CHAPTER 78

A STUDY ON PHENOMENA OF FLOW AND THERMAL DIFFUSION CAUSED BY OUTFALL OF COOLING WATER

Akira Wada

Technical Laboratory, Central Research
Institute of Electric Power Industry, Tokyo, Japan

SYNOPSIS

This report concerns with the investigation of recirculation of cooling water discharged from outlet of thermal (or atomic) power stations sited on a bay under the tidal and wind effects.

In this study, the author describes the effect of some dominant factors on the recirculation and shows a method to estimate the temperature distribution and velocity field in the bay by numerical experiment. Especially, numerical experiments in the vertical section are examined by taking account of the interaction between the flow and thermal diffusion.

I INTRODUCTION

In designing thermal power station, it is essential to take the water of suitable temperature into the cooling system of condenser.

When the water which has been warmed through the cooling system is discharged from the outlet into the bay, it usually returns to the intake in the same basin in the form of surface layer and is partly taken into the system again. Such recirculation of water has great influence on the efficiency of operation of power stations. It is, therefore, an important problem to predict the variation of the sea surface temperature in the bay for the future development of the plant

The main factors that affect the recirculation of the cooling water are as follows. the distance between intake and outlet, the spreading behavior of the warmed water from the outlet, the heat budget in the bay, the tidal action, the wind stress and its direction, the configuration and the depth of the bay, the stratified state of water temperature off the outlet, and so on. In order to solve the problem of recirculation which depends on these various factors, it is necessary to analyze the effects of these factors separately, and then to synthesize all the results obtained. In this manner, we can find the design criteria of hydraulic structures of cooling system

As a part of investigations on this problem, the authors[1)] have previously conducted the study on the method of taking the colder bottom water with relation to the design of the intake structure of cooling water for the Sakai-Port Thermal Power Station sited on the east coast of Osaka Bay in Japan. The quantity of the cooling water for this station amounts to 100 m³/sec in maximum at the ultimate output of 2000 MW.

As the fact that the sea water in the forebay forms the remarkable stratification of temperature was verified from the result of field observations extending over two years, it was decided to construct the intake structure of submerged curtain-wall type in order to take the cold bottom water to achieve a higher efficiency of the power station. Model experiments on density flow have been carried out to examine the hydraulic behavior of bottom water intake and the results obtained were applied to determine the location and the dimensions of the intake. After completion of this intake structure, its efficacy was proved by the field tests.

The above study on the bottom water selective intake was carried out on the assumption that the vertical distribution of water temperature is steady. But in case that a power station is located at the innermost of a bay, the fetch length of wind is long, and the winds blow from the mouth to the innermost of the bay, the effect of winds on a stratified sea bay becomes a subject of discussion.

Recently, Wada (1965)[2] has discussed the effect of wind stress on the velocity of flow and the stratified distribution of water temperature in a bay As the result, it was concluded that the effect of winds should be taken into consideration in the design of intake structures of cooling water.

In the present report, a few factors of great importance in the determination of the degree of recirculation were chosen for theoretical analysis.

First, the author describes the effect of some dominant factors on the recirculation and shows a method to estimate the temperature distribution and velocity field in a bay by numerical experiment. In especially, numerical experiments in the vertical section are examined by taking account of the interaction between the flow and thermal diffusion.

In this paper, an iteration method (accelerated Liebmann method) using finite differences has been developed for the analysis of the water temperature distribution, the biharmonic equation governing the dynamic behaviour of discharged cooling water under the adequate boundary conditions such as the vertical density variation in the stratified sea basin, outflow velocity and temperature at the outlet, and process of cooling the sea surface. The above equations in two or three dimensional space were transformed into difference equation system, which was solved numerically with the aid of an electronic computer IBM 7090.

II FORMULATION OF THE PROBLEM

For the purpose of predicting the degree of recirculation, it is essential to know in what pattern of the warmed cooling water discharged from the outlet spreads into the sea basin and how it returns to the intake.

In order to obtain the temperature distribution in the sea basin off the outlet, it is necessary to consider both dynamic movement of released water and thermal diffusion of water temperature. In addition, many other factors should be taken into consideration, such as the effect of wind and tidal current, topographical feature of sea basin and so on. As shown in Fig. 1, take the Cartesian coordinates in three dimensional space, the origin of which is taken as the center of outlet. The direction of three axis of the coordinates are as seen in the definition sketch (Fig. 1). Let us assume that the outlet has a rectangular section, 2B in breadth and H in height, from which the cooling water with a initial constant temperature To is released into the sea in the direction of the y-axis.

In general, the discharged water has to keep a small velocity in the bay. Because the small relative velocity between an upper and a lower layers keep the stability of the stratification in the bay.

Therefore, the eddy viscosities are predominant in a field of the flow in the bay, and the field of flow are strongly subject to the influence of the coastal boundaries near the outlet. This point differs from the general phenomena of jet flow. Neglecting inertia terms and any tidal effects, the equations of motion in the i direction and the equation of continuity can be written as

$$\frac{\partial}{\partial x_j}\left(A_j \frac{\partial u_i}{\partial x_j}\right) = \frac{\partial p}{\partial x_i} - \rho g \lambda_k \qquad \text{....... ....(1)}$$

and

$$\frac{\partial (\rho u_i)}{\partial x_i} = 0 \ , \qquad \text{..... ... .(2)}$$

where $j = 1, 2, 3$ corresponds to the x, y, z direction respectively. λ_k is the unit vector along the z axis. u_i $(i = 1, 2, 3)$ are the velocity components u, v, w along the x, y, z direction and the eddy viscosities corresponding to those directions are A_x, A_y, A_z respectively. p the pressure, ρ the density.

On the other hand, the equation for the thermal diffusion is

$$u_j \frac{\partial T}{\partial x_j} = \frac{\partial}{\partial x_j}\left(\frac{K_j}{\rho} \cdot \frac{\partial T}{\partial x_j}\right) + \frac{Q_o}{\rho C_w H_w} \qquad \text{................(3)}$$

where K_j are eddy thermal diffusivities, Q_o represents the heat gain or loss for the surface layer of sea basin, C_w is specific heat of water and H_w is the thickness of layer between the sea surface and the atmosphere, across which process of momentum and heat transfer occur.

On the other hand, an approximate relation between density and water temperature is

$$\rho = \rho_0 \ (1 - \alpha T) \qquad \text{... .. ..(4)}$$

where the density, ρ_0 , is the standard density of the fluid. To estimate accurately the water temperature variation near the sea surface owing to discharge of cooling water, incoming and outgoings of heat energies must be added to the equation of thermal diffusion.

The main processes for the heat balance in any part of the coastal region are as follows. The schematic diagram of these processes is shown in Fig. 2.

1. Absorption of radiation from the sun and the sky, Q_s

Radiation from the sun and sky which penetrates the sea surface is mostly absorbed either directly or after multiple scattering. Only a small fraction of the incident energy is scattered back into the atmosphere. Both absorption and scattering vary with the wave-length.

Net short-wave radiation penetrating the sea surface, Q_s, will be approximated by

$$Q_s = Q_o \, (1 - \bar{r}) \qquad \ldots\ldots\ldots (5)$$

where Q_o is the solar and short wave sky radiation which reaches the sea surface and $\bar{r}$ is the average reflectance over the integration period

2. Effective back-radiation from the sea surface, Q_b

Effective back-radiation rate, the difference between rates of long-wave radiation from the sea surface and long-wave radiation from the atmosphere, can be approximated by the following formula:

$$Q_b = s\sigma \, (T + 273)^4 \{ 1 - a - b\sqrt{e(T_a)} \} \qquad \ldots\ldots\ldots (6)$$

where the following definitions obtain:

- T : the water temperature near the sea surface in $^\circ$C;
- a, b : the constants;
- $e(T_a)$: the saturation vapour pressure at the sea surface in mbs;
- σ : the Stefan-Boltzman's constant $= 1.367 \times 10^{-12}$ cal$\cdot$cm$^{-2}\cdot$sec$^{-1}\cdot$ $^\circ$K^{-4};
- s : the ratio of emittance of the sea surface to that of a black-body $= 0.97$.

