CHAPTER 12

THE EFFECT OF ROUGHNESS ON THE MASS-TRANSPORT OF PROGRESSIVE GRAVITY WAVES

by

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INTRODUCTION

On the basis of non-viscous small amplitude first-order theory the maximum value of the horizontal orbital motion at the bed in water of constant depth h is given by

$$U_{max} = \frac{\pi H}{T \sinh kh}$$

where $k = 2\pi/L$, H is the wave height crest to trough, T is the period, and L the wave length ($L = gT^2/2\pi \tanh 2\pi h/L$).

On the basis of finite amplitude wave theory where the particle orbits are not closed and by the insertion of the viscous laminar boundary layer (the conduction solution) the mean drift velocity or mass transport velocity on a perfectly smooth bed is given by Longuet-Higgins (1952) as

$$U_B = \frac{K H^2 k\sigma}{\sinh^2 kh}$$

where $\sigma = 2\pi/T$ and K has a maximum value of 0.344 within the boundary layer and a value of 0.313 (i.e. 5/16) just outside the boundary layer. This mass transport current offers a mechanism whereby bed material outside the breaking zone may be transported.

The latter mass transport relationship has been verified experimentally and good agreement attained for laminar conditions and a limited amount of turbulence within the boundary layer. It appears, however, that as might be expected, a theory developed for essentially laminar conditions will not apply for increasing turbulence within the boundary layer. Accordingly the limiting condition of applicability may be defined by a limiting Reynolds Number, R_δ, of the form $R_\delta = U_{max}\delta/\nu$ where ν is the kinematic viscosity of the water and δ a boundary layer parameter given by $\sqrt{2\nu/\sigma}$ or $\sqrt{\nu T/\pi}$ (If the thickness of the boundary layer is δ_1, then $\delta_1 = 4{\cdot}6\,\delta$).

Previous work on a smooth boundary, Brebner and Collins (1961), has shown that up to a limiting R_δ of about

160 the value of U_B is as shown theoretically but beyond this value the variation of U_B with H is nolonger quadratic.

All the parameters involved in the theory and the defined Reynolds Number may be brought together in the form

$$U_B = \frac{5}{16} \frac{H^2 k\sigma}{\sinh^2 kh} = \frac{5\pi^2}{4L} \left\{ \frac{H}{\sqrt{T} \sinh kh} \right\}^2$$

Now $$R_\delta = \frac{U_{max}\,\delta}{\nu} = \sqrt{\frac{\pi}{\nu}} \left\{ \frac{H}{\sqrt{T} \sinh kh} \right\}$$

Thus $$U_B L = \frac{5\pi\nu}{4} \left\{ R_\delta \right\}^2 \quad \text{i.e} \quad U_B L \propto \left\{ R_\delta \right\}^2$$

The results of tests carried out in a 150 ft. long wave-flume with periods varying from 0 to 2.5 secs., depths from 0.5 to 3.0 ft., and wave heights from 0.1 to 0.5 ft. approx. are shown in Figure 1. This figure shows, as has been reported previously using a different experimental apparatus, that at a value of R_δ of about 160, the boundary layer on a smooth bed becomes quite turbulent, and the turbulence decreases the theoretical mass transport velocity based on a laminar boundary layer. (Distortion of dye into turbulent streaks or plumes commences about $R_\delta = 120$).

On a perfectly smooth flat bed the degree of turbulence required for the transition is developed from the instability of the velocity profile within the boundary layer. However, perfectly smooth beds seldom exist so that the effect of roughness elements upon the transition assumes some importance.

MASS TRANSPORT ON A ROUGH BED.

