CHAPTER 11

LAMINAR DAMPING OF OSCILLATORY WAVES DUE TO BOTTOM FRICTION

Yuichi Iwagaki
Professor, Disaster Prevention Research Institute
Kyoto University, Kyoto, Japan
and
Yoshito Tsuchiya
Assistant Professor, Ditto

ABSTRACT

The purpose of this paper is to discover the mechanism of the laminar damping of oscillatory waves due to bottom friction with the aid of the theory of the laminar boundary layer due to waves and of the measurements of instantaneous shearing stresses exerted on a smooth bottom, resulting from wave motion and wave amplitude attenuation with distance. In a theoretical approach the effects of convective terms involved in the basic equations of laminar boundary layers developing both on the bottom and the side walls of a wave channel, are considered on the basis of an approximate solution of the equation, and a theory of the laminar damping of Airy waves is established. In experimental studies, furthermore, direct measurements of instantaneous stresses and observations of wave amplitude attenuation were performed, and the experimental results are compared with both the above theory and the linearized one.

INTRODUCTION

The phenomenon of wave damping due to bottom friction is of interest and also of practical significance in the determining of design waves for coastal structures in shallow water.

This paper presents part of the results obtained from basic studies on the wave damping due to bottom friction which have been carried out for several years at the Ujigawa Hydraulic Laboratory, Disaster Prevention Research Institute.

With regard to the transformation of waves by the processes of internal friction and bottom friction, Lamb(1945) investigated theoretically the damping characteristics of deep water waves by applying the small amplitude wave theory, and also Hough(1896) and Biesel(1949) established a theory of wave damping for shallow water waves. It was concluded from their studies that the rate of wave damping due to internal friction has little effect on the waves treated in this paper. As regards the wave damping due to bottom friction, on the other hand, Putnam and Johnson(1949) made practical studies, but it seems that their results are not exact because they fail to provide an adequate description of the characteristics of flow near a sea bottom which is the result of oscillatory wave motion. In addition, some experiments on the wave damping due to bottom friction and percolation over a permeable bed were performed by Savage(1953), and the experimental results were compared with the theory. Moreover, the relation between the formation of sand waves and the wave energy dissipation was also investigated. With regard to the energy dissipation on a sea bottom, there is the phenomenon of percolation as discussed in Savage's paper. On this subject, Putnam(1949) made a theoretical investigation, and later, the same problem was re-examined by Reid and Kajiura(1957), using a more rigorous approach than that employed by Putnam and a misinterpretation in Putnam's paper was discovered. In recent times, such problems were investigated theoretically by Hunt(1959) and Marray(1965), taking a viscous flow on the permeable boundary surface into account, and the result obtained by Hunt agreed well with the results of Savage's experiment on waves over a smooth sand bed in cases where the values of viscosity and permeability were small.

On the other hand, in Japan some investigations on wave damping were made by Kishi(1954), and Nagai and Kubo(1960), using the same procedure as that used by Putnam, but it appears that there are many problems to clarify. Friction facors along actual coasts were measured by the authors(1965).

In studying the phenomenon of wave damping due to bottom friction, it is necessary to analyze the behavior of boundary layer developing on a sea bottom. Regarding the development of the boundary layer, Eagleson (1959, 1962), Grosch and Lukasik(1960, 1963), and the authors(1961, 1964, 1965) have carried out experimental studies on the wave damping due to bottom friction and the results were compared with the formula of wave damping derived on the basis of the linearized, laminar boundary layer theory. It was found from the comparison that there are wide differences between the theoretical and experimental values. Jonsson(1963) has attempted to estimate the bottom friction by measuring the velocity profiles in a turbulent boundary layer developing on a rough sea bottom due to wave motion. Most recently Van Dorn(1966) has carried out the precise experiments of laminar wave damping for dispersive oscillatory waves and compared with the theoretical results in good agreement.

The purpose of the present studies is to discover the mechanism of the laminar damping of oscillatory waves. For this an approximate solution of the non-linear laminar boundary layer equations is derived by means of the perturbation method, and the effects of the convective terms in the equations on the bottom shearing stress and wave energy dissipation are clarified. The theoretical results for bottom shearing stresses are compared with the results of the direct measurement of them. With regard to the wave damping, a theory of laminar damping based on the above laminar boundary layer theory is established and the theoretical result is compared with the results of the experiment of wave amplitude attenuation and the linearized theory.

THEORY OF WAVE DAMPING DUE TO BOTTOM FRICTION

LAMINAR BOUNDARY LAYER THEORY

With regard to the boundary layer growth resulting from wave motion, for a solitary wave Iwasa(1959) made an analytical investigation applying the momentum integral equation of the boundary layer and obtained interesting results on the laminar boundary layer growth and the wave damping. Also, for uniform oscillatory waves, there is only a linearized theory of the laminar boundary layer, based on Stokes's solution, of which the validity has been examined by comparing with the experimental results obtained by Eagleson(1959, 1962), Grosch and Lukasik(1960, 1963), and the authors(1961, 1964). However, it has not yet been made clear how the convective terms involved in the basic equation of laminar boundary layer influence the boundary layer growth. Grosch(1962) has already derived a solution of the non-linear boundary layer equation in the form of a power series by using Glauert's method, but it seems that the solution is inadequate because it is impossible to examine the phenomenon over a whole period. Therefore, the authors derive an approximate solution of the laminar boundary layer equation written in dimensionless forms by means of Lighthill's method. With regard to the boundary layer developing both on the bottom and the side walls of a wave channel, the effects of the convective terms on the shearing stress are investigated on the basis of the above solution.

Laminar boundary layer developing on the bottom of a wave channel

Taking the axis of x in the direction of the wave propagation and the axis of z perpenducular to the bottom and denoting the velocity components in these directions by u and w respectively, the two-dimensional laminar boundary layer equations for the unsteady, incompressible fluid are written as:

$$\left.\begin{aligned}&\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+w\frac{\partial u}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\frac{\partial^2 u}{\partial z^2}\\&\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}=0,\quad -\frac{1}{\rho}\frac{\partial p}{\partial x}=\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial x}\end{aligned}\right\}\tag{1}$$

in which t is the time, p the pressure, ν the kinematic viscosity of water, ρ the density and U the velocity just outside the boundary layer, to which the relation derived from the wave theory is applied. Now introducing a representative velocity u_o, and the wave celerity c and using the dimensionless quantities defined as follows:

$$\left.\begin{aligned}&u=\bar{u}u_0,\ w=u_0\bar{w}/\sqrt{R},\ U=u_0\bar{U}\\&p=\rho u_0^2\bar{p},\ R=cL/2\pi\nu,\ x=(L/2\pi)\xi\\&z=(L/2\pi\sqrt{R})\zeta,\ t=(L/2\pi c)\tau\end{aligned}\right\}\tag{2}$$