When the clouds reduce the effective outgoing radiation, the equation (6) is multiplied by a correction factor

$$1 - Kn$$

where K is the coefficient depending on the cloud height (K = 0.083 for the cloud height 1.5 km ~ 2 km) and n is the cloudiness.

3. Convection of sensible heat, Q_h

Conductive heat-exchange rate, Q_h, is given by

$$Q_h = h_a \, (T_a - T) \qquad \ldots\ldots\ldots (7)$$

where h_a is a heat transfer coefficient. The value of h_a is calculated by

$$h_a = 2.77 \times 10^{-4} \, (0\ 48 + 0\ 272V)$$

where V is wind speed in m/s.

4. Heat-exchange rate by evaporation, Q_e

Assuming that the rate of energy gain due to condensation, dE/dt, follows the Dalton's equation, Q_e in cal/cm$^2\cdot$s is given

$$Q_e = L \cdot \frac{dE}{dt} = Lk \{ e(T_a) - e(T)\} \qquad \ldots\ldots\ldots\ldots\ldots(8)$$

where the following definitions obtain,

L : the latent heat of evaporation = 585 cal$\cdot g^{-1}$,
k : the mass transfer coefficient,
e(T): the saturation pressure for the water temperature.

Within the range of difference between water and air temperature likely to be encountered in practice, the saturation pressure vs, the water terperature curve may be regarded as a straight line, say $e(t) = mT - n$ approximately. The coefficient k may be expressed in terms of a heat transfer coefficient h_a, since the laws governing heat-and mass-transfer between phases are essentially similar, provided that the temperature is expressed in degree of centigrade and the vapour pressure in mm of mercury head, the relation between h_a and k for water becomes $Lk = 2h$.

The net-exchange rate across the sea surface to the above processes is then represented by the linear combination,

$$Q = Q_s + Q_b + Q_h + Q_e = b_0 - b_1 T \qquad \ldots\ldots\ldots\ldots(9)$$

wherein the positive sense denotes heat gained by the sea. In the fundamental equations which govern phenomena of flow and thermal diffusion, the density enters explicitly. It might therefore be expected that variations of density in a vertical direction would modify the results, but the variations of the density in the sea are too small to be of importance in this respect. If the variations of the density are related to gravitational effect, the product term of ρg, would play an important role in the interaction between the flow and thermal diffusion and should therefore be taken into account in the equations of motion. From this assumption, the equation of continuity can be replaced by the Boussinesq approximation.

Boundary conditions on the velocity are taken to be : at the free surface, flow parallel to the surface, at the fixed boundaries, velocity equal to zero, net transport equal to cooling water flow. The thermal flux must be zero normal to the boundaries except the sea surface or at the mouth of bay The thermal gradient at the sea surface or at the mouth of the bay should be remain constant.

The problem now is to solve this nonlinear set of equations subject to the boundary conditions. Since the set is not tractable by any analytical methods known at present, it is necessary to resort to approximate methods for the finite difference solution.

III ACTUAL STATE OF THERMAL DIFFUSION OF COOLING WATER AND STABILITY OF STRATIFIED DISTRIBUTION IN DENSITY

Vertical profile of water temperature upper the thermocline presents nearly constant temperature. This part consists of a homogeneous sea water not only in the water temperature but also in the salinity and therefore in the density, and forms, what is called, the mixing layer of the surface. The thickness of this layer varies with place, season and wind or tidal current.

In general, the surface layer has thin one from spring to summer, but thick one from autumn to winter. It has large thickness at the place of strong wind and current. The vertical distribution of the density presents continuous profile by development of the intermediate layer when the mixing between two layers is promoted by the wind action.

During summer, there exists 4 $^{\circ}C \sim 5$ $^{\circ}C$ of temperature difference between the surface and the lower layer in the bay, and the interface of two water masses of different density locates at the layer of $3 \sim 4$ m below the surface of the sea.

This interface is stable in spite of tidal changes, direction of wind, and velocity of wind. When the surface-waters are heated during summer, the density is decreased and hence $\partial\rho/\partial z$ is increased When $\partial\rho/\partial z$ is very great, there will be very little vertical turbulence, and hence very little vertical mixing across layer of large temperature gradient because the vertical stability $E = (1/\rho)\cdot(\partial\rho/\partial z)$ is very great at its place In the limiting case of a surface of discontinuity of density, it will, for the sake of simplicity, often bc assumed that there is neither mixing nor friction across the surface

The stability in this situation is represented by a parameter known as the Richardson number $R_i = g\,|\partial\rho/\partial z|/\rho(\partial u/\partial z)^2$, and it follows that the intensity of the turbulence is reduced if $R_i > 1$. When the inlet is formed by reclamation of the foreshore, the water temperature in this inlet is supposed to be undergone a small change so far. It is expected that the surface water temperature of the bay after reclamated works differs a little from that of the open sea in the past owing to the poor mixing with the water masses of the bottom layer. According to field measurements about vertical profile of water temperature carried out in each bay of Japan, it is made clear that the surface layer temperature of the bay is 1 $^{\circ}C \sim 2$ $^{\circ}C$ higher than that of the open sea. From this fact, the following questions arise:

(1) The bay water differs from one of the open sea about the water temperature of bottom layer,

(2) The rise of the water temperature in whole depths of the bay occurs by the thermal diffusion into the lower layer.

But this is not true. From the results of the observation for the vertical distribution of water temperature in the Sakai-Port, the Yawata-Port, the Kawasaki-Port and the Mizushima Bay, the face of the discontinuity of the density extending from the innermost to the mouth of the bay presents the horizontal stratification of the density and the water temperature of the bottom layer in the bay shows the same one of the open sea. The depth of this thermocline locates at 3 ～ 5 m below the sea surface and can't be appreciated a large change for each bay.

To know the aspect of the thermal diffusion by the outfall of heated cooling water, we can not but rely on the methods such as the field survey, the hydraulic model test and the numerical experiment by computers.

Field observations with respect to water temperature and velocity of the flow issuing from the outlet of the cooling system were carried out in 1964 at the Mizushima Steam Power Plant located on Mizushima Bay facing the Inland Sea of Seto, in order to obtain the state of thermal diffusion influenced by the outflow of warmed colling water Measurements were made at each station in the bay to obtain the variation of the water temperature distribution due to the tidal current The observed results of the water temperature distribution in the sea basin off the outlet are illustrated in Fig. 3 (profile), Fig. 4 (plan) and Fig. 5 (sections).

The results show that the temperature of the discharged cooling water decreases gradually and high temperature content is observed in the surface layer about 5 m thick in the region 60 m or 80 m distant from the outlet. The temperature of water at the depth below 5 m from the sea level decreases remarkably, which is probably due to the poor vertical mixing with the upper layer owing to the large stability. As shown in these figures, the temperature distribution in the upper layer is apparently governed by the mixing state of water masses in the vicinity of the interface between upper and lower layer. In addition, such distribution is not always corresponding to the tidal current pattern, because the effect of the diffusion is frequently stronger than that of the advection by the current. Further, it is found by Figs 3, 4 and 5 that the horizontal diffusion seems to be more effective than the vertical.

Based on the data of the field survey, the thermal diffusion coefficients were calculated. It is found that the effect of the y-direction diffusion is of the same order as the x-direction diffusion under the tidal current action, and the horizontal thermal diffusibility is 50 times greater than the vertical, the order of which seems to be about 0.01 m^2/sec. Accordingly, it is concluded that the decrease of the water temperature observed in the field tests should be due to the horizontal mixing with surrounding waters of lower temperature, with addition to the process of cooling the sea surface.