For uniform steady flow conditions it is traditional to characterise roughness by the relative roughness, ε/δ_1, where ε is the size of roughness element and δ_1 the boundary layer thickness. The possibility of using the concept of hydraulically smooth and rough for oscillatory flows depending on the value of ε/δ_1 has been used by Li (1954) and Vincent (1957). On such a basis it can be postulated that if δ_1/ε is greater than a certain value S_1, then the boundary layer is hydraulically smooth and the Longuet-Higgins theory should hold up to a limiting value of R_δ using $U_{max}\delta/\nu$ as the Reynolds Number. On the other hand if δ_1/ε is less than another value S_2 $(S_2 < S_1)$ then the bed is hydraulically rough and the mass transport might be controlled by roughness and the transition from laminar to turbulent controlled by a Reynolds Number of the form $U_{max}\, f(\varepsilon_1, \delta)/\nu$. Between S_1 and S_2 might be a no-man's transition zone.

Above a value of R_δ of 160 extensive turbulence is

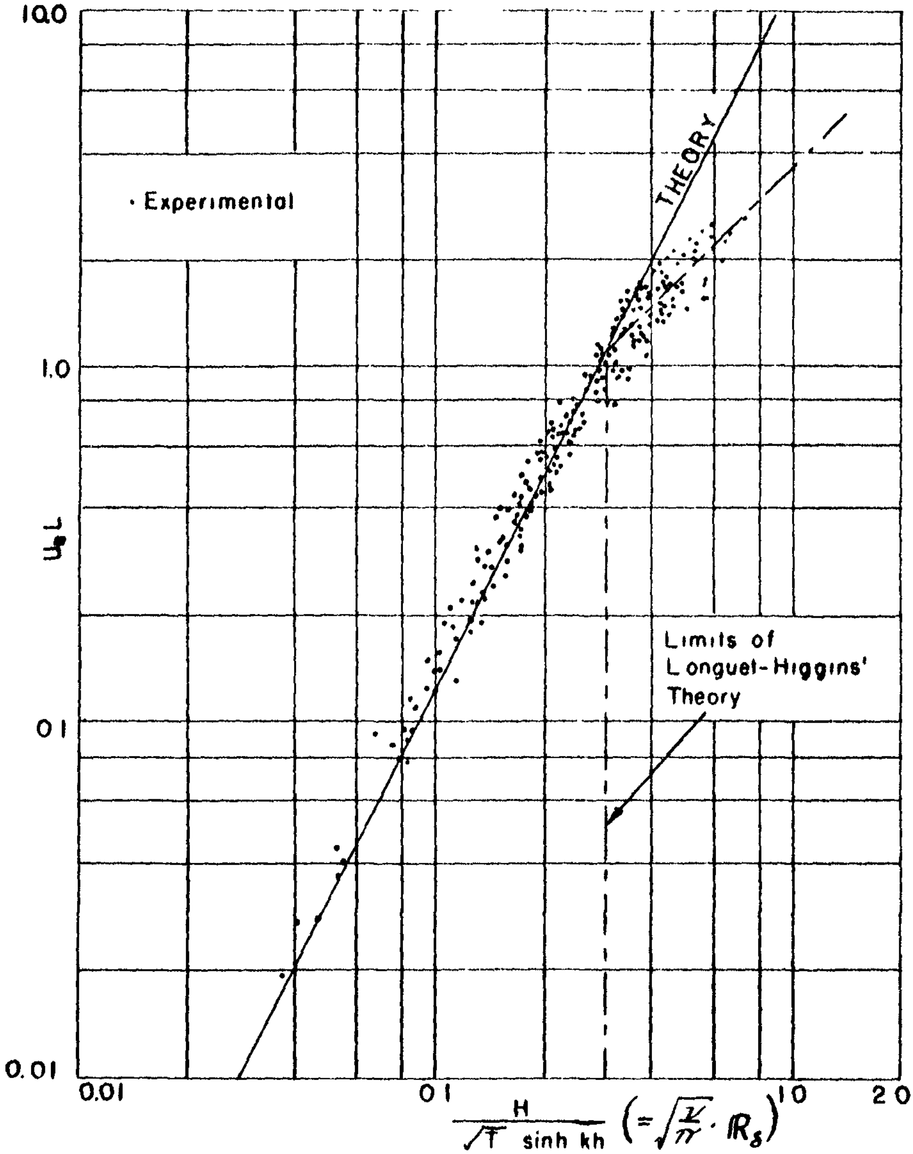

Fig. 1. U_BL vs $H/\sqrt{T}$ sinh kh for the smooth bed.

