CHAPTER 47
ON THE EFFECT OF BREAKWATERS AGAINST TSUNAMI

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ABSTRACT

As well known the coast of Japan is often attacked by tsunami. As a kind of disaster preventive facilities, breakwaters have been built or are now under construction in several ports in order to protect the harbour area by diminishing the energy of rushing water and by reducing the water level elevation inside the basin. The biggest work of this sort is found at the port of Ofunato, Iwate Prefecture.

The effect of breakwaters against such long-period wave as tsunami was investigated by the authors, using electronic computers. The principle is to solve the equations of motion and of continuity. Not only for the stationary state but also for the transient state. One of the most important items to be studied is the response of basin water to the incoming tsunami, of which the period is variable in a wide range from several minutes to more than one hour.

The result of calculation has clarified the behaviour of tsunami affected by breakwaters located in a bay open to the outer sea. The computing method, together with an approximate method devised from fundamental investigations made in advance, has also been applied to several other ports, supplying a plenty of data for harbour planning and design.

METHOD OF CALCULATION

BASIC EQUATIONS

Basic Equations - Tsunami can be treated as a long wave. The equations of motion and of continuity are as follows, higher order terms being neglected.

\[
\frac{\partial u}{\partial t} = -g \frac{\partial z}{\partial x} \\
\frac{\partial v}{\partial t} = -g \frac{\partial u}{\partial y} \\
\frac{\partial z}{\partial t} = -\frac{2}{\partial x} (\kappa u) - \frac{2}{\partial y} (\kappa v)
\]

(1)
where, \( u, v \) = velocity component in \( x \)- and \( y \)-direction, respectively
\( \zeta \) = water level elevation with respect to mean water level
\( h \) = water depth below mean water level

These differential equations are transformed into difference equations, with the distribution of velocity and water level computing points shown in Figure 1.

\[
\begin{align*}
\frac{\partial u_{2m, 2k+1}}{\partial t} (2n+1) &= u_{2m, 2k+1} (2n) \\
- \frac{\partial \zeta_{2m, 2k+1}}{\partial x} (2n+1) &= \zeta_{2m, 2k+1} (2n) - \zeta_{2m, 2k+1} (2n+1) \\
\frac{\partial v_{2m+1, 2k}}{\partial t} (2n+1) &= v_{2m+1, 2k} (2n) \\
- \frac{\partial \zeta_{2m+1, 2k}}{\partial y} (2n+1) &= \zeta_{2m+1, 2k} (2n) - \zeta_{2m, 2k+1} (2n+1)
\end{align*}
\]

(2)

where, \( m, k \) = index for the position of computing points
\( n \) = time step

Figure 2 shows the layout of the tsunami breakwater in the port of Ofunato. Ofunato Bay is situated along the Pacific coast of north-eastern Japan. This district has been frequently attacked by tsunamis, especially the damages in 1896, 1933 and 1960 were remarkable. The construction of the tsunami breakwater was started after the disaster due to Chilean Earthquake Tsunami in 1960. Ofunato Bay is 1.7 km wide and 40 m deep at the mouth and has a length of about 7.7 km. For the purpose of calculation, the configuration of actual bay is modified and divided into mesh as shown in Figure 3, where the mesh interval \( \Delta s (=\Delta x, \Delta y) \) is equal to 280 m. The water depth distribution is given in Figure 4. Time step \( \Delta t \), 10 seconds in this case, is determined from the mesh interval and the maximum water depth of 40 m.

The outer sea is represented by a channel of constant depth and width, where the computation is only aimed at supplying the incoming tsunami and may not necessarily coincide with the actual phenomenon. Moreover, this imaginary outer sea is divided into rough meshes in order to save the amount of the computation, namely, \( \Delta x = L/10 \) and \( \Delta y = 6 \times 280 \) m, where \( L \) is the wave length of the incident tsunami.

Equations at the bay-mouth – The difference of mesh interval in and outside the bay requires special equations around the bay-mouth.

