CHAPTER 9

HYPERBOLIC WAVES AND THEIR SHOALING

Yuichi Iwagaki
Professor, Department of Civil Engineering
Kyoto University, Kyoto, Japan

ABSTRACT

Is is very difficult for engineers to deal with the cnoidal wave theory for practical application, since this theory contains the Jacobian elliptic functions, their modulus k, and the complete elliptic integrals of the first and second kinds, K and E respectively.

This paper firstly proposes formulae for various wave characteristics of new waves named "hyperbolic waves", which are derived from the cnoidal wave theory under the condition that k = 1 and E = 1 but K is not infinite and are a function of $T\sqrt{g/h}$ and H/h, so that cnoidal waves can be approximately expressed as hyperbolic waves by primary functions only, in which T is the wave period, h the water depth and H the wave height.

Secondly, as an application of the hyperbolic wave theory, the present paper deals with wave shoaling, that is, changes in the wave height, the wave crest height above still water level, and the wave velocity, when the waves proceed into shallow water from deep water.

INTRODUCTION

Cnoidal waves, which were discovered by Korteweg and de Vries (1895), are not familiar to engineers, despite a long history, because of mathematical treatments including the Jacobian elliptic function and the complete elliptic integrals of the first and second kinds. Since, however, it has been noticed that the theory of Stokes waves is not appropriate to periodic waves progressing in shallow water in cases where $h/L < 1/10$ (Keulegan, 1950), $h/L < 1/8$ (Laitone, 1962) or $h/L < (\eta_0/L)^{1/3}$ (Wilson, Webb and Hendrickson, 1952), and that the theory of cnoidal waves should be applied to those waves, it is necessary to modify the theoretical results of cnoidal waves and provide graphs to easily find the wave characteristics, where L is the wave length and η_0 the wave crest height above the still water level. For this purpose, graphs were presented by Wiegel (1960) and tables published by Masch and Wiegel (1961) to obtain the profile and velocity of cnoidal waves from a given wave height, water depth, wave length or wave period. These graphs and tables, however, were based on the theoretical results of the first approximation by Keller (1948) and Keulegan and Patterson (1940), and furthermore the mean water depth was replaced by the water depth below the wave trough, so that they seem to be unsatisfactory compared with those of Stokes waves of the third order approximation presented by Skjelbreia (1959).

For this reason, the author tried to provide graphs to easily obtain the wave profile, wave velocity, wave length and wave steepness from a given wave period, still water depth and wave height for practical use based on Laitone's cnoidal wave theory of the second approximation (Laitone, 1961, 1962), and confirmed the validity of the cnoidal wave theory by making a comparison with the results of an experiment conducted for a case where $T\sqrt{g/h} \geq 15$ (Iwagaki, 1964, 1965; Iwagaki and Hosomi, 1966a, 1966b).

Theoretical curves of the wave height change in shoaling water for a large depth to wave length ratio were presented by Le Méhauté and Webb (1964) based on the Stokes wave theory of the third order approximation and by Koh and Le Méhauté (1966) based on that of the fifth order approximation. On the other hand, the computation of wave height change in shoaling water for a small depth to wave length ratio using the cnoidal wave theory was attempted by Masch (1964), but it was not successful because of the difficulty of numerical computation for complicated expressions including the Jacobian elliptic functions.

In this paper, after firstly defining the new waves named "hyperbolic waves" (Iwagaki, 1967), which are derived from the cnoidal wave theory, formulae for various wave characteristics of hyperbolic waves are presented, which are expressed by primary functions only. Secondly, changes in the wave height, the wave crest height above still water level and the wave velocity in shoaling water are computed based on the hyperbolic wave theory and compared with experimental results to confirm the validity of the hyperbolic wave theory.

DEFINITION OF HYPERBOLIC WAVES

Wave characteristics of cnoidal waves in dimensionless form Φ are generally expressed as follows:

$$\Phi = f\left(\frac{h_t}{H},\ \frac{z}{h_t},\ K,\ \frac{E}{K},\ k,\ \mathrm{cn}(v,k),\ \mathrm{sn}(v,k),\ \mathrm{dn}(v,k)\right) \quad \cdots\cdots (1)$$

in which $v = 2K(x-ct)/L$, h_t is the water depth below the wave trough, cn, sn and dn are the Jacobian elliptic functions, c the wave velocity, t the time, x the distance in the direction of wave propagation, and z the distance taken upwards from the wave trough as shown in Fig. 1.

Since the solitary wave theory can be derived with the limitation that $k = 1$ or $K \to \infty$, $E = 1$ and $L \to \infty$, and in this case $\mathrm{cn}(v,k)$ and $\mathrm{dn}(v,k)$ are written as $\mathrm{sech}\,v$ and $\mathrm{sn}(v,k)$ as $\tanh v$, wave characteristics of a solitary wave in dimensionless form Φ_s are expressed as

$$\Phi_s = f_s\left(\frac{h_t}{H},\ \frac{z}{h_t},\ \mathrm{sech}\,v,\ \tanh v\right) \quad \cdots\cdots (2)$$

and

$$v = 2\alpha(x-ct)/h_t \quad \cdots\cdots (3)$$

in which α is a function of h_t/H.

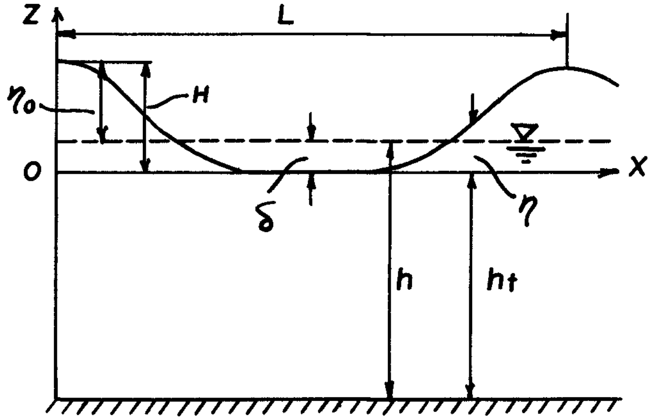

Fig. 1 Coordinate system.