Eq. (1) can be written as:

$$\left.\begin{aligned}&\frac{\partial\bar{u}}{\partial\tau}+\left(\frac{u_0}{c}\right)\left\{\bar{u}\frac{\partial\bar{u}}{\partial\xi}+\bar{w}\frac{\partial\bar{u}}{\partial\zeta}\right\}=-\frac{\partial\bar{p}}{\partial\xi}+\frac{\partial^2\bar{u}}{\partial\zeta^2}\\&\frac{\partial\bar{u}}{\partial\xi}+\frac{\partial\bar{w}}{\partial\zeta}=0,\quad -\frac{\partial\bar{p}}{\partial\xi}=\frac{\partial\bar{U}}{\partial\tau}+\left(\frac{u_0}{c}\right)\bar{U}\frac{\partial\bar{U}}{\partial\xi}\end{aligned}\right\}\tag{3}$$

with the initial and boundary conditions that $u = 0$ at $\tau = 0$, $u = 0$ at $\zeta = 0$ and $\bar{u} = U$ at $\zeta \to \infty$.

Taking account of progressive waves on the basis of Airy's wave theory and applying the maximum velocity component at a bottom $u_{b\ max}$ to u_o, the following relationships are obtained:

$$\left.\begin{aligned}&\bar{U}=\sin(\xi-\tau)\\&-\partial\bar{p}/\partial\xi=-\cos(\xi-\tau)+(1/2)(u_0/c)\sin 2(\xi-\tau)\\&u_0=u_{b\ max}=(\pi H/T)/\sinh kh,\ k=2\pi/L\\&u_0/c=u_{b\ max}/c=\pi(H/L)/\sinh kh\ll 1\end{aligned}\right\}\tag{4}$$

Expressing the solutions of $\bar{u}$ and $\bar{w}$ respectively by

$$\left.\begin{aligned}\bar{u}&=\bar{u}_0+\varepsilon\,\bar{u}_1+\varepsilon^2\,\bar{u}_2+\quad ,\\ \bar{w}&=\bar{w}_0+\varepsilon\,\bar{w}_1+\varepsilon^2\,\bar{w}_2+\quad ,\end{aligned}\right\}\tag{5}$$

the solution of Eq. (3) can be obtained by the perturbation method with a parameter of ε which is equal to u_0/c. Substituting these expressions into Eq. (3) and satisfying the relation between each coefficient of terms on both sides, multiplying the ascending powers of ε, a family of equations is obtained as follows: For $\bar{u}_0$ and $\bar{w}_0$,

$$\left.\begin{aligned}&\frac{\partial\bar{u}_0}{\partial\tau}-\frac{\partial^2\bar{u}_0}{\partial\zeta^2}=-\cos(\xi-\tau)\\ &\frac{\partial\bar{u}_0}{\partial\xi}+\frac{\partial\bar{w}_0}{\partial\zeta}=0\end{aligned}\right\}\tag{6}$$

which is identical with that for one-dimensional heat conduction. The initial and boundary conditions for Eq. (6) are: $\bar{u}_0 = 0$ at $\tau = 0$ and $\zeta = 0$, and $\bar{u}_0 = U = \sin(\xi - \tau)$ at $\zeta \to \infty$. Eq. (6) is for the so-called linearized theory and its solution has been derived by Grosch(1962) in the form

$$\begin{aligned}\bar{u}_0=&\sin(\xi-\tau)-e^{-\frac{1}{\sqrt{2}}}\sin\left(\xi-\tau+\frac{1}{\sqrt{2}}\zeta\right)\\ &+\frac{2}{\pi}\int_0^\infty e^{(\xi-\tau)\sigma^2}\frac{\sigma\sin(\zeta\sigma)}{1+\sigma^4}\,d\sigma\end{aligned}\tag{7}$$

In the above equation, the third term on the right vanishes when τ becomes sufficiently large. Therefore, taking only the so-called steady state solution into account, the third term can be omitted.

Next, the equation for $\bar{u}_1$ and $\bar{w}_1$ can be written together with the initial and boundary conditions as follows:

$$\left.\begin{aligned}&\frac{\partial\bar{u}_1}{\partial\tau}-\frac{\partial^2\bar{u}_1}{\partial\zeta^2}=-\left(\bar{u}_0\frac{\partial\bar{u}_0}{\partial\xi}+\bar{w}_0\frac{\partial\bar{u}_0}{\partial\zeta}\right)+\bar{U}\frac{\partial\bar{U}}{\partial\xi}\\ &\frac{\partial\bar{u}_1}{\partial\xi}+\frac{\partial\bar{w}_1}{\partial\zeta}=0,\ \bar{u}_1=\bar{w}_1=0,\ \tau=\zeta=0,\ \zeta\to\infty\end{aligned}\right\}\tag{8}$$

In general, the expression for u_1 can formally be written in the form

$$\left.\begin{aligned}&\frac{\partial\bar{u}_i}{\partial\tau}-\frac{\partial^2\bar{u}_i}{\partial\zeta^2}=F_i(\xi,\tau),\ \frac{\partial\bar{u}_i}{\partial\xi}+\frac{\partial\bar{w}_i}{\partial\zeta}=0\\ &\bar{u}_i=\bar{w}_i=0,\ \tau=\zeta=0,\ \zeta\to\infty\end{aligned}\right\}\tag{9}$$

together with the initial and boundary conditions.

Since this is a heat conduction type equation, the solution for $\bar{u}_1$ can be found by applying Green's function $H(\zeta,\tau;q,s)$ and the solution for $\bar{u}$ by the perturbation method can formally be expressed as:

$$\begin{aligned}\bar{u}(\xi,\zeta,\tau)=\bar{u}_0+&\varepsilon\int_0^\tau ds\int_0^\infty H(\zeta,\tau,q,s)F_1(q,s)dq\\ &+\varepsilon^2\int_0^\tau ds\int_0^\infty H(\zeta,\tau,q,s)F_2(q,s)dq\\ &+\quad ,\end{aligned}\tag{10}$$

in which

$$
\begin{aligned}
H(\zeta,\tau,q,s) &= \{1/2\sqrt{\pi(\tau-s)}\} \\
&\quad \times [\exp\{-(\zeta-q)^2/4(\tau-s)\} \\
&\quad -\exp\{-(\zeta+q)^2/4(\tau-s)\}], \tau > s, \\
&= 0,\ \tau < s
\end{aligned} \tag{11}
$$