But the method by the hydraulic model test comes into question in the similitude of real phenomena. For that reason, the state of the thermal diffusion by the outfall of the warmed cooling water of real operating power plant must be correctly grasped. And after that, it is necessary to conduct the numerical experiment or the hydraulic model test.
In general, the surface layer of the sea near the outlet in the bay is greatly influenced by the cooling water flow discharged. To estimate the aspect of mixing of the water temperature in front of the outlet, it is necessary to know the mechanism of inflow and mixture of the cooling water into the sea basin. Next, let us consider the stability of vertical distribution of water temperature by wind stress.

The rate at which the mixing process caused by the wind penetrates toward greater depths will depend upon the gradient of density with depth In nearly homogeneous sea of a small density difference, the mixing by the wind will, in short time, reach a considerable depth. When the surface-waters are heated, remarkable stratification of density is formed. In this case the rate of generation of turbulent energy may no longer be great enough to overcome the stability effect of a transition region, which is known as the thermocline

This homogeneous upper layer, therefore, is hard to expect for the more development. Even if the wind would stir up the water near the surface and by a mixing process create a homogeneous surface layer, the increase of the stability will be expected at its lower boundary of an upper layer, so that the eddy viscosity will have low values.

It is not expected that the warmed water in the surface layer reach lower layer for some causes and is uniformalized from the top layer to the layer of the bottom.

Recently, Wada, A (1965)[2] has discussed the effect of wind stress on the velocity of flow and the stratified distribution of water temperature in a bay. As the result, it was concluded that the effect of winds should be taken into consideration in the design of intake structure of cooling water at the innermost of the bay

IV. THERMAL DIFFUSION OF COOLING WATER IN THE THREE DIMENSIONAL SPACE[3]

The equation (3) for thermal diffusion in the three dimensional space, taking account of the heat loss from the sea surface, may be written in terms of different coordinates as follows:

$$\rho_0(1-\alpha T)\lambda\frac{\partial T}{\partial\xi}=\frac{\partial^2 T}{\partial\xi^2}+\frac{\partial^2 T}{\partial\eta^2}+\frac{\partial^2 T}{\partial\zeta^2}-\left(\frac{\alpha}{1-\alpha T}\right)\cdot$$

$$\left[\left(\frac{\partial T}{\partial\xi}\right)^2+\left(\frac{\partial T}{\partial\eta}\right)^2+\left(\frac{\partial T}{\partial\zeta}\right)^2\right]+B_0-B_1T \quad \cdots\cdots\cdots\cdots(10)$$

where $\lambda=U/\sqrt{K_x}$, $\xi=x/\sqrt{K_x}$, $\eta=y/\sqrt{K_y}$, $\zeta=z/\sqrt{K_z}$, $B_0=b_0/C_w\cdot H_w$, $B_1=b_1/C_w\cdot H_w$,

The solution of this problem divides naturally into two parts, one of which corresponds to the radiation effect and diffusion effect, and the other the effect of internal diffusion mechanism which results from non-uniform temperature distribution in the water. The temperature distributions due to these effects are found separately by solving the basic equation for two different sets of initial and boundary conditions.

In solving the equation (10), the boundary conditions are

(1) $T=T_0$ at $\eta=0$, $-B/\sqrt{K_x}\leqq\xi\leqq B/\sqrt{K_x}$ and $0\leqq\zeta\leqq H/\sqrt{K_z}$,

(ii) $\partial T/\partial\eta = 0$ at $\eta = 0$, $\xi > B/\sqrt{K_x}$ or $\xi < -B/\sqrt{K_x}$ and for all ζ or at $\eta = 0$, $-B/\sqrt{K_x} < \xi < B/\sqrt{K_x}$, $H < \zeta$,

(iii) The vertical distribution of water temperature, as shown in Fig. 6, is given for the sea region infinitely far off the outlet.

(iv) $\partial^2 T/\partial\zeta^2 = 0$ at $\zeta = 0$

In order to calculate the water temperature distribution with the aid of electronic digital computer IBM 7090, we transfer the equation (10) into difference equation We subdivide the xyz-plane into meshes of three dimensional net ($x = mh$, $y = nh$, and $z = ph$, where m, n, p = 0, ± 1, ± 2, . . .).

It is generally found that the convergence of the elliptic type equation can be sped up by using the method of successive over-relaxation This method means that, instead of correcting each value of T (ξ, η, ζ) by adding the residual, we overcorrect by adding a quantity obtained by multiplying the residual by a suitable factor.

If we let $T^{(k)}(\xi, \eta, \zeta)$ denote the k-th approximation to the solution, this method is represented by the following equation:

$$T^{(k+1)}(\xi,\eta,\zeta) = T^{(k)}(\xi,\eta,\zeta) + A \times R^{(k)}(\xi,\eta,\zeta) \qquad \ldots \quad (11)$$

$$\begin{aligned} R^{(k)}(\xi,\eta,\zeta) = [&F1 \times T(\xi+1,\eta,\zeta) + F2 \times T(\xi-1,\eta,\zeta) \\ &+ F4 \times \{T(\xi,\eta+1,\zeta) - T(\xi,\eta-1,\zeta) + T(\xi,\eta,\zeta+1) \\ &\qquad + T(\xi,\eta,\zeta-1)\} \\ &+ F5 \times B_0 + F6[\{T(\xi+1,\eta,\zeta) - T(\xi-1,\eta,\zeta)\}^2 \\ &\qquad + \{T(\xi,\eta+1,\zeta) - T(\xi,\eta-1,\zeta)\}^2 \\ &\qquad + \{T(\xi,\eta,\zeta+1) - T(\xi,\eta,\zeta-1)\}^2] \\ &\times \{10 - \alpha \times T(\xi,\eta,\zeta)\}^{-1}] \\ &- [10 - F3 \times \{T(\xi+1,\eta,\zeta) - T(\xi-1,\eta,\zeta)\}] - T(\xi,\eta,\zeta) \ldots (12) \end{aligned}$$

where A is relaxation factor $1 \leqq A \leqq 2$,

$F1 = (1.0 - \rho_0 \lambda h/2.0)/(6.0 + B_1 h^2)$,
$F2 = (1.0 + \rho_0 \lambda h/2.0)/(6.0 + B_1 h^2)$,
$F3 = (\rho_0 \alpha \lambda H/2\ 0)/(6.0 + B_1 h^2)$,
$F4 = 1.0/(6.0 + B_1 h^2)$,
$F5 = h^2/(6.0 + B_1 h^2)$,
$F6 = \alpha/4.0\ (6.0 + B_1 h^2)$.

If A = 1, the procedure is reduced to the Gauss-Seidel method. The proper choice of the relaxation factor determines the rate of convergence of the iteration method. Starting with the initial approximation of temperature distribution at all nodes inside the sea region, the calculations were carried out by a high-speed automatic digital computer, IBM 7090

The numerical calculations involving the effect of tidal current parallel to the coastal line, consist of two cases in which one corresponds to the case that the vertical distribution of water temperature is homogeneous in the sea, the other does to the case that there is the remarkable stratification at the depth 4 m below the sea surface.

The aim of this calculation is to know how the thermal diffusion is governed by the state of the vertical distribution of water temperature in the coastal region

The results of calculation are shown in Fig 6 through Fig.8. Fig 6 represents the vertical distribution of water temperature along the η (y)-axis perpendicular to the coast line Fig. 7 represents the horizontal distributions of water temperature at the sea surface ($\zeta = 0$) and the layer of $\zeta = 4$ in depth. Further, Fig. 8 shows the vertical distribution of water temperature at various sections parallel to the coast line, at $\eta = 0$ and $\eta = 4$.

In this numerical experiment, the remarkable stratified distribution of water temperature formed in the summertime is taken into consideration in one of the boundary conditions. From these figures, we can obtain the distribution of water temperature for field of each diffusion if the eddy viscosities are set up and the coordinates on these figures are adequately expanded and contracted After the coordinates are returned to normal system, we conducted the numerical experiments on the thermal diffusion by substituting the values obtained by the field survey into the coefficients of diffusion.

