CHAPTER 43

THE INCREASE OF BED SHEAR IN A CURRENT DUE TO WAVE MOTION

by
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1 INTRODUCTION

As early as 1948 Einstein [1] suggested that the approach to the calculation of the sand transportation by waves could be similar to that for uniform flow. Tests conducted by author proved that for a current, with waves being propagated in a direction perpendicular or almost perpendicular to this current, the sand transportation is a function of the intensity of bed shear in the direction of the current.

Therefore, an attempt has been made to study the increase of the bed shear of a current due to wave motion. The results of this study are presented in this paper.

This problem has also been studied by Jonsson and Lundgren [2],[3] but they have assumed that current and wave-propagation occurs in the same direction. As for normal beach conditions, the angle between current and wave crests is between 0° and 20°, tests have been executed in the Delft Hydraulics Laboratory for angles of 0° and 15°. For an angle of 0° between the wave crests and the direction of the current, results have been presented by Bijker in the proceedings of the seminars at the I.A.H.R. conference which was held in Leningrad in 1965.

Because it is not feasible to measure the bed shear directly an indirect method had to be chosen. Determination of the bed shear by means of the velocity profile in the vicinity of the bed is not feasible in this case as the combined velocity profile is of a rather complicated nature. The bed shear is therefore determined by means of the energy slope.

2 EQUIPMENT USED FOR THE TESTS AND ACCURACY OF THE MEASUREMENTS.

The tests were executed in a basin which was 27 m long and 17 m wide (diagram 1). On one of the longer sides a wave generator was installed, and on the opposite side a bank with a slope of 1:7 was constructed in order to dissipate the energy of the waves. The reflection was reduced to an acceptable degree.

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The wave heights were measured by means of a resistance wave height-meter.

A maximum discharge of 0.7 m^3/sec could be adjusted with a degree of accuracy of 3% by an automatically governed inlet sluice. This discharge was distributed evenly over that part of the model which has a constant depth by means of an overflow weir and a grid.

As the flow was practically uniform, the energy level could be determined by measuring the slope of the waterlevel. This was done by measuring the differences of the waterlevel at two points at a distance of 10 m along the centre line of the model. The waterlevels were recorded by means of floats placed in drums next to the model. The drums were connected by means of a pipe to measuring points at the bottom of the model. Special precautions were taken in order to guarantee that the waterlevel was recorded without any velocity effect. By means of potentiometers attached to the floats the difference in waterlevel at the two points was recorded with an accuracy of 0.05 mm.

The accuracy of the determination of the shear from the slope of the waterlevel is limited due to the fact that the slope is calculated from a very small difference of two piezometric heights which can be measured only with limited accuracy.
In order to see what results can be obtained the roughness values will be analysed. Variations in the roughness of the sand bed may be contributed not only to inaccuracy of the measuring method but also to changes in the ripple height and form. Therefore only the k-values for the bed covered with stones will be considered in this respect. The different values for k as calculated from the tests range from 2 to 6.7 10^{-2} m. The mean value is 3.7 10^{-2} m, whereas the standard deviation is 1.8 10^{-2} m, which is about 50% of the actual value.

From the inaccuracy of the single records of the piezometric heights it can be judged whether this inaccuracy is acceptable. The inaccuracy of a single reading of the piezometric height is 0.05 10^{-3} m. Therefore the inaccuracy in the difference from which the slope is calculated is 2 0.05 mm = 0.07 mm. The difference in waterlevel is in the order of magnitude of 1.4 mm, consequently the inaccuracy of this difference is about 5%. From this follows for the inaccuracy for C, about 6% when the inaccuracy of the velocity is estimated at 3%.

For the calculation of the bed-roughness by means of the resistance coefficient C, the logarithmic formula ($C = 18 \log 12\, h{:}k$) has been used. For the estimation of the inaccuracy of k the Manning-Strickler formula can also be used. From this formula ($C = 25 \left(\frac{h}{k}\right)^{1/6}$) follows that the inaccuracy of k will be 6 times that of C, that is about 40%. This is of the same order of magnitude as the standard deviation which is found from the tests, so that there are at any rate no hidden sources of errors in the tests.

It is regrettable that the accuracy of the test results is so low. Compilation of the test results will show a clear tendency which is sufficient as a base for the scale laws.

3 CALCULATION OF THE RESULTANT SHEARSTRESS

The bed shear of the current together with wave motion can be obtained by calculating the gradient of the resultant velocity vector of the main current and the orbital motion.

According to Prandtl the intensity of the bed shear in a turbulent current may be written as

$$\tau = \varrho l^2 \left(\frac{\partial v_y}{\partial y}\right)^2_{bottom} \tag{1}$$

in which l = mixing length
v_y = velocity at height y above the bed
ϱ = density
τ = intensity of bed shear
y = distance from the bed.

According to the theory of Prandtl for a rough bed l is determined by the roughness of this bed and the distance to the bed so that;

$$l = \kappa y \text{ , for small values of } y \tag{2}$$

in which κ is a universal constant with value 0.4.

For a normal fully turbulent current the differential quotient of the velocity distribution (the velocity gradient) outside the laminar sublayer to the bottom can be written as

$$\frac{\partial v_y}{\partial y} = \frac{v_*}{\kappa y} \text{ ,} \tag{3}$$

where

$$v_* = \sqrt{\frac{\tau}{\varrho}} = \sqrt{ghI} = \frac{v\sqrt{g}}{C} \text{ ,} \tag{4}$$

in which v = shear velocity
h = waterdepth
I = slope of energy level
v = mean velocity
C = resistance coefficient of Chezy
g = acceleration of earth gravitation

Integration of equation 3 gives the vertical distribution of the velocity,

viz.

$$v_y = \frac{v_*}{\kappa} \ln \frac{33y}{k} \text{ ,} \tag{5}$$

in which k is a value for the bed roughness.

For the calculation of the bed shear $\left(\frac{\partial v_y}{\partial y}\right)_{bottom}$ should be known.

According to diagram 2 it will be assumed that

$$\left(\frac{\partial v_y}{\partial y}\right)_{bottom} = \frac{v_{y'}}{y'} = \frac{v_*}{\kappa y'} \tag{6}$$

So that in this case

$$v_{y'} = \frac{v_*}{\kappa} \tag{7}$$

After substituting this value in equation 5 for the vertical distribution of the velocity one finds

$$y' = \frac{ke}{33} \quad , \quad \text{in which} \tag{8}$$

e = base of the natural logarithm .