probably present in the boundary layer in both rough and smooth beds. Admittedly the prime cause of turbulence may differ for differing boundary roughnesses but it would seem logical to assume that the resulting values of U_B at $R_s > 160$ would be similar for all roughnesses. The postulated behaviour of U_B with varying values of ϵ and H for constant values of T and h is shown on Figure 2A based on the foregoing argument. However, the variation of U_B with H for a constant T and h could equally well have the form shown in Figure 2B, based on the use of the parameter $f(\epsilon, \delta_1)$ to define the transition on a rough bed.

In the following section, discussions are classified into A and B corresponding to the two postulations as suggested above.

EXPERIMENTAL RESULTS AND CONCLUSIONS

The experimental roughness used to establish the relationship between U_B and the other wave parameters consisted of attaching sand with varnish to aluminum sheets on the bed in a manner analogous to the Nikuradse pipe roughness. Mass transport velocities were measured using fluorescent tracers and neutral density beads. Six sand roughnesses were used, with a mean diameter ranging from 0.00165 ft. to 0.00717 ft.

A typical variation of U_B with H for a given value of T and h is shown in Figure 3, exhibiting the behaviour pattern suggested by either Figure 2A or 2B. A complete account of the experimental study for a typical value of T and h is shown in Figure 4.

A. The turbulent portion of Figure 3 shows a relationship for all bed roughnesses (including smooth) of $U_B \propto H^{1.2}$ whereas the laminar portion exhibits the theoretical relationship. For a smooth bed the Longuet-Higgins value of 5/16 (or .313) is reasonably correct as has been demonstrated also in Figure 1, whereas for even a very slightly roughened bed (i.e. sand of mean diameter 2.6×10^{-3} ft.) the value is approximately 0.45 showing that the mass transport for identical wave parameters is higher than in the smooth bed case in a similar laminar range. Apparently "hydraulically smooth" is not the same as "physically smooth" in this case. For the coarsest sand, mean diameter 7.2×10^{-3}ft., no laminar region was found and the mass transport was considerably greater than in the laminar case of a smooth boundary for identical wave conditions.

From Figure 3 it is evident that U_B is a function of T, h, H and ν (wave properties) for a smooth boundary with the additional parameter ϵ (boundary property) for roughened boundaries. Assuming that the function is linear, depending only on ϵ or ϵ/δ_1, a parallel pattern as shown in Figure 2A is drawn.

