$

If the total thickness of the bottom frictional layer is assumed to be very small compared with the wave length, and the nonlinear effect is also assumed to be negligible (except turbulence), the equation of oscillatory motion in the boundary layer is given by

$$\frac{\partial}{\partial t}(u-U)=\frac{\partial}{\partial z}\left(\frac{\tau}{\rho}\right) \tag{7}$$

Combination of Eqs. (2) and (7) gives the following basic equation:

$$\frac{\partial^2 u^*}{\partial z^2}-\frac{i\sigma}{K_z}u^*=0 \tag{8}$$

where τ/ρ is replaced by u^* with the aid of Eq. (2). Substituting the hypothetical expression of K_z, Eq. (4) or Eq. (5) corresponding to the bottom condition, into Eq. (8), K. Kajiura solved the basic equation to obtain the generalized equations for u and u^*. He also defined the friction coefficient C by $\tau_B/\rho=C\hat{U}U$ and found that the amplitude $\hat{C}$ and phase θ of friction coefficient were expressed as functions of $R=\hat{U}\delta_L/\nu$ (where $\delta_L=(\nu/\sigma)^{1/2}$ $=\delta_L/\sqrt{2}$) for smooth bottom or $\hat{U}/(\sigma z_o)$ for rough bottom.

He took tentatively the following criterions to classify the bottom conditions in the hydraulic sense and the flow regimes.

For the transitional region from smooth to rough:

$$0.4\leq D/D_L\leq 5 \tag{9}$$

For the transitional region from laminar to turbulent:

$$0.4\leq \delta^*/D_L\leq 5 \tag{10}$$

Rewriting Eqs. (9) and (10), the transitional region from laminar to turbulent for smooth bottom is expressed by

$$25\leq R\leq 650 \tag{11}$$

while that rough bottom is given by

$$10^2\leq M\leq 10^3 \tag{12}$$

where $M=\hat{U}D/\nu$.

The description mentioned above is the outline of Kajiura's

theory, in which thereare several questions to be clarified. These are:

(1) Whether the assumption for K_z is appropriate to the present problem or not?

(2) Whether the basic equation (7) is adequate to express the flow characteristics in the vicinity of bottom with ripples or not? The question of this kind arises from the fact that a relatively large vortex exists behind a ripple.

(3) How should we determine the origin of z axis when the bottom is rough?

(4) Is the assumption for the estimate of D_R and z_0 as shown in Eq. (6) reasonable?

In order to clarify the above questions the systematic and careful measurements of the flow characteristics in a boundary layer are basically required.

In addition to the above discussion we will consider the critical condition for the disappearance of overlap layer. For the smooth bottom the criterion is given by $D_L > d$. According to K. Kajiura, the height of the outeredge of overlap layer is expressed by

$$d = 0.05\hat{C}^{\varkappa}\hat{U}/\sigma \tag{13}$$

Considering the relations of $D_L = \nu N/\hat{u}_B^*$ and $R^2 = (\hat{U}\delta_L'/\nu)^2 = \hat{U}^2/(\sigma\nu)$, we will obtain the following inequality as the critical condition for the disappearance of overlap layer:

$$\hat{C} < 240/R \tag{14}$$

Taking into consideration the given relationship between $\hat{C}$ and R, the criterion of Eq. (14) is rewritten as

$$R < 217 \tag{15}$$

Comparing Eq. (15) with Eq. (11), we may conclude that the overlap layer for the smooth bottom exists always in the range of turbulent flow, but not in the range of laminar flow.

On the other hand, the criterion for rough bottom is expressed by $D_R > d$, which is equivalent to Eq. (16) or Eq. (17).

$$\hat{C} < 9.0\times10^4(\hat{U}/\sigma z_0)^2 \tag{16}$$

$$\hat{U}/(\sigma z_0) < 3.5\times10^3 \tag{17}$$

The above criterion is again rewritten as follows by using the total excursion distance of horizontal particle motion just outside of boundary layer d_0:

$$d_0/D < 230 \tag{18}$$

As a result of the above discussion it is known that when the roughness elements with relatively large size exist along the bottom, the overlap layer may always disappear.

LABORATORY APPARATUS AND MEASURING TECHNIQUES

The present authors have recognized that the hydrogen bubble technique is quite powerful to visualize the stream line and velocity distribution in a laboratory flume even in the case of oscillatory flow.[4] The principle of this instrument is to take moving pictures of hydrogen bubble line which is generated from a cathode of a fine platinum wire (50μ in diameter). The schematic diagram of the instrumentation system is shown in Fig. 1. The voltage charged between the cathode and anode is as high as about 1 KV, and the input pulse is discharged at intervals of about 50 cps. An iodine lamp (200 V and 1 KW) is used as a light source for illuminating the hydrogen bubble lines, and a 16mm camera (Canon Scoopic-16) is used for taking clear and accurate pictures with negligible aberration of camera lens.