It is assumed that the thickness of the laminar sublayer is y'.

For the calculation of the bed shear of the combination of the main current and the orbital velocity above the bed the procedure described above will be followed. For the orbital velocity at the boundary of the viscous sublayer a value of $p\ u_b$ will be introduced as illustrated in diagram 2. In par. 4 the derivation of p will be given.

The orbital velocity at the bottom u_b is a function of the time according to the equation

$$u_b = u_o \sin \omega t \tag{9}$$

in which $\omega = \frac{2\pi}{T}$, where ω is the frequency and T is the period of the wave.

and
$$u_o = \frac{\omega H}{2 \sinh \frac{2\pi h}{L}} \tag{10}$$

in which H = wave height
L = wave length

In the case where the orbital velocity makes an angle of φ with the perpendicular to the main current the resultant velocity on the outside of the laminar boundary layer can be written as

$$v_r = \sqrt{v_{y'}^2 + p^2 u_b^2 + 2\ v\ p\ u_b \sin \varphi} \tag{11}$$

(vide diagram 3).

The angle α between the resultant instantaneous bed shear and the main current is in this case defined by

$$\cos \alpha_{(t)} = \frac{v_{y'} + p u_b \sin \varphi}{\sqrt{v_{y'}^2 + p^2 u_b^2 + 2\ v_{y'}\ p u_b \sin \varphi}} \tag{12}$$

The component of the resultant bed shear in the direction of the main current is in this case, using equation 1 and 6, given by

$$\tau'_{(t)} = \frac{v_{y'} + p\,u_b \sin\varphi}{\sqrt{v_{y'}^2 + p^2u_b^2 + 2\,v_{y'}\,p\,u_b \sin\varphi}}\,\varrho l^2 \cdot \frac{v_{y'}^2 + p^2u_b^2 + 2\,v_{y'}\,p\,u_b \sin\varphi}{y'^2} \tag{13}$$

With $l = \kappa y'$ this can be written as

$$\tau'_{(t)} = \varrho v_*^2 \,(1 + \xi \frac{u_o}{v} \sin\omega t \sin\varphi) \cdot \sqrt{1 + \xi^2 \frac{u_o^2}{v^2} \sin^2\omega t + 2\,\xi \frac{u_o}{v} \sin\omega t \sin\varphi} \tag{14}$$

So that

$$\frac{\tau'_{(t)}}{\tau_c} = (1 + \xi \frac{u_o}{v} \sin\omega t.\ \sin\varphi) \cdot \sqrt{1 + \xi^2 \frac{u_o^2}{v^2} \sin^2\omega t + 2\,\xi \frac{u_o}{v} \sin\omega t \sin\varphi} \tag{15}$$

in which $\xi = \frac{p\kappa C}{\sqrt{g}}$.
The mean value can again be obtained via integration. The integration should be done over half the period as the resultant shear-stress is not symmetrical.

$$\frac{\tau'_{(t)}}{\tau_c} = \frac{2}{T}\int_{-\frac{T}{4}}^{+\frac{T}{4}} \left[(1 + \xi \frac{u_o}{v} \sin\omega t \sin\varphi) \cdot \sqrt{1 + \xi^2 \frac{u_o^2}{v^2} \sin^2\omega t + 2\,\xi \frac{u_o}{v} \sin\omega t.\sin\varphi} \right] dt \tag{16}$$

This integral is of the elliptic type and has been computed numerically. For $\varphi = 0^o$ the result is shown in diagram 4 and for $\varphi = 15^o$ in diagram 5. All results can be written in the form

$$(\frac{\tau'}{\tau_c} - 1) = N(\xi \frac{u_o}{v})^{1.5}, \tag{17}$$

in which N is a function of φ.

The value of N is calculated by a computer and can be written with sufficient accuracy as

$$N = 0.36 - 0.14 \cos 2\varphi \tag{18}$$

The component of the bed shear perpendicular to the main current can be written as

$$\tau''_{(t)} = \frac{\varrho v_*^2 \, \xi \, \frac{u_o}{v} \sin \omega t \cos \varphi}{\sqrt{1 + \xi \, \frac{2 \, u_o^2}{v^2} \sin^2 \omega t + 2 \, \xi \, \frac{u_o}{v} \sin \omega t \sin \varphi}} \qquad (19)$$

So that

$$\frac{\tau''}{\tau_c} = \frac{2}{T} \int_{-\frac{T}{4}}^{+\frac{T}{4}} \left[\frac{\xi \, \frac{u_o}{v} \sin \omega t \cos \varphi \, .}{\sqrt{1 + \xi^2 \, \frac{u_o^2 \sin^2 \omega t}{v^2} + 2 \, \xi \, \frac{u_o}{v} \sin \omega t \sin \varphi}} \right] dt \qquad (20)$$

For $\varphi = 15^o$ the result is shown on diagram 6. For other angles similar lines are obtained.

In analogy with the case for the component parallel to the main current the results can be written in the form

$$\frac{\tau''}{\tau_c} = M \, (\xi \, \frac{u_o}{v})^{1.25} \qquad (21)$$

For small values of φ the agreement is quite good, but for large values of φ the deviation between equation (21) and the exact values calculated by means of equation (20) are larger.

M can be approximated by

$$M = 0.205 \sin 2 \, \varphi \qquad (22)$$

The mean value of the ratio between the total bed shear and the bed shear due to current only can be written according to the same derivation as

$$\frac{\tau_r}{\tau_c} = \frac{2}{T} \int_{-\frac{T}{4}}^{+\frac{T}{4}} (1 + \xi^2 \, \frac{u_o^2}{v^2} \sin^2 \omega t + 2 \, \xi \, \frac{u_o}{v} \sin \omega t \sin \varphi) \, dt = (1 + \frac{1}{2} \, \xi^2 \, \frac{u_o^2}{v^2}) \qquad (23)$$

4 DISCUSSION OF THE VALUE p

In order to determine which values p is a function of the boundary layer near the bottom due to the orbital motion will be considered.