Now, let us consider a case where k can be approximately put as unity, which is not the limiting case k = 1 as in the solitary wave theory. The value of K does not become as large as infinity even if the values of k and E tend to unity and can be put approximately as unity, as seen in Fig. 2. In other words, when K = 3, k = 0.98 and E = 1.05, then k and E can be put approximately as unity. Therefore, if k and E are put as unity on condition that

$$K \geq 3 \quad \cdots\cdots (4)$$

the following expression for wave characteristics of new waves in dimensionless form Φ_h can be derived corresponding to Eq. (1):

$$\Phi_h = f_h\left(\frac{h_t}{H}, \frac{z}{h_t}, K, \operatorname{sech} v, \tanh v\right) \cdots\cdots (5)$$

The waves are not periodic because they are expressed by hyperbolic functions. However, if the wave troughs are connected with each other for every wave, they can be treated as periodic waves, since the wave length is not infinite. The author has named the new waves "hyperbolic waves", because the characteristics of the waves can be expressed by hyperbolic functions with finite wave lengths (Iwagaki, 1967).

Since the dimensionless quantity for wave characteristics of hyperbolic waves Φ_h contains the complete elliptic integral of the first kind K as shown in Eq. (5), it is necessary to obtain the relationship between the wave period, wave height, water depth and K. The relationship was already presented graphically by the author (Iwagaki, 1965) based on Laitone's cnoidal wave theory of the second approximation (Fig. 3). Then, the relationship between K, $T\sqrt{g/h}$ and h/H is formulated using Fig. 3. Fortunately in the range of K ≧ 3, the curves for each value of a parameter h/H can be expressed approximately by parallel straight lines, which are shown by a solid line in Fig. 4. The formula to fit this solid line was proposed as follows (Iwagaki, 1967; Iwagaki and Sakai, 1968);

$$\frac{K}{T\sqrt{g/h}} = \frac{\sqrt{3}}{4}\left(\frac{H}{h}\right)^{1/2}\left\{1 - a\left(\frac{H}{h}\right)^n\right\}^m \cdots\cdots (6)$$

in which

a = 1.3, n = 2 and m = 1/2 for H/h = 0.55 ····· (7a)

and a = 0.54, n = 3/2 and m = 1 for H/h > 0.55 ···· (7b)

The plotted points are values computed by Eq. (6), which are in good agreement with the solid line.

The limit condition for application of the hyperbolic wave theory can be obtained from Eqs. (4) and (6). Fig. 5 shows a graphical expression of the limit.

CHARACTERISTICS OF HYPERBOLIC WAVES

Formulae for various characteristics of hyperbolic waves can be derived by putting k and E as unity and replacing the Jacobian elliptic functions with hyperbolic functions in the cnoidal wave theory as mentioned previously.

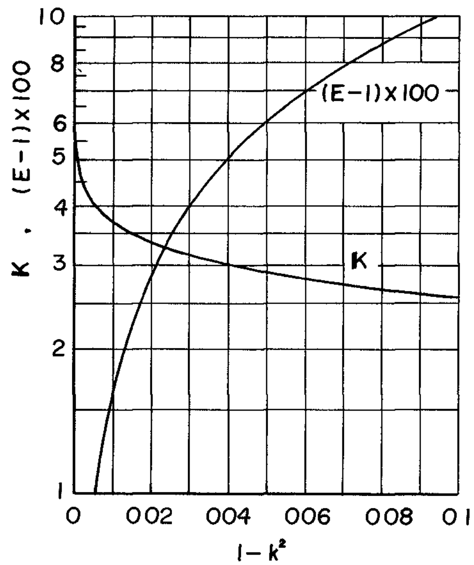

Fig. 2 Relationships between the complete elliptic integrals of the first and second kinds and the modulus of the Jacobian elliptic function.

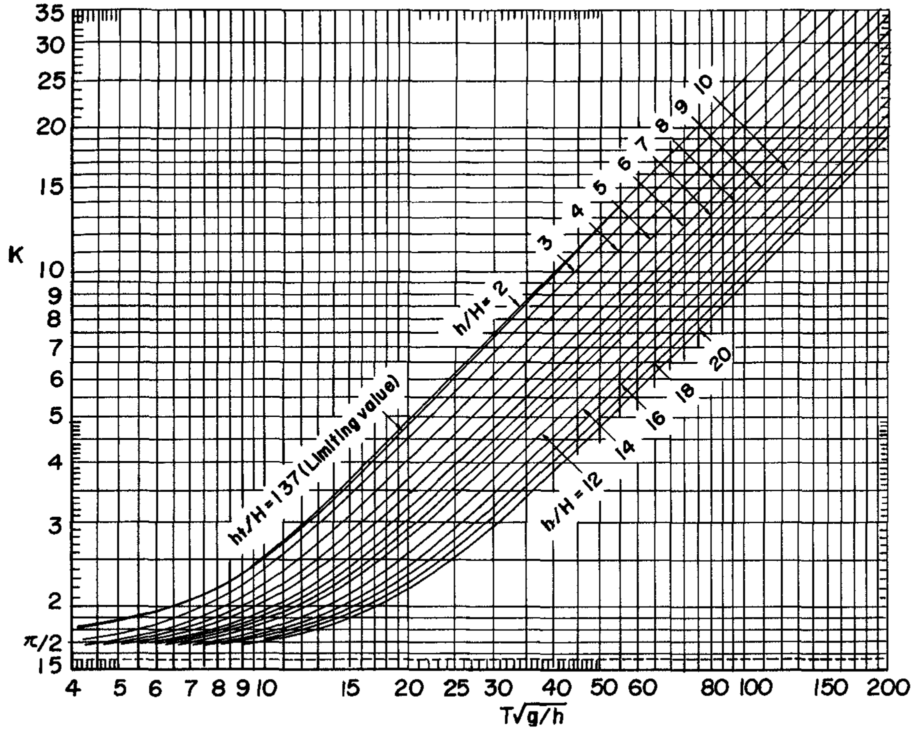

Fig. 3 Relationship between K and $T\sqrt{g/h}$ with a parameter of h/H.