The wave flume used for the present investigation is 50m long, 60cm wide and 90cm high. The bottom conditions tested are two; one is smooth bottom made of hard vinyl plate and the other is rough bottom made of plastic wavy board as shown in Fig. 2. The rise and pitch of the artificial ripple are 0.8cm and 3.2cm respectively. The test waves for the rough bottom are adjusted as much as possible to have the combination of wave height and period which suits to the size of the ripple. In order to adjust the wave conditions the generalized diagrams among wave characteristics, water depth, ripple size and grain diameter are applied under the assumption of $M_d=200\mu$, where M_d is the medium grain diameter of sediment.

The test waves are generated by a flap type wave generator which is installed at the end of wave flume. The 5th to 7th waves are selected for the measurement of velocity by the reason that these waves are almost in a steady state and that the waves reflected from the other end of flume have not arrived at the measuring section yet. The hydraulic conditions of the present investigations are given in Table 1 together with that of Jonsson's experiment.

In order to compare the laboratory data with the theory, the time histories of surface profile and of velocity at each elevation are analyzed in Fourier series and the first mode of corresponding data is only, in general, taken into consideration. Therefore in Table 1, H is the wave height corresponding to the fundamental period T, and $\hat{U}$ is calculated by the relation of $\hat{U}=\pi H/(T \sinh kh)$. The Reynolds number $R=\hat{U}\delta'/\nu$ or $M=\hat{U}D/\nu$ is also given in Table 1 as an index of flow regions such as laminar, transitional and turbulent. The computation of M is based on the assumption of $D=4\eta$, where η is the rise of ripple.

The close-up of hydrogen bubble rows is taken by the 16mm cine camera with the speed of 48 flames per second. Analysis of each flame of film gives the time history of horizontal velocity component at each elevation. In the case of rough bottom with artificial ripples, the measuring section is fixed at the trough of a certain ripple.

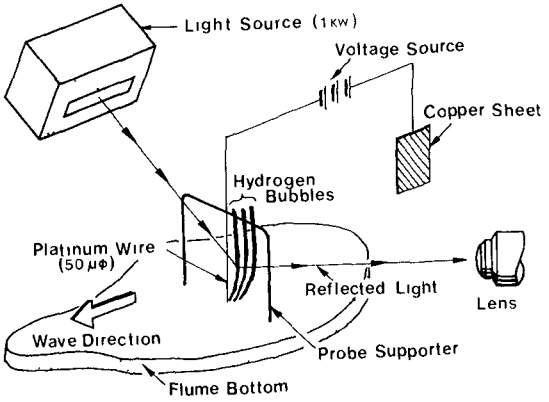

Fig. 1. Instrumentation system.

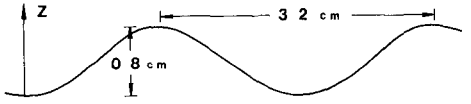

Fig. 2. Artificial ripple.

Table 1. Experimental conditions.

Investigators	Experimental Method	Bottom Condition	No	T (sec)	h (cm)	H (cm)	ν (cm²/sec)	$\hat{U}$ (cm/sec)	$R = \hat{U}\delta_s/\nu$	$M = \hat{U}0/\nu$
Authors	Progressive Waves	Smooth	—	1 50	41 0	3 71	$9\,23\times10^{-3}$	6 82	35	—
		Artificially Roughened with Ripples	1	1 07	39 4	8 16	$1\,00\times10^{-2}$	10 9	—	$3\,5\times10^{3}$
			2	1 60	34 6	11 8	$1\,37\times10^{-2}$	25 6	—	$6\,0\times10^{3}$
Jonsson	Oscillatory Tank	Artificially Roughened	—	8 39	—	—	$9\,33\times10^{-3}$	213 1	—	$3\,4\times10^{4}$

ANALYSIS OF DATA AND DISCUSSION

SMOOTH BOTTOM

1) Velocity Distribution

The horizontal velocity at each elevation read from film as well as the surface profile recorded by a parallel wire resistance gauge is, as mentioned in the previous chapter, analyzed in Fourier series, and the data corresponding to the fundamental period T are plotted as shown in Fig. 3. Figure 3(a) is for the amplitude ratio between u and U, $\hat{u}/\hat{U}$, and Fig. 3(b) is for the phase difference ϵ between u and U. The reader should notice that the velocity at the elevation of z=0.018cm is measurable. In the same figures the theoretical curves calculated through the Kajiura's theory are also shown by solid line for the comparison between the theory and the experimental results. In this particular case the Reynolds number R is equal to 35; this fact indicates that the flow is in the transition between laminar and turbulent (see Eq. (11)). Under such condition the rate of discrepancy in $\hat{u}/\hat{U}$ and ϵ between Eq. (1) for laminar flow and the Kajiura's theory is at the most 1% respectively.