According to Lamb (art. 328) the motion near the bed can be described by

$$\frac{\partial u}{\partial t} = X - \frac{1}{\varrho}\frac{\partial \tau}{\partial y} , \tag{24}$$

τ is in this case the bed shear at a distance y from the bottom as acting from the upper on the lower layer and u is the velocity at a distance y from the bed , y being the ordinate drawn vertically upwards from the bed

$$X = f \cos \omega t , \quad \text{and} \tag{25}$$

$$u_b = \frac{f}{\omega} \sin \omega t = u_o \sin \omega t , \quad \text{so that} \tag{26}$$

$$f = \omega\, u_o \tag{27}$$

From this follows that

$$X = f \cos \omega t = \omega\, u_o \cos \omega t = \frac{\partial u_b}{\partial t} , \tag{28}$$

where $u_b = u_o \sin \omega t$ (9)

So that equation 24 can be written in the form

$$\frac{\partial (u_b - u)}{\partial t} = \frac{1}{\varrho} \frac{\partial \tau}{\partial y} \tag{29}$$

In order to be able to solve this equation either τ or u should be known as f(y).
For the entire viscous case,

$$\tau = \varrho \nu \frac{\partial u}{\partial y} , \tag{30}$$

in which ν = kinematic viscosity coefficient.
Equation (24) or (29) can in this case be written in the form

$$\frac{\partial (u_b - u)}{\partial t} = \nu \frac{\partial^2 u}{\partial y^2} \tag{31}$$

From this follows for the velocity distribution near the bed

$$u_b - u = u_o\, e^{-\beta y} \sin (\omega t - \beta y) \tag{32}$$

(Lamb art. 347) [4].

in which $\beta = \sqrt{\frac{\omega}{2\nu}}$, and (33)

u_o is the amplitude of the orbital motion near the bottom as calculated from equation

$$u_o = \frac{\omega H}{2 \sinh \frac{2\pi h}{T}} \tag{10}$$

The order of magnitude of ω for the tests was 6 sec^{-1} and $\nu = 10^{-6}$ m^2/sec.

Therefore β is in order of magnitude 2.10^3. For $\beta y = 3$, u will be almost (95%) equal to u_b.

From this follows that the thickness of the viscous sublayer will be in order of magnitude of some millimeters.

For almost all tests this is much smaller than the bed roughness. It is therefore reasonable to assume a turbulent boundary layer from the bed to the normal orbital velocity.

In anology with the fully developed turbulent boundary layer the normal formula will be applied for the bed shear

$$\tau = \varrho\, l^2 \left(\frac{\partial u}{\partial y}\right)^2 , \tag{1}$$

Outside the laminar boundary layer

$$l = \kappa\, y \tag{2}$$

So

$$\tau = \varrho\, \kappa^2\, y^2 \left(\frac{\partial u}{\partial y}\right)^2 \tag{34}$$

Equation 38 can be written as

$$\frac{\partial (u_b - u)}{\partial t} = \kappa^2 \left[\frac{\partial}{\partial y}\, y^2 \left(\frac{\partial u}{\partial y}\right)^2\right] \tag{35}$$

In analogy with the viscous case the following velocity distribution in the boundary layer close to the bed will be assumed.

$$u_b - u = u_o\, e^{-Y} \sin\,(\omega t - Y) \tag{36}$$

where Y is an unknown function of y. This function will be determined from equation (35).

$$\frac{\partial (u_b - u)}{\partial t} = \omega\, u_o\, e^{-Y} \cos\,(\omega t - Y) \tag{37}$$

$$\frac{\partial}{\partial y}\left[y^2 \left(\frac{\partial u}{\partial y}\right)^2\right] = 2\, y \left(\frac{\partial u}{\partial y}\right)^2 + 2\, y^2\, \frac{\partial u}{\partial y}\, \frac{\partial^2 u}{\partial y^2} \tag{38}$$

$$\frac{\partial u}{\partial y} = u_o\, e^{-Y}\, Y'\, \sqrt{2}\, .\, \sin\,(\omega t - Y + \frac{\pi}{4}) \tag{39}$$

$$\frac{\partial^2 u}{\partial y^2} = u_o\, e^{-Y}\left[Y''\, \sqrt{2}\, \sin\,(\omega t - Y + \frac{\pi}{4}) - 2\, Y'^2 \cos\,(\omega t - Y)\right] \tag{40}$$

$$\frac{\partial u}{\partial y}\, \frac{\partial^2 u}{\partial y^2} = u_o^{\,2}\left[e^{-2Y}\quad 2Y'Y''\, \sin^2\,(\omega t - Y + \frac{\pi}{4}) - 2\quad 2Y'^3\, .\right.$$
$$\left.\sin\,(\omega t - Y + \frac{\pi}{4})\, .\, \cos\,(\omega t - Y)\right] \tag{41}$$

Equation (35) now becomes

$$\omega\, u_o\, e^{-Y} \cos\,(\omega t - Y) = 4\kappa^2\, y\, u_o^{\,2} e^{-2Y}\, Y'^2 \sin^2\,(\omega t - Y + \tfrac{\pi}{4})$$

$$+ 2\,\kappa^2\, y^2\, u_o^{\,2}\, e^{-2Y} \Big[2Y'Y'' \sin^2\,(\omega t - Y + \tfrac{\pi}{4})$$

$$- 2\sqrt{2}\, Y'^3 \sin\,(\omega t - Y + \tfrac{\pi}{4}).\ \cos\,(\omega t - Y)\Big] \qquad (42)$$

Or

$$\omega \cos\,(\omega t - Y) = 4\kappa^2\, y\, u_o\, e^{-Y}\, Y'^2 \sin^2\,(\omega t - Y + \tfrac{\pi}{4})$$

$$+ 2\kappa^2\, y^2\, u_o\, e^{-Y} \Big[2Y'Y'' \sin^2\,(\omega t - Y + \tfrac{\pi}{4})$$

$$- \sqrt{2}\, Y'^3 \sin\,(2\,\omega t - 2Y + \tfrac{\pi}{4}) + \tfrac{1}{2}\sqrt{2})\Big] \qquad (43)$$

In order to obtain a relationship independent of t the mean values over a full period T will be taken.
In this case all terms with sin and cos can be omitted, and the mean value of $\sin^2\,(\omega t - Y + \pi/4)$ equals $\frac{1}{2}$.
So

$$2\,\kappa^2\, y\, u_o\, e^{-Y}\, Y'^2 + 2\,\kappa^2\, y^2\, u_o\, e^{-Y} \Big[Y'Y'' - Y'^3\Big] = 0 \qquad (44)$$

or
$$Y'' = Y'^2 - \frac{Y'}{y} \qquad (45)$$

The solution of this equation is

$$Y = -\ln\ln Ay + B \qquad (46)$$

or
$$Y = -\ln\ln\frac{A}{y} + B \qquad (47)$$

The equation for the velocity distribution now becomes

$$u = u_o\Big[\sin\omega t - e^{-B}\ln\frac{A}{y}\,(\sin\omega t + \ln\ln\frac{A}{y} - B)\Big] \qquad (48)$$

This result shows a logarithmic profile as assumed by Jonsson and Lundgren [2] [3] but differs in details.
As boundary conditions will be taken

$$u = u_o \sin\omega t \quad \text{at} \quad y = \delta\ ,$$

where δ is the thickness of the boundary layer, and $u = 0$ at $y = \frac{k}{33}$ in accordance to the velocity distribution of the main current. (vide diagram 2).