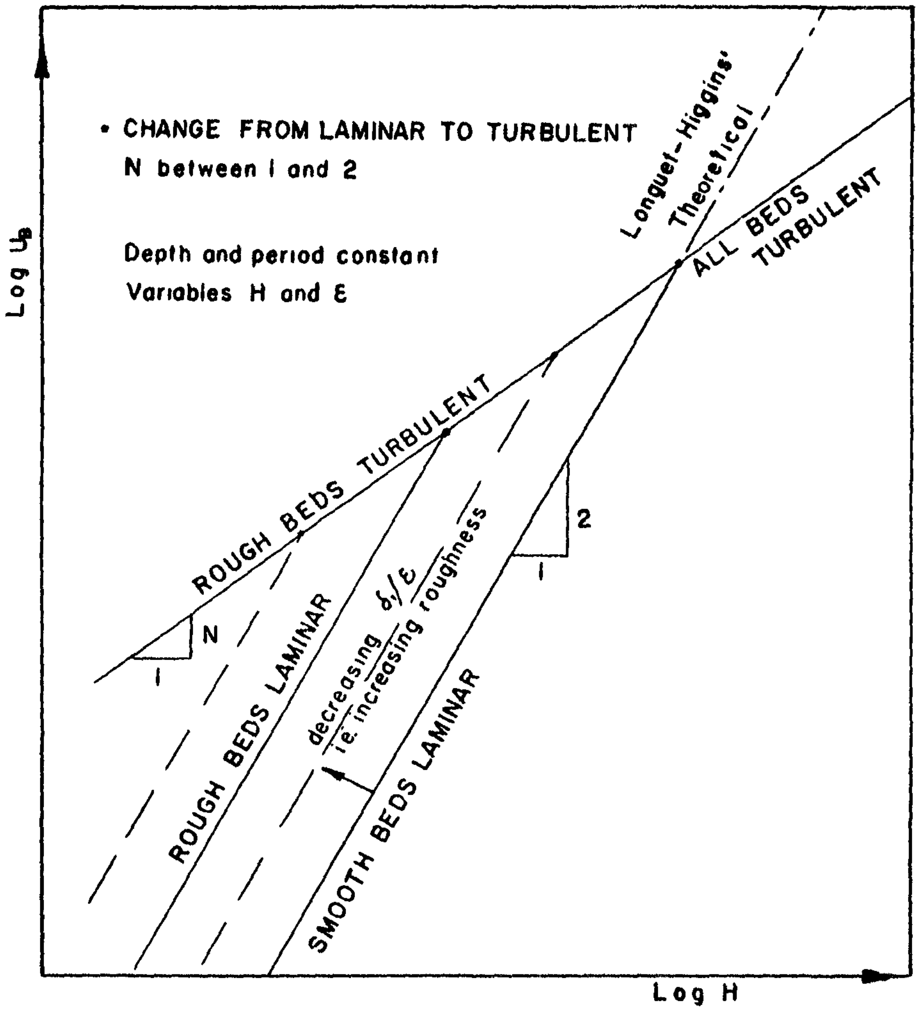

Fig. 2A. Postulated behaviour for rough and smooth beds.