The comparison of the experimental data with the theoretical curves indicates that:

(1) The rate of discrepancy in velocity amplitude between them is in the order of 1 to 5%, but increases up to about 10% in the very vicinity of bottom (z=0.018cm).

(2) The agreement of velocity phase difference is very good in the region of $z \geq 0.09$cm, but is not satisfactory at the elevation of z=0.018cm. The phase of experimental data at the latter elevation is about 12° smaller than the theoretical one. It is still uncertain whether such systematic differences mentioned above are caused by the nonlinear effect of higher harmonics or by the inaccuracy of the measuring techniques and of the data analysis.

2) Eddy Viscosity

Combining Eqs. (2) and (7), we obtain the following expression for the vertical eddy viscosity K_z:

$$K_z = -\int_{\delta}^{z} \frac{\partial}{\partial t}(u-U)\,dz \Big/ \frac{\partial u}{\partial z} \tag{19}$$

where δ is the thickness of frictional layer.

By using the analyzed data of velocity distribution, the value of K_z at each elevation for each phase is calculated numerically on the basis of Eq. (19) and plotted in Fig. 4. In the same figure the vertical distribution of K_z assumed by K. Kajiura is also shown by solid lines. The reader should be mentioned that the overlap layer disappears in this case because of R=35 (see Eq. (15)). The data of K_z estimated from the experimental values are distributed in the vicinity of solid lines

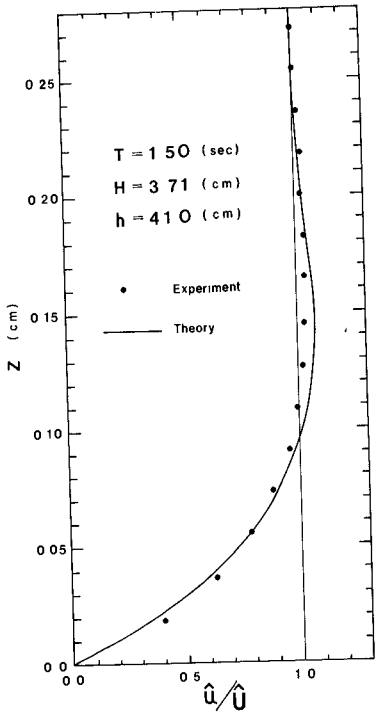

Fig. 3(a). Amplitude ratio of u to U. (Smooth bottom)

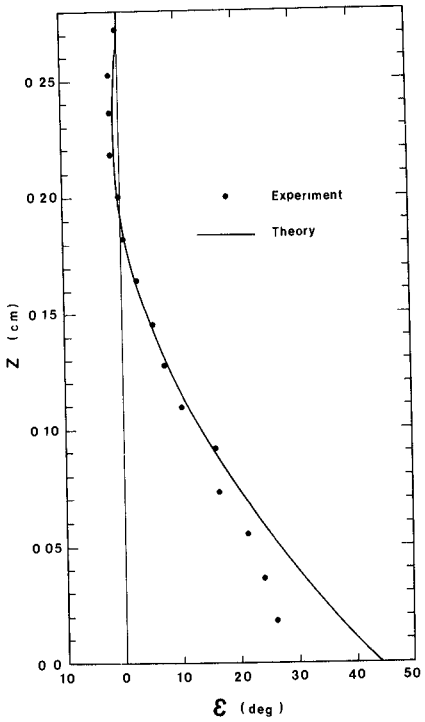

Fig. 3(b). Phase difference between u and U. (Smooth bottom)

with great scattering especially at the level of $z \gtrsim 0.05$cm. As an example the variation of K_z at $z=0.091$cm with the wave phase φ is shown in Fig. 5. The origin of abscissa φ in this figure is taken at the moment when the corresponding crest of surface wave arrives at the measuring station. According to the assumption taken in the Kajiura's theory, $K_z=\nu$ in the inner layer ($z \leq D_L$), and $K_z=\kappa \hat{u}_* d$ in the outer layer ($z>D_L$), therefore there is a discontinuity of K_z at the boundary between the inner and outer layers in the present case. On the other hand, the estimated values of K_z at each elevation indicate a periodic fluctuation and also the mean of K_z averaged over one wave cycle seems to be expressed quite naturally by a certain continuous function of z.

3) Shear Stress

Figure 6 gives the value of τ/ρ estimated through Eq. (7) by using the analyzed data of velocity distribution. Figure 6(a) is for the amplitude of τ/ρ and Fig. 6(b) is for the phase difference γ between τ/ρ and U. In these figures the solid circle indicates the data calculated from the fundamental mode of velocity u_1, while the open circle indicates the first mode data of τ/ρ calculated from the combined velocity $u_{1\sim6}$. Where $u_{1\sim6}$ is the combined velocity which includes 1st to 6th modes. The mode higher than 6th is practically negligible. The good agreement between the open circle and closed circle suggests us the applicability of superposition of shear stresses as a simple approximation. In Fig. 6 the theoretical curves of $\hat{\tau}/\rho$ and γ are also shown for the comparison. The agreement between the estimated values and the theory is satisfactory.