For the velocity points at the bay-mouth;
\[ U_{2k+1} (2n+2) = U_{2k+1} (2n) \]

\[ \frac{\partial}{\partial s} \left\{ \sum_{m=0}^{\infty} \left[ \frac{d}{d s} \left\{ \frac{\partial}{\partial s} \left( U_{2k+1} (2n) \right) - \sum_{k=0}^{n} h_{2k+1} (2n) \right\} \right] \right\} \]

For the outside water level points adjacent to the bay-mouth:

\[ \sum_{m=0}^{\infty} \left\{ \frac{d}{d s} \left\{ \frac{\partial}{\partial s} \left( U_{2k+1} (2n+1) \right) - \sum_{k=0}^{n} h_{2k+1} (2n+1) \right\} \right\} \]

where, \( h = \) constant water depth in the imaginary outer sea

Equation at the breakwater opening - Although the higher order terms are neglected in the above equations, the term of head loss is added to the equation of motion at the breakwater opening, assuming that the head loss is represented in the following form.

\[ \Delta \sum = \int \frac{u^2}{2g} \]

Adding this term, the equation of motion is modified into;

\[ U_{2k+1} (2n+2) = U_{2k+1} (2n) \]

\[ \frac{\partial}{\partial s} \left\{ \sum_{m=0}^{\infty} \left[ \frac{d}{d s} \left\{ \frac{\partial}{\partial s} \left( U_{2k+1} (2n+1) \right) - \sum_{k=0}^{n} h_{2k+1} (2n+1) \right\} \right] \right\} \]

The following approximation is made of use to solve this equation;

\[ u |u| = \frac{1}{2} \left\{ U_{2k+1} (2n+2) |U_{2k+1} (2n+2)| \right\} \]

Thus, the final formula for the velocity at the breakwater opening is given by;

\[ U_{2k+1} (2n+2) = \frac{1 - \sqrt{1 + fR}}{f \frac{\partial}{\partial s}} \]

\[ R \geq 0 \]

\[ \frac{1 + \sqrt{1 - fR}}{f \frac{\partial}{\partial s}} \]

\[ R < 0 \]
where,
\[ R = \frac{g}{4} \left( \frac{a t}{d s} \right)^2 \left\{ \sum_{2m,2k+1} (2m+1) - \sum_{2m,2k} (2m+1) \right\} \]

The head loss at the breakwater opening is mainly due to contraction and expansion of flow. As the coefficient for sudden contraction or expansion in the limiting case is 0.5 or 1.0 respectively, the coefficient \( f \) of 1.5 was used in our calculations.

The actual breakwater opening is to be of the water depth of 16.9 m below mean water level and a width of 200 m, which differs from the mesh interval. In the calculation an equivalent water depth of 12 m ( = 16.9 x 200 / 280 ) is used, as indicated in Figure 4.

**INITIAL CONDITIONS**

The profile of incident tsunami is assumed to be a train of sinusoidal wave with constant amplitude and period. These waves invade the bay where the water is originally at rest. The induced motion of basin water will reach a stationary state after a certain period of transient state. The highest water level elevation is found either during the transient state or in the stationary state.

The computation is started when the front of tsunami reaches the bay-mouth and the initial conditions are given by the velocity distribution at \( t=0 \) and by the water level distribution at \( t=\Delta t/2 \).

Inside the bay:
\[ \zeta_{2m,2k+1} (0) = \varphi_{2m+1,2k} (0) = 0 \]

Outside the bay:
\[ \varphi_{2m,2k+1} (0) = \sqrt{\frac{\sigma}{\xi}} \cos \left( \frac{2 \pi m - 2 \delta}{2 \xi} \Delta x \right) \]

\[ \zeta_{2m+1,2k} (0) = 0 \]

where, \( \alpha \) = amplitude of the incident tsunami in the imaginary outer sea

As shown in these equations, the incident tsunami was assumed to begin with retreating water in case of the calculation for Ofunato Bay.
As a boundary condition, velocity components normal to the shoreline or breakwater are put to be zero. Besides this, an imaginary boundary is placed at a certain offshore part of the outer sea, where the velocity variation due to succeeding tsunami is given as a function of time. This velocity variation is obtained by solving the difference equations (2) under the initial condition of (11). The following formula gives the final solution with a sufficient accuracy.