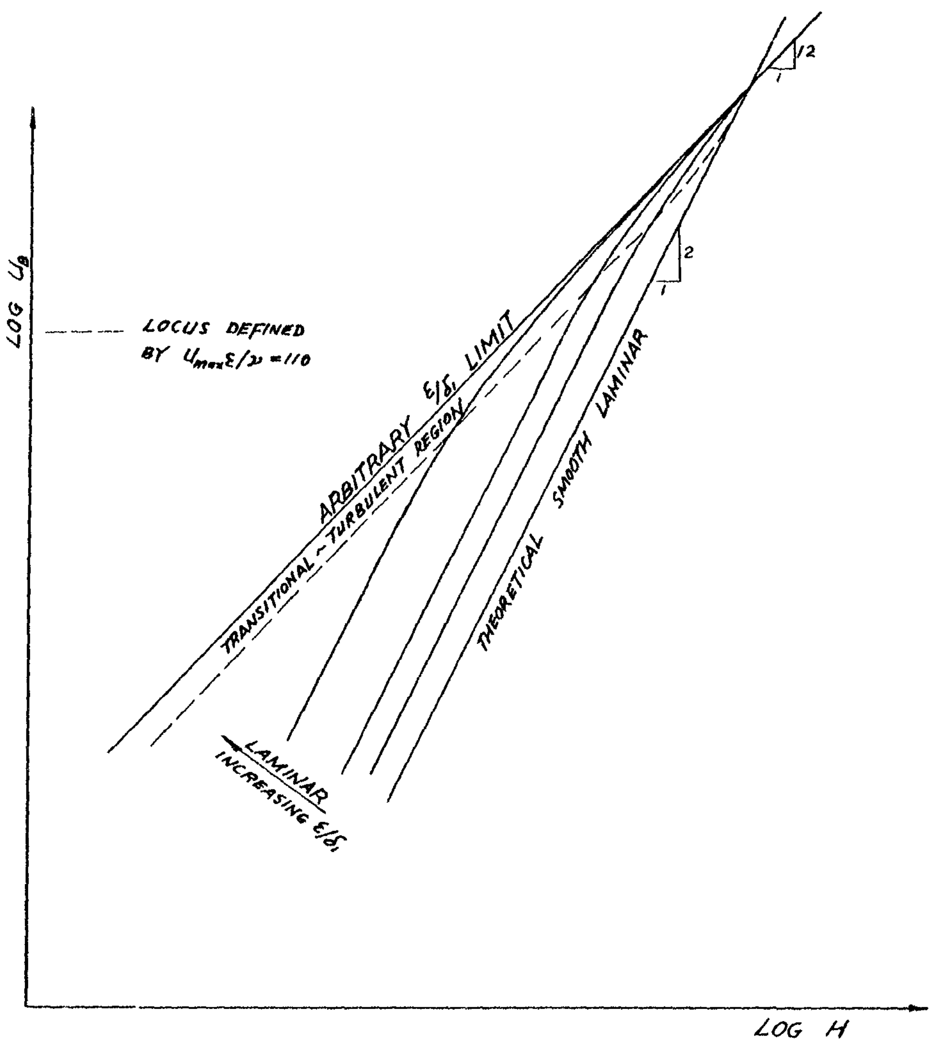

Fig. 2B. Postulated behaviour for rough and smooth beds.

B. It has been established above that the relationship $U_B \propto H^{1.2}$ exists for all bed roughnesses, when a turbulent boundary layer is fully developed. It is argued that this slope of 1.2 on the Log scale plot of U_B versus H forms also the limiting slope when $\varepsilon \rightarrow \delta_1$ for U_B has little meaning when $\varepsilon > \delta_1$. It follows that,

when $\mathbb{R}_\delta > 160$ all beds are turbulent and the slope (= log U_B/Log H) is 1.2

when $\mathbb{R}_\delta < 160$, the smooth bed ($\varepsilon/\delta_1 \rightarrow 0$) is laminar and the slope is 2, confirming the Longuet-Higgins theory,

when $\mathbb{R}_\delta < 160$, the rough bed ($\varepsilon/\delta_1 \rightarrow 1$) is fully turbulent, and the slope approaches 1.2 asymptotically.

The states of intermediate rough beds with $0 < \varepsilon/\delta_1 < 1$ depend on a Reynolds number of the form $U_{max}\varepsilon/\nu$. The critical value of $U_{max}\varepsilon/\nu$ is about 110 (Kalkanis 1964, Askew 1965). For given values of T and h, this critical value always falls in the range of $\mathbb{R}_\delta < 160$.

Thus two regions can be distinguished in the plot of U_B against H with $\mathbb{R}_\delta < 160$. One depicts laminar condition on all beds ($U_{max}\varepsilon/\nu < 110$) and the parallel ε/δ_1 lines pattern revealed in section (A) applies. The other region represents transitional to fully turbulent flow on all rough beds. In this region, the ε/δ_1 lines form a family of curves fanning out from a common point (or a region) designated by the condition of $\mathbb{R}_\delta = 160$. Beyond this point (achieved by increasing the wave heights), all beds are turbulent and the flow is represented by a common line of slope about 1.2. The situation as discussed above is shown in Figure 2B.

Based on the foregoing experimental studies, the following conclusions may be drawn:

1) At values of $\mathbb{R}_\delta$ above 160 all boundary layers are turbulent and the mass transport is less than the theoretical value for a laminar boundary layer.

2) The presence of turbulence within the boundary layer reduces the power of the wave height to which mean transport velocities are proportional.

Apparently, under fully turbulent conditions, the Reynolds stresses near the mean bottom surface assume a negative sense. The layer of fluid close to the mean surface then tends to starve the turbulent eddies of their energy supply with a consequent reduction in the turbulence level. This condition applies to cases when $\mathbb{R}_\delta > 160$.