From this follows

$$A = \delta \quad \text{and} \quad B = \ln \ln \frac{33\delta}{k}$$

The equation for this case becomes

$$u = u_o \left[\sin \omega t - \frac{\ln \frac{\delta}{y}}{\ln \frac{33\delta}{k}} \cdot \sin\left(\omega t + \ln \ln \frac{\delta}{y} - \ln \ln \frac{33\delta}{k}\right) \right] \qquad (49)$$

and $$\frac{\partial u}{\partial y} = \frac{u_o}{y} \frac{\sqrt{2}}{\ln \frac{33\delta}{k}} \cdot \sin\left(\omega t + \ln \ln \frac{\delta}{y} - \ln \ln \frac{33\delta}{k} + \frac{\pi}{4}\right) \qquad (50)$$

According to equations 1 and 6 the bed shear at the bottom due to the orbital motion can be written as

$$\tau = \varrho l^2 \frac{p^2 u_o^2 \sin^2 \omega t}{(\frac{ke}{33})^2}, \qquad (51)$$

so that for $\frac{\partial u}{\partial y}$ is written $$\frac{p\, u_o \sin \omega t}{\frac{ke}{33}} \qquad (52)$$

According to equation 50

$\frac{\partial u}{\partial y}$ at $\frac{ke}{33}$ above the bottom is:

$$\frac{\partial u}{\partial y} = \frac{u_o}{\frac{ke}{33}} \cdot \frac{\sqrt{2}}{\ln \frac{33\delta}{k}} \cdot \sin\left(\omega t + \ln \ln \frac{\delta}{\frac{ke}{33}} - \ln \ln \frac{33\delta}{k} + \frac{\pi}{4}\right) \qquad (53)$$

Comparing the moduli we find that

$$p = \frac{\sqrt{2}}{\ln \frac{33\delta}{k}} \qquad (54)$$

When it is assumed that δ is proportional to k, which is the case when the boundary layer for the waves is determined by the main current, p is found to be constant.

From the measurements which will be discussed in par. 4, p is found to be 0.45, which results in a value for the boundary layer thickness of

$$\delta = \frac{k}{33} e^{2.2\sqrt{2}} = 22.4 \frac{k}{33} \qquad (55)$$

$$\delta \approx \frac{2}{3} k .$$

5 MEASUREMENTS

The measurements were executed for $\varphi = 0^o$ and $\varphi = 15^o$. In the case where $\varphi = 0^o$, viz. direction of current and wave propagation perpendicular to each other, firstly tests were executed with waves of 1.57 sec period. Two different bed conditions were used, viz. a bed covered with stones with a mean diameter of 3 to 4 cm and a sand bed covered with ripples of a few cm's height. Afterwards some tests were conducted with $\varphi = 0^o$ and a wave period of 0.68 sec. The bed in this case was a sand layer covered with ripples.

The tests with $\varphi = 15^o$ were conducted with two periods, viz. 0.68 and 2 sec. The sand bed was in this case covered with ripples too.

For the calculation of the bed shear with $\varphi = 15^o$ the influence of the stream refraction has to be taken into account. Firstly the angle φ is increased to about 16^o and the orbital velocity at the bottom will increase with about 10 to 25%. All data have been corrected for this effect.

Under the assumption that the theory developed in the preceeding paragraphs is valid the factor p has been calculated for the formula

$$\left(\frac{\tau'}{\tau_c} - 1\right) = N\left(\frac{C\,\kappa\,p\,u_o}{\sqrt{g}\,v}\right)^{1.5} \qquad (17)$$

τ' and τ_c have been determined from the measurements, and N is determined from equation 18 and the actual value of φ. C has been determined from the data with current only. The tests have been executed in such a way that the bottom configuration with current only, was equal to that with current and waves.

Equation 61 can also be written as

$$\frac{\frac{\tau'}{\tau_c} - 1}{N} = \left(p\,\frac{C\,\kappa u_o}{\sqrt{g}\,v}\right)^{3/2}, \qquad (61)$$

or

$$Y = p^{3/2} X \qquad (62)$$

With the assumption that Y and X are both stochastic variables with a normal distribution, p can be calculated. As Y has a smaller accuracy than X, the regression of Y on X will be the best value.
This is

$$p_1 = \left(\frac{\Sigma XY}{\Sigma XX}\right)^{2/3}. \qquad (63)$$

The other regression coefficient is

$$p_2 = \left(\frac{\Sigma YY}{\Sigma XY}\right)^{2/3} \qquad (64)$$

In order to determine whether the assumption which is the base of formula 61 is valid the correlation coefficient for the assumption that equation 62 represents a straight line is determined. The results of this calculation are summarized in the table on page 12. Apart from the data for series V, there is a very good correlation.

On diagram 7 all data are reproduced.

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Tests	n	correlation coefficient for linear regression	confidence	p_1	p_2
I Stones $\varphi = 0^o$ T = 1.57 sec	15	0.938	> 99.9 %	0.46	0.48
II Sand with ripples $\varphi = 0^o$ T = 1.57 sec	19	0.958	> 99.9 %	0.53	0.54
III Sand with ripples $\varphi = 0^o$ T = 0.68 sec	8	0.919	99.86%	0.43	0.46
IV Sand with ripples $\varphi = 15^o$ T = 0.68 sec	13	0.608	97.4 %	0.38	0.49
V Sand with ripples $\varphi = 15^o$ T = 2.0 sec	12	0.513	91 %	0.40	0.50
All data	67	0.821	> 99.9 %	0.45	0.51

