CHAPTER 13

WAVE REFLECTION AND TRANSMISSION IN CHANNELS OF VARIABLE SECTION

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SUMMARY

Reflection and transmission phenomena were investigated in a laboratory channel for transitions of linearly varying depth and/or width terminating in channels of reduced cross-section. Upstream wave characteristics were varied from deep to shallow water waves by changing wave frequencies, amplitudes and channel depths over a wide spectrum of conditions. Experimental values were corrected to correspond to transmission in an endless channel. The reflection and transmission coefficients are given as functions of the pertinent dimensionless parameters such as: group velocity ratio, channel depth ratio and wave steepness. Associated wave energies were also evaluated.

THE PROBLEM

Ocean waves arriving on beaches and in estuaries undergo a process of shoaling, i.e., they are transformed by the variation in the bottom topography of these areas. Changes in wave length, amplitude and phase angle are produced, and the transmission of wave energy is affected; part of the energy is transmitted, a part is reflected, and in addition processes of dispersion and of dissipation are active. Detailed knowledge concerning these phenomena is important in the design of maritime structures and harbors.

This study deals with an experimental program on reflection and transmission of waves in channel transitions of linearly varying bottom elevations and/or width for three geometries of the channel transition as given in Figure 1. labelled A, B and C. The depth in the approach channel was varied to give ratios of h_1/h_3 between 1.80 and 3 to 5. The periods of the incoming waves could be varied from .8 to 12.0 seconds resulting in a range of characteristics from deep water to shallow water waves. Wave steepness also was considered as a significant parameter and varied accordingly from 1/1000 to 1/20.

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The limits of the theory for such transitions are well established. On the one end of the spectrum is Green's Theorem which gives the amplitude of the transmitted wave on the basis of frictionless flow for shallow water waves in long, gradual transitions while reflection is zero. On the other end we find the theoretical transmission and reflection coefficients for abrupt transitions in width and/or depth given by Lamb (2) for linearized shallow water waves. There have been a number of attempts to generalize the theory for transitions between these limits by various investigators such as Rayleigh (3), Carrier (4), Takano (5), Kajiura (6) and Dean (7). An extensive review of this literature is given in references (8) and (9). The theory for the amplitude and phase changes for a wave of discrete frequency encountering a gradual transition is rather complex.

The mass flux varies with position and time over the transition length, such that the net storage within this length is variable and is zero only over a full wave period.

THE EXPERIMENTAL STUDY

In view of the rather limited scope of the theoretical results available so far, the present study adopted for the experimental program a systematic variation of the basic wave parameters within the possibilities of the available laboratory wave channel. These parameters are in dimensionless form: the group velocity ratio C_{G3}/C_{G1}, the reflection coefficient $K_r = a_2'/a_1'$ the transmission coefficient $K_t = a_3'/a_1'$, the depth ratio h_3/h_1 and the incoming wave steepness H_1/L_1.

The experimental wave channel is 100 feet long and is of rectangular cross-section, 2.5 feet wide and 3.0 feet deep as shown in Figure 2. The sides and 40 feet of the bottom consist of plate glass. Both a piston-type and a flap-type wave maker are available at one end followed by an expanded aluminum wave filter. At the other end energy absorbers of several types were utilized. Data were obtained by means of resistance wire wave gages mounted on carriages travelling over the channel sections upstream and downstream of the transition. The wave envelopes were recorded on Sanborn oscillographs.

Several transitions with linearly varying depth were tested during the overall program, the first attempts being made with relatively steep transition of 1:0.58 ($\alpha = 60°$) and 1:2.75 ($\alpha = 20°$). The depth ratios h_1/h_3 employed varied from 2.5 to 7.0. The short and intermediate waves were kept to small amplitudes. The channel following the transition section terminated abruptly. The results obtained for this initial study for reflection and transmission are reported in reference (8) and are employed for the present study primarily for comparison. This is true also for the second study with a transition of 1:16 ($\alpha = 3.57°$), however, wave steepness was varied in this program as an important parameter. Depth variations extended over a more limited range of h_1/h_3 = 2.0 to 3.0. The wave periods corresponding to the range of short waves to shallow water waves were T = .67 seconds to 7.25 seconds. The wave steepness varied correspondingly from a maximum of .058 for short waves to a minimum of .00072 for long waves (8).

The test program to be emphasized here is the one specified for the three transitions A, B and C, indicated in Figure 1. As for the transition 1:16 these transitions were also exposed to the full range of waves from short to shallow water waves with depth ratios and wave steepnesses as given before.