Since the integration of the above equation is complicated, taking into consideration the form of fuction $F_1(q, s)$, only the steady state solution is considered in the subsequent descriptions. The steady state solutions for $\bar{u}_0$ and $\bar{w}_0$ can be written in the form

$$
\left.
\begin{aligned}
\bar{u}_0 &= \sin(\xi-\tau) - e^{-\zeta/\sqrt{2}} \sin\left(\xi-\tau+\frac{1}{\sqrt{2}}\zeta\right) \\
-\bar{w}_0 &= \zeta\cos(\xi-\tau) - e^{-\zeta/\sqrt{2}} \sin \\
&\quad \times\left(\xi-\tau+\frac{1}{\sqrt{2}}\zeta-\frac{\pi}{4}\right)+\sin\left(\xi-\tau-\frac{\pi}{4}\right)
\end{aligned}
\right\} \tag{12}
$$

Substituting these relationships for $\bar{u}_0$ and $\bar{w}_0$ into Eq. (8), the equation for $\bar{u}_1$ becomes finally

$$
\begin{aligned}
\frac{\partial \bar{u}_1}{\partial \tau} - \frac{\partial^2 \bar{u}_1}{\partial \zeta^2} &= \frac{1}{2}\left\{e^{-\zeta/\sqrt{2}}\cos\left(\frac{1}{\sqrt{2}}\zeta\right)+\zeta\, e^{-\zeta/\sqrt{2}}\cos\left(\frac{1}{\sqrt{2}}\zeta-\frac{\pi}{4}\right)\right\}\sin 2(\xi-\tau) \\
&\quad +\frac{1}{2}\left\{e^{-\zeta/\sqrt{2}}\sin\left(\frac{1}{\sqrt{2}}\zeta\right)-\zeta\, e^{-\zeta/\sqrt{2}}\sin\left(\frac{1}{\sqrt{2}}\zeta-\frac{\pi}{4}\right)\right\}\cos 2(\xi-\tau) \\
&\quad +\frac{1}{2}\left\{\zeta\, e^{-\zeta/\sqrt{2}}\sin\left(\frac{1}{\sqrt{2}}\zeta-\frac{\pi}{4}\right)+e^{-\zeta/\sqrt{2}}\cos\left(\frac{1}{\sqrt{2}}\zeta\right)-e^{-\zeta/\sqrt{2}}\right\}
\end{aligned} \tag{13}
$$

Following Schlichting's procedure(1960), the solution of Eq. (13) which satisfies the boundary conditions that $\bar{u}_1 = 0$ at $\zeta = 0$ and $\partial\bar{u}_1/\partial\zeta = 0$ at $\zeta \to \infty$, can easily be derived, and an approximate solution for $\bar{u}$ finally be written in the form

$$
\begin{aligned}
\bar{u} &= \sin(\xi-\tau) - e^{-\zeta/\sqrt{2}}\sin\left(\xi-\tau+\frac{1}{\sqrt{2}}\zeta\right) + \epsilon\Bigg[\left\{\frac{11}{18}e^{-\zeta}\sin\zeta-\frac{7}{18}e^{-\zeta/\sqrt{2}}\sin\left(\frac{1}{\sqrt{2}}\zeta\right)\right. \\
&\quad \left.-\frac{1}{6}\zeta\, e^{-\zeta/\sqrt{2}}\sin\left(\frac{1}{\sqrt{2}}\zeta-\frac{\pi}{4}\right)\right\}\sin 2(\xi-\tau) - \left\{-\frac{11}{18}e^{-\zeta}\cos\zeta+\frac{11}{18}e^{-\zeta/\sqrt{2}}\cos\left(\frac{1}{\sqrt{2}}\zeta\right)\right. \\
&\quad \left.+\frac{1}{6}\zeta\, e^{-\zeta/\sqrt{2}}\cos\left(\frac{1}{\sqrt{2}}\zeta-\frac{\pi}{4}\right)\right\}\cos 2(\xi-\tau) + \left\{\frac{1}{4}e^{-\zeta/\sqrt{2}}+\frac{1}{2}e^{-\zeta/\sqrt{2}}\sin\left(\frac{1}{\sqrt{2}}\zeta\right)\right. \\
&\quad \left.-e^{-\zeta/\sqrt{2}}\cos\left(\frac{1}{\sqrt{2}}\zeta\right)-\frac{1}{2}\zeta\, e^{-\zeta/\sqrt{2}}\sin\left(\frac{1}{\sqrt{2}}\zeta+\frac{\pi}{4}\right)+\frac{3}{4}\right\} + O(\epsilon^2)\Bigg]
\end{aligned} \tag{14}
$$

From this result, it is found that only the constant term on the right of Eq. (14) remains, taking the average of $\bar{u}$ with respect to time at $\zeta \to \infty$, just outside the boundary layer, and that there exists a certain mass transport velocity which can be expressed by

$$
\bar{u}_m = (3/4)\epsilon \tag{15}
$$

This can be written in the form

$$
u_m = (3/16)H^2 k(2\pi/T)/\sinh^2 kh \tag{16}
$$

which is identical with that obtained by Longuet-Higgins(1953).

Applying the above result, a theoretical formula for the bottom shearing stress can be derived. The shearing stress on the bottom is generally given by the relationship $\tau_o = \mu(\partial u/\partial z)_{z=0}$ for laminar flows. Expressing this in the dimensionless form and using Eq. (5), the shearing stress can be written as.

$$\tau_0/\rho u_0^2 = R_e^{-1/2}\Big\{\sin\Big(\xi-\tau-\frac{\pi}{4}\Big)+\varepsilon(\partial\bar{u}_1/\partial\zeta)_{\zeta=0} + \varepsilon^2(\partial\bar{u}_2/\partial\zeta)_{\zeta=0}+\ \Big\} \tag{17}$$

Calculating the above equation with the relationship of Eq. (14), the following equation is obtained as an approximate solution for τ_o.

$$\tau_0/\rho u_0^2 \approx R_e^{-1/2}\Big[\sin\Big(\xi-\tau-\frac{\pi}{4}\Big)+\varepsilon\Big\{\frac{1}{2\sqrt{2}}+\Big(\frac{11}{18}-\frac{5\sqrt{2}}{18}\Big)\sin 2(\xi-\tau) + \Big(\frac{11}{18}-\frac{4\sqrt{2}}{18}\Big)\cos 2(\xi-\tau)\Big\}+O(\varepsilon^2)\Big] \tag{18}$$

in which

$$R_e = \frac{\pi}{2\sinh^2 kh}\Big(\frac{CH}{\nu}\Big)\Big(\frac{H}{L}\Big) = \frac{1}{2\pi}\frac{u_0^2 T}{\nu} \tag{19}$$

The first term in the brackets of Eq. (18) indicates the results based on the linearized theory and the second term indicates the effect of the convective terms. In Fig. 1, the calculated results of Eq. (18) are shown by using the value of $\varepsilon = \pi(H/L)/\sinh(kh)$ as a parameter and also the time variation of the dimensionless water profile $\bar{\eta}$ for the purpose of comparison. It is found from the figure that the characteristics of the shearing stress vary slightly with the value of ε, but the effect of ε may be negligible because the value will not exceed about 0.15 for the waves treated in practice.