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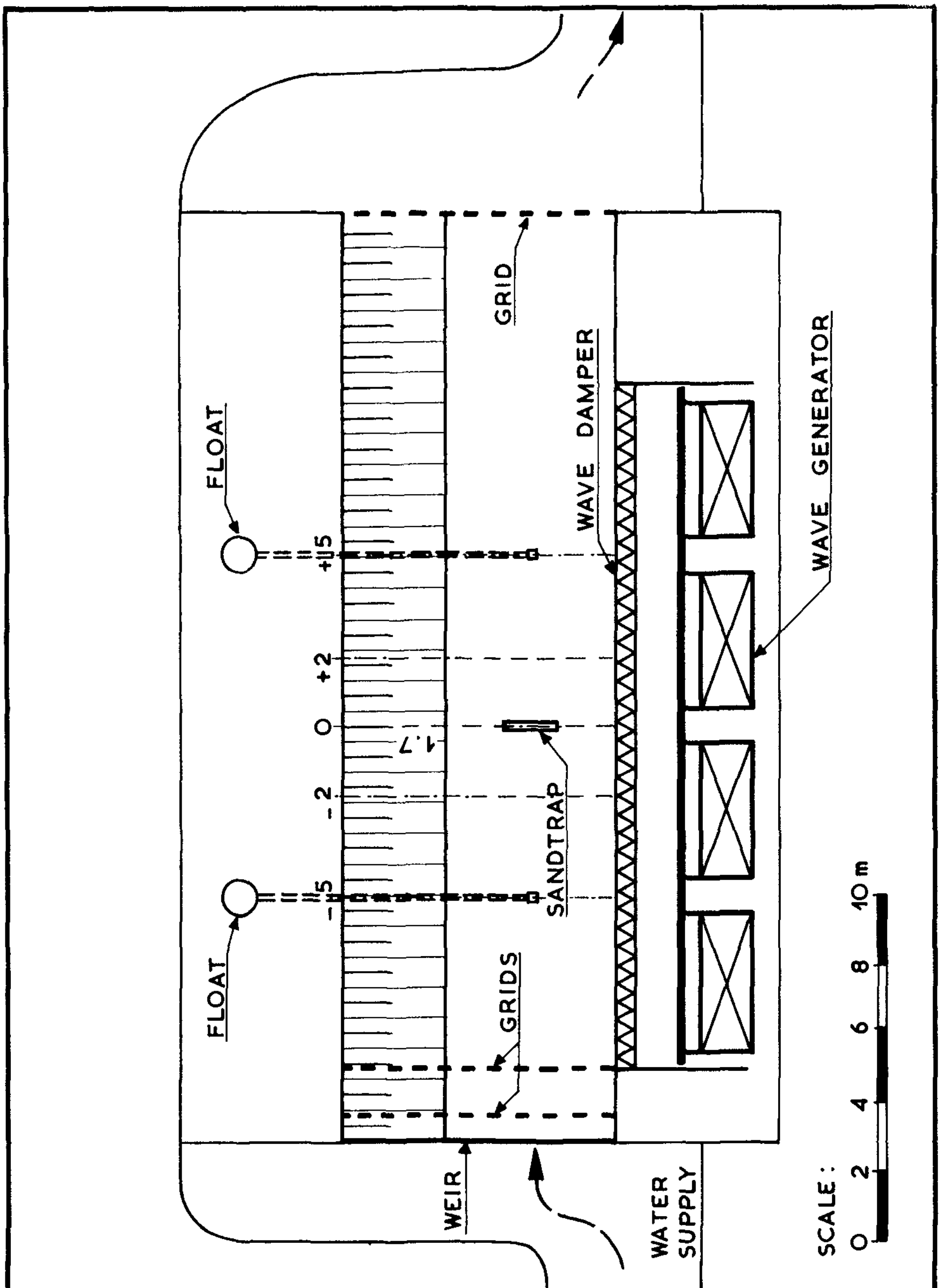
LIST OF SYMBOLS

C	resistance coefficient
H	wave height
L	wave length
N, M	coefficients
T	wave period
Y	function of y
Y', Y''	first and second derivatives of Y
e	base of the natural logarithme
g	gravitational acceleration
h	waterdepth
k	bed roughness
l	mixing length
p	coefficient
u	orbital velocity at a distance y above the bed
u_b	orbital velocity at a distance δ above the bed
u_o	amplitude of the orbital velocity at a distance δ from the bed
v	mean velocity of the main current
v_*	shear velocity of the main current
v_y	velocity at a distance y above the bed
v_r	resultant velocity due to main current and orbital velocity at a distance y' above the bed
y'	thickness of the laminar boundary layer above the bed
α	angle between the resultant instantaneous shearstress and the main current
β	coefficient
δ	thickness of turbulent boundary layer of the orbital motion above the bed
κ	constant of von Karman
ν	kinematic viscosity coefficient
ϱ	density of water
τ	bed shear
τ_r	resultant bed shear
τ'	resultant bed shear in the direction of the main current
τ''	resultant bed shear perpendicular to the main current
τ_c	bed shear only due to the main current
$\ldots(t)$	value at the time t
ξ	coefficient = $\frac{p \kappa C}{g}$
ω	wave frequency
φ	angle between wave crest and main current