EXPERIMENTAL RESULTS

Although the channel following the transition terminated in wave absorbers at the end of the wave channel some reflection remained as determined from the wave amplitude envelope. This necessitated the reduction of the data again as originally developed by Ursell and Dean (10) for an endless channel. This was done by a computer program for the wave systems S_1 and S_3 shown in Figure 3. The corrected emplitudes a'_1, a'_2 and a'_3 are related to the measured wave amplitudes a_1, a_2, a_3 and a_4 for the linearized waves assumed as follows:

$$a_1' = a_1 \sqrt{1 + \left(\frac{a_2}{a_1}\right)^2 \left(\frac{a_4}{a_3}\right)^2 - 2 \left(\frac{a_2}{a_1}\right) \left(\frac{a_4}{a_3}\right) \cos\left(\delta_1 - \delta_2 + \delta_4\right)} \qquad (1)$$

$$a_2' = a_2 \sqrt{1 + \left(\frac{a_1}{a_2}\right)^2 \left(\frac{a_4}{a_3}\right)^2 - 2 \left(\frac{a_1}{a_2}\right) \left(\frac{a_4}{a_3}\right) \cos\left(\delta_1 - \delta_2 + \delta_4\right)} \qquad (2)$$

$$a_3' = a_3 \left[1 - \left(\frac{a_4}{a_3}\right)\right]^2 \qquad (3)$$

Hence, the experimental reflection and transmission coefficients are defined by:

$$K_r = \frac{a_2'}{a_1'} \quad (4) \qquad K_t = \frac{a_3'}{a_1'} \qquad (5)$$

These coefficients derived from experimental data are compared in the graphical presentations of results with those given by Lamb for abrupt transitions for long waves (2). Since in the short and intermediate wave range Lamb's analysis is not applicable the coefficients of reflection and transmission were restated from the energy balance quite generally for a channel of constant width:

$$(1 - K_r^2)\, C_{G1} = K_t^2\, C_{G3} \qquad (6)$$

and
$$K_r = \left[1 - K_t^2 \left(\frac{C_{G3}}{C_{G1}}\right)\right]^{1/2} \tag{7}$$

wherein
$$C_G = nC = \frac{1}{2} C \left[\frac{2\ kh + \sinh\ 2kh}{\sin\ 2\ kh}\right] \tag{8}$$

Values for the group velocity ratios are readily obtained from wave tables (11).

Lamb has derived the reflection and transmission coefficients for abrupt transitions and shallow water waves as:

$$K_r = \frac{1 - \left(\frac{B_3}{B_1}\right)\sqrt{\frac{h_3}{h_1}}}{1 + \left(\frac{B_3}{B_1}\right)\sqrt{\frac{h_3}{h_1}}} \tag{9}$$

$$K_t = \frac{2}{1 + \left(\frac{B_3}{B_1}\right)\sqrt{\frac{h_3}{h_1}}} \tag{10}$$

Noting that the terms $\sqrt{h_3/h_1}$ represent group velocity ratios for shallow water waves and since C = L/T these coefficients may be stated for our purposes as:

$$K_r = \frac{1 - \left(\frac{B_3}{B_1}\right)\left(\frac{C_{G3}}{C_{G1}}\right)}{1 + \left(\frac{B_3}{B_1}\right)\left(\frac{C_{G3}}{C_{G1}}\right)} \tag{11}$$

and
$$K_t = 1 + K_r = \frac{2}{1 + \left(\frac{B_3}{B_1}\right)\left(\frac{C_{G3}}{C_{G1}}\right)} \tag{12}$$

Values of the coefficients computed from equations (11) and (12) were used for comparison with the experimental results defined by equations (4) and (5).

Figure 4 gives these experimental values for transition A and short and intermediate waves versus the group velocity ratio. Previous experimental investigations for linearly varying bottom transitions 1:2.75 and 1:16 are shown by respective average lines. As in the previous studies the scatter for this transition of slope 1:8 is very large and a good physical explanation is difficult. A possible reason is the restricted length of channel, within which only a few waves can be generated ahead of the transition. Damping effects also make an accurate evaluation of amplitude changes difficult, since wave envelopes extend over a considerable portion of the channel. Generally, reflection coefficients for gradual transitions are considerably higher than those established for Lamb's case. The most abrupt transition 1:2.75 is closest to this case, the transition 1:8 giving the highest values of K_r. The transition 1:16 shows the reflection coefficient lower again as one would expect for decreasing bottom slopes, the reflection eventually becoming negligible.

The trend for both the reflection and transmission coefficients is a decreasing one with an increase in the group velocity ratio. Differences in the transmission coefficients for the various transitions are relatively small in the significant range of group velocity ratios near unity and all K_t values are close to unity. Maximum values of $K_t = 1.25$ were reached for the lowest group velocity ratios. To be noted in connection with the scatter of the results in Figure 4 is the fact that they are not differentiated with regard to wave steepness, which was found however, to have relatively little effect on reflection and transmission for the short and intermediate wave range.

Figure 5 gives the experimental data for transition B for short and intermediate waves. Within the scatter the average experimental trend shows surprising agreement with the equivalent curves computed from Lamb's theory for shallow water waves. No previous results are available for this type of transition.

Figures 6 and 7 show a comparison of the reflection and transmission coefficients for transitions A and B for shallow water waves with Lamb's theory for abrupt changes in section as a function of group velocity ratio. The reflection coefficients for transition A are again higher than Lamb's values and the transmission coefficients are lower, but the change is not great. Transition B gives higher values than transition A, as expected, in view of the additional side contraction. However, it is seen that Lamb's theory overcompensates for this effect and the reflection and transmission coefficients are now lower and higher respectively than those from this theory.