\[ u_{2n+1} = \sqrt{\frac{g}{\kappa}} \alpha \sin n \theta \]

The distance between this offshore boundary and the bay-mouth is so determined that any reflected waves from shoreline or breakwater may not affect the phenomena in the bay until the end of the calculation, after being re-reflected from the offshore boundary. When four cycles of tsunami are taken into computation, for example, the distance should be at least two times of the wave length, as understood from Figure 5. Thus, eliminating the influence of reflected waves which is impossible to be known beforehand, the effect of breakwaters can be examined under exactly the same incident tsunami before and after its construction.

FUNDAMENTAL TWO-DIMENSIONAL CALCULATION

In order to examine the influence of various factors step by step, the above-mentioned computing method was first applied to simplified cases, such as a semi-infinite channel of a uniform rectangular cross section with a couple of breakwater in it (Figure 6) or a rectangular bay connected to the outer sea (Figure 7), etc. The factors considered are natural period of the basin, width of the breakwater gap, shape of the bay (Figure 8), reflection from the innermost end of the bay (Figure 9) and so on. After analyzing these two-dimensional calculations for 42 cases in total conducted by an electronic computer, an approximate one-dimensional method was proposed. When the variation of the phenomena in the lateral direction is not so big, this method is very useful to interpret or interpolate the two-dimensionally computed results and is also applicable to preliminary studies. The details of the method is mentioned hereafter.

ONE-DIMENSIONAL APPROXIMATE METHOD

WATER LEVEL BEFORE BREAKWATER CONSTRUCTION

Transmission and reflection at the bay-mouth - The principle of this method is to solve one-dimensional difference equations analytically. When the incident waves arrive at the bay-mouth, a certain part of its energy is transmitted into the bay while the rest
is reflected offshore. The profile and velocity of each wave are assumed as follows:

Incident wave: \[ \zeta_i = a \sin (kx + \omega t) \]
\[ u_i = -\sqrt{\frac{g}{k}} a \sin (kx + \omega t) \]

Reflected wave: \[ \zeta_r = -a \sin (kx - \omega t - \phi_i) \]
\[ u_r = \sqrt{\frac{g}{k}} g a \sin (kx - \omega t - \phi_i) \]

Transmitted Wave: \[ \zeta_t = p a \sin (kx + \omega t - \phi) \]
\[ u_t = -\sqrt{\frac{g}{k}} p a \sin (kx + \omega t - \phi) \]

where, \( p, q \) = transmission and reflection coefficient, respectively
\( \phi, \phi_i \) = phase variation due to reflection or transmission
\( k = \frac{2\pi}{L} \)
\( \omega = \frac{2\pi f}{T} \)

The water level and velocity in and outside the bay are:

Inside: \[ \zeta = \zeta_t, \quad u = u_t \]
Outside: \[ \zeta = \zeta_i + \zeta_r, \quad u = u_i + u_r \]

One-dimensional difference equations including the velocity at the bay-mouth \( u(l, t) \) are (Figure 10);

\[
\left. \begin{array}{l}
\frac{\partial \zeta}{\partial t} = -g \frac{\partial u}{\partial x} \\
\frac{\partial u}{\partial t} = -\frac{B}{B_i} \left\{ u(l + ax, t) - \frac{B}{B_i} u(l, t) \right\} \\
\frac{\partial \zeta}{\partial t} = -\frac{B}{B_i} \left\{ u(l - ax, t) - \frac{B}{B_i} u(l, t) \right\}
\end{array} \right\}
\]

where, \( l \) = length of the bay
\( B \) = width of the bay
\( B_i \) = width of the imaginary outer sea

The mesh interval in onedimensional difference equation can be taken as:
Substituting Equations (13) and (14) into (15) and eliminating \( u(l, t) \), the following transmission coefficient is obtained:

\[
\begin{align*}
\frac{\Delta p}{\Delta z} &= \frac{B_1}{B} \left( \frac{B_1}{B} + 1 \right) \left( 1 + \cos \frac{\pi \Delta x}{L} \right) \\
&= \left( \frac{B_1}{B} \right)^2 + 2 \frac{B_1}{B} \cos \frac{\pi \Delta x}{L} + 1
\end{align*}
\]