ROUGH BOTTOM

1) Velocity Distribution

Figures 7 and 8 give the amplitude $\hat{u}/\hat{U}$ and phase ϵ of u/U corresponding to the fundamental mode of different waves. As stated in the previous chapter the velocity was measured at the section of ripple trough. The flow characteristics in both cases are in a fully turbulent region judging from the values of M given in Table 1 (see Eq. (12)). The overlap layer is also disappeared in both cases. In order to compare the above data with the Kajiura's theory, we have to consider the following two questions: One is how to determine the origin of z axis (apparent bottom level) and the other is how to assume the thickness of inner layer $D_R (=15z_0)$. H. Motzfeld[5] conducted his laboratory experiments by using a ripple model, which had the shape of crest angle being 120° and of the trough with circular arc, under the condition of steady turbulent flow, and determined the Nikuradse's equivalent roughness D as 4 times the rise of ripple η. Supposing that the Motzfeld's result is applicable to the present unsteady flow also, we determined the thickness D_R as follows:

$$D_R = D/2 = 2\eta = 1.6cm$$

In Figs. 7 and 8 are shown the theoretical curves calculated on

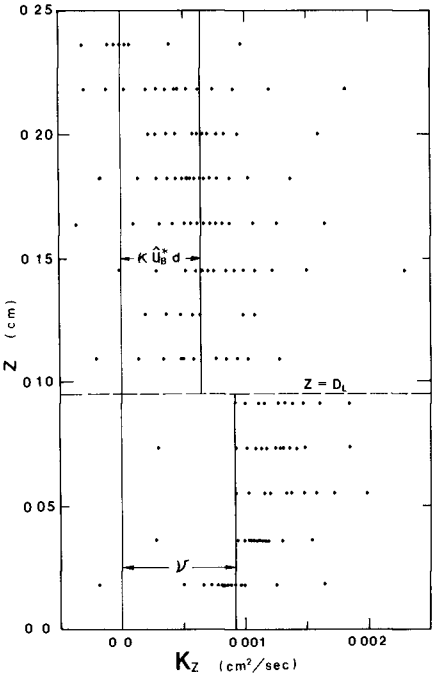

Fig. 4. Vertical distribution of eddy viscosity. (Smooth bottom)

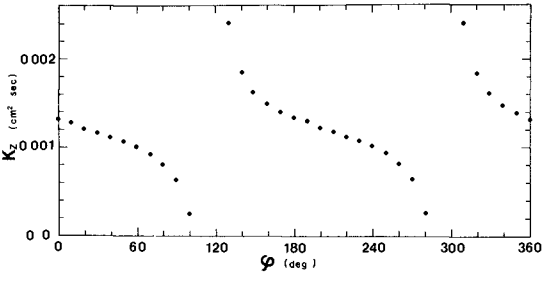

Fig. 5. Variation of eddy viscosity with wave phase. (Smooth bottom, z = 0.191cm)

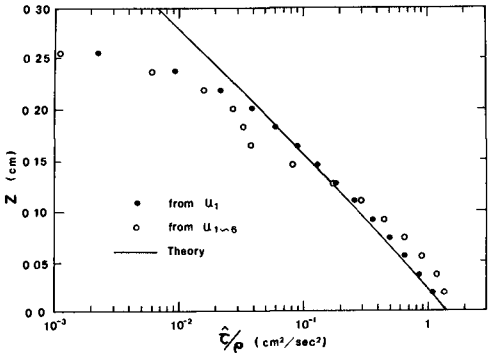

Fig. 6(a). Amplitude of shear stress. (Smooth bottom)

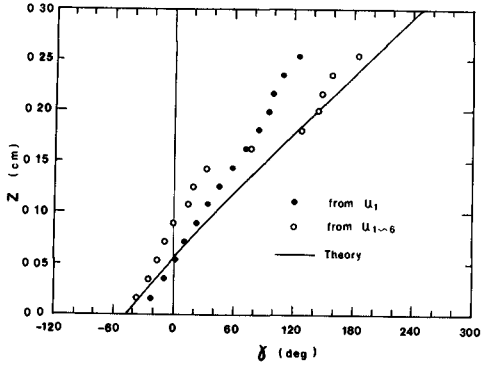

Fig. 6(b). Phase difference between shear stress and U. (Smooth bottom)