Reflection and transmission coefficients both decrease with increasing group velocity ratio. The scattering for shallow water waves for each change in the depth ratio is more distinctly attributable to wave steepness as is seen from the following figures 8 and 9. In view of the absence of any theory including wave steepness as a parameter results for the reflection and transmission coefficients are compared to the mean curve of the transition 1:16 for shallow water waves for transition A and show higher values for A than obtained for the transition 1:16 (8). Transitions A and B both show a considerable decrease in the K_r values with increasing wave steepness. Some of this effect is undoubtedly associated with the markedly increased energy dissipation in the transition process as the wave steepness increases. This, although subject to extensive experimental error, was verified from energy balance considerations in reference (9). Energy losses for the range of wave steepnesses tested on transitions A and B show increasing dissipation rates for shallow water waves to maxima of 5 and 8% respectively. For the intermediate wave range these dissipation rates are somewhat higher, increasing from 2% for the lowest steepnesses of $H_1/L_1 = 1/1000$ to 6% and 12% for transitions A and B for the highest steepnesses of 6/100.

Transmission coefficients seem very little affected by wave steepness changes for both transitions. This trend is confirmed by the previous study on transition 1:16.

Figure 10 shows the effect of wave steepness on the K_r and K_t values for transition C of constant depth, hence of constant group velocity ratio. While again the scatter is sizeable a generally decreasing value of K_r with rising wave steepness is noticeable, while the value of K_t is constant.

CONCLUSIONS

An extensive experimental program was conducted in a rectangular wave channel to determine the reflection and transmission coefficients for transitions of linearly varying depth and/or width. Experimental results were obtained for three transitions: a transition A of constant width and linearly sloping bottom (1:8), a transition B with a linear reduction in width to one half (1:12.8) and a linearly sloping bottom (1:8), a transition C with a linear reduction in width to one half (1:25).

The variables were: the wave frequency, which could be adjusted from deep water to shallow water waves in the approach channel, the wave amplitude, to give a maximum possible range of wave steepness, the channel depth resulting in different depth ratios for the upstream and downstream channels. The experimental results obtained in this program were compared to results of previous experiments on transitions of constant width with linearly varying depth of 1:2.75 and 1:16 where applicable.

Data reduction was on the basis of linearized, small amplitude theory with systematic reference to the theory for abrupt transitions by H. Lamb for shallow water waves. Generally, the coefficients of reflection and transmission were related to the resulting dimensionless parameters: group velocity ratio, wave steepness and channel depth ratio. Only very general conclusions are possible due to the relatively large scatter of the data in spite of considerable care exercised in the measurements.

1. For transitions of constant width and linearly rising bottom experimental reflection coefficients for all slopes are materially above the values predictable from Lamb's theory for abrupt transitions. This holds for waves from deep water to shallow water in the approach channel.

2. For these transitions transmission coefficients are somewhat lower than predicted from Lamb's equations, but generally quite close.

3. For the case of the transition of linearly decreasing width and depth Lamb's theory seems to predict the transmission and reflection coefficients quite well in the shorter wave range. For shallow water waves the reflection coefficient was found to drop lower and the transmission coefficient to rise above the values computed from Lamb's relations.

4. All values of reflection and transmission coefficients tend to decrease with increasing group velocity ratios as is predicted from Lamb's expressions.

5. Wave steepness was confirmed to have a material effect on reflection coefficients for all transitions tested, as had been established in a previous study. It does not seem probable that viscous effects alone are responsible for this. Non-linear wave characteristics may account for part of this effect. Transmission coefficients were found essentially constant with wave steepness within experimental accuracy.

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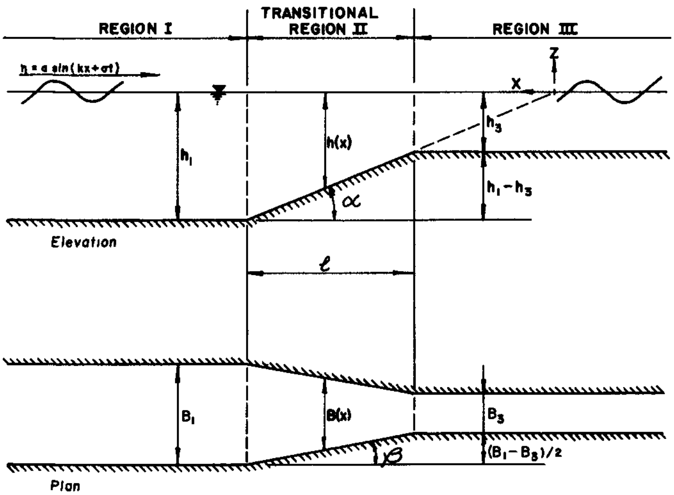

TRANSITION	VARIATION OF DEPTH	VARIATION OF WIDTH
A	$(h_1-h_3)/\ell = 1/8$ (α= 7 16°)	$B_1 = B_3$ = const
B	$(h_1-h_3)/\ell = 1/8$	$B_3/B_1 = 1/2$ $\frac{B_1-B_3}{2\ell} = 1/12.8$ (β= 4 46°)
C	$h_1 = h_3$ = const	$B_1/B_3 = 1/2$ $\frac{B_1-B_3}{2\ell} = 1/25$ (β= 2 29°)