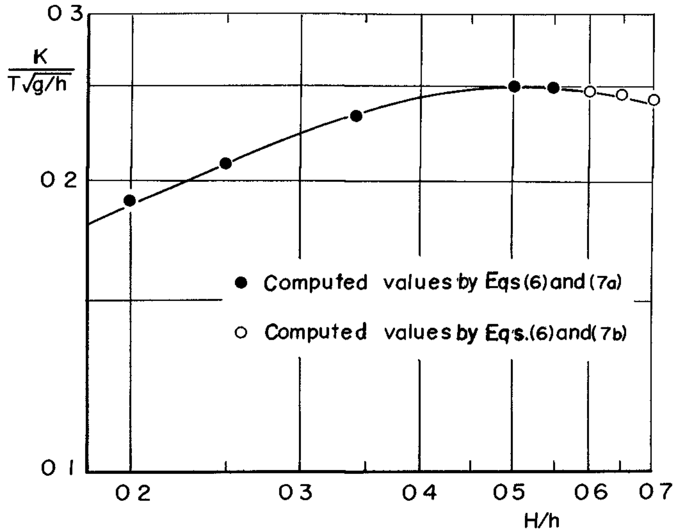

Fig. 4 Relationship between $K/T\sqrt{g/h}$ and H/h.

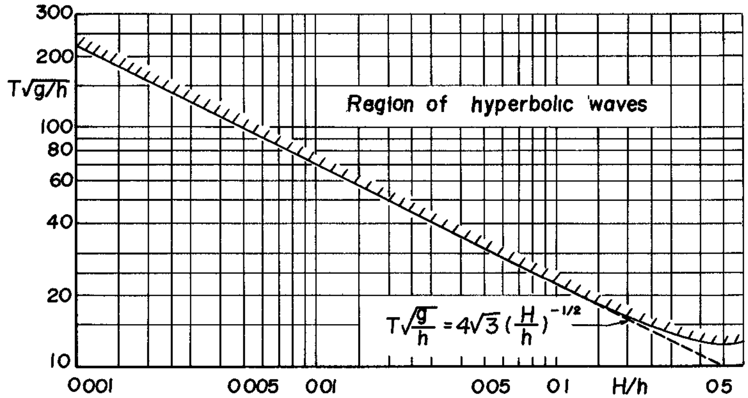

Fig. 5 Limit for application of hyperbolic wave theory.

HEIGHT OF STILL WATER LEVEL ABOVE WAVE TROUGH

$$\frac{\delta}{H}=\frac{1}{K}\left\{1-\frac{1}{12}\frac{H}{h}-\frac{1}{12}\frac{1}{K}\left(\frac{H}{h}\right)^2\right\} \qquad (8)$$

WAVE PROFILE

$$\frac{\eta}{H}=\operatorname{sech}^2\left(\frac{2K}{L}X\right)-\frac{3}{4}\frac{H}{h}\operatorname{sech}^2\left(\frac{2K}{L}X\right)\left(1+\frac{1}{K}\frac{H}{h}\right)\left\{1-\operatorname{sech}^2\left(\frac{2K}{L}X\right)\right\} \qquad (9)$$

in which $X=x-ct$ (10)

WAVE VELOCITY

$$\frac{c}{\sqrt{gh}}=\left(1-\frac{1}{2}\frac{1}{K}\frac{H}{h}\right)\left[1+\left(1+\frac{1}{K}\frac{H}{h}\right)\frac{H}{h}\left(\frac{1}{2}-\frac{1}{K}\right)+\left(1+\frac{2}{K}\frac{H}{h}\right)\left(\frac{H}{h}\right)^2 \times\left\{\frac{1}{K}\left(\frac{1}{K}-\frac{1}{4}\right)-\frac{3}{20}\right\}\right] \qquad (11)$$

WAVE LENGTH

$$\frac{L}{h}=\left(1-\frac{3}{2}\frac{1}{K}\frac{H}{h}\right)\frac{4K}{\sqrt{3}}\left(\frac{H}{h}\right)^{-1/2}\left\{1-\left(1+\frac{1}{K}\frac{H}{h}\right)\frac{5}{8}\frac{H}{h}\right\}^{-1} \qquad (12)$$

PRESSURE

$$\frac{p}{\rho g H}=\operatorname{sech}^2\left(\frac{2K}{L}X\right)-\frac{3}{4}\frac{H}{h}\operatorname{sech}^2\left(\frac{2K}{L}X\right)\left(1+\frac{1}{K}\frac{H}{h}\right)\left\{1-\operatorname{sech}^2\left(\frac{2K}{L}X\right)\right\}$$
$$-\frac{3}{4}\frac{H}{h}\operatorname{sech}^2\left(\frac{2K}{L}X\right)\left\{2\frac{z}{h_t}+\left(\frac{z}{h_t}\right)^2\right\}\left(1+\frac{1}{K}\frac{H}{h}\right)\left\{2-3\operatorname{sech}^2\left(\frac{2K}{L}X\right)\right\}$$
$$-\frac{z}{H} \qquad (13)$$

PARTICLE VELOCITIES

Horizontal velocity

$$\frac{u}{\sqrt{gh}}=A_1\operatorname{sech}^2\left(\frac{2K}{L}X\right)+A_2\operatorname{sech}^4\left(\frac{2K}{L}X\right)-A_3,$$
$$A_1=\left(1+\frac{1}{2}\frac{1}{K}\frac{H}{h}\right)\frac{H}{h}\left[1-\frac{5}{4}\left(1+\frac{1}{K}\frac{H}{h}\right)\frac{H}{h}-\frac{3}{2}\left(1+\frac{1}{K}\frac{H}{h}\right)\frac{H}{h}\left\{2\frac{z}{h_t}+\left(\frac{z}{h_t}\right)^2\right\}\right]$$
$$A_2=\left(1+\frac{3}{2}\frac{1}{K}\frac{H}{h}\right)\left(\frac{H}{h}\right)^2\left[\frac{5}{4}+\frac{9}{4}\left\{2\frac{z}{h_t}+\left(\frac{z}{h_t}\right)^2\right\}\right]$$
$$A_3=\left(1+\frac{1}{2}\frac{1}{K}\frac{H}{h}\right)\frac{H}{h}\frac{1}{K}\left\{1-\left(1+\frac{1}{K}\frac{H}{h}\right)\frac{H}{h}\left(\frac{1}{K}-\frac{1}{4}\right)\right\} \qquad (14)$$

Vertical velocity

$$\frac{w}{\sqrt{gh}}=\left(1+\frac{1}{K}\frac{H}{h}\right)\left(1+\frac{z}{h_t}\right)\sqrt{3}\left(\frac{H}{h}\right)^{3/2}\operatorname{sech}^2\left(\frac{2K}{L}X\right)\tanh\left(\frac{2K}{L}X\right)$$
$$\times\left[1-\frac{7}{8}\left(1+\frac{1}{K}\frac{H}{h}\right)\frac{H}{h}-\frac{1}{2}\left(1+\frac{1}{K}\frac{H}{h}\right)\frac{H}{h}\left\{2\frac{z}{h_t}+\left(\frac{z}{h_t}\right)^2\right\}\right.$$
$$\left.-\frac{1}{2}\left(1+\frac{1}{K}\frac{H}{h}\right)\frac{H}{h}\operatorname{sech}^2\left(\frac{2K}{L}X\right)\left\{1-6\frac{z}{h_t}-3\left(\frac{z}{h_t}\right)^2\right\}\right] \qquad (15)$$