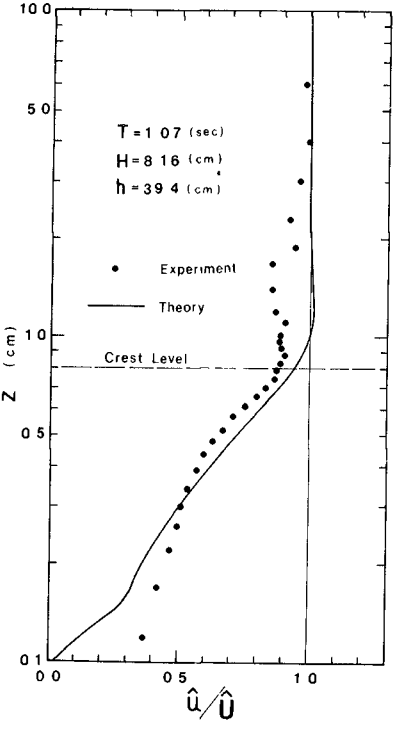

Fig. 7(a). Amplitude ratio of u to U. (Rippled rough bottom, Test series No. 1)

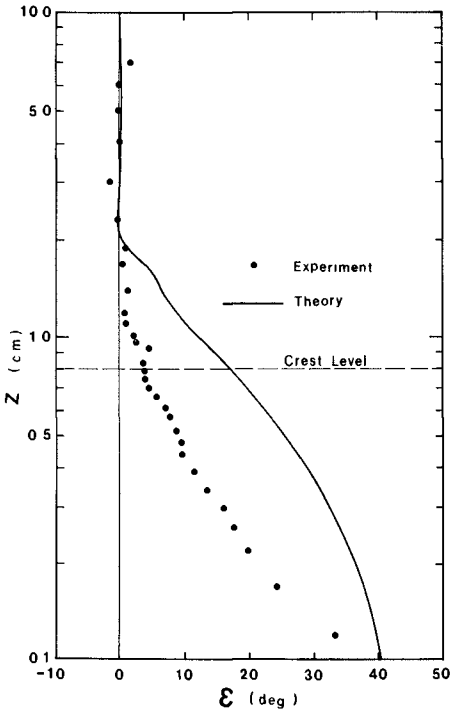

Fig. 7(b). Phase difference between u and U. (Rippled rough bottom, Test series No. 1)

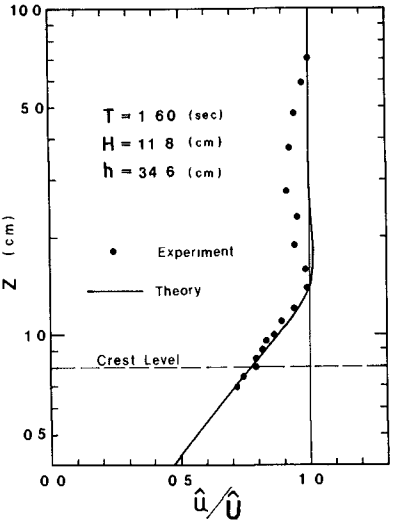

Fig. 8(a). Amplitude ratio of u to U. (Rippled rough bottom, Test series No. 2)

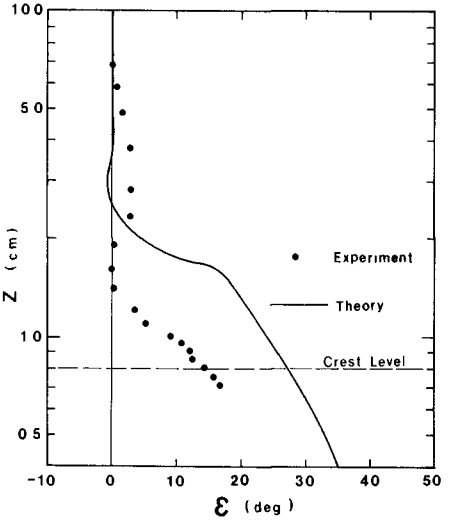

Fig. 8(b). Phase difference between u and U. (Rippled rough bottom, Test series No. 2)

the basis of the above assumption and also by taking the zero level of z at the trough of artificial ripple. The agreement between the laboratory data of $\hat{u}/\hat{U}$ and the theoretical curve is fairly good in general. But the data at the elevation a little higher than the ripple crest (z=0.8cm) have a consistent tendency of taking smaller values compared with the theoretical ones. This tendency may be caused by the influence of vortex appeared behind a ripple. On the other hand the laboratory data of ϵ are about 10° to 20° smaller than the theoretical ones.