Figure I

Schematic of Channel Transitions Tested

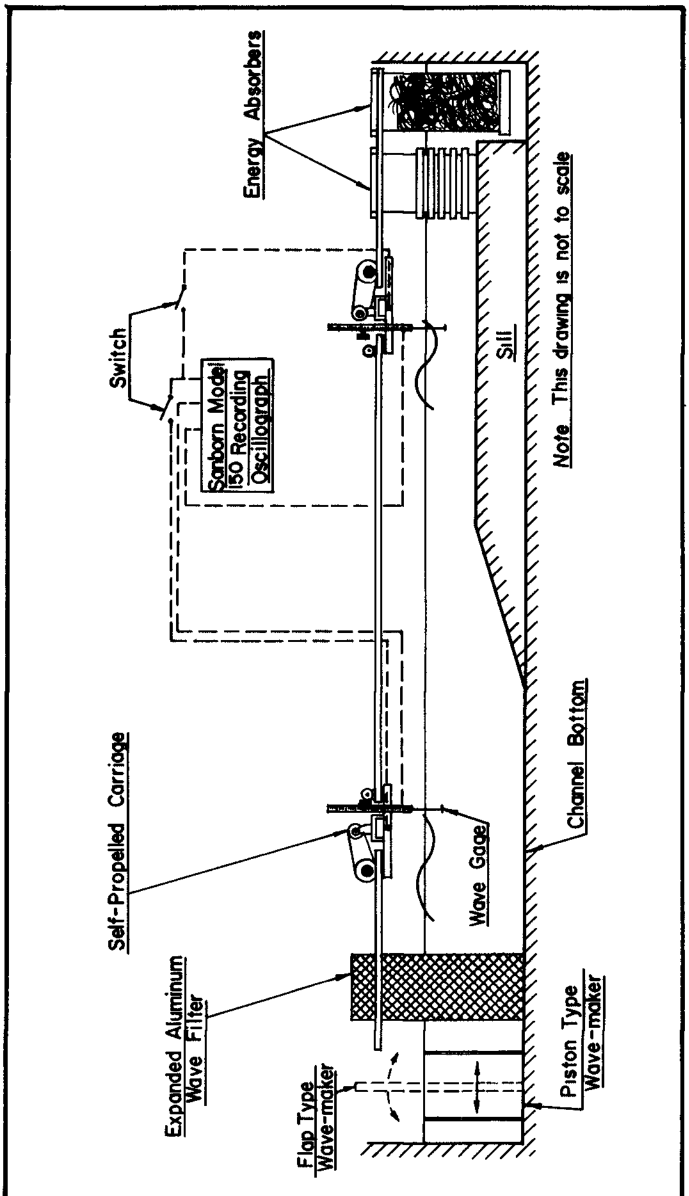

Figure 2

Schematic Diagram of Experimental Equipment

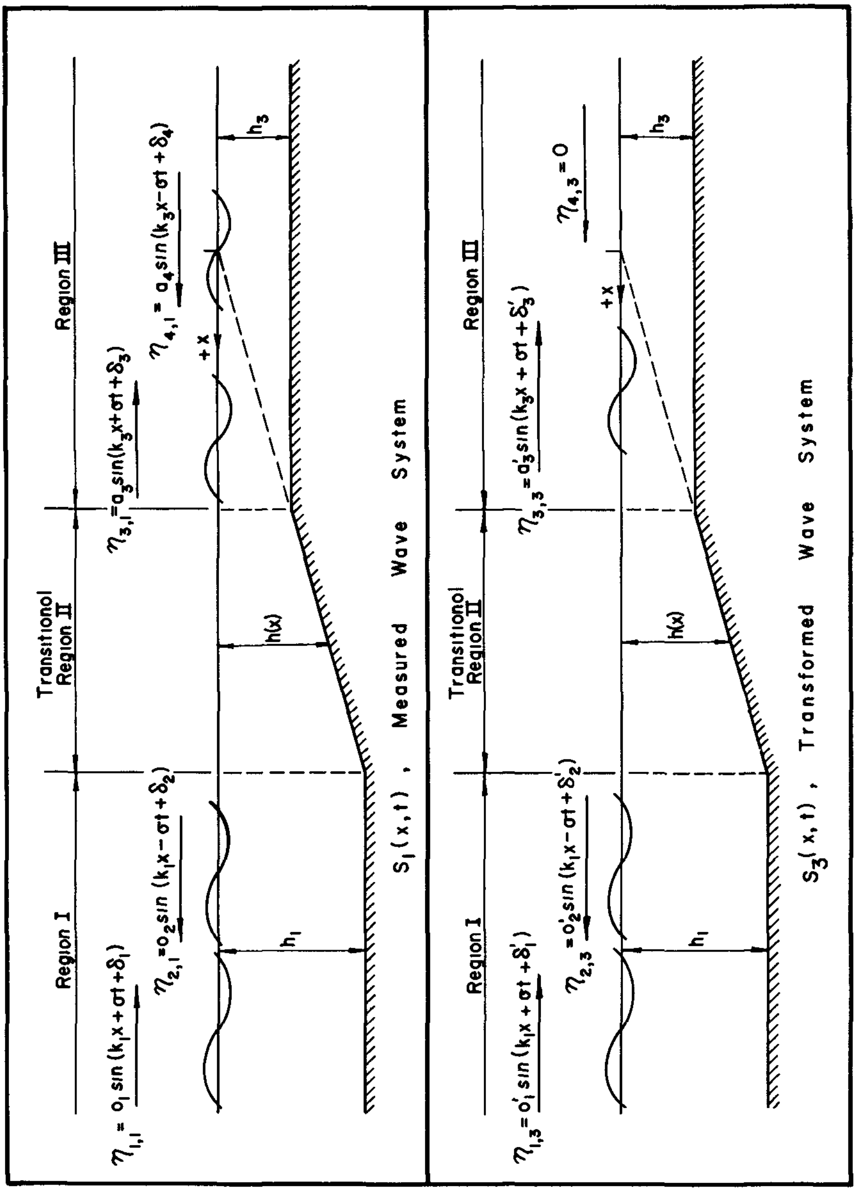

Figure 3