The following points were made clear as the result of the numerical calculation conducted under the consideration of field results:

(1) Where there is a remarkable stratified profile of water temperature in the coastal region, high temperature content of the discharged cooling water is not diffused into the lower layer through the layer of thermocline with the large stability. The discharged water has a tendency to mix with the water masses near the surface layer.

(2) Where the water temperature is homogeneous in the whole sea basin, the discharged warm water, on the other hand, seems to mix uniformaly with the water masses of relatively bottom layer.

V. THERMAL DIFFUSION TAKING DYNAMIC BEHAVIOR OF RELEASED WATER INTO ACCOUNT[4)]

From the results of the field observation and the numerical analysis, it was made clear that the high temperature content is restricted to upper thermocline. Thus, a two-dimensional horizontal motion can be assumed. Applying the Boussinesq approximation, Equation (2) is satisfied by introducing the stream function, P, in the form, $u = \partial P/\partial y$ and $v = -\partial P/\partial x$. If the pressure, p, is eliminated by cross differentiation, we obtain the governing equation for the flow in a bay as:

$$\frac{\partial^4 P}{\partial x^4} + (1+\delta)\frac{\partial^4 P}{\partial x^2 \partial y^2} + \delta\frac{\partial^4 P}{\partial y^4} = 0 \qquad \text{.... ... .(13)}$$

where $\delta = A_z/A_x$

The field of flow in a bay with voluntary shape can be obtained by the directional differentiation of P (x, y) determined by the equation (13) in place of the solution derived from the simultaneous equations (1), (2) and (3). In order to calculate the velocity distribution, we introduce SOR method explained in the foregoing section. We obtain the following expression as an approximation to the equation (13) of elliptic type:

$$\begin{aligned} P(\xi,\eta) = \; & 040(P(\xi+1,\eta)+P(\xi,\eta+1)+P(\xi-1,\eta)+P(\xi,\eta-1)) \\ & -010(P(\xi+1,\eta+1)+P(\xi-1,\eta+1)+P(\xi-1,\eta-1)+P(\xi+1,\eta-1)) \\ & -005(P(\xi+2,\eta)+P(\xi,\eta+2)+P(\xi-2,\eta)+P(\xi,\eta-2)) \qquad \text{.......(14)} \end{aligned}$$

The steps of procedure of calculation are the same as that in the preceeding section The boundary condition for the stream function on each point of the coastal line can be set up by the following relation:

$$P = \int\frac{\partial P}{\partial x}\,dx + \int\frac{\partial P}{\partial y}\,dy = -V_0\int dx + U_0\int dy \; . \quad \text{... .... (15)}$$

where U_0 and V_0 are the velocity components of outfall at the outlet section.

The rate of convergence of the biharmonic equation is very slow. The speed of convergence is not so slow in a relatively simplified sea basin (see Fig. 9), but as for the S Bay with irregular shape its speed is very slow (see Fig. 10). In the latter case, the repetition of convergence exceeds 360 sweeps for the value (10^{-2}) of allowance error. The speed of the rate of convergence is high when the frequency of repetition is a few, but its speed makes slow progress as the residuals are running short[5]. After the stream function is determined, we will be able to deal with the thermal diffusion by means of velocity distribution obtained by equation (14)

The equation (3) can be replaced by introducing an apparent temperature T_2 defined by

$$T_2 = T_1 + \frac{B_1 T_s - B_0}{\Delta T - B_1}$$

into the following equation:

$$u\frac{\partial T_2}{\partial x} + v\frac{\partial T_2}{\partial y} + B_1 T_2 = \frac{\partial}{\partial x}\left(K_x\frac{\partial T_2}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_y\frac{\partial T_2}{\partial y}\right)$$

where T_0 and T_s represent respectively the water temperatures at the outlet and infinite sea basin being not affected by outflow, $T_1 = (T - T_s)/(T_0 - T_s)$, $B_0 = b_0/C_w \cdot H_w$ and $B_1 = b_1/C_w \cdot H_w$.

It can be assumed that the thermal flux is zero normal to the coastal boundaries. From the relation $T_2 = (T - B_0/B_1)/\Delta T$, we can take the boundary conditions as follows:

(i) $T_2 = 1.0$ at the outlet,

(ii) $T_2 = 0.0$ at the sea basin being not affected by the discharged warm water.

Therefore, we can obtain the distribution of water temperature in the bay, regardless of the values such as the discharged water temperature and the water temperature at the infinitely far off the sea. But, from the character of the equation for the thermal diffusion, it is not always appropriate to establish the boundary condition, assuming that the infinitely far off the sea is not influenced by the discharged warm water. The thermal gradient should be rather remain constant at the mouth of the bay (see Fig. 11).

The numerical solution of the equation of thermal diffusion with the dynamic behaviour of the outflow can be provided quantitative predictions of the water temperature distribution in a bay with voluntary shape with the aid of digital and analogue computers. On the basis of these studies, the design criteria of intake and outlet structures can be determined.

As an example of the most fundamental application of this calculation method, we consider the model of a simple semi-infinite sea basin, into which a large quantities of warmed cooling water are discharged. Fig. 12 represents the distribution of surface velocity in the semi-infinite sea basin. Fig. 13 represents the distribution of water temperature in the surface layer corresponding to the velocity distribution of Fig. 12. As you can see in these figures, it shows that the field of flow in the sea basin in which the eddy viscosities are predominant differs greatly from the general phenomenon of jet flow and are apt to subject to the influences of boundary near the outlet and the configuration of the bay On the basis of these results, the range of the influence of the water temperature rise in the surface layer by the discharged warm water was estimated. These values give quantitative relations of the cooling water flow and the range of influence of the water temperature rise or the flow in the surface layer of the sea. A part of the results are shown in Tables 1 and 2. Table 1 shows the maximum range of influence of the flow by the outfall of warmed cooling water. Table 2 shows also the maximum range of influence of the rise of water temperature.

Table 1. The maximum range of influence of the flow by the outfall of warmed cooling water.

Cooling water flow Q (m^3/s)	20	40	60	80	100
the maximum range of influerce of the flow (distance from the outlet m)	60	120	180	240	300

Table 2. The maximum range of influence of the water temperature rise by the outfall of warmed cooling water.

Cooling water flow Q (m^3/s)	20	40	60	80	100
the maximum range of influence in which the water temperature of the sea basin rises 1 °C	460	920	1380	1840	2300
its influenced area : m^2	17.8×10^4	71.3×10^4	160.5×10^4	285.4×10^4	446.0×10^4

These values were obtained under the assumption that the discharged cooling water flows in the surface layer only of the sea forming a two layer system. In this numerical calculation, the advective effect by the tidal current is not involved. In especial, as to the special site where some other factors must be taken consideration, it is again necessary to execute the numerical experiments according to the respective conditions on the basis of the above mentioned method.

As we state later on, it is found to give rise to upwelling phenomena from the lower layer near the outlet accompanied by outfall of cooling water. Therefore, we must evaluate the distribution of water temperature in the bay, to some extent, taking the supply of the lower water into account. But, it is not necessary to make modifications so much if the outfall velocity woudl be hold as small as possible After all, the values represented by Table 1 and 2 present the maximum range of influence of the water temperature rise in the calm sea, from the standpoint of separation of two layers with a warm water in the surface layer and with a cold water in the lower layer

VI. INTERACTION BETWEEN FLOW AND THERMAL DIFFUSION

Generally speaking, phenomena of flow and thermal diffusion by outfall of cooling water seem to be composed of complex processes The field of flow would change that of water temperature distribution in the vertical section. The diffusion process of warmed water would also change the field of flow. Thus, these processes of two phenomena can not be considered independently each other, but must be taken into account of the interaction between these two phenomena. In order to confirm the realization of the above mentioned matters, the numerical experiments on the thermal diffusion in the vertical section were conducted by taking into consideration of the thermal diffusion-velocity correlation. By applying the Boussinesq approximation, and therefore introducing the stream function, we obtain the following equation, taking into account of the thermal diffusion flow interaction.

$$\frac{\partial^2}{\partial x^2}\left(A_x \frac{\partial^2 P}{\partial x^2}\right)+\frac{\partial^2}{\partial z^2}\left(A_z \cdot \frac{\partial^2 P}{\partial z^2}\right)+\frac{\partial^2}{\partial x \partial z}\left((A_x+A_z)\frac{\partial^2 P}{\partial x \partial z}\right) = -g\alpha\rho_0\frac{\partial T}{\partial x} \quad . \quad . \quad (16)$$

The term of thermal horizontal gradient is contained in the right-hand side of equation (16). And this term could probably be interpreted as one having power of binding the flow associated with addition of cooling water. The effect of thermal horizontal gradient is omitted, in the first order approximation, to simplify the mathematical development Solutions of the first order approximation are applicable only to bay having negligible stratification or gravitational convection A more general solution is obtained by the combination of (3) and (16), by applying these first order values previously obtained. If the processes are repeated, the required solution will be obtained finally Solutions thus obtained give the values which are influenced to some extent by the thermal distribution-velocity interaction.