According to Eagleson's study, the average bottom friction coefficient is defined by

$$\bar{C}_f = 2\bar{\tau}_0/\rho\bar{U}^2 \tag{20}$$

in which $\bar{\tau}_o$ and $\bar{U}^2$ are the average values of τ_o and U^2 expressed by Eqs. (18) and (4) respectively. Since it is complicated to calculate $\bar{C}_f$ directly by Eq. (17), the results obtained by the graphical integration are shown in Fig. 2. Each of the curves (a), (b) and (c) indicates how the phase interval is to be chosen in taking the time average; (a) resulted when the absolute value of τ_o was averaged with respect to time from the phase when $\tau_o = 0$ to $\vartheta + 2\pi$, and (b) and (c) resulted when τ_o was averaged over the phase intervals corresponding to the positive and negative values of τ_o respectively. In the figure, the wave Reynolds number R_{eT} is expressed as

$$R_{eT} = 2\pi R_e = u_0^2 T/\nu \tag{21}$$

In the case of the linearized theory where the value of ε vanishes, the friction coefficient $\bar{C}_f$ is expressed as

$$\bar{C}_f = 8\sqrt{2/\pi}R_{eT}^{-1/2} \tag{22}$$

<u>Laminar boundary layer developing on the side wall of a wave channel</u> In the experiment on wave damping, the energy dissipation due to the

friction acting on the side walls of a wave channel must be considered when the width of a wave channel is small compared with the water depth, so that it is necessary to describe the behavior of boundary layers developing on the side wall.

Taking the axis of z vertically along the side wall and the axis of y perpendicular to it, and using the same notations as those in Eq. (1), the boundary layer equations for this case are expressed as

$$\left.\begin{aligned} &\partial u/\partial t+u\,\partial u/\partial x+v\,\partial u/\partial y+w\,\partial u/\partial z \\ &\quad=-(1/\rho)\partial p/\partial x+\nu\partial^2 u/\partial y^2 \\ &\partial w/\partial t+u\,\partial w/\partial x+v\,\partial w/\partial y+w\,\partial w/\partial z \\ &\quad=g-(1/\rho)\partial p/\partial z+\nu\,\partial^2 w/\partial y^2 \\ &\partial u/\partial x+\partial v/\partial y+\partial w/\partial z=0 \end{aligned}\right\} \tag{23}$$

in which g is the acceleration of gravity and v the velocity component in the direction of y. Using the dimensionless quantities, $v = u_0\bar{v}/\sqrt{R}$, $p = \rho u_0^2\bar{p} - (\rho gL/2\pi)\zeta$, and $y = (L/2\pi)\eta/\sqrt{R}$ in addition to Eq.(2), the above equation can be written as

$$\left.\begin{aligned} &\partial \bar{u}/\partial\tau+\varepsilon(\bar{u}\,\partial \bar{u}/\partial\xi+\bar{v}\,\partial \bar{u}/\partial\eta+\bar{w}\,\partial \bar{u}/\partial\zeta) \\ &\quad=-\partial \bar{p}/\partial\xi+\partial^2 \bar{u}/\partial\eta^2 \\ &\partial \bar{w}/\partial\tau+\varepsilon(\bar{u}\,\partial \bar{w}/\partial\xi+\bar{v}\,\partial \bar{w}/\partial\eta+\bar{w}\,\partial \bar{w}/\partial\zeta) \\ &\quad=-\partial \bar{p}/\partial\zeta+\partial^2 \bar{w}/\partial\eta^2 \\ &\partial \bar{u}/\partial\xi+\partial\bar{v}/\partial\eta+\partial \bar{w}/\partial\zeta=0 \end{aligned}\right\} \tag{24}$$

To derive the solution of Eq. (24) by the perturbation method, using the forms

$$\left.\begin{aligned} &\bar{u}=\bar{u}_0+\varepsilon\,\bar{u}_1+\varepsilon^2\,\bar{u}_2+\cdots, \\ &\bar{v}=\bar{v}_0+\varepsilon\,\bar{v}_1+\varepsilon^2\,\bar{v}_2+\cdots, \\ &\bar{w}=\bar{w}_0+\varepsilon\,\bar{w}_1+\varepsilon^2\,\bar{w}_2+\cdots, \end{aligned}\right\} \tag{25}$$

a family of the equations corresponding to Eqs. (6) and (8) together with the boundary conditions can be written as: for $\bar{u}_0$ and $\bar{w}_0$,

$$\begin{aligned} &\partial \bar{u}_0/\partial\tau-\partial^2 \bar{u}_0/\partial\eta^2=\partial \bar{U}/\partial\tau \\ &\partial \bar{w}_0/\partial\tau-\partial^2 \bar{w}_0/\partial\eta^2=\partial \bar{W}/\partial\tau \\ &\partial \bar{u}_0/\partial\xi+\partial \bar{v}_0/\partial\eta+\partial \bar{w}_0/\partial\zeta=0 \\ &\bar{u}_0=\bar{w}_0=0,\ \eta=0 \\ &\bar{u}_0=\bar{U}\ \text{ and }\ \bar{w}_0=\bar{W},\ \eta\to\infty \end{aligned} \tag{26}$$

and for $\bar{u}_1$ and $\bar{w}_1$,

$$\left.\begin{aligned} &\partial \bar{u}_1/\partial\tau-\partial^2 \bar{u}_1/\partial\eta^2=\bar{U}\partial \bar{U}/\partial\xi+\bar{W}\,\partial \bar{U}/\partial\zeta \\ &\quad-(\bar{u}_0\,\partial \bar{u}_0/\partial\xi+\bar{v}_0\,\partial \bar{u}_0/\partial\eta+\bar{w}_0\,\partial \bar{u}_0/\partial\zeta) \\ &\partial \bar{w}_1/\partial\tau-\partial^2 \bar{w}_1/\partial\eta^2=\bar{U}\partial \bar{W}/\partial\xi+\bar{W}\partial \bar{W}/\partial\zeta \\ &\quad-(\bar{u}_0\,\partial\bar{w}_0/\partial\xi+\bar{v}_0\,\partial\bar{w}_0/\partial\eta+\bar{w}_0\partial\bar{w}_0/\partial\zeta) \\ &\partial \bar{u}_1/\partial\xi+\partial \bar{v}_1/\partial\eta+\partial \bar{w}_1/\partial\zeta=0 \\ &\bar{u}_1=\bar{w}_1=0,\ \eta=0,\ \partial \bar{u}_1/\partial\eta=\partial \bar{w}_1/\partial\eta=0,\ \eta\to\infty \end{aligned}\right\} \tag{27}$$