Measured and Transformed Wave System

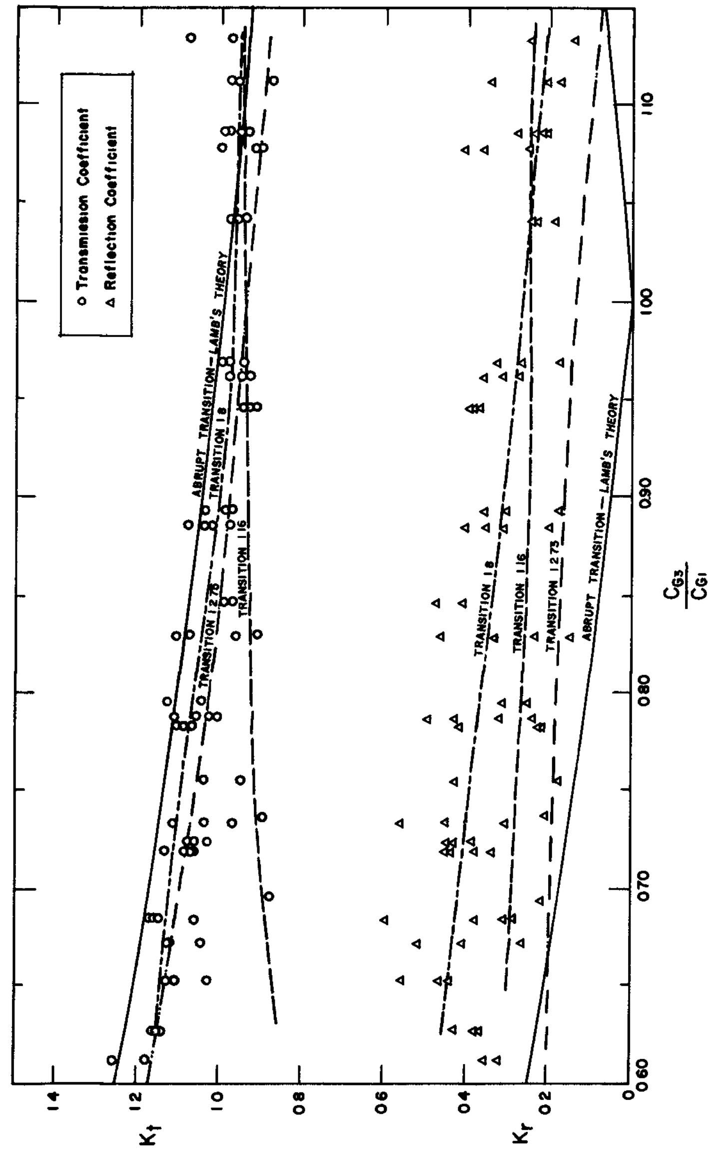

Figure 4

Reflection and Transmission Coefficients vs Group Velocity Ratio — Short and Intermediate Waves — Transition A