LAYOUT OF MODEL BASIN

DIAGRAM 1

HYDRAULICS LABORATORY DELFT

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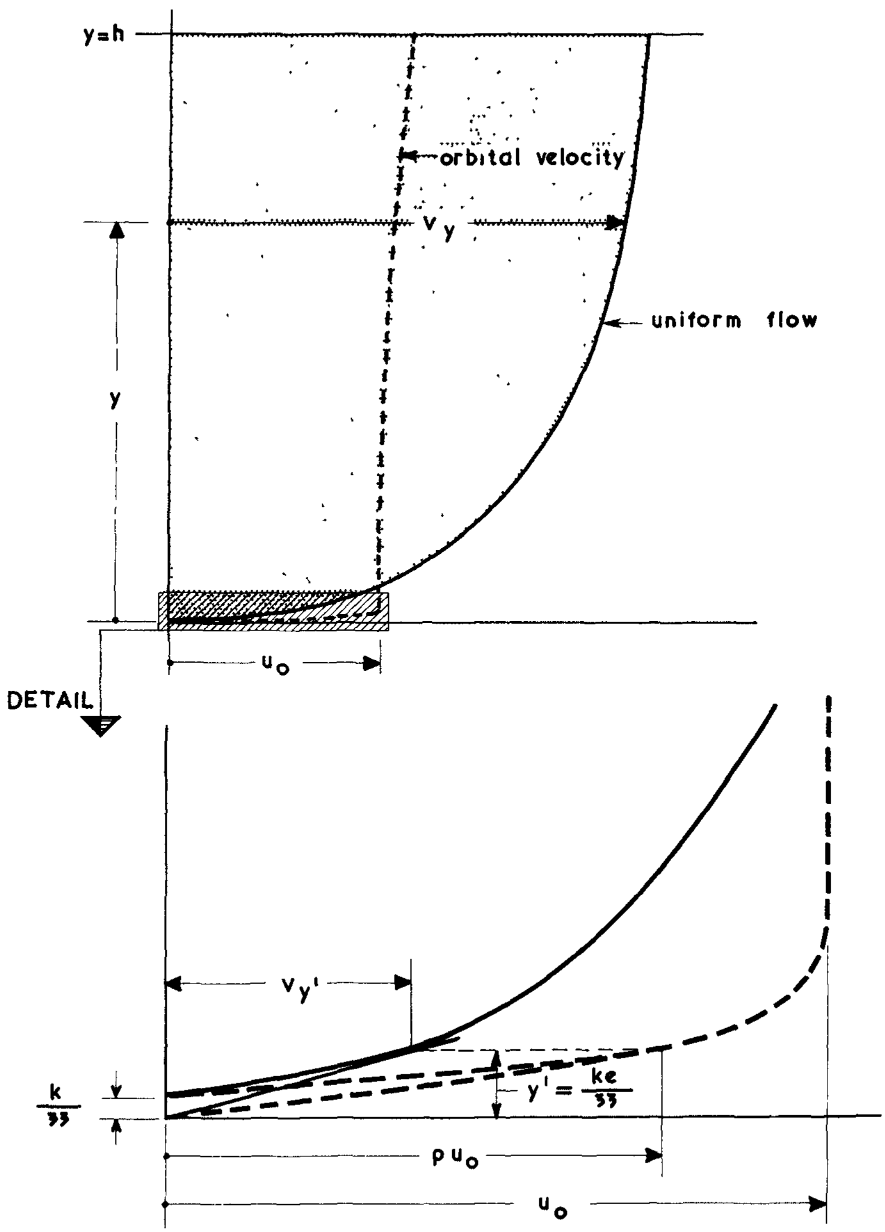
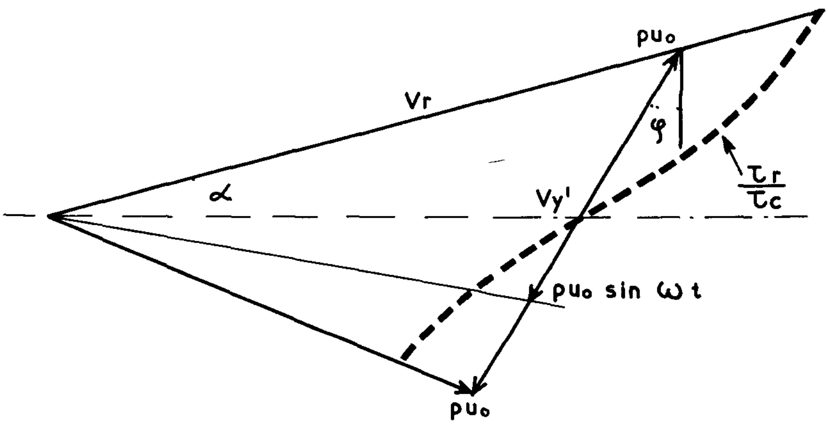
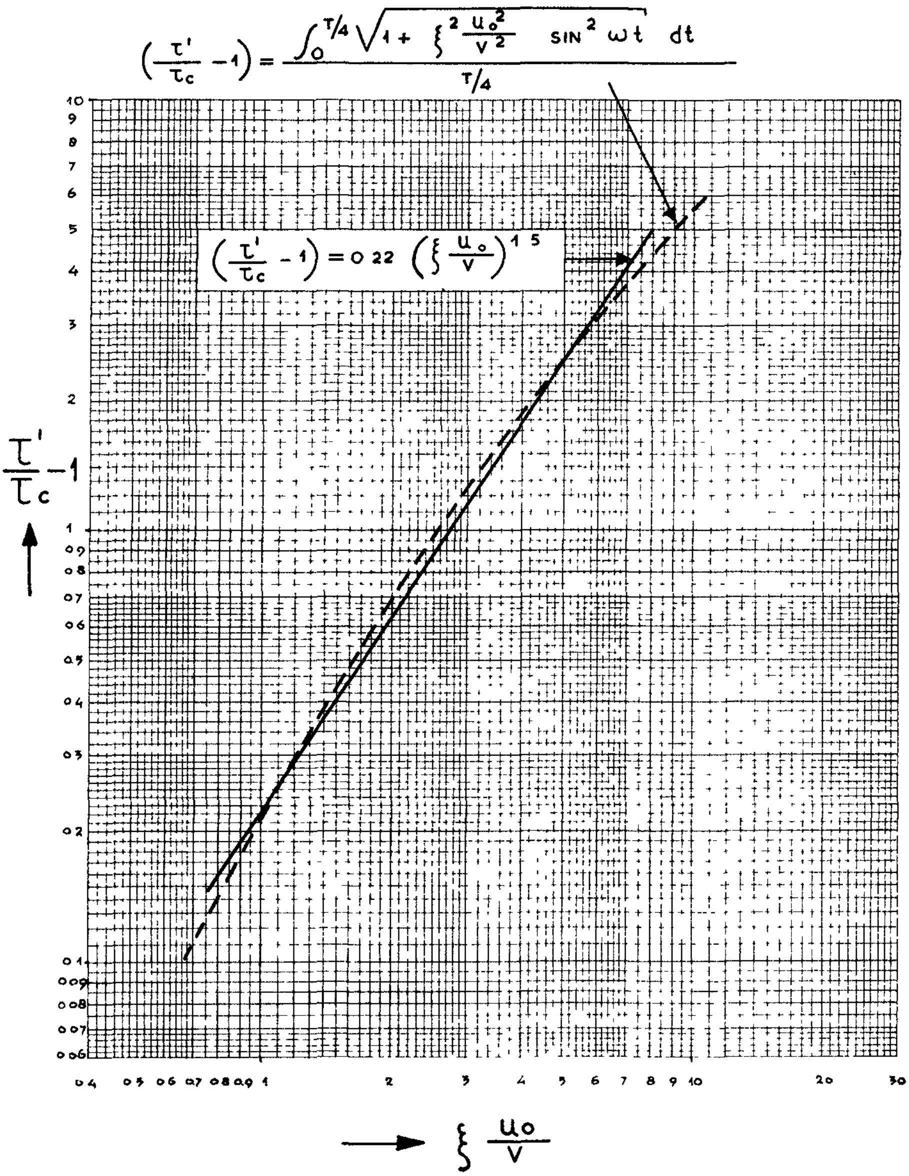