As the term of \( \Delta x/L \) has little influence when the mesh interval is relatively small compared with the wave length, this formula is replaced by:

\[
\frac{p \cos \beta_2}{\Delta z} = \frac{B_1}{B} \left( \frac{B_1}{B} - 1 \right) \sin \frac{\pi \Delta x}{L} \\
= \left( \frac{B_1}{B} \right)^2 + 2 \frac{B_1}{B} \cos \frac{\pi \Delta x}{L} + 1
\]

In the case \( B_1/B = 3 \), for example, the formula gives \( p = 1.5 \). This means that the incident wave of amplitude \( a \) invades the bay after being amplified to \( 1.5a \) at the bay-mouth. The transmitted wave is then reflected from the end of the bay, where the composed amplitude is equal to \( 2 \times 1.5a = 3a \). This reflected wave returns again partly from the bay-mouth, as shown in Figure 10. The inversed reflection coefficient is obtained in a similar way:

\[
\begin{align*}
p &= \frac{2 \frac{B_1}{B}}{\frac{B_1}{B} + 1} , \\
\beta_2 &= 0
\end{align*}
\]

Water level during transient state - The water level in the bay is obtained as the superposition of the incident wave and the successive reflected waves (Figure 10). The followings give the variation of water level at the end of the bay:

\[
\begin{align*}
t &= 0 \sim T_0 , & \xi &= 2pa \sin \alpha t \\
t &= T_0 \sim 2T_0 , & \xi &= 2pa \sin \alpha t + 2p' a \sin (\alpha t - \pi - \lambda) \\
t &= 2T_0 \sim 3T_0 , & \xi &= 2pa \sin \alpha t + 2p' a \sin (\alpha t - \pi - \lambda) \\
& & & + 2p' a \sin (\alpha t - 2\pi - 2\lambda)
\end{align*}
\]
where, \( T_0 \) = natural period of the bay as a closed basin
\[ \lambda = \frac{2 \pi}{T} \]

Figure 11 shows some applications of this method to the amplitude variation in the bay, together with the values from two-dimensional calculation for Case-B. It is proved that one-dimensional approximate method has a sufficient accuracy to get the tendency from transient to stationary state. The same procedure is also applicable to the basin restricted by breakwaters.

Water level in the stationary state - Equation (20) approaches the following limit with the increase of time;

\[
\zeta = \frac{2 \rho a}{1 + \frac{\rho}{2} + 2 \frac{\rho}{2} \cos \lambda} \left\{ \left( \frac{\rho}{2} \cos \lambda \right) \sin \omega t + \frac{\rho}{2} \sin \lambda \cos \omega t \right\} \tag{21}
\]

The amplitude equals to;

\[
A_s = \frac{2 \rho a}{\sqrt{1 + \frac{\rho}{2} + 2 \frac{\rho}{2} \cos \lambda}} = \frac{2 \sqrt{2} \frac{B}{B} a}{\sqrt{(\frac{B}{B})^2 + 1} + \left( \frac{B}{B} - 1 \right) \cos 2 \pi \frac{T_0}{T}} \tag{22}
\]

This is a function of \( T_0/T \) and the maximum is, in the case \( B_i/B = 3 \) for example, equal to 6a when \( T_0/T = 1, 3 \), etc., while the minimum is 2a when \( T_0/T = 2, 4 \), etc.

The highest water level before breakwater construction - The highest water level occurs either during the transient state or in the stationary state, as shown in Figure 11. Therefore, the bigger value of either state should be considered as the highest. In the above-mentioned example, the first wave crest of 3a represents the water level at the end of the bay during the transient state and the stationary amplitude less than 3a must be replaced by it, as later shown in Figure 15.