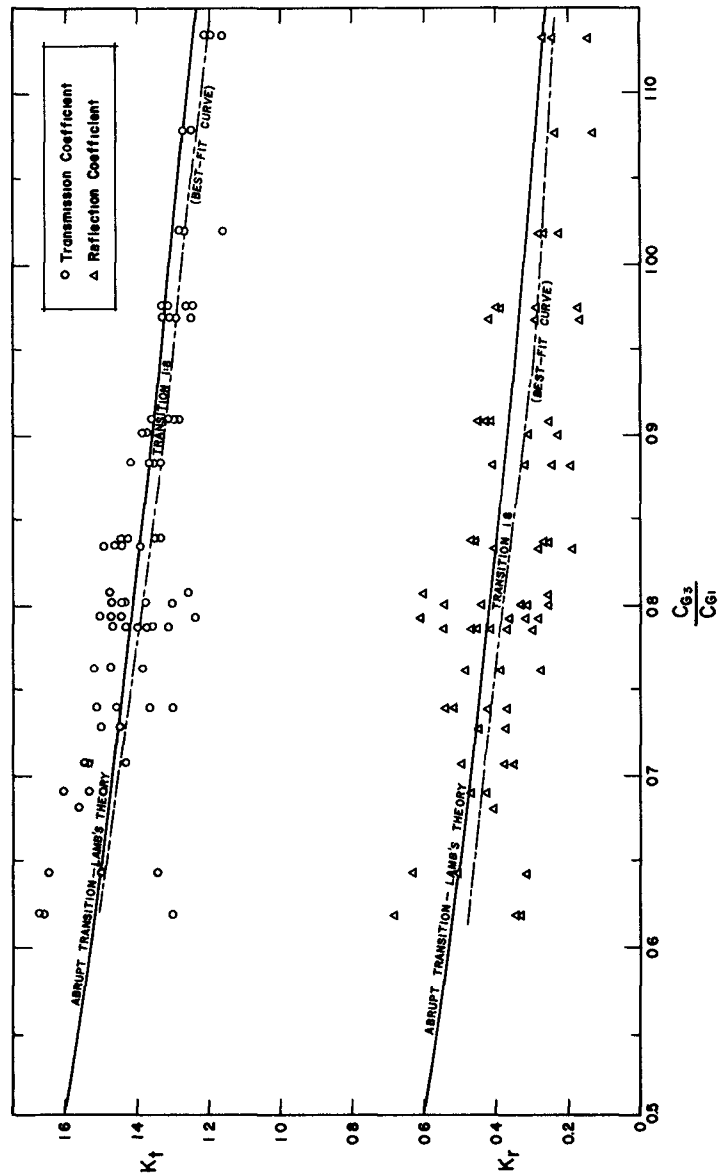

Figure 5.

Reflection and Transmission Coefficients vs Group Velocity Ratio — Short and Intermediate Waves — Transition B

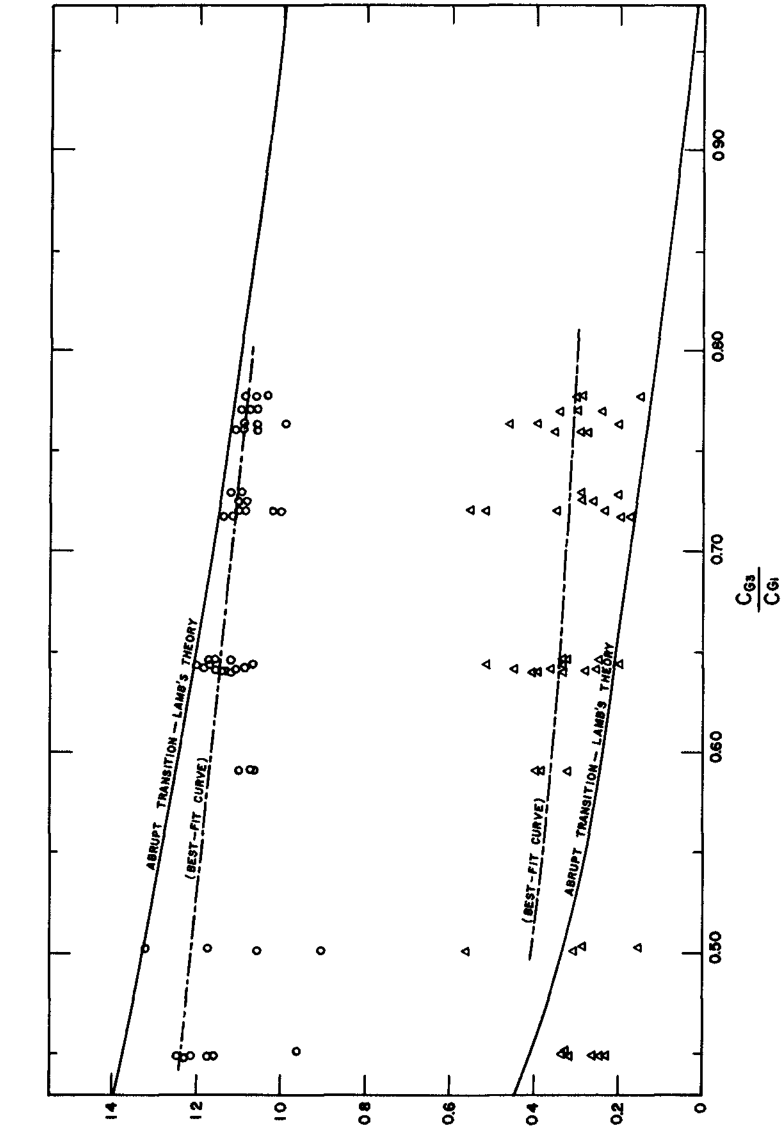

Figure 6.

Reflection and Transmission Coefficients vs Group Velocity Ratio–Shallow Waves–Transition A