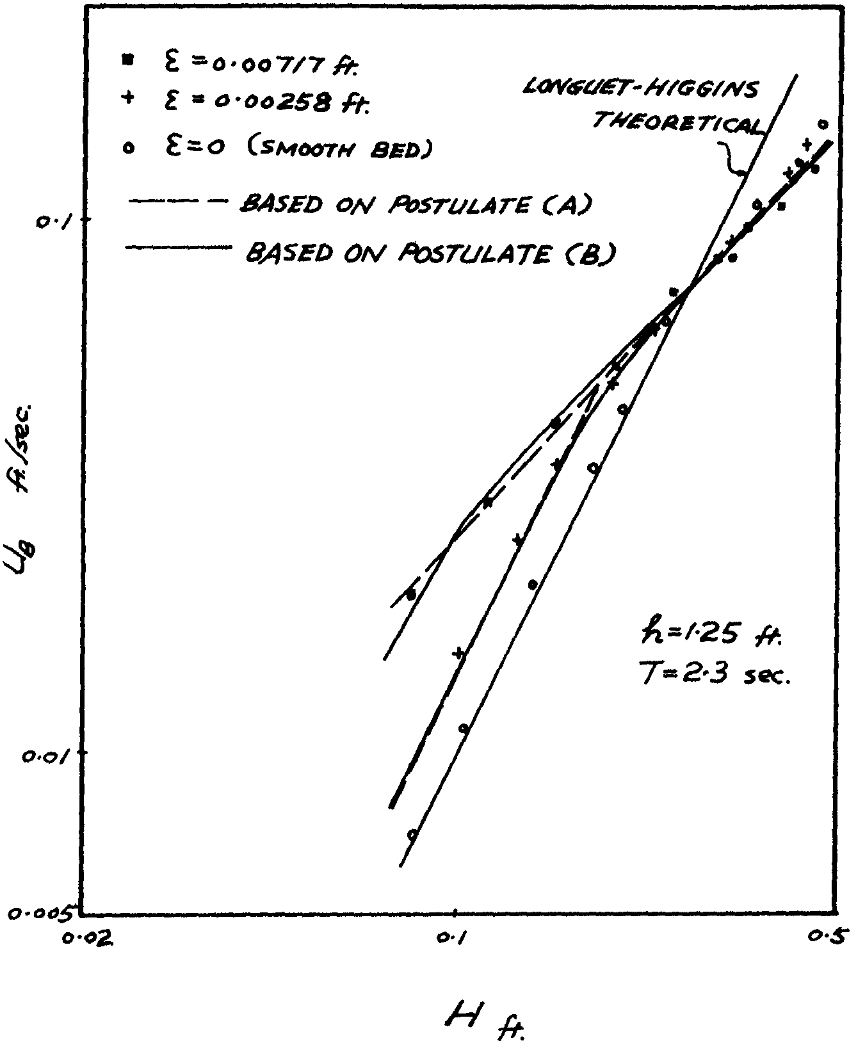

Fig. 3. Typical results, U_B vs. H, all beds.