WAVE ENERGIES

Potential energy per unit area

$$E_p=\frac{1}{2}\rho g\frac{1}{L}\int_{-L/2}^{L/2}(\eta-\delta)^2dx$$

$$= \frac{1}{3}\rho g H^2 \frac{1}{K}\left\{1-\frac{3}{2}\frac{1}{K}+\frac{H}{h_t}\left(-\frac{3}{10}+\frac{3}{4}\frac{1}{K}\right)+\left(\frac{H}{h_t}\right)^2\left(\frac{27}{560}-\frac{15}{288}\frac{1}{K}\right)\right\} \quad\cdots\cdots\cdots(16)$$

Kinetic energy per unit area

$$E_k=\frac{1}{2}\rho\frac{1}{L}\int_{-L/2}^{L/2}\int_{-h_t}^{\eta}(u^2+w^2)dx\cdot dz$$

$$=\frac{1}{3}\rho g H^2\frac{1}{K}\left\{1-\frac{3}{2}\frac{1}{K}+\frac{H}{h_t}\left(\frac{1}{10}-\frac{3}{4}\frac{1}{K}+\frac{3}{2}\frac{1}{K^2}\right)\right.$$

$$\left.+\left(\frac{H}{h_t}\right)^2\left(\frac{23}{560}-\frac{113}{160}\frac{1}{K}+\frac{3}{8}\frac{1}{K^2}\right)\right\} \quad\cdots\cdots\cdots(17)$$

Total energy per unit area

$$E=E_p+E_k$$

$$=\frac{2}{3}\rho g H^2\frac{1}{K}\left\{1-\frac{3}{2}\frac{1}{K}+\frac{H}{h_t}\left(-\frac{1}{10}+\frac{3}{4}\frac{1}{K^2}\right)+\left(\frac{H}{h_t}\right)^2\left(\frac{5}{112}+\frac{157}{480}\frac{1}{K}+\frac{3}{16}\frac{1}{K^2}\right)\right\}\cdots(18)$$

RATE OF WAVE ENERGY TRANSMISSION

$$W=\frac{\rho}{T}\int_{-T/2}^{T/2}\int_{-h_t}^{\eta}\left\{\frac{1}{2}(u^2+w^2)+\frac{p}{\rho}+gz\right\}u\,dz\,dt$$

$$=\frac{2}{3}\rho g H^2\sqrt{gh_t}\cdot\frac{1}{K}\left\{1-\frac{3}{2}\frac{1}{K}+\frac{H}{h_t}\left(\frac{2}{5}-\frac{5}{2}\frac{1}{K}+\frac{3}{K^2}\right)\right.$$

$$\left.+\left(\frac{H}{h_t}\right)^2\left(-\frac{31}{112}-\frac{29}{160}\frac{1}{K}+\frac{13}{4}\frac{1}{K^2}\right)\right\} \quad\cdots\cdots\cdots(19)$$

CHANGE IN WAVE HEIGHT ON SLOPING BEACH

The change in the wave height on a sloping beach can be calculated from the condition that the rate of wave energy transmission is constant at each section if the wave motion on a sloping beach is assumed to be the same as that on a horizontal sea bottom having the corresponding water depth. If wave energy dissipation due to bottom friction and wave refraction and reflection due to variable water depths are neglected, the relationship described above is expressed as follows;

$$\overline{W}=\overline{W}_0 \quad\cdots\cdots\cdots(20)$$

Eq. (19) is applied to $\overline{W}$ at the left side of Eq. (20) and the following relationships derived by Le Méhauté and Webb (1964) based on Stokes wave theory of the third order approximation by Skjelbreia (1959) is used for $\overline{W}_0$ at the right side of Eq. (20), in which the suffix o denotes the quantity in deep water:

$$W_0=\frac{\rho T^5}{32\pi}\left(\frac{L_0}{T^2}\right)^4\lambda_0^2\left\{4\left(1+\frac{3}{4}\lambda_0^2\right)\right\}\cdots\cdots\cdots(21)$$

$$\frac{3}{8}\lambda_0^3+\lambda_0=\pi\frac{H_0}{L_0} \quad\cdots\cdots\cdots(22)$$

$$\frac{L_0}{T^2}=\frac{g}{2\pi}(1+\lambda_0^2) \quad\cdots\cdots\cdots(23)$$

Finally the following equation of H/H_0 is derived as a function of H_0/L_0 and h/L_0:

$$\frac{H}{H_0}=\frac{3}{16}\left(\frac{1}{4}\right)^{1/3}\left(\frac{h}{L_0}\right)^{-1}\left(\frac{H_0}{L_0}\right)^{1/3}\left\{1-\pi^2\left(\frac{H_0}{L_0}\right)^2\right\}\left\{1-\frac{1}{K}\frac{H}{h}+\frac{1}{12}\frac{1}{K}\left(\frac{H}{h}\right)^2\right\}^{-1/3}$$

$$\times\left\{1-a\left(\frac{H}{h}\right)^n\right\}^{2m/3}\left\{1-\frac{3}{2}\frac{1}{K}+\frac{H}{h_t}\left(\frac{2}{5}-\frac{5}{2}\frac{1}{K}+\frac{3}{K^2}\right)+\left(\frac{H}{h_t}\right)^2\left(-\frac{31}{112}-\frac{29}{160}\frac{1}{K}+\frac{13}{4}\frac{1}{K^2}\right)\right\}^{-2/3} \quad (24)$$

because

$$\frac{h_t}{H}=\frac{h}{H}-\frac{\delta}{H}=\frac{h}{H}\left\{1-\frac{1}{K}\frac{H}{h}+\frac{1}{12}\frac{1}{K}\left(\frac{H}{h}\right)^2\right\} \quad (25)$$

$$\frac{H}{h}=\frac{H}{H_0}\frac{H_0}{L_0}\left(\frac{h}{L_0}\right)^{-1} \quad (26)$$

and $T\sqrt{g/h}$ in Eq. (6) is a function of h/L_0 and H_0/L_0 from Eqs. (22) and (23).