WATER LEVEL AFTER BREAKWATER CONSTRUCTION

Transmission coefficient at the breakwater gap - Although the principle used for the bay-mouth is applicable to the breakwater gap, two-dimensionally computed data for Case-C is directly used in order to take into account the effect of head loss. Figure 12 shows the transmission coefficient at the breakwater gap including head loss, where the influence of \( T/T_0 \) is due to the difference of mesh interval in each period. In the breakwaters are constructed not in a semi-infinite channel but at a certain place in an open bay, the incident
wave to the breakwater gap has to be replaced by the transmitted \( \text{w} \) through the bay-mouth, which is obtained by Formula (18). In the case \( B/ \beta = 3 \) and \( B/b = 5 \) for example, the first wave crest at the end of the basin is equal to \( 2 \times (0.4 \times 0.5) \times 1.5 = (1.2 \times 1.5) \lambda \).

**Stationary state after breakwater construction** - The modification of one-dimensional calculation without considering the head loss by two-dimensional data for Case-C is taken as an approximate method to estimate the stationary state water level after breakwater construction. The wave profile and velocity in the stationary state are assumed as (Figure 13);

Inside the basin; 
\[
\zeta = r_0 \cos kx \sin \alpha t
\]
\[
U = -\frac{2}{k} r_0 \sin kx \cos \alpha t
\]

Outside the basin in the bay;
\[
\zeta = -r_i \cos k(x-x_0) \sin \alpha t
\]
\[
U = \frac{2}{k} r_i \sin k(x-x_0) \cos \alpha t
\]

where, \( r_0 \) = amplitude of stationary wave at the end of the basin

\( r_i \) = amplitude of stationary wave outside the basin

\( x_0 \) = position of the first loop of the outside stationary wave measured from the end of the basin.

One-dimensional difference equations including the velocity at the breakwater gap are;

\[
U(l_i,t+\Delta t) - U(l_i,t)
= -\frac{B}{a} \frac{\Delta t}{\Delta x} \left\{ \zeta(l_i+\Delta x/2,t+\Delta t) - \zeta(l_i-\Delta x/2,t+\Delta t) \right\}
\]

\[
\zeta(l_i+\Delta x/2,t+\Delta t) - \zeta(l_i+\Delta x/2,t-\Delta t/2)
= -\frac{k}{\Delta x} \left\{ B \left( U(l_i,\Delta x,t) - U(l_i+\Delta x,t) \right) \right\}
\]

\[
\zeta(l_i-\Delta x/2,t+\Delta t/2) - \zeta(l_i-\Delta x/2,t-\Delta t/2)
= -\frac{k}{\Delta x} \left\{ \frac{b}{B} U(l_i,t) - U(l_i-\Delta x,t) \right\}
\]

where, \( l_i \) = length of the basin

\( b \) = width of the breakwater gap

Substituting Equation (23) into (24) and eliminating \( U(l_i,t) \), the following solution is obtained.
The stationary wave outside the bay has an amplitude of $2a$. The ratio of $r_1$ to $2a$ is obtained from Equation (22), regarding the loop between the breakwater and the bay-mouth as the innermost boundary of the bay, namely;

$$\frac{r_1}{2a} = \sqrt{\frac{2 B_1}{B}} \sqrt{\left\{ \left( \frac{B_1}{B} \right)^2 + 1 \right\} \left( \frac{B_1}{B} \right)^2 - 1 \cos \frac{2 \pi T_r}{T}}$$

(26)

where,

$$T_r = \frac{z(l - x_o)}{\sqrt{g \theta}}$$

(27)

If $x_o$ exceeds $\frac{l}{2}$, no actual loop is found between the breakwater and the bay-mouth. In this case, an imaginary loop situated at $x_o = -\frac{l}{2}$ is regarded as the innermost boundary.