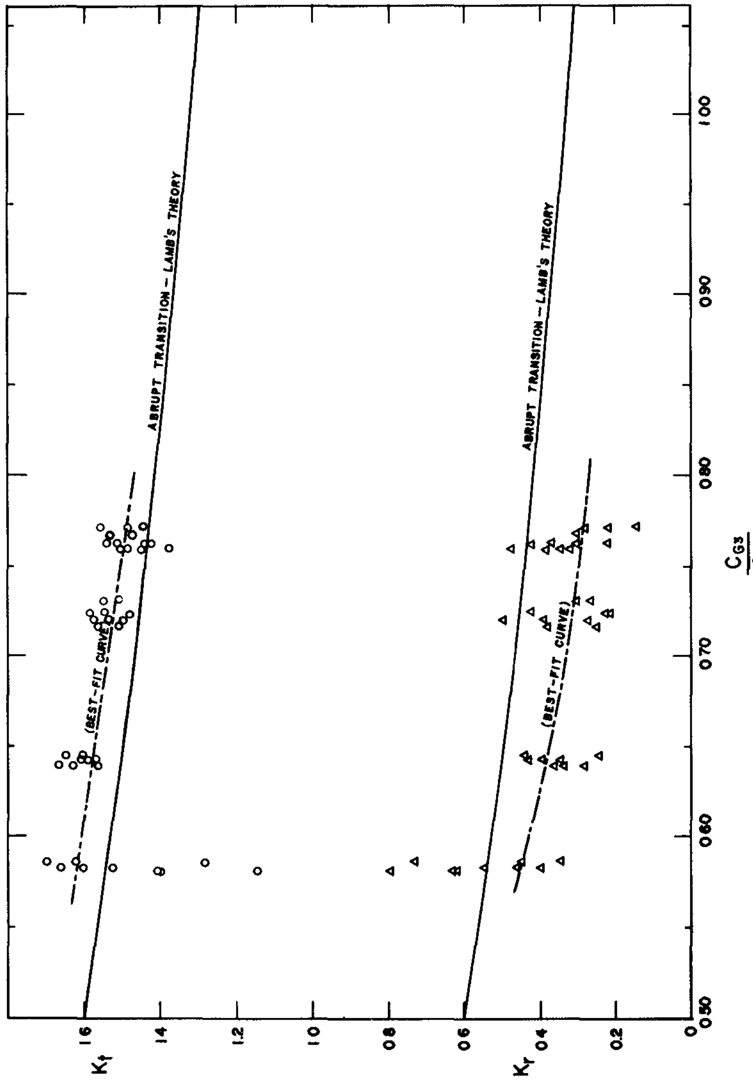

Figure 7.

Reflection and Transmission Coefficients vs Group Velocity Ratio—Shallow Waves—Transition B

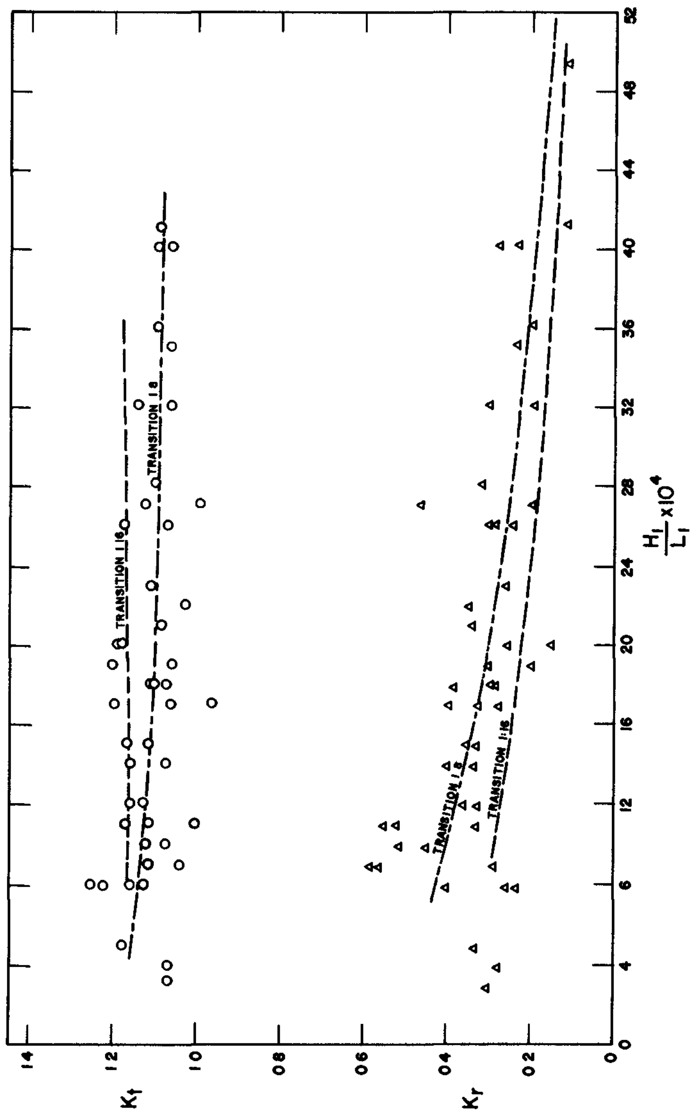

Figure 8

Reflection and Transmission Coefficients vs Wave Steepness — Shallow Waves — Transition A.