COMPARISON BETWEEN LOGARITHMIC VELOCITY DISTRIBUTION OF UNIFORM FLOW AND ORBITAL VELOCITY DISTRIBUTION OF WAVES	DIAGRAM 2	
HYDRAULICS LABORATORY DELFT	S. 55	

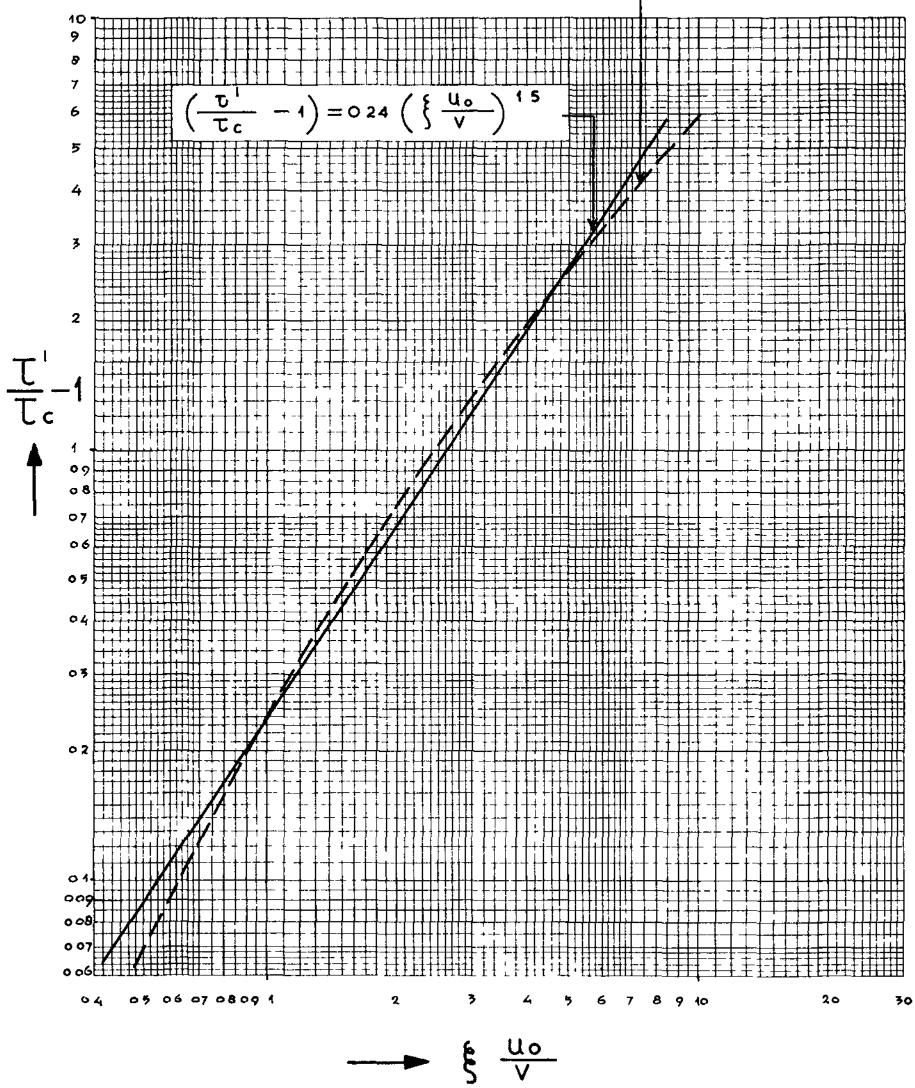


COMBINATION OF ORBITAL VELOCITY AND MAIN CURRENT		
	DIAGRAM 3	
HYDRAULICS LABORATORY DELFT	S 55	

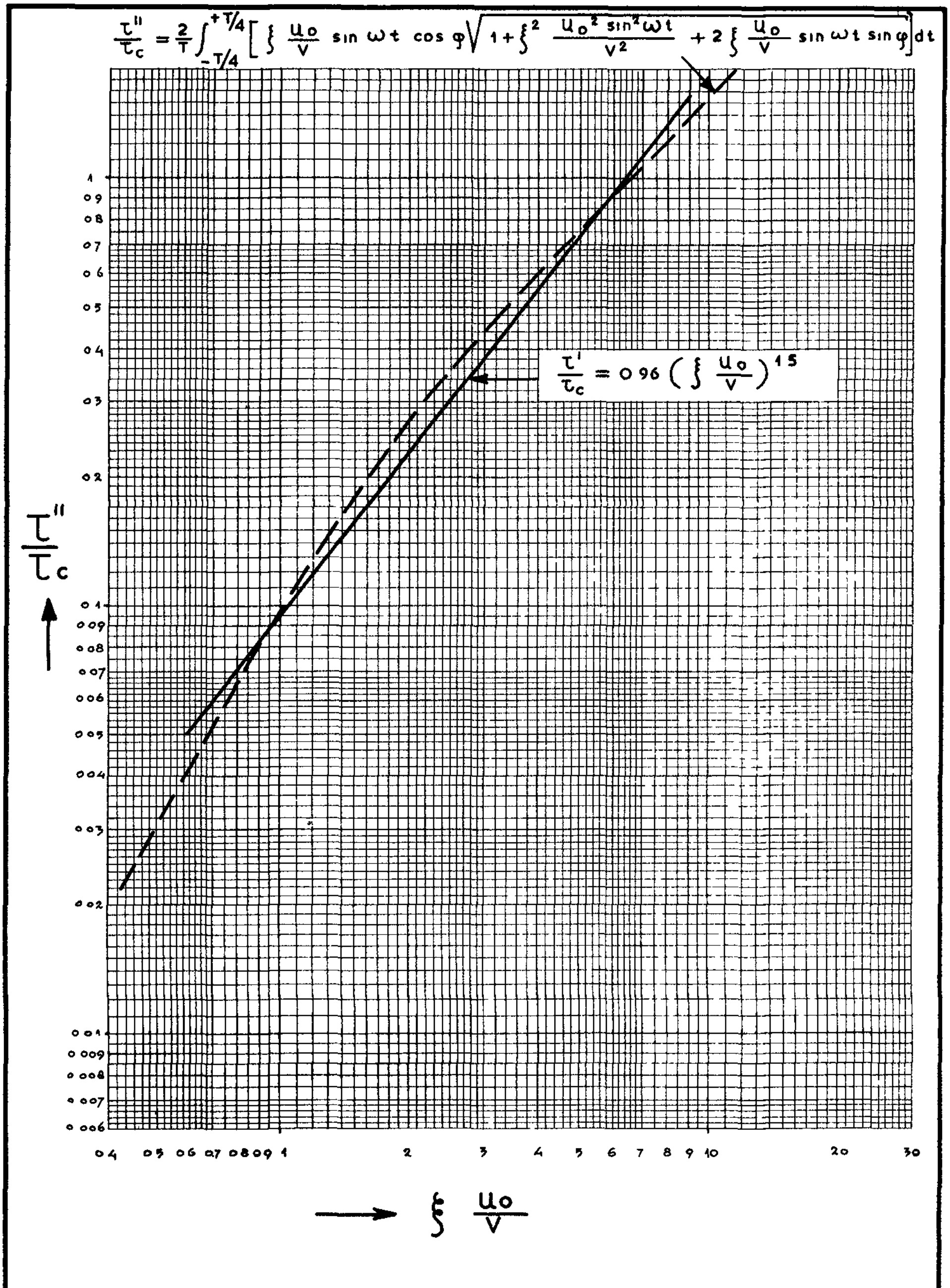


$\left(\frac{\tau'}{\tau_c}-1\right)$ VERSUS $\left(\xi\frac{u_o}{V}\right)$ FOR $\varphi = 0°$	DIAGRAM 4
HYDRAULICS LABORATORY DELFT	S. 55

$$\left(\frac{\tau'}{\tau_c}-1\right)=\frac{\int_{-T/4}^{+T/4}\left[\left(1+\xi\frac{u_o}{V}\sin\omega