Generally, discharged cooling water in a bay flows as an upper layer current partly because of its inertial momentum and partly because of its lower density. The upper layer current thus exerts the tangential stress on the lower layer which favours the compensation current along the sea bottom.

It therefore seems to give rise to upwelling motions from the lower layer to the upper layer. Until recently the investigation of these phenomena was not enough to afford any insight into the inner mechanism of this phenomenon Detailed systematic field surveys of the upwelling phenomena in the coastal region have been made since 1964 at power plants located on the Mizushima Bay and the Miike Port.

The author found in the field test at the outlet of Mizushima Thermal Power Plant that the surface outflow of cooling water in the bay was accompanie by an inflow of sea water in the lower layer.

Fig. 14 represents the state of occurrence of the return flow in the botto layer of the coastal region by the outfall of cooling water at the Mizushima Thermal Power Plant The upwelling motion may be also understood by the vertical structure on the thermal diffusion of Fig. 3, in which the uniform rise of the isothermal line towards the coast is a particularly marked feature of the thermocline structure of the upwelling region.

The occurrence of lower currents of this type in a bay could also be shown by numerical experiment Figs. 15 and 16 represent respectively the vertical profiles of velocity and water temperature along the longitudinal section obtained by the numerical experiment

The general nature of the solution by numerical experiment is shown by Fig. 17, where the value, $A_z/A_x = 0.1$, has been used. This figure utilizes only the first and fourth order velocity These curves show a surface outflow and a deeper inflow of water. Higher order approximation, taking into accout of interaction, has a considerable effect on the shape of the vertical profile of water temperature and velocity. An interaction solution in the vicinity of the outlet gives the velocity layer or smaller than that give by the 1st order solution at surface or at bottom. However, off the outlet, differential advection between two layers will tend to develop a density instability in these layers, giving a much increased vertical coefficient of diffusivity. The specification of the vertical variation in density shows that sea water in the lower layer is sucked into the upper layer through the boundary surface, and that just below this surface there is the offshore flow, a portion of which is sucked into the upper layer On the other hand, the velocity of outflow decreases with the depth and under a certain depth the bottom sea water flows toward the shore It seems that these phenomena do suggest one mechanism of cooling water flow in a bay.

In the above treatments the coefficients of eddy diffusion were assumed to be constant despite the fact that the magnitude of the coefficients depends strongly on the local turbulence. Generally speaking, the effect of a thermal stratification on the state of turbulence is described by means of the local Richardson number. But when the vertical gradient of velocity are small or constant; and the vertical gradient of density strong, the state of turbulence depends on the stability

The calculation in consideration of the variation of eddy diffusivity in the stratified sea is now in progress. In this treatment, the coefficients of eddy diffusion are assumed to be functions of the static stability The results will be shown in the future paper

CONCLUSIONS

The following points were made clear as the results of the field surveys and the numerical experiments. Those were carried out to make clear the possibility of intake and outfall of cooling water in the same basin of bay.

1) The high temperature content of the discharged cooling water is observed i the surface layer, and the water temperature decreases remarkably at the depth below layer of thermocline (layer of large temperature gradient) The latter fact is probably due to the poor vertical mixing with the upper layer owing to the large stability.

2) Based on the observed data of the temperature distribution of warmed cooli water discharged into the sea basin, the thermal diffusion coefficients were calculated. It is found that the horizontal diffusivity is 50 ~ 80 times greater than the vertical, the order of which seems to be about 0.01 m^2/sec.

3) Accordingly, it is concluded that the decrease of the water temperature observed in the field tests should be due to the horizontal mixing with surrounding waters of low temperature in addition to the cooling process of the sea surface.

4) From the numerical solution of the equation for the thermal diffusion with dynamic behavior of the outflow, the quantitative relation between cooling water discharge and the range of influences of the water temperature rise or the flow in the surface layer of the sea was obtained.

5) Systematic field surveies with respect to velocity and water temperature off the outlet were made. From the result, the occurrence of the return flow in the bottom layer of the coastal region was confirmed. On the other hand, the mechanism of development of upwelling phenomenon was made clear from the numerical experiment, taking into account of the interaction between the flow and the thermal diffusion

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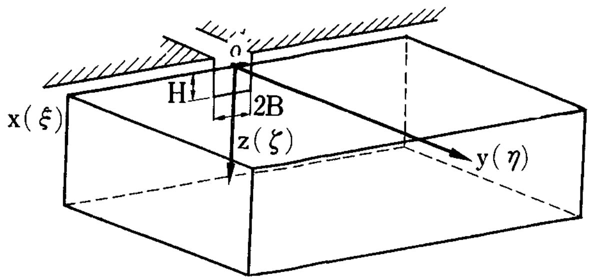

Fig. 1. Definition sketch for thermal spread.

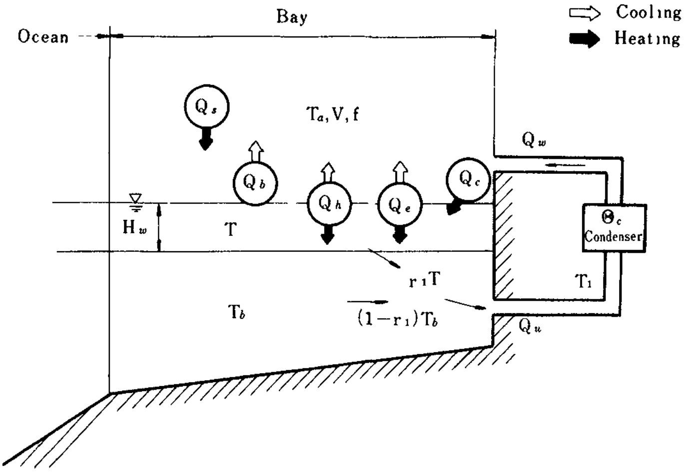

Fig. 2. Schematic diagram of main processes for heat balance.

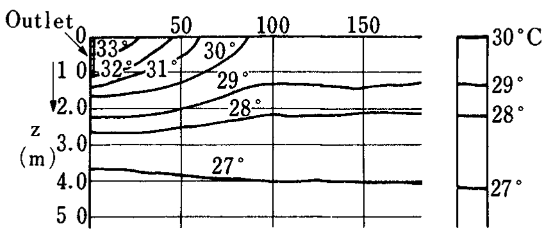

Fig. 3. Vertical distribution of water temperature along the y-axis perpendicular to the shore in M. Bay.