It is well known that I. G. Jonsson conducted his valuable experiment on oscillatory flow by using the ingenious apparatus of oscillatory tank. He installed artificial roughness elements as shown in Fig. 9 on the bottom of oscillatory tank and measured the velocity distribution by means of a miniature current meter of propeller type. Figures 10(a) and 10(b) were taken from the figures appeared in the Kajiura's paper. In this case K. Kajiura took z=0 at the height of 0.35cm above the trough of roughness elements and also assumed as z_0=0.05cm , which was equivalent to the condition of $D=30z_0=2.5\eta$, η being the height of artificial roughness. In this experiment the total excursion length d_0 was fairly large compared with the scale of roughness elements, hence the value of d_0/D=380 was large enough to maintain the overlap layer. The agreement between the theoretical curves and the experimental data for the first mode is quite satisfactory.

The agreement in the authors' cases (T=1.07sec and 1.60sec) is not so good as in Jonsson's case (T=8.39sec). The above fact may be explained by the following reason: The artificial roughness elements of the authors' experiment are relatively large compared with the scale of oscillatory motion, hence the turbulence induced by vortex is predominate. As a result of this situation Eq. (17), in which the vertical component of velocity is neglected, is incomplete to represent the flow characteristics of the present case.

3) Eddy Viscosity

Figure 11 gives the distribution of K_z determined through Eq. (19) as for the case of smooth bottom by using the velocity distribution of the first mode oscillation. The scattering of data indicates the time variation of K_z as in the case of smooth bottom. There happens a curious result such as negative value of K_z. This may be caused partly by the error of numerical computation for K_z, but mainly by the following fundamental reason. That is, Eq. (7) is inadequate to express the present phenomena due to the existance of vortex with relatively large scale. The vortex induces the vertical velocity component which is comparative to the horizontal one.

Figure 12 shows the vertical distribution of K_z estimated from the Jonsson's data. The fluctuation of K_z in the vicinity of bottom is small compared with that in Fig. 11, but it seems relatively large in the range of $z \gtrsim 2$cm. The agreement between

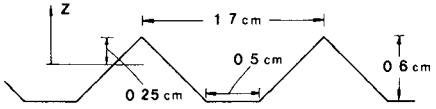

Fig. 9. Artificial roughness in Jonsson's experiment.

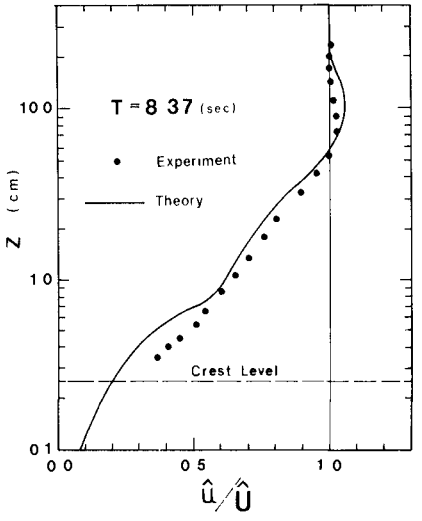

Fig. 10(a). Amplitude ratio of u to U based on Jonsson's data. (After K. Kajiura[2])

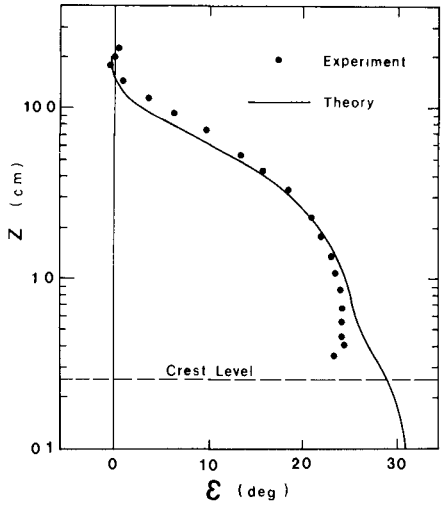

Fig. 10(b). Phase difference between u and U based on Jonsson's data. (After K. Kajiura[2])

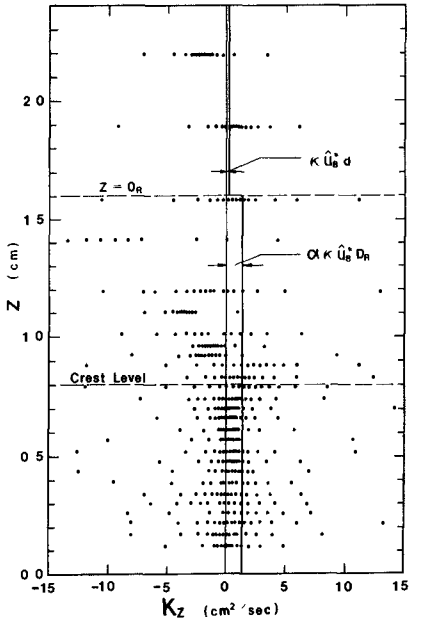

Fig. 11. Vertical distribution of eddy viscosity. (Rippled rough bottom, Test series No. 2)