From Equations (25) and (26), the following formula is obtained as a relation between the stationary wave amplitude at the end of the basin and that outside the bay.

$$\frac{\alpha}{\alpha_s} \left( \frac{B_1}{B} \right)^2 = \left( \frac{B_1}{B} \right)^2 \left\{ 2 \left( \frac{B_1}{B} - 1 \right) \sin \frac{\pi T}{T} \cos \frac{\pi (T-T_0)}{T} \tan \frac{\pi \theta \alpha}{L} - \cos \frac{\pi T_0}{T} \right\}^2 \left( \frac{B_1}{B} \right)^2 \left\{ 2 \left( \frac{B_1}{B} - 1 \right) \sin \frac{\pi T}{T} \sin \frac{\pi (T-T_0)}{T} \tan \frac{\pi \theta \alpha}{L} - \sin \frac{\pi T_0}{T} \right\}^2$$

(28)

where, $T_1$ = natural period of the basin $= 2 \frac{l}{\sqrt{g \theta}}$

The effect of head loss at the breakwater gap is given by Figure 14, which indicates the ratio of stationary wave amplitude at the end of the basin computed with head loss (Case-C) to that without head loss (Case-A). The variation of the ratio with $T/T_0$ shows that the difference of velocity at the gap regulates the effect of head loss. In the case $T/T_0$ is nearly equal to unity, for example, the velocity is so small that little effect is found in the ratio. The final stationary wave amplitude is given as the value from Equation (28) multiplied by the ratio in Figure 14.

The highest water level after breakwater construction - The bigger value either during the transient or in the stationary state is the highest water level elevation.
EXAMPE OF CALCULATION

The period of tsunami for two-dimensional calculation were selected to be 10, 15, 25, 40 and 60 minutes. The natural period of Ofunato Bay is approximately 40 minutes and is nearly equal to the estimated period of Chilean Earthquake Tsunami in 1960, while the period of tsunami in 1896 or 1933 is considered to be 10 minutes or so.

The values required for one-dimensional computation were taken as $B/L = 3$, $B/b = 5$, $T_0/T = 0.75$ and $T = 20$ minutes (natural period as a closed basin). Two curves in Figure 15 are the computed results, from which the general response characteristics of the bay is clearly obtained. In the region of horizontal or nearly horizontal parts, the highest water level occurs during the transient state.

The amplitude of the incident tsunami for calculation was so determined that the computed highest water level elevation for 40 minutes tsunami might coincide with that actually caused by Chilean Earthquake Tsunami, namely, about 6 m at the innermost part of the bay. In case of one-dimensional calculation, the incident tsunami amplitude of 1 m corresponds to the highest water level of 6 m. For two-dimensional calculation, however, the incident amplitude was determined to be 0.5 m taking into account the gradual contraction around the bay-mouth.

Figure 16 is, as an example, the distribution of the highest water level during four cycles of 40 minutes tsunami, before and after the construction of the breakwater. In Figure 15 are also plotted the results of two-dimensional calculation. It is clear that the highest water level elevation is much reduced by the breakwater for all the probable period of tsunami. The distribution of current velocity is also obtained and is generally reduced in the bay except at the breakwater gap, where the maximum speed reaches 3 or 4 m/sec. Although the period, amplitude or wave profile of the incident tsunami cannot be predicted exactly, the investigation shows that the breakwater under construction is expected to serve as an efficient protective work for future disasters.

REFERENCES

Design Section, Port and Harbour Bureau (1962): Calculations on the effect of breakwaters against long-period waves; Ministry of Transport, Japan

Fig. 1. Computing points.

Fig. 2. Layout of Tsunami Breakwater in Ofunato Bay.
Fig. 3. Distribution of computing points.
Fig. 5. Reflected wave and offshore boundary.

Fig. 6. Case-A (without head loss).
Case-C (with head loss).

Fig. 7. Case-B.

Fig. 8. Case-D (without breakwater).
Case-E (with breakwater).

Fig. 9. Case-G.
Fig. 10. Transmission and reflection at bay-mouth.

Fig. 12. Transmission coefficient at breakwater gap.
EFFECT OF BREAKWATER

One-Dimensional Comp.
Two-Dimensional Comp.

Fig. 11. Water level variation in the bay.
Fig. 13. Stationary wave profile.

Fig. 14. Amplitude correction factor.
Fig. 15. Response of Ofunato Bay.

Fig. 16. Highest water level for 40 min. Tsunami.

(a) Without Breakwater  
(b) With Breakwater