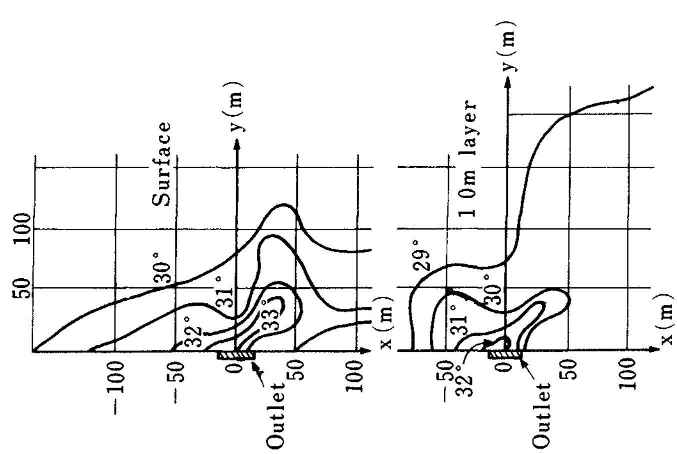

Fig. 4. The horizontal distributions of water temperature at the surface and 1.0m layer in M. Bay.

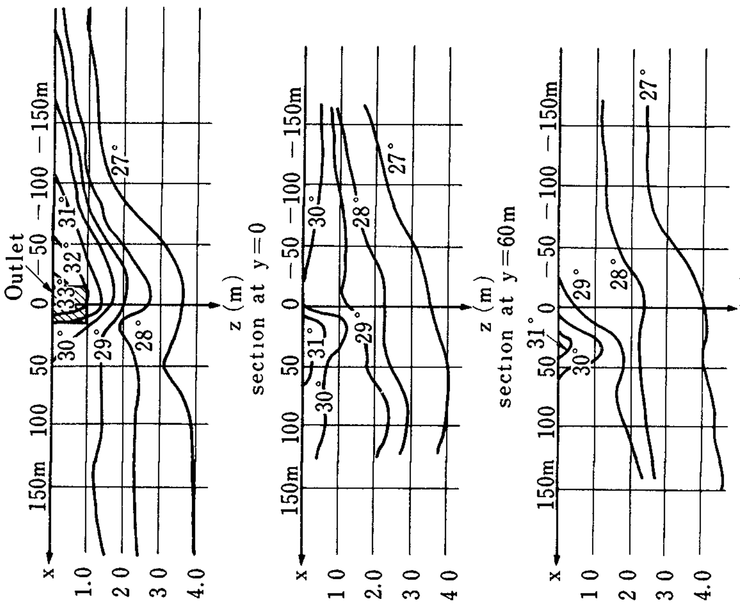

Fig. 5. Vertical distributions of water temperature at various sections parallel to the shore at y = 0.60m and 90m.

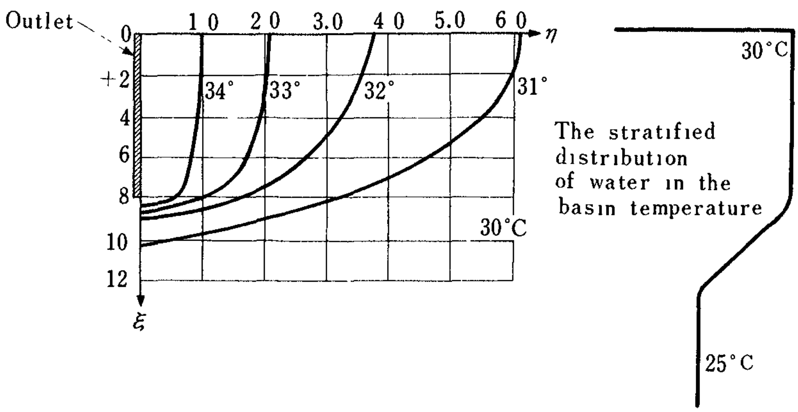

Fig. 6. Vertical distribution of water temperature along the η-axis perpendicular to the shore ($\lambda = 0$).

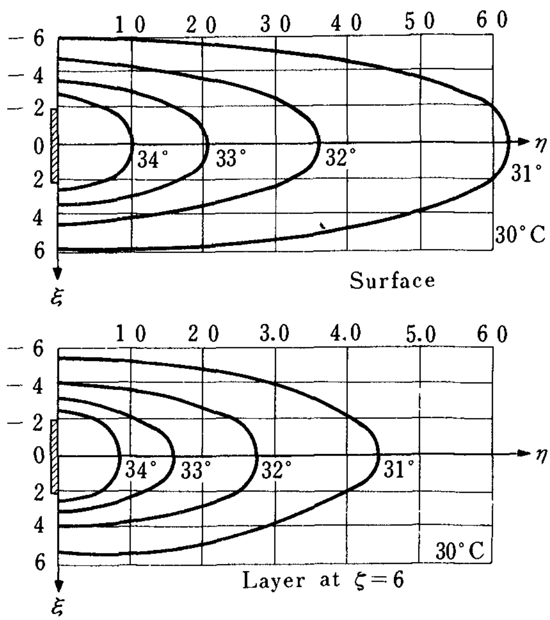

Fig. 7. Horizontal distributions of water temperature at the surface and layer of $\zeta = 6$ ($\lambda = 0$).

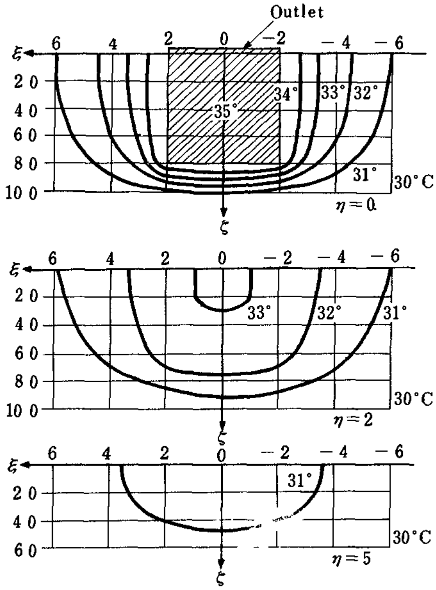

Fig. 8. Vertical distributions of water temperature at various sections parallel to the shore at $\eta = 0,2$ and 5 ($\lambda = 0$).

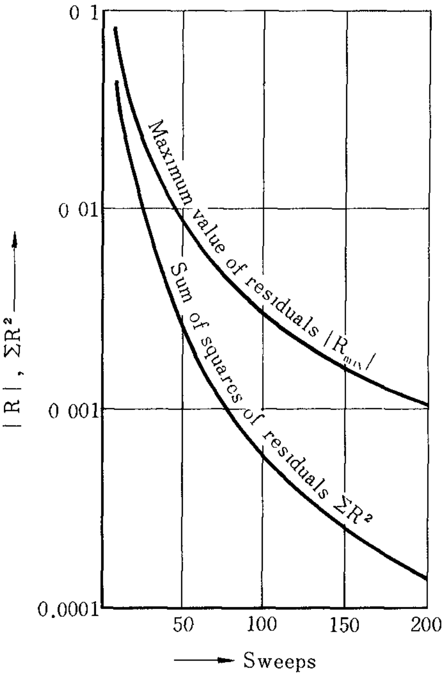

Fig. 9. Rate of convergence of equation of biharmonic type for semi-infinite sea basin.

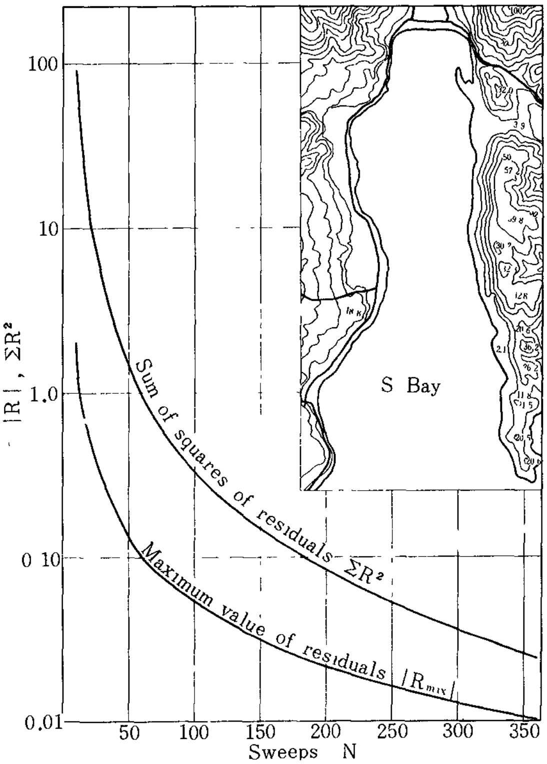

Fig. 10. Rate of convergence of equation of biharmonic type for S Bay with irregular shape.