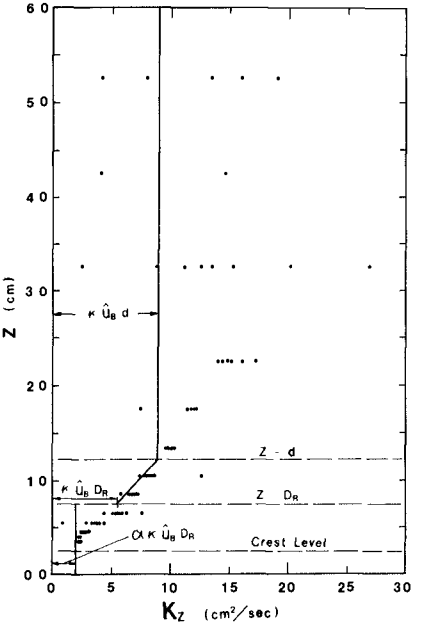

Fig. 12. Vertical distribution of eddy viscosity based on Jonsson's data.

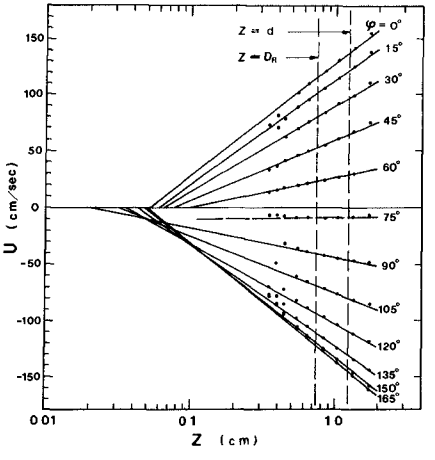

Fig. 13. Velocity distribution for each wave phase based on Jonsson's data.

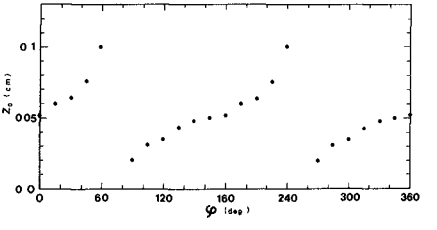

Fig. 14. Variation of roughness length with wave phase based on Jonsson's data.

the theoretical assumption (Eq. (5)) and the estimated value is fairly well, but the expression of $K_z=\kappa\hat{u}_R^* z$ for $D_R<z<d$ and $0\leq z\leq D_R$ too seems better than that of Eq. (5) at least in the questioned case.

4) Roughness Length

The horizontal velocity distribution in the overlap layer is well approximated by the logarithmic distribution curve. From this point of view in Fig. 13 the analyzed data of Jonsson are plotted in each phase. The roughness length z_o is determined as an intersection between the axis of $u=0$ and the extended straight line of velocity distribution curve in the overlap layer on a semilog paper. The value of z_o thus determined fluctuates with the phase of surface wave motion as shown in Fig. 14. But it can not be determined at the phases near 75° and 255°, where shear stress becomes zero.

5) Shear Stress

Similar to the case of smooth bottom, the vertical distribution of shear stress τ/ρ is calculated numerically by using the Jonsson's data and is shown in Fig. 15. Figure 15(a) is for $\hat{\tau}/\rho$ and Fig. 15(b) is for the phase difference γ between τ/ρ and U. The definition of open and closed circles is the same as in the smooth bottom case. The agreement between the two results is fairly well for $\hat{\tau}/\rho$, but is not well for γ. The discrepancy of γ is about 5°~6° between the two cases. At any rate the superposition of shear stress seems to be a good approximation in the rough bottom case as well as in the smooth bottom case.

SUSPENDED SEDIMENT CONCENTRATION

During the last decade the senior author has devoted his continuous efforts to study the suspended sediment concentration and reported a part elsewhere.[6] In the following a further study will be described briefly.

The suspended particle is actually heavier than the fluid, hence the following will be given as the fundamental equation of the suspended sediment concentration.[7]

$$\frac{d}{dz}\left(K_z\frac{d\bar{c}}{dz}\right)+(\beta w_o)\frac{d\bar{c}}{dz}=0 \tag{20}$$

where $\bar{c}$: sediment concentration averaged over one wave cycle, w_o: fall velocity of sediment particle, and β: a coefficient usually taken as unity.

The senior author has analyzed the suspended sediment phenomena due to surface waves and proposed a method to determine the vertical distribution of suspended sediment concentration.[6] The basic assumption of the above treatment was that the diffusion process was governed by the turbulence associated with the orbital motion of water particle, and that the potential theory of surface waves was applicable as a first approximation to

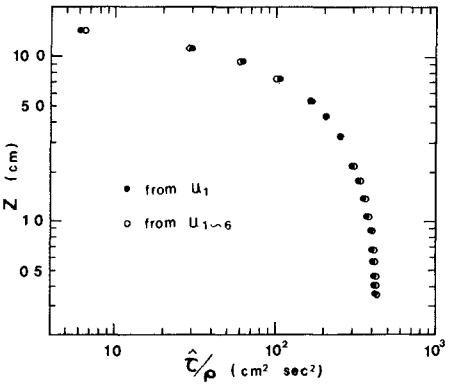

Fig. 15(a). Amplitude of shear stress based on Jonsson's data.