in which on the basis of the wave theory $\bar{U}$ and $\bar{W}$ are the velocity components of water particles just outside the boundary layer on the side wall. Applying the relationships derived from Airy's wave theory in this case, solutions for $\bar{u}$ and $\bar{w}$ calculated to the second approximation become finally

$$\left.\begin{aligned}
\bar{u} =& \left\{\sin(\xi-\tau) - e^{-\eta/\sqrt{2}} \sin\left(\xi-\tau+\frac{1}{\sqrt{2}}\eta\right)\right\}\cosh\zeta + \varepsilon\left[-\left\{e^{-\eta/\sqrt{2}}\sin\left(\frac{1}{\sqrt{2}}\eta\right)\right.\right. \\
&+ (1/4)e^{-\sqrt{2}\eta}\sin(\sqrt{2}\eta) + e^{-\eta}\sin\eta\Big\}\sin 2(\xi-\tau) + \left\{e^{-\eta/\sqrt{2}}\cos\left(\frac{1}{\sqrt{2}}\eta\right)\right. \\
&\left.+ (1/4)e^{-\sqrt{2}\eta}\cos(\sqrt{2}\eta) - (5/4)e^{-\eta}\cos\eta\Big\}\cos 2(\xi-\tau)\right] + O(\varepsilon^2), \\
\bar{w} =& -\left\{\cos(\xi-\tau) - e^{-\eta/\sqrt{2}}\cos\left(\xi-\tau+\frac{1}{\sqrt{2}}\eta\right)\right\}\sinh\zeta + \varepsilon\Big\{(1/4)e^{-\sqrt{2}\eta} \\
&+ e^{-\eta/\sqrt{2}}\sin\left(\frac{1}{\sqrt{2}}\eta\right) - (1/4)\Big\}\sinh 2\zeta + O(\varepsilon^2)
\end{aligned}\right\} \quad (28)$$

With regard to the mass transport, it can be seen from the above results that it does not exist in the direction of the wave propagation, but that in the vertical direction there exists a mass transport velocity expressed as

$$\bar{w}_m = -(\varepsilon/4)\sinh 2\zeta \quad (29)$$

which is rewritten as

$$w_m = -(1/16)\{H^2k(2\pi/T)/\sinh^2 kh\}\sinh 2\zeta \quad (30)$$

In the above equation, w_m vanishes at $\zeta = 0$ and becomes maximum at the water surface.

Consider the shearing stresses acting on the side walls in a wave channel. Using Eq. (28), the relationships for these are derived as follows:

$$\begin{aligned}
\tau_{0x}/\rho u_0^2 = R_e^{-1/2}[&\sin(\xi-\tau-\pi/4)\cosh\zeta + \varepsilon\{(1-3\sqrt{2}/4)\sin 2(\xi-\tau) \\
&+ (5/4-3\sqrt{2}/4)\cos 2(\xi-\tau)\} + O(\varepsilon^2)]
\end{aligned} \quad (31)$$

in the x direction and

$$-\tau_{0z}/\rho u_0^2 = R_e^{-1/2}[\cos(\xi-\tau-\pi/4)\sinh\zeta - \varepsilon(\sqrt{2}/4)\sinh 2\zeta + O(\varepsilon^2)] \quad (32)$$

in the z direction, in which Re is expressed by Eq. (19).

The above method of analysis in applying Airy's wave theory is also applicable in the case of waves accompanying the mass transport on a substantial scale, Stokes's waves for example, and the authors have already made some calculations regarding it which will be published at a late date.

THEORY ON WAVE DAMPING

In the subsequent descriptions, the wave damping due to friction is considered after the wave energy dissipation due to viscosity within the boundary layers on the bottom and side walls has been estimated on the basis of the non-linear laminar boundary layer theory.

Wave energy dissipation within boundary layers It is assumed that the wave energy is dissipated only by bottom friction due to viscosity. The rate of energy dissipation in an incompressive fluid due to viscosity per second per unit volume is written in terms of velocity gradients through Rayleigh's laminar dissipation function as

$$\phi=\mu\left[2\left(\frac{\partial u}{\partial x}\right)^2+2\left(\frac{\partial w}{\partial z}\right)^2+\left\{\left(\frac{\partial w}{\partial x}\right)+\left(\frac{\partial u}{\partial z}\right)\right\}^2\right] \tag{33}$$

in which ϕ is the rate of energy dissipation, known as the dissipation function. Neglecting the terms including w and $\partial\bar{u}/\partial x$, which are quite small compared with $\partial\bar{u}/\partial z$, the average rate of energy dissipation per unit area in a boundary layer $\bar{E}_{fb}$ can be approximately expressed as

$$\bar{E}_{fb}\approx\mu u_0^2\sqrt{R}/L\int_0^{2\pi}\int_0^{\delta_\zeta}(\partial\bar{u}/\partial\zeta)^2\,d\zeta\,d\xi \tag{34}$$

in which μ is the dynamic viscosity of water and δ_ζ the dimensionless expression $(2\pi\sqrt{R}\ \delta/L)$ of the boundary layer thickness δ. Calculating Eq. (32) with the aid of Eq. (14) yields

$$\bar{E}_{fb}\approx\frac{\mu}{2}\beta\left(\frac{\pi H}{T}\right)^2\operatorname{cosech}^2 kh\ \left\{1-\frac{8\sqrt{2}}{3\pi}\left(\frac{11}{18}-\frac{91\sqrt{2}}{288}\right)\varepsilon+O(\varepsilon^2)\right\} \tag{35}$$

This shows, needless to say, the energy dissipation when the effect of the convective terms involved in the boundary layer equation is taken into account. In the equation, the first term on the right is identical with that derived from the linearized theory and the second term indicates the effect of the convective terms. From this result, it is found that the rate of the wave energy dissipation is about 2 % less than that in the linearized theory when the value of ε is assumed to be 0.2.

Since the average rate of energy dissipation per unit area of the side wall of a water tank E_{fw} can be calculated by

$$\begin{aligned}2\bar{E}_{fw}=\mu u_0^2(\sqrt{R}/\pi h)&\int_0^{kh}\int_0^{2\pi}\int_0^{\delta_\zeta}\\&\times\{(\partial\bar{u}/\partial\eta)^2+(\partial\bar{w}/\partial\eta)^2\}\,d\eta\,d\xi\,d\zeta\end{aligned} \tag{36}$$

substituting Eq. (28) into this, the integration yields

$$\begin{aligned}2\bar{E}_{fw}\approx\frac{\mu}{kh}\beta\left(\frac{\pi H}{T}\right)^2&\coth kh\\&\times\left\{1+\frac{8\sqrt{2}}{3\pi}\left(\frac{11}{12}-\frac{\sqrt{2}}{120}\right)\varepsilon\operatorname{sech} kh+O(\varepsilon^2)\right.\end{aligned} \tag{37}$$

From the above result, it is considered that the effect of the convective terms on the rate of energy dissipation on the side wall is of the order of ε sech(kh). However in the authors' experiment the maximum value becomes as much as 20 % of that of the linearized theory.