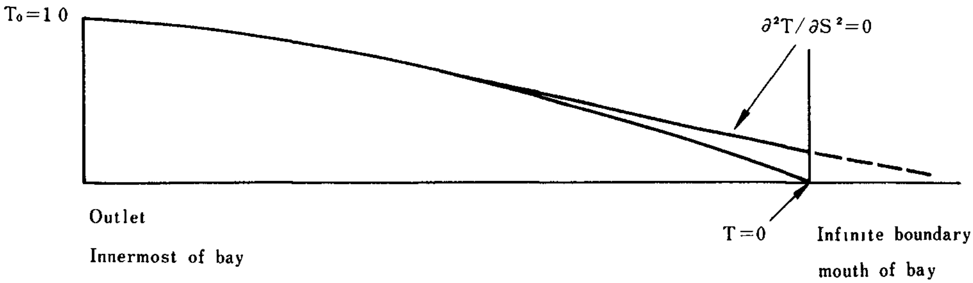

Fig. 11.

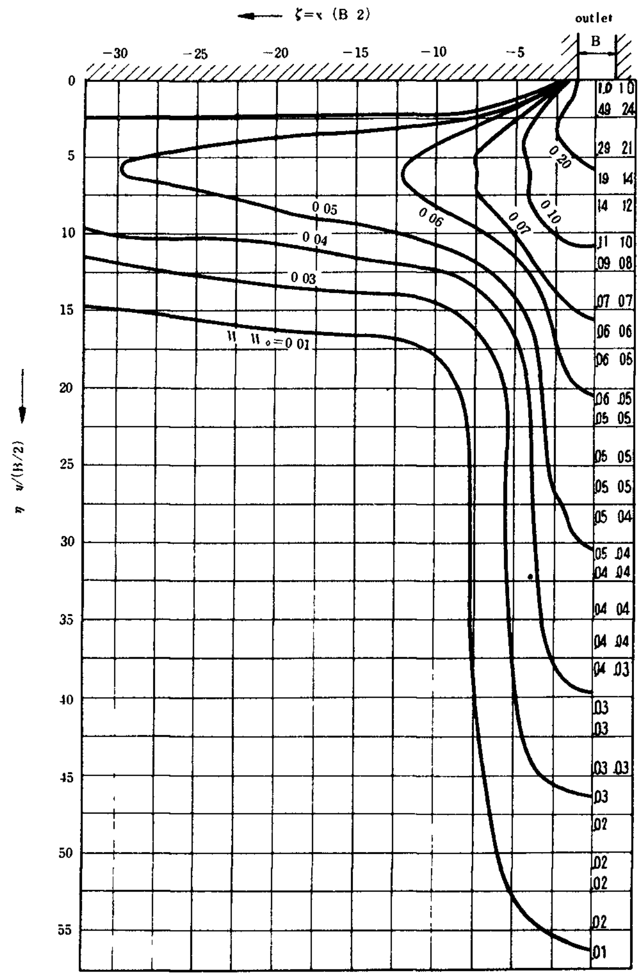

Fig. 12. Distribution of surface velocotiy (W_O = 1 m/sec, W_O:outfall velocity).

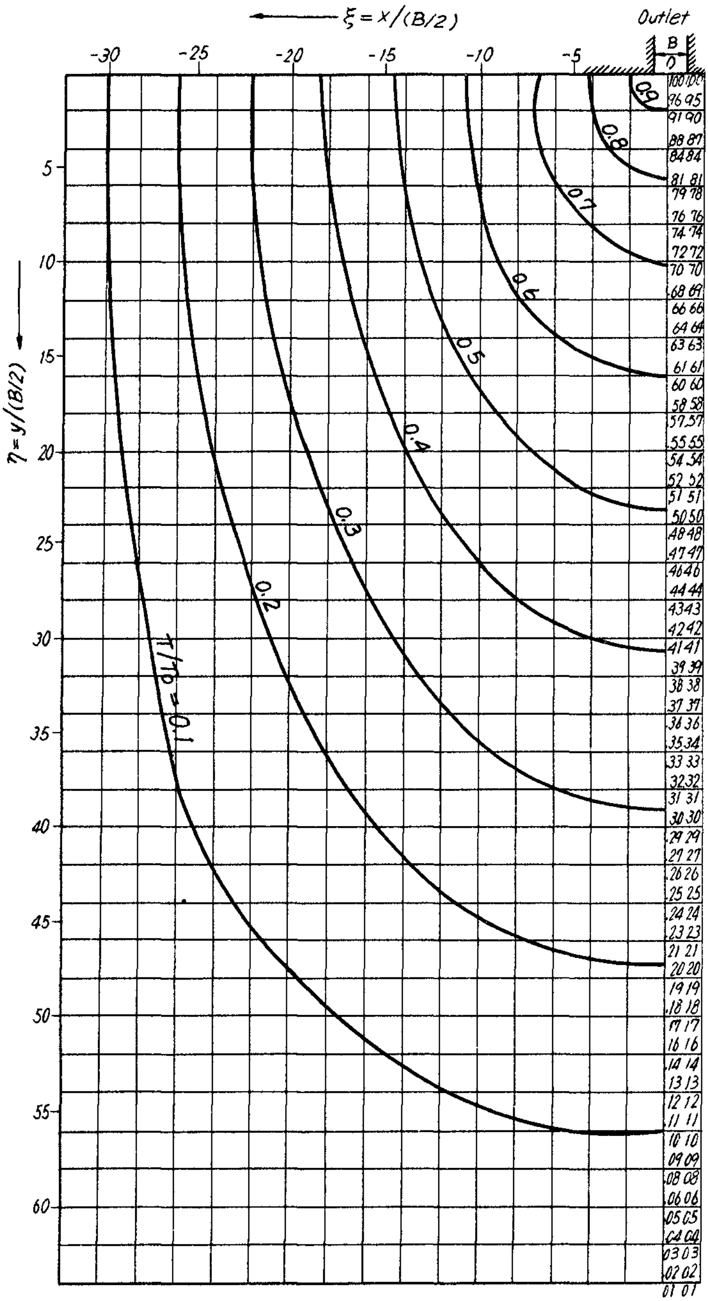

Fig. 13. Distribution of water temperature in the surface layer.

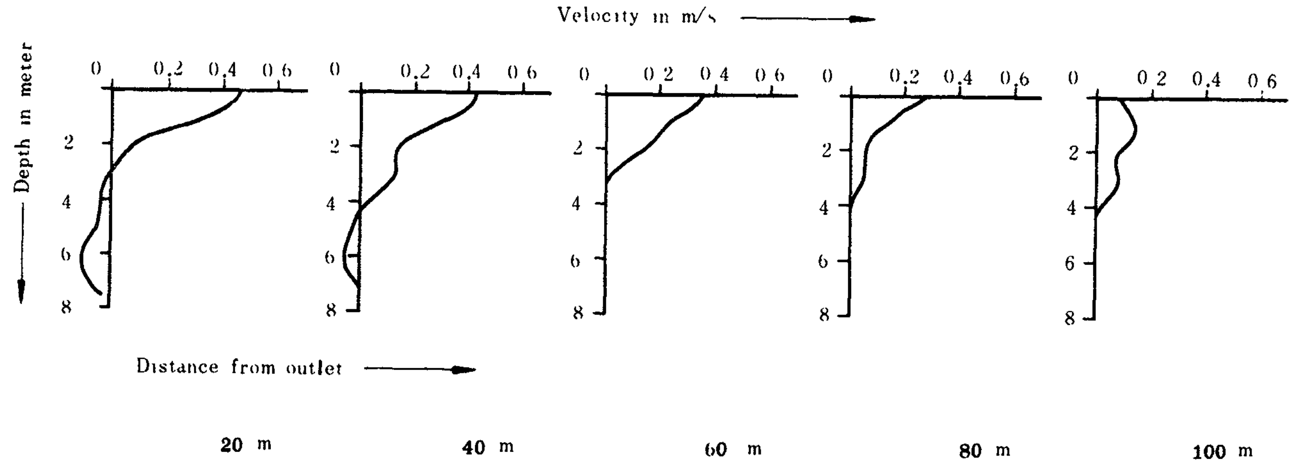

Fig. 14. Vertical variation of velocity off the outlet by discharged cooling water (section perpendicular to the shore).