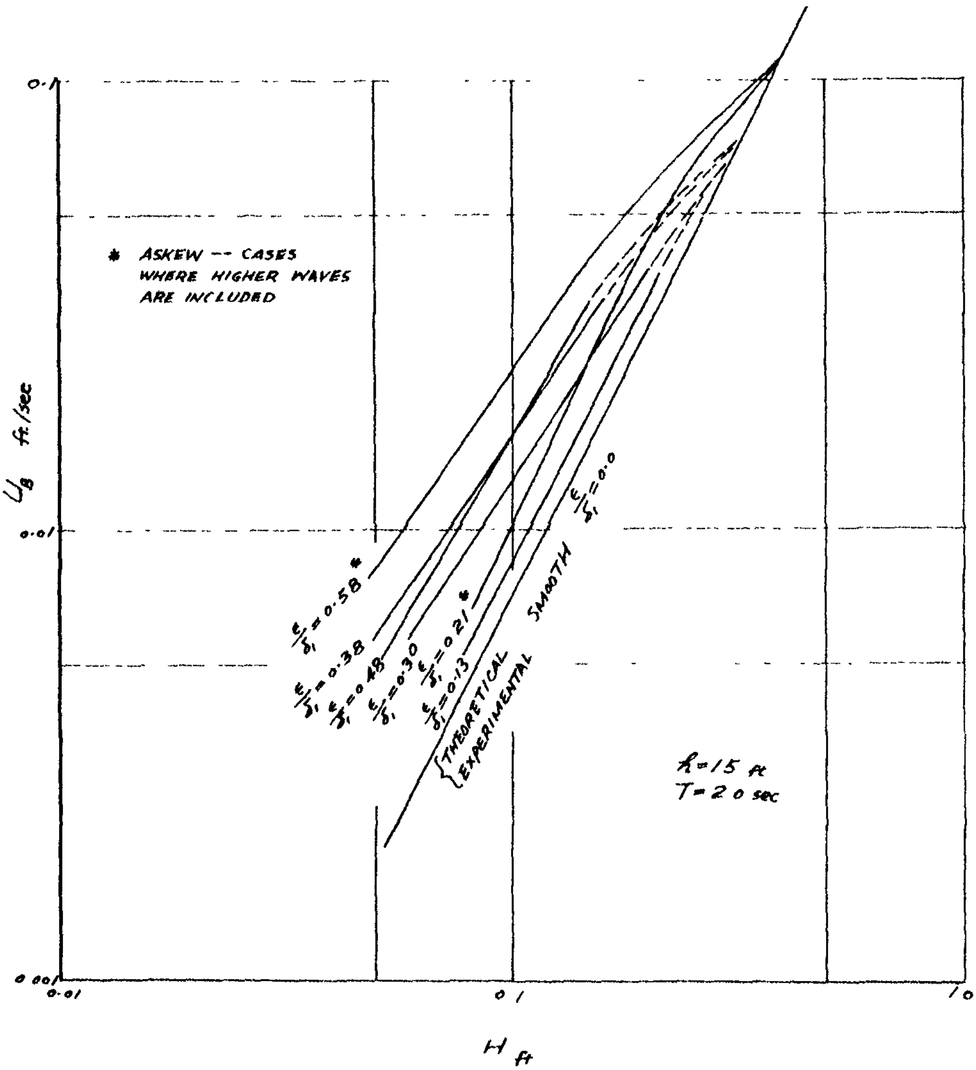

Fig. 4. Typical results for given values of T and h.

When $\mathbb{R}_\delta < 160$, the presence of the roughness on the bed increases the mass transport velocity irrespective of whether the boundary layer is turbulent or laminar. The turbulence level induced by the roughness only (proportional to $U_{max}\,\varepsilon/\nu$) is in general much weaker than fully turbulent conditions designated by $\mathbb{R}_\delta$. Thus, below $\mathbb{R}_\delta = 160$, the effect of the roughness predominates and above that value, the roughness effect becomes negligible.

3) At values of $\mathbb{R}_\delta$ below 160, the roughness elements produce a turbulent boundary layer which results in higher values of mass transport than would occur on a smooth boundary.

With a smooth bed, the boundary layer will always be laminar. With rough beds however, two régimes may be distinguished depending on the parameter $U_{max}\,\varepsilon/\nu$.

Below the critical value of $U_{max}\,\varepsilon/\nu$ (≈ 110), all beds are laminar and a parallel pattern of ε/δ, lines to the smooth laminar case is assumed. These ε/δ, lines extend into the turbulent region ($U_{max}\,\varepsilon/\nu > 110$) and converge to a single point (or region) defined by $\mathbb{R}_\delta = 160$.

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