Since the right side of Eq. (24) contains H/H_0, successive approximate computation using a digital computer has been adopted. A group of curves on the left side of Fig. 6 shows the result of computation by Eq. (24). In addition, a group of curves on the right side of the figure is that by Le Méhauté and Webb (1964) for the third order approximation of Stokes waves. As a condition of breaking inception for hyperbolic waves,

$$\frac{H_b}{h_t}=0.73 \quad (27)$$

has been used, which was proposed by Laitone (1961) for cnoidal waves.

CHANGE IN WAVE CREST HEIGHT ABOVE STILL WATER LEVEL

From Eq. (8), the wave crest height above still water level of hyperbolic waves η_0 is expressed as

$$\frac{\eta_0}{H}=\frac{H-\delta}{H}=1-\frac{\delta}{H}=1-\frac{1}{K}\left(1-\frac{1}{12}\frac{H}{h}\right) \quad (28)$$

which is a function of H_0/L_0 and h/L_0 as seen from Eq. (26) and Fig. 6. Fig. 7 shows the relationship between η_0/H and h/L_0 with a parameter of H_0/L_0 besides the curves for Stokes waves.

CHANGE IN WAVE LENGTH AND WAVE VELOCITY

From Eqs. (6), (12), (22) and (23), the expression for the wave length ratio L/L_0 is derived as follows:

$$\frac{L}{L_0}=\sqrt{2\pi}\left(\frac{h}{L_0}\right)^{1/2}\left\{1-\frac{\pi^2}{2}\left(\frac{H_0}{L_0}\right)^2\right\}\left(1-\frac{3}{2}\frac{1}{K}\frac{H}{h}\right)\times\left\{1-a\left(\frac{H}{h}\right)^n\right\}^m\left\{1-\left(1+\frac{1}{K}\frac{H}{h}\right)\frac{5}{8}\frac{H}{h}\right\}^{-1} \quad (29)$$

Since L/L_0 is also a function of H_0/L_0 and h/L_0 and Fig. 8 is a result of numerical calculation. Of course, the wave velocity ratio c/c_0 is equal to L/L_0.

EXPERIMENT

EQUIPMENT AND PROCEDURE

The wave tank used for the experiment of wave shoaling on a beach was 63 m long, 50 cm wide and 65 cm deep, with a wave

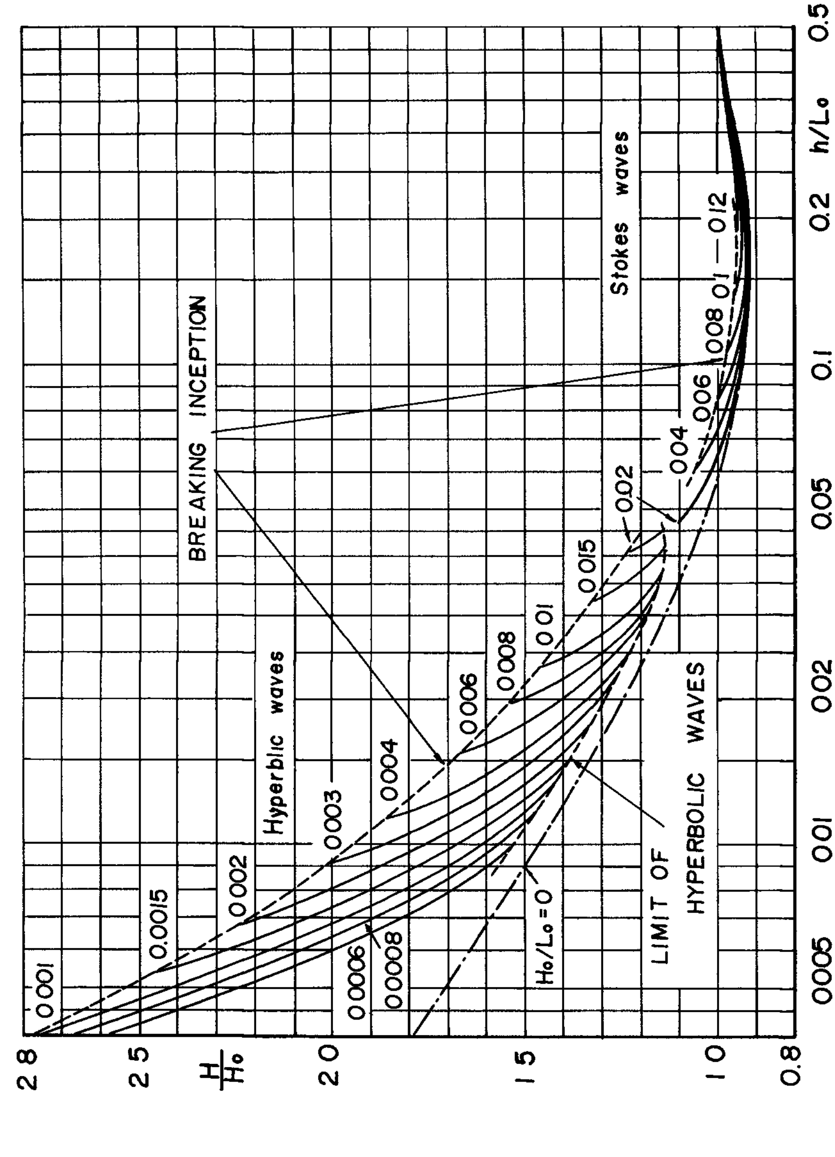

Fig. 6 Wave height change in shoaling water based on hyperbolic wave and Stokes wave theories.