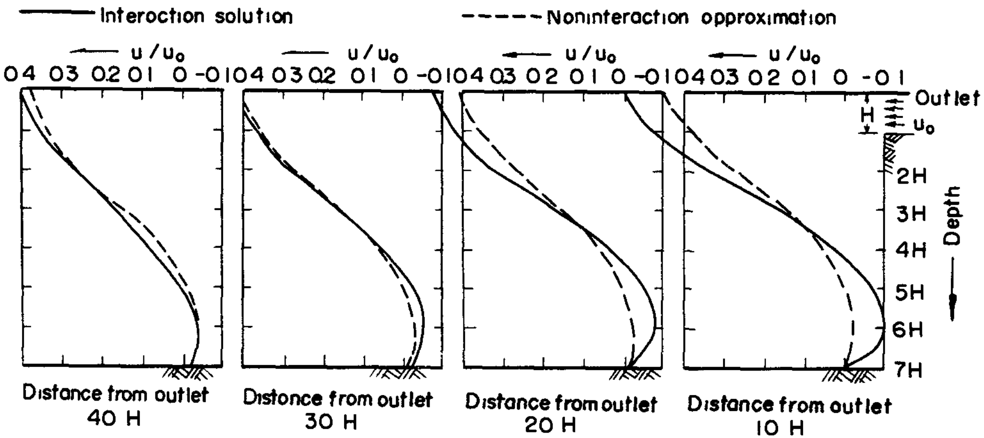

Fig. 17. Variation of velocity profile taking the water temperature-velocity interaction into account.

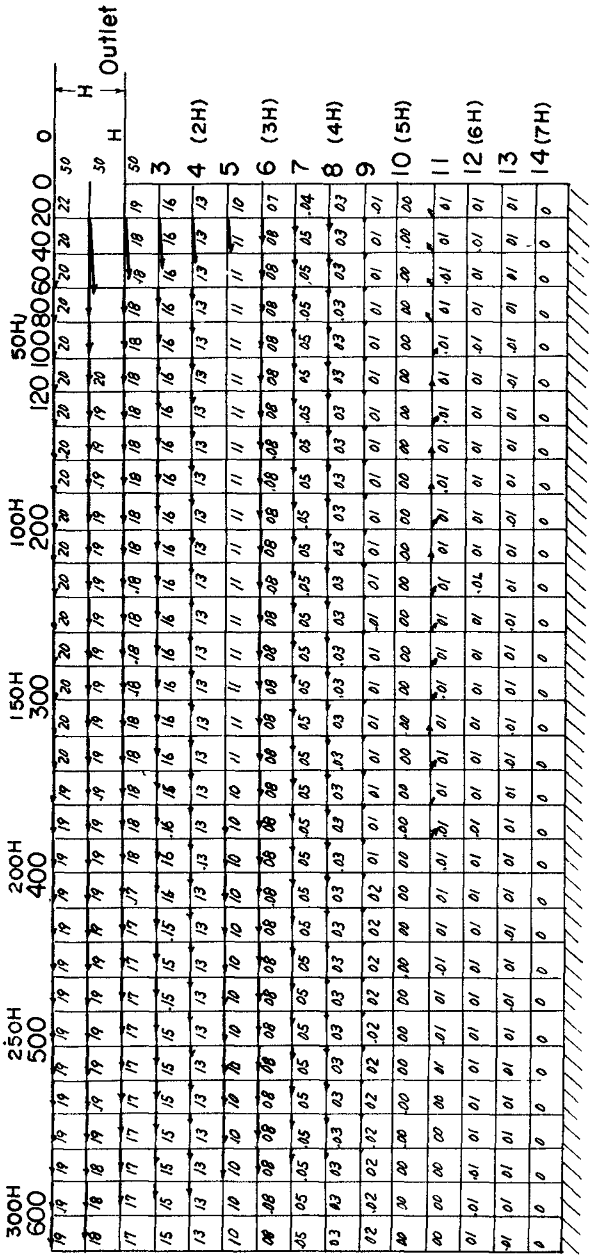

Fig. 15. Velocity distribution along a longitudinal section.

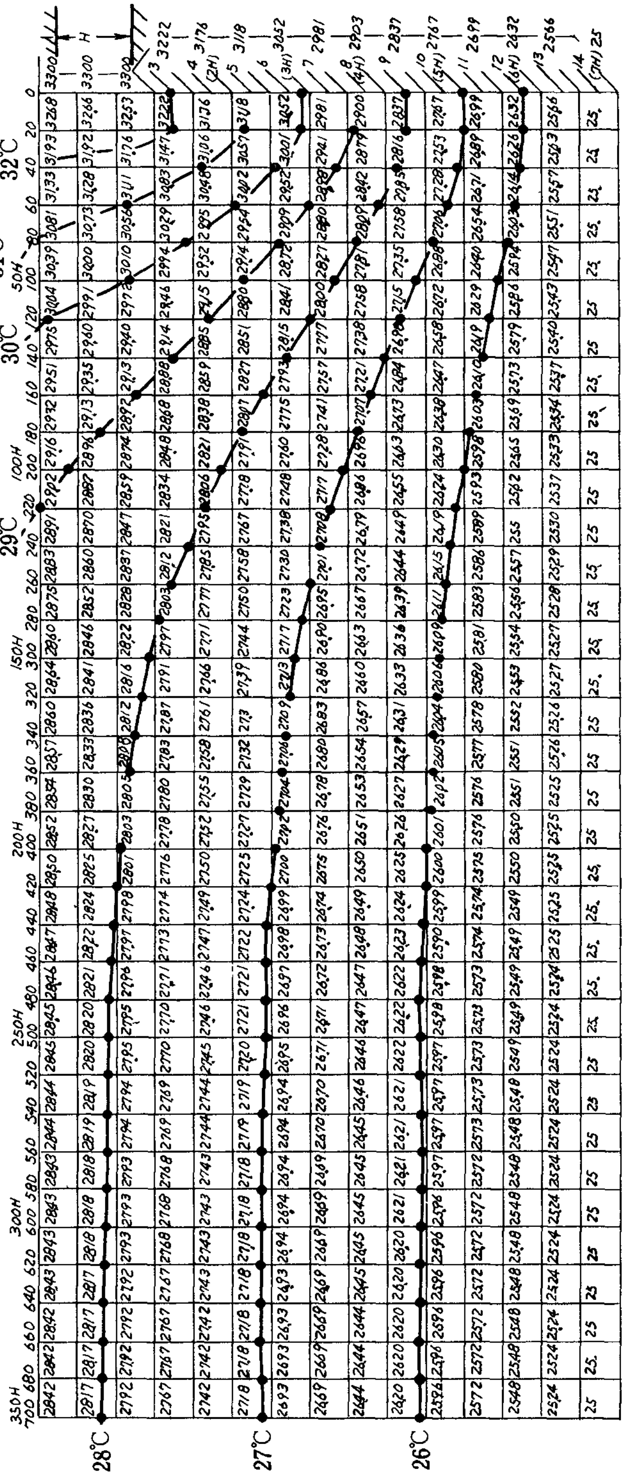

Fig. 16. Water temperature distribution along a longitudinal section.