Now, denoting the width of a water tank by B and the ratio of the energy dissipation in boundary layers on the bottom to that on the side walls by ψ, the following approximate relationship can be obtained.

$$\psi = \frac{\bar{E}_{fb}B}{2\bar{E}_{fw}h} \approx (kB/\sinh 2kh)\{1-(1.086\operatorname{sech}kh+0.197)\varepsilon\} \tag{38}$$

In the above equation, the relation corresponding to the case when ε vanishes is identical with what is called Keulegan's method presented in Savage's paper which is derived from the linearized theory.

Mechanism of wave damping The relationship of the wave energy conservation for a two-dimensional case, under the assumption that the energy is dissipated by bottom friction only, is given by

$$\frac{d}{dx}(C_g E) = -\bar{E}_{fb} \tag{39}$$

in which Cg is the group velocity and E the wave energy per unit area.

Substituting the relationships for Cg and E derived from Airy's wave theory into Eq. (37), and integrating under the assumptions that $H = H_0$ at $x = 0$ and ε is taken to be constant, yield

$$H = H_0 \exp(-\epsilon_b x/L) \tag{40}$$

in which

$$\begin{aligned} &\epsilon_b \approx (4\pi^2/\beta L)(1-0.197\varepsilon)/(\sinh 2kh + 2kh), \\ &\beta = (\pi/\nu T)^{1/2} \end{aligned} \tag{41}$$

It is concluded from the above equation that the effect of the convective terms on ϵ_b becomes at most 3 % for the waves made in the authors' experiment. In addition, the expression for ϵ_b in the case when $\varepsilon = 0$ agrees with that obtained by Eagleson and is called the dimensionless decay modulus. On the other hand, another expression for the relationship of Eq. (38) was established by one of the authors(1961).

Instead of Eq. (37), the following equation must be used when the energy dissipation due to side wall friction is taken into account in addition to that due to bottom friction:

$$\frac{d}{dx}(C_g EB) = -(\bar{E}_{fb}B + 2\bar{E}_{fw}h) \tag{42}$$

Thus, denoting the decay modulus based on both bottom and side wall friction by $\epsilon_{(b+w)}$, the relationship of wave damping corresponding to Eq. (38) becomes

$$\left.\begin{aligned} &H = H_0 \exp(-\epsilon_{b+w} x/L) \\ &\epsilon_{b+w} = (4\pi^2/\beta L)(1+1/\psi)/(\sinh 2kh + 2kh) \end{aligned}\right\} \tag{43}$$

From the comparison between the above equation and Eq. (39), the relationship between ϵ_b and $\epsilon_{(b+w)}$ can be expressed as

$$\epsilon_b = \{\psi/(\psi+1)\}\epsilon_{(b+w)} \tag{44}$$

so that if $\epsilon_{(b+w)}$ is established by experiment the wave decay modulus ϵ_b due only to bottom friction without the effect of side walls can be calculated by Eq. (42). Referring to Eq. (36), the effect of the convective terms on ϵ_b is expected to be fairly large when the energy dissipation due to side wall friction is taken into account.

Relationships between bottom friction factor, bottom friction coefficient and wave decay modulus In studies on wave damping by wave observations, the estimation of the bottom friction factor has been usually made by means of the following relationship for bottom shearing stress, defined by Breschneider(1954),

$$\tau_0=\rho f u_b^2 \tag{45}$$

in which f is the so-called bottom friction factor, and u_b the velocity component of water particles on the bottom, which is equivalent to U presented before. The rate of wave energy dissipation E_{fb}' derived by the definition of Eq. (43) can be written as

$$\bar{E}'_{fb}=(4/3\,\pi)\rho f u_0^3 \tag{46}$$

Then, assuming that the rate of energy dissipation based on the linearized theory equalizes Eq. (44), the following expression is obtained for the bottom friction factor.

$$f=(3\pi\sqrt{\pi}/8)R_{eT}^{-1/2} \tag{47}$$

Consequently, from the comparison between Eqs. (22) and (46), the relation between f and C_f can be expressed as

$$f=(3\pi^2/64\sqrt{2})\bar{C}_f \tag{48}$$

and from Eqs. (40) and (46) the relationship between ϵ_b and f can be written as

$$f=(3/32\pi)\epsilon_b(H/L)^{-1}\{\sinh kh(\sinh 2kh+2kh)\} \tag{49}$$

In addition, the damping characteristics of waves based on Eq. (44) is expressed by

$$H/H_0=1-(8/3\pi\sqrt{\pi})f\epsilon_b R_{eT}^{1/2}(x/L) \tag{50}$$

EXPERIMENTS ON BOTTOM SHEARING STRESS AND WAVE DAMPING

MEASUREMENT OF BOTTOM SHEARING STRESS

There are two methods for determining experimentally the bottom shearing stress in the case of laminar boundary layer. One is to find the value of τ_0 indirectly from the measurement of the velocity distribution, and the other is by the direct measurement of the shearing stress on a bottom surface. The latter was adopted in the present study and a measuring device similar to that used by Eagleson(1959) was made.

Characteristics of the measuring device Fig.3 shows a schematic view of the measuring device. It consists of three main parts, which are a moment meter, a supporting rod and a flat plate called a shear plate. Basic investigations of the characteristics of the device were carried out. Although the details are omitted here, this investigation yields the following results: (1) It is desirable to reduce the mass of the shear plate and the supporting rod as much as possible. (2) If the shear plate slips upward from the bottom, the shearing stress is overestimated owing to the drag force acting on the edges of the shear plate;conversely, if the shear plate slips downward, there is not such a marked effect. (3) The larger the clearance Δ h under the shear plate, the smaller the experimental value of the shearing stress becomes, and the more the value tends to approach the theoretical one. (4) The less clearance gap between the shear plate and the bottom surface Δ b, the smaller the experimental value becomes, and the more the value tends to approach the theoretical one. (5) The thinner the shear plate, the smaller the experimental value becomes, and the more the value tends to approach the theoretical one. (6) If the shear plate is made smaller and the supporting rod is made lighter, the experimental value becomes small and approaches the theoretical one. (7) The width of the shear plate b has little effect, but the influence of the shield pipe on the shearing stress will appear if the width is too small.

On the basis of these results, a shear plate 8.1 cm long, 5 cm wide, 0.2 mm thick and made of stainless steel was finally chosen to be used. Furthermore, to prevent flow through the clearance under the plate, a small channel 3 mm wide, running from wall to wall of the recess in the transverse direction, similar to that used by Eagleson, was made and filled with mercury until the meniscus touched the underside of the plate.