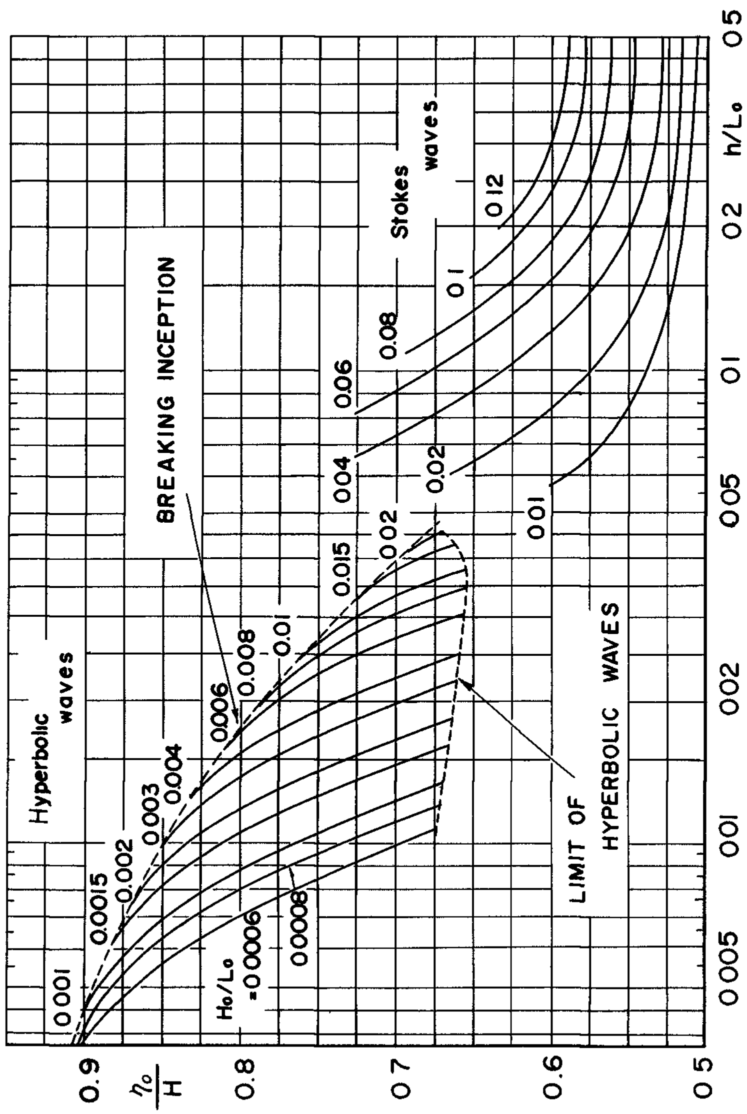

Fig. 7 Wave crest height change in shoaling water based on hyperbolic wave and Stokes wave theories.

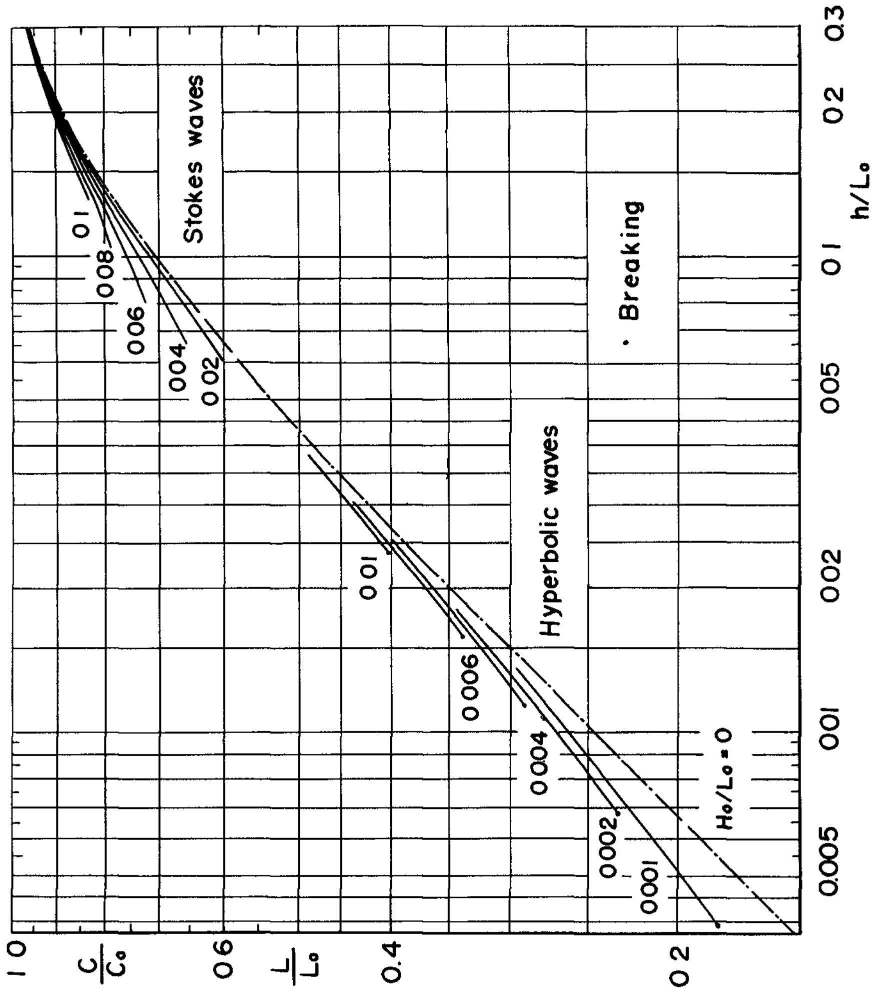

Fig. 8 Wave velocity change in shoaling water based on hyperbolic wave and Stokes wave theories.

generator of piston type. A model beach of uniform slope 1/20, which was made of steel plate, was set 8 m long about in the middle of the wave tank. The top of the slope was just 40 cm high from the bottom, and the water depth above the horizontal bottom was always controlled at 40.5 cm, so that breaking waves overtopped on the upper edge of the slope and reflected waves were reduced as much as possible. Periods of waves used were 1, 1.2, 1.5 and 2 sec, and wave heights at uniform depth were from 1.5 to 12.0 cm. Wave records were taken instantaneously by six wave meters of electric resistance type set at uniform depth and on the slope. The wave height in deep water H_0 was obtained from the water depth and the wave height at uniform depth using theoretical curves of wave height change based on the third order approximation of Stokes waves by Le Méhauté and Webb (1964). The wave length L_0 and wave velocity c_0 in deep water were computed approximately using $gT^2/2\pi$ and $gT/2\pi$ respectively. Wave velocity data were taken by measuring the time necessary for each wave crest to pass between two adjacent wave meters, as the wave velocity in the middle of the wave meters. Several waves were adopted as data, which were recorded until waves reflected by the beach returned again from the wave generator after the wave height became constant.