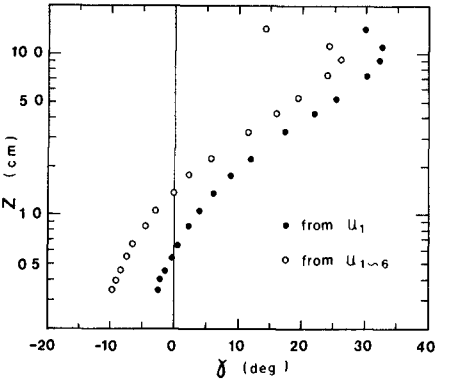

Fig. 15(b). Phase difference between shear stress and U based on Jonsson's data.

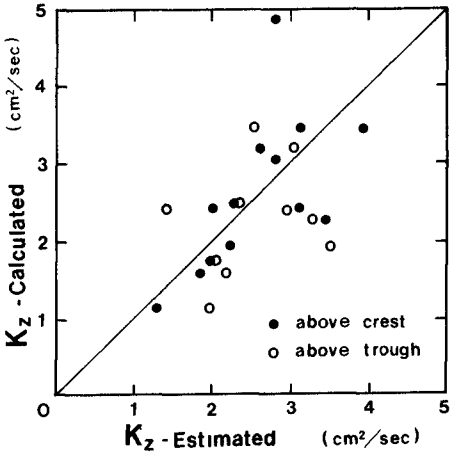

Fig. 16. Comparison between estimated and calculated eddy viscosity.

estimate the eddy viscosity K_z. Therefore the above treatment may be applicable only to the suspended sediment phenomena in the upper layer of water beyond the outer layer.

While Kishi et al.[8] applied the expression of eddy viscosity such as $K_z=\alpha' \kappa u_B^* (z+z_0)$ to the suspended sediment problem, where α' is a coefficient. The above expression of eddy viscosity was originally proposed by K. Kajiura[9] in his early paper. Considering the assumption of K_z, we may recognize that their result on the vertical distribution of suspended sediment concentration is applicable to a certain limited layer near the bottom. Here the authors will concentrate their attention on the effect of ripples on the sediment motion.

According to Eq. (5a) the eddy viscosity for the inner layer is expressed by $K_z=\alpha\kappa\hat{u}_B^* D_R$, hence the integration of Eq. (20) gives the following solution:

$$\bar{c}=\bar{c}_0 \exp[(\beta w_0/K_z)z] \tag{21}$$

where $\bar{c}_0$ is the concentration at the level of $z=0$. Under the assumption of $\beta=1.0$ the value of K_z for each run of experiment reported previously was estimated,[10] while the theoretical one was calculated under the assumption of $D_R=2\eta$, η being the rise of sand ripple. Figure 16 shows the comparison between K_z (estimated) and K_z (calculated), where the laboratory data are grouped into two; one is the data measured above ripple crest and the other above ripple trough. Even a large scattering of data, the majority of them falls near the 45° straight line. From the above fact, the dimensionless number β, which was introduced to express the influence of relative scale between grain size and the scale of turbulence, may be taken as unity at least in the very vicinity of sand bottom.

CONCLUSIONS

The characteristics of oscillatory boundary layer flow have been investigated from many aspects on the basis of laboratory data of velocity distribution in a boundary layer. The hydrogen bubble technique is surprisingly powerful to measure the velocity in the very vicinity of bottom or in the space behind ripple. In order to increase the accuracy of measurement and of data processing, it is still required to improve more the present instrumentation system.

The Kajiura's theory is a very important treatment to clarify the characteristics of oscillatory boundary layer flow, but involves some questionaries to be solved in future. These are as follows:

1) According to the present investigation, the eddy viscosity seems to be a time dependent function with the same period as the surface wave, and also a continuous function of z. Based on the above fact, the Kajiura's assumption is not always satisfactory to the present phenomena.

2) There is no rule how to determine the zero level of z axis

in the case of rough bottom.

3) It is still uncertain whether the estimation of D_R, D and z_0 introduced into the Kajiura's theory is appropriate to the present phenomena.

More accurate data are required to make the above problems clear. Besides, it is necessary to introduce another basic equation which is applicable to the phenomena in the very vicinity of sand ripple. The flow is very complicative owing to the existence of a large vortex. Further studies are truely needed to advance our knowledge on the oscillatory flow phenomena.

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