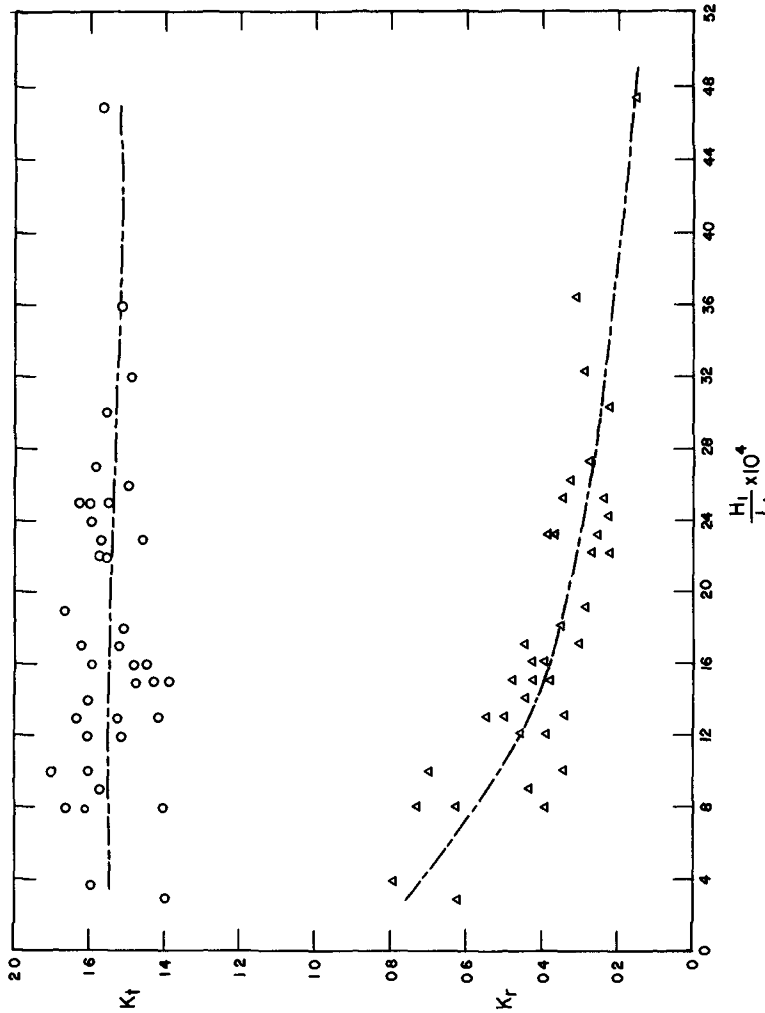

Figure 9

Reflection and Transmission Coefficients vs Wave Steepness—Shallow Waves—Transition B

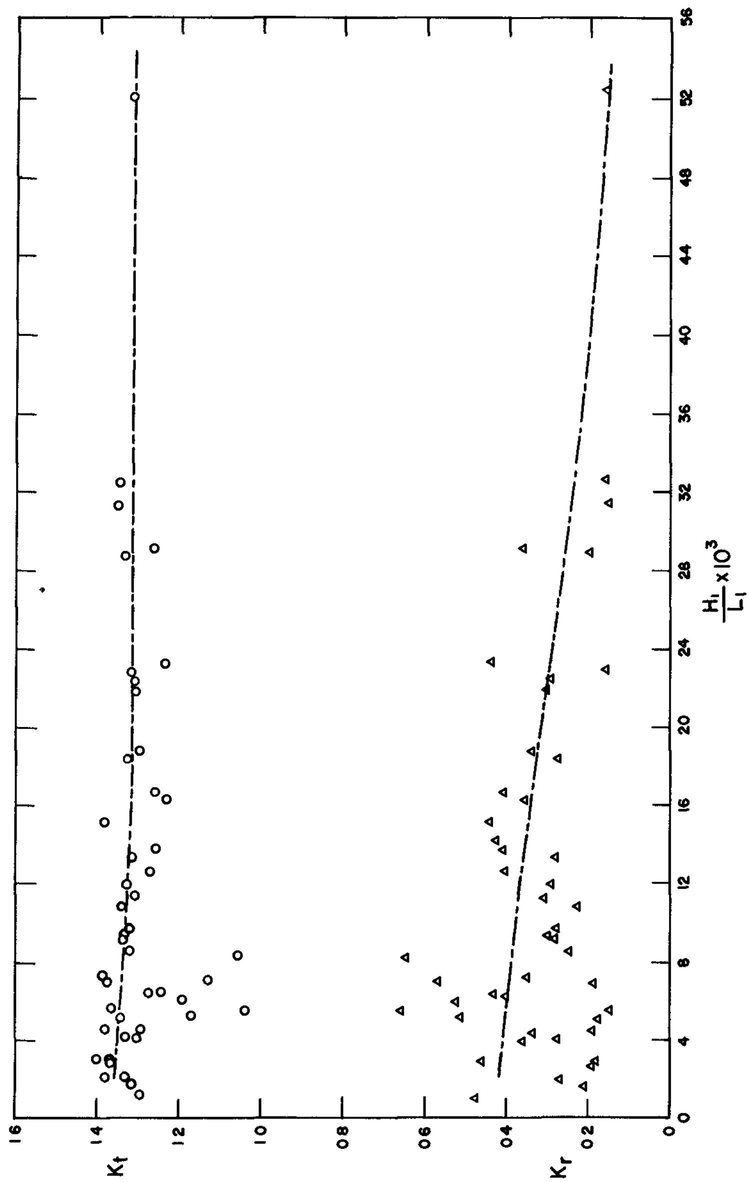

Figure 10.

Reflection and Transmission Coefficients vs Wave Steepness—Short and Intermediate Waves—Transition C