EXPERIMENTAL RESULTS AND CONSIDERATIONS

Fig. 9 shows a comparison between experimental values and the theoretical curves on wave height change in shoaling water. It is found that the experimental data in the figure agree well with both theoretical curves based on the hyperbolic and Stokes wave theories. According to the plots of experimental data taken by Wiegel (1950), Iversen (1952) and Eagleson (1 956), their experimental results are not in agreement with the theoretical curves except Eagleson's data in the case of $H_0/L_0 = 0.02$ (Iwagaki and Sakai, 1967). This fact can not be explained even if the effect of bottom friction is taken into account. As far as the present experiment by the author is concerned, the effect of bottom friction is estimated as approximately 1 % when the wave period is equal to two seconds.

Experimental data of the wave crest height are plotted in Fig. 10 with the theoretical curves of hyperbolic and Stokes waves for comparison. Agreement between them is very poor. The reason will be due to asymmetric deformation of the wave profile in shoaling water. This fact suggests that it is necessary to study not only wave height change but also wave profile deformation in shoaling water including breaking of waves over a sloping beach.

Fig. 11 shows experimental results of wave velocity change in shoaling water compared with the theoretical curves. Agreement of the experimental data with the theoretical curves is as poor as in the case of the wave crest height. The reason may also be due to asymmetric deformation of the wave profile in progress.

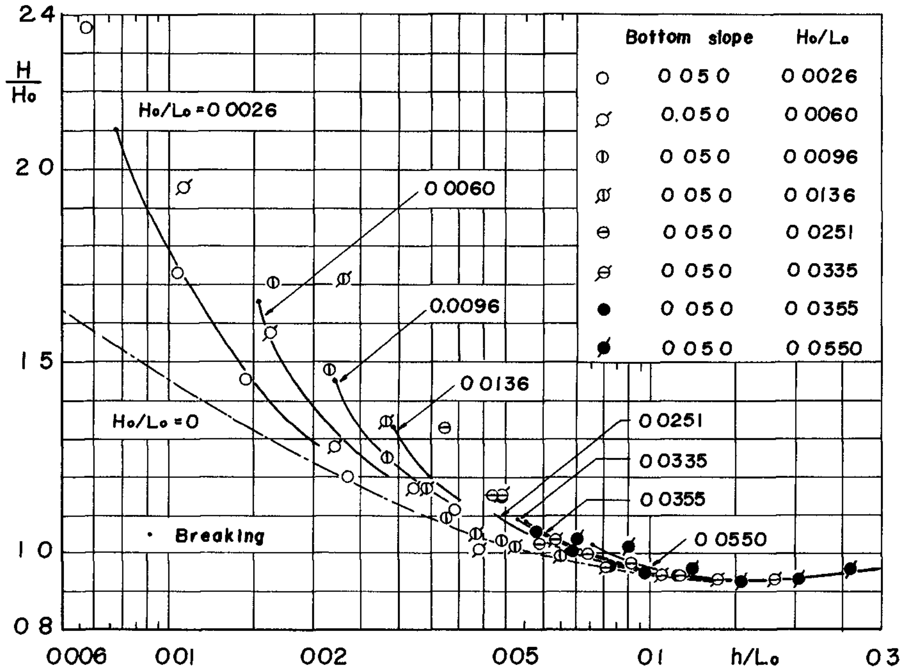

Fig. 9 Comparison of experimental data of wave height change with theoretical curves.

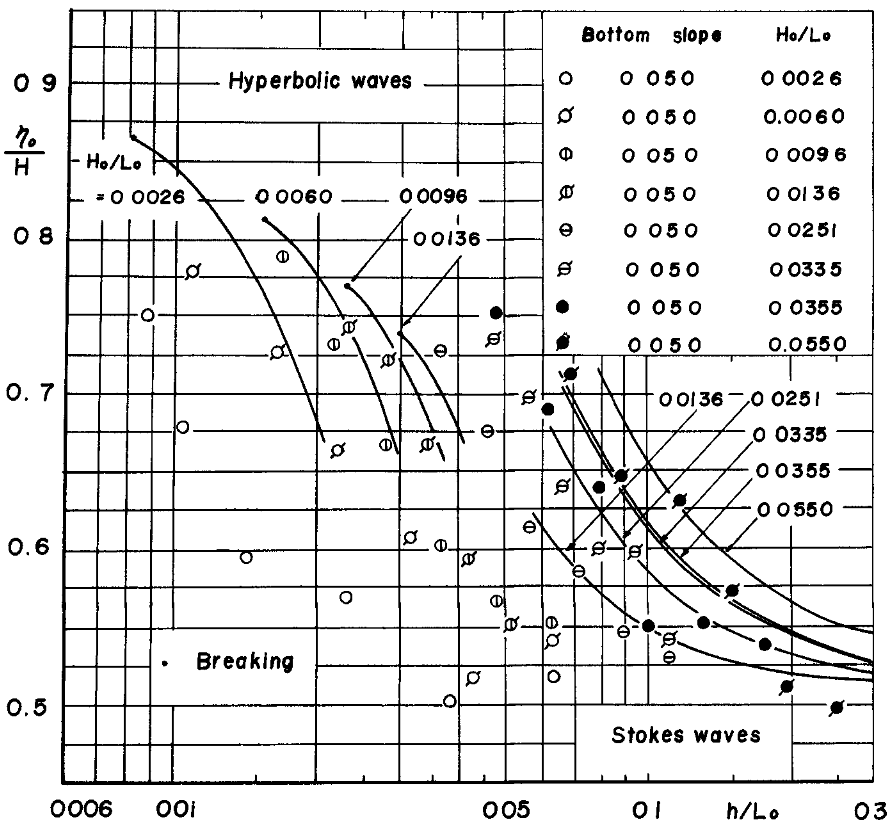

Fig. 10 Comparison of experimental data of wave crest height change with theoretical curves.