t\sin\varphi\right)\sqrt{1+\xi^2\frac{u_o^2}{V^2}\sin^2\omega t+2\xi\frac{u_o}{V}\sin\omega t\sin\varphi}\,\right]dt}{T/2}$$



$\left(\frac{\tau'}{\tau_c}-1\right)$ VERSUS $\left(\xi\frac{u_o}{V}\right)$ FOR $\varphi = 15^\circ$	DIAGRAM 5
HYDRAULICS LABORATORY DELFT	S. 55



$\left(\frac{\tau''}{\tau_c}\right)$ VERSUS $\left(\xi \frac{u_o}{V}\right)$ FOR $\varphi = 15°$

DIAGRAM 6

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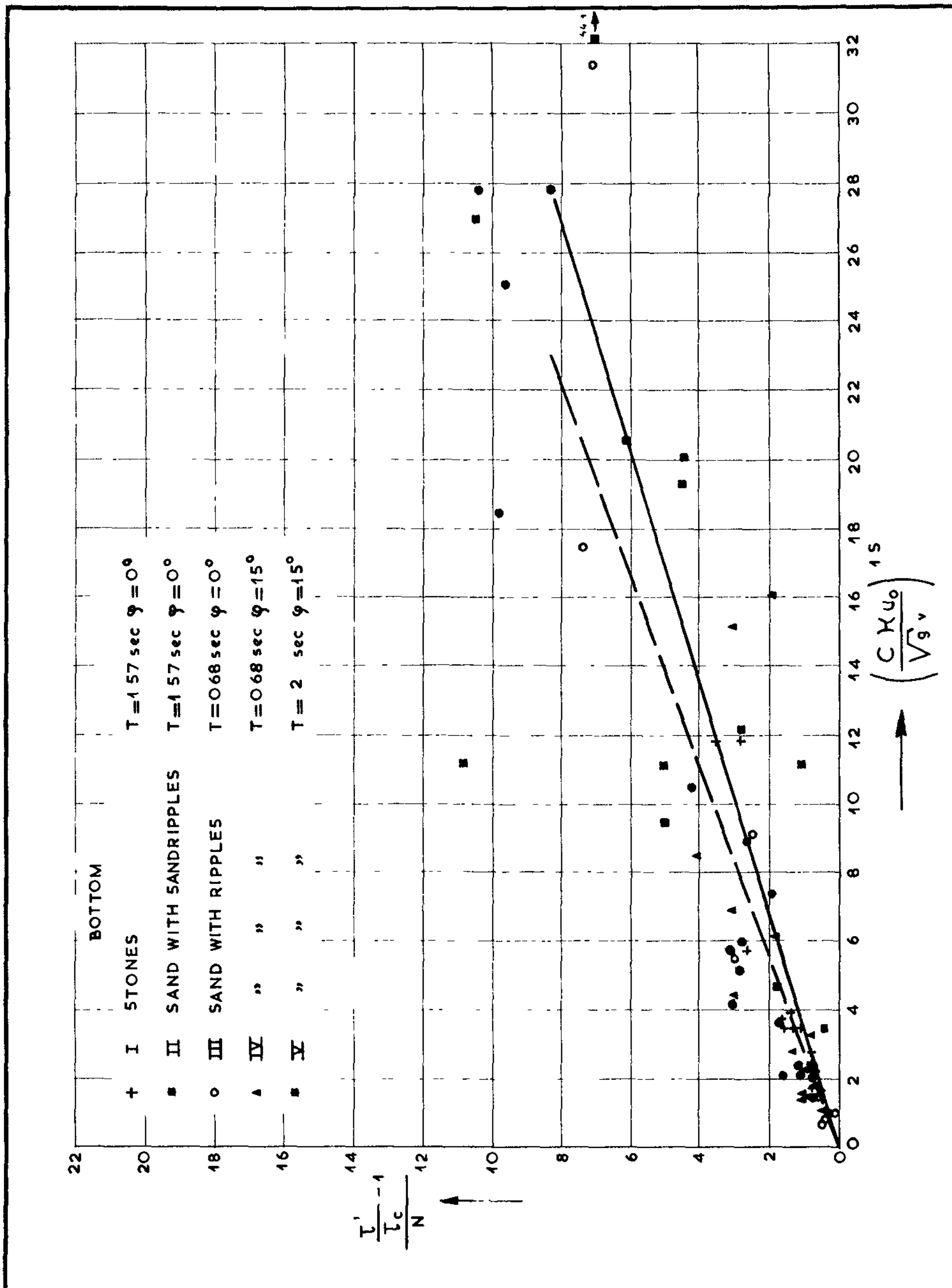

$\left(\dfrac{\frac{\tau'}{\tau_c}-1}{N}\right)$ VERSUS $\dfrac{C K u_0}{\sqrt{g}\, V}$	DIAGRAM 7	
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