Experimenatal procedures The characteristics of waves and water depths used in the experiment are shown in Table 1, in which (1) indicates the experiment made in 1964 with the use of a plunger-type wave generator and (2) that made in 1965 with the use of a fluter-type one. The shearing stress acting on the bottom was measured for various wave characteristics. Wave heights were recorded by electric resistance type wave meters into a penwriting oscillograph.

Results of experiment and considerations In order to estimate exactly the shearing stress from the measurement of the force acting on the shear plate, a correction for the forces acting on the plate other than the shearing force is necessary. The external force F' is assumed to be equal to the sum of three forces: the shearing force, the force resulting from pressure gradients acting on both sides of the plate and the virtual mass force. Since, however, the flow under the shear plate is prevented by injecting mercury into the small channel, it is doubtful whether the virtual mass force acts on the plate; therefore, the virtual mass force is neglected in this case, and the experimental values are examined on the basis of the linearized theory, assuming that the effect of the convective terms presented before is omitted.

Denoting the surface area of the shear plate by A, and the thickness by d, the horizontal force F per unit area acting on the shear plate is finally written as

$$F=F'/A=-\sqrt{C^2+(C+D)^2}H\sin(\xi-\tau+\acute{\varepsilon}) \tag{51}$$

in which

$$\left.\begin{array}{l} C=\mu kc\beta/2\sinh kh,\ D=\rho gkd/2\cosh kh, \\ \acute{\varepsilon}=\tan^{-1}\{1+(D/C)\},\ \ D/C=2\beta d \end{array}\right\} \tag{52}$$

Therefore, the relationship between the maximum measured horizontal force per unit area F_{max} and the maximum shearing stress $\tau_{o\ max}$ can be expressed from Eqs. (50) and (18) as

$$\tau_{0\max}=\left[\frac{2}{1+\{1+(D/C)\}^2}\right]^{1/2}F_{\max} \tag{53}$$

Fig.4 shows the relation between $\tau_{o\ max}/\rho gH$ and h/L_o with the wave period, in which L_o is the deep water wave length. The experimental data were corrected by applying Eq. (52). In this figure, arrows at each experimental value indicate the range of scatter and circular points are the corresponding mean values. It may be seen from the figure that experimental results agree well with the theoretical values from the linearized theory.

The comparison between the theoretical bottom friction coefficients and the experimental values obtained from the results of shearing stresses is shown in Fig.5 against the wave Reynolds number of R_{eT}. The experimental values were corrected by Eq. (52). Eagleson's data are also shown in the same figure. They are very much larger than the authors' results and scatter considerably. A possible reason for this is that the shear plates used by Eagleson were much larger than those used by the authors, so that the effective forces other than the shearing force would act on the shear plate, and that therefore the correction method for these forces was inadequate. It may be seen from these figures that the experimental values agree sufficiently well with the theoretical ones. This is also to be expected from the theoretical consideration taken into account of the effect of the convective terms, and it is concluded that the effect can be neglected within the range of the present experiments.

EXPERIMENT ON WAVE DAMPING

Experimental equipment and procedures The wave channel, the wave generators and the wave meters used in the experiments were the same as those used in the experiment on shearing stresses. Characteristics of waves and water depths in the experiment are presented in Table 2, in which (1) and (2) indicate the experiments carried out in 1964 and 1965 respectively.

Wave heights at the five or six stations at intervals of 7 m or 9 m were recorded at the same time. Owing to the limitations of the experiment, the wave heights were measured simultaneously from all the stations although not from all the stations at any one time, the wave period being kept constant, and then determined by taking an average of five to ten wave heights when the wave train was uniform, or twenty wave heights when the wave train was somewhat scattered.

Results of the experiment By changing the water depth for each wave period as presented in Table 2, and plotting the experimental values of wave height on semi-log scale paper, the following relationship already derived theoretically in Eq. (42), could be verified:

$$H/H_0=\exp(-\alpha_{(b+w)}x) \tag{54}$$

in which $\alpha_{(b+w)}$ is the damping coefficient including the influences of the bottom and side walls of the wave channel. From Eqs. (39), (42) and (54), the relationships between $\alpha_{(b+w)}$ and $\epsilon_{(b+w)}$, α_b and ϵ_b are expressed respectively as

$$\alpha_{(b+w)}L=\epsilon_{(b+w)}\ , \qquad \alpha_b L=\epsilon_b \tag{55}$$

Therefore, by drawing a fitted straight line in the figure, the wave decay modulus can be calculated from Eq. (54). As the value of $\epsilon_{(b+w)}$ varies widely according to the manner of drawing a straight line, however, the following method was used. For practical purposes, the value of $\epsilon_{(b+w)}$ must be obtained from the wave heights at stations, H and H_1, and the distance x between them. Wave heights were accordingly taken at several pairs of stations. Thus the damping coefficients were calculated from Eqs. (54) and (55) and these were averaged. And then the values of ϵ_b were obtained by applying Eq. (55).

Fig.6 shows the comparison between the experimental values of the wave decay modulus and the theoretical ones for the two cases; one is based on the linearized theory and the other on the non-linear theory in which the effect of the convective terms is taken into account. In this figure, the experimental results obtained by Watson and Martin, Grosch and Lukasik, and Eagleson are plotted, in addition to the authors' results. It is found from the comparison that the experimental values of ϵ_b are nearly as much as 40 % larger than the theoretical ones based on the linearized theory, but when corrected theoretically for the side wall effect based on the non-linear theory, the experimental values decrease by as much as 10 % and approach more closely to the theoretical ones. The data of Grosch and Lukasik were obtained from the experiment on wave damping, which was performed in a wave channel whose width was negligible as far as the side wall effect was concerned, while Eagleson's data are calculated values obtained from the results of the direct measurement of bottom shearing stresses. As mentioned previously, it may be seen that Eagleson's data give much larger values for ϵ_b than those obtained by the authors and by Grosch and Lukasik.

From the above results, it is found that the effect of the convective terms on wave damping is approximately as much as 10 % and yet the experimental values are as much as 30 % larger than the theoretical ones.

Although the reasons why the experimental values of wave damping appear slightly larger than the theoretical ones, are not yet perfectly clear, the following suggestions may be put forward: One of the reasons may be in the application of the wave theory to the theory of wave damping, though Airy's wave theory was applicable to the authors' experiment, and it is necessary to analyze the damping characteristics of finite amplitude waves, such as Stokes's waves. The authors intend to perform successive experiments in order to derive the theoretical formula of wave damping in the case of Stokes's waves and to compare with the experimental results. Secondly, there may be a problem of the transition from laminar to turbulent boundary layers resulting from wave motion. Although most of the authors' data described above were laminar under the criterion of Collins(1963) for the transition. Since, however, there are wide differences between the criteria of different authorities, this problem must be investigated in detail on the basis of further experimental work. Thirdly, the wave energy dissipation on the water surface resulting from a wave should be taken into account as Van Dorn(1966) has treated. The authors wish to investigate such problems through further detailed experiments and to discover the mechanism of wave damping due to bottom friction.