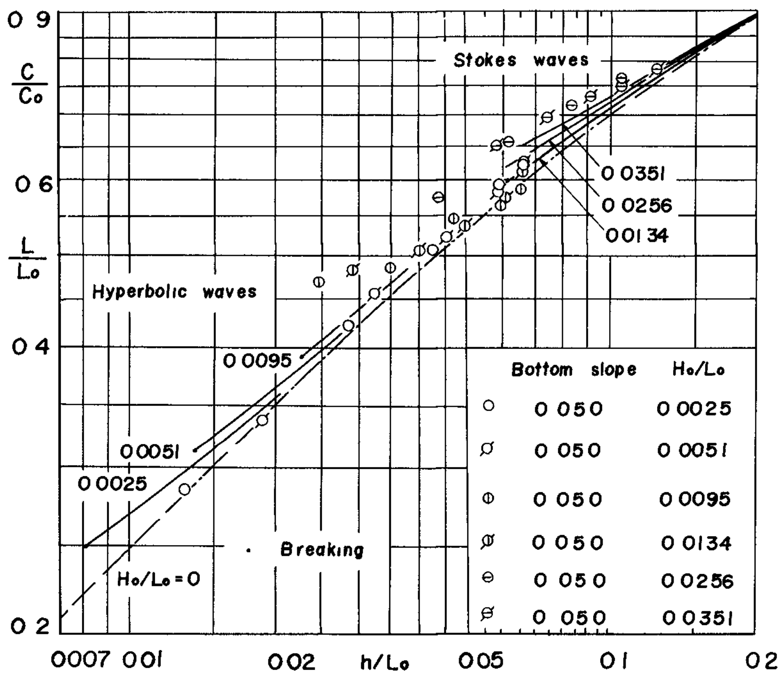

Fig. 11 Comparison of experimental data of wave velocity change with theoretical curves.

CONCLUSIONS

The author proposed new waves named "hyperbolic waves", which are derived from the cnoidal wave theory under the condition that k = 1 and E = 1 but $K \neq \infty$. Since the theory of the waves does not contain the Jacobian elliptic functions but primary functions only, it is possible to easily compute various wave characteristics of finite amplitude long waves. The computation of wave shoaling presented here is an application of the hyperbolic wave theory to coastal engineering problems. A combination of the hyperbolic wave theory and Stokes wave theory now makes it possible to give a qualitative description of finite amplitude wave characteristics all over the region of shallow water before breaking, subject to the limitation that the waves are of permanent type.

ACKNOWLEDGEMENT

Part of this investigation was accomplished with the support of the Science Research Fund of the Ministry of Education, for which the author expresses his appreciation. Thanks are due to Mr. Sakai for his help during this investigation and Professor Tsuchiya for his help in preparing the paper.

REFERENCES

Eagleson, P. S. (1956). Properties of shoaling waves by theory and experiments: Trans. AGU, Vol. 37, pp. 568 -572.

Iversen, H. W. (1952). Waves and breakers in shoaling water: Proc. 3rd Conf. on Coastal Eng., pp. 1 - 12.

Iwagaki, Y. (1964). Studies on cnoidal waves (First report) -- On the wave steepness and profile --: Dis. Pre. Res. Inst. Annuals, No. 7, Kyoto University, pp. 373 - 386 (in Japanese).

Iwagaki, Y. (1965). Studies on cnoidal waves (Second report) -- On the wave velocity and wave length --: Dis. Pre. Inst. Annuals, No. 8, Kyoto University, pp. 343 - 351 (in Japanese).

Iwagaki, Y. (1967). Studies on cnoidal waves (Fourth report) -- On hyperbolic waves (1) --: Dis. Pre. Res. Inst. Annuals, No. 10B, Kyoto University, pp. 283 - 294 (in Japanese).

Iwagaki, Y. and Hosomi, M. (1966a). Studies on cnoidal waves (Third report) -- Experiments on the wave profile, wave velocity and wave length --: Dis. Pre. Res. Inst. Annuals, No. 9, Kyoto University, pp. 637 -645 (in Japanese).

Iwagaki, Y. and Hosomi, M. (1966b). Some experiments on cnoidal waves: Proc. 13th Conf. on Coastal Eng. in Japan, pp.

1 - 8 (in Japanese).

Iwagaki, Y. and Sakai, T. (1967). On the shoaling of finite amplitude waves: Proc. 14th Conf. on Coastal Eng. in Japan, pp. 1 - 7 (in Japanese).

Iwagaki, Y. and Sakai, T. (1968). Studies on cnoidal waves (Fifth report) -- on hyperbolic waves (2) --: Dis. Pre. Res. Inst. Annuals, No. 11B, Kyoto University, pp. 463 - 476 (in Japanese).

Keller, J. B. (1948). The solitary wave and periodic waves in shallow water: Communication on Pure and Applied Math., Vol. 1, pp. 323 - 339.

Keulegan, G. H. (1950). Wave motion: Engineering Hydraulics; Wiley, New York, ed by H. Rouse, Chapt. 11, pp. 711 - 768.

Keulegan, G. H. and Patterson, G. W. (1940). Mathematical theory of irrotational translation waves: Jour. of Res. of N. B. S., U. S. Dept. Commerce, Vol. 24, pp. 47 - 101.

Koh, R. C. Y. and Le Méhauté, B. (1966). Wave shoaling: Jour. of Geophy. Res., Vol. 71, pp. 2005 - 2012.

Korteweg, D. L. and de Vries, G. (1895). On the change of form of long waves advancing in a rectangular canal and on a new type of long stationary waves: Phil. Mag., Ser. 5, Vol. 39, pp. 422 - 443.

Laitone, E. V. (1961). The second approximation to cnoidal and solitary waves: Jour. of Fluid Mechanics, Vol. 9, pp. 430 - 444.

Laitone, E. V. (1962). Limiting conditions for cnoidal and Stokes waves: Jour. of Geophy. Res., Vol. 67, pp. 1555 - 1564.

Le Méhauté, B. and Webb, L. M. (1964). Periodic gravity waves over a gentle slope at a third order approximation: Proc. 9th Conf. on Coastal Eng., pp. 23 - 40.

Masch, F. D. (1964). Cnoidal waves in shallow water: Proc. 9th Conf. on Coastal Eng., pp. 1 - 21.

Masch, F. D. and Wiegel, R. L. (1961). Cnoidal waves, tables of functions: Council on Wave Research, The Engineering Foundation, pp. 1 - 129.

Skjelbreia, L. (1959). Gravity waves, Stokes' third order approximation, tables of functions: Council on Wave Research, The Engineering Foundation, pp. 1 - 337.

Wiegel, R. L. (1950). Experimental study of surface waves in shoaling water: Trans. AGU, Vol. 31, pp. 377 - 385.

Wiegel, R. L. (1960). A presentation of cnoidal wave theory for practical application: Jour. of Fluid Mechanics, Vol. 7, pp. 273 - 286.

Wilson, B. W., Webb, L. M. and Hendrickson, J. A. (1952). The nature of tsunamis, their generation and dispersion in water of finite depth: National Science Co., Tech. Rep. No. SN57 - 2, pp. 1 - 150.