CONCLUSION

As described above, the authors established a theory of the laminar damping of oscillatory waves based on an approximate solution of the boundary layer equation, and measured the bottom shearing stress and the decay modulus of oscillatory waves. It was concluded that the influence of the convective terms in the basic equation on the bottom shearing stress can be negligible, but that on the side wall it becomes quite considerable. With regard to wave damping it was concluded that the experimental values are approximately 30 % larger than the theoretical ones. It would seem that the discrepancy is due to the existence of some other energy dissipation.

Part of this investigation was accomplished with the support of the Science Research Fund of the Ministry of Education, for which the authors express their appreciation. Thanks are due to Messrs. M. Sakai and H. Chen for their help during this investigation and Miss S. Ichiju for her help in preparing the paper.

REFERENCES

Bisel, F. (1949). Calcul de l'amortissement d'une houle dans un liquide visqueux de profondeur finie: La Houille Blanche, pp.630-634.

Bretschneider, C.L. (1954). Field investigation of wave energy loss of shallow ocean waves: BEB, Tech. Memo. No.46, pp.1-21.

Collins, J.I. (1963). Inception of turbulence at the bed under periodic gravity waves: Jour. Geophs. Res., Vol.68, pp.6007-6014.

Eagleson, P.S. (1959). The damping of oscillatory waves by laminar boundary layers: MIT, Hydr. Lab., Tech., Rep., No.32, pp.1-37.

Eagleson, P.S. (1962). Laminar damping of oscillatory waves: Proc. ASCE, Vol.88, pp.155-181.

Grosch, C.E., Ward, L.W. and Lukasik, S.J. (1960). Viscous dissipation of shallow water waves: Phys. Fluids, Vol.3, No.3, pp.477-479.

Grosch, C.E.(1962). Lminar boundary layer under a wave: Phys. Fluids, Vol.5, No.10, pp.1163-1167.

Grosch, C.E. and Lukasik, S.J. (1963). Discussion of "Laminar damping of oscillatory waves": Proc. ASCE, Vol.89, No.HY1, pp.232-239.

Hough, S.S. (1896). On the influence of viscosity on waves and currents: Proc. London Math. Soci., Vol.28, pp.264-288.

Hunt, J.N. (1959). On the damping of gravity waves propagated over permeable surface: Jour. Geophs. Res., Vol.64, No.4, pp.437-442.

Iwagaki, Y., Tsuchiya, Y. and Sakai, M. (1964). Basic studies on the wave damping due to bottom friction(2): Proc. 11th Conf. on Coastal Eng., pp.41-49(in Japanese) and (1965). Coastal Eng. in Japan, Vol.8, pp.37-49.

Iwagaki, Y. and Kakinuma, T. (1965). On the transformation of ocean wave spectra in shallow water and the estimation of the bottom friction factor: Dis. Pre. Res. Inst. Annuals, No.8, Kyoto Univ., pp.379-396. (in Japanese).

Iwasa, Y. (1959). Attenuation of solitary waves on a smooth bed: Trans. ASCE, Vol.124, pp.193-206.

Jonsson, I.G. (1963). Measurements in the turbulent wave boundary layer: IAHR Cong. London, pp.85-92.

Kishi, T. (1954). Studies on sea dikes(5)---Energy dissipation of shallow water waves due to bottom friction---: Report, Pub. Works Res. Inst., No.93, pp.1-9(in Japanese).

Lamb, H. (1945). Hydrodynamics, 6th edition, New York, Dover Publications, p.619.

Longuet-Higgins, M.S. (1953). Mass transport in water waves: Phil. Trans. Royal Soci., London, Series A, No.903, pp.535-581.

Marry, J.D. (1965). Viscous damping of gravity waves over a permeable bed: Jour. Geophs. Res., Vol.70, NO.10,pp.2325-2331.

Nagai, S. and Kubo, H. (1960). Hindcast of maximum waves generated by Ise-wan typhoon at the north seashore of Aichi and Mie Prefectures: Jour. JSCE, Vol.45, No.5, pp.15-26(in Japanese).

Putnam, J.A. and Johnson, J.W. (1949). The dissipation of wave energy by bottom friction: Trans. AGU, Vol.30, pp.67-74.

Putnam, J.A. (1949). The dissipation of wave energy by flow in a permeable sea bottom: Trans. AGU, Vol.30, pp.349-356.

Reid, R.O. and Kajiura, K. (1957). On the damping of gravity waves over a permeable sea bed: Trans. AGU, Vol.38, pp.662-666.

Savage, R.P. (1953). Laboratory study of wave energy by bottom friction and percolation: BEB, Tec. Memo., No.31, pp.1-25.

Schlichting, H. (1960). Boundary layer theory, McGraw Hill, New York, p.207.

Tsuchiya, Y. and Inoue, M. (1961). Basic studies on the wave damping due to bottom friction (1): Proc. 8th Conf. on Coastal Eng., pp.19-24. (in Japanese).

Van Dorn, W.G. (1966). Boundary dissipation of oscillatory waves: Jour. Fluid Mech., Vol.24, pp.769-779.

Table 1. Characteristics of waves and water depths used in the measurement of bottom shearing stress.

(1) Experiment made in 1964

Water depth h(cm)	Wave period T(sec)	Wave height H(cm)
8.2-29.3	0.85	0.63-3.64
9.0-34.3	0.95	0.48-3.64
9.0-29.0	1.10	0.77-3.75
11.0-34.1	1.30	0.65-3.03

(2) Experiment made in 1965

Water depth h(cm)	Wave period T(sec)	Wave height H(cm)
7.0	0.99-1.50	0.26-0.31
10.0	0.99-1.49	0.21-0.95
15.0	0.95-2.50	0.39-3.49
20.0	0.88-3.00	0.61-6.45
25.0	1.01-2.00	4.67-6.84
30.0	1.01-2.58	0.81-10.0

Table 2. Characteristics of waves and water depths used in the experiment on wave damping.

(1) Experiment made in 1964

Water depth h(cm)	Wave period T(sec)	Wave height H(cm)
10.8-24.6	0.80	1.66-6.00
12.0-28.5	1.00	1.53-7.03
25.9	1.10	4.50-6.85
16.3-30.0	1.30	2.40-6.05

(2) Experiment made in 1965

Water depth h(cm)	Wave period T(sec)	Wave height H(cm)
5.6	0.80	0.099-0.117
9.9	0.99-1.23	2.22 -2.89
10.0	1.00-1.47	0.27 -1.31
11.0	0.80	1.74
13.6	0.85-1.53	0.969-1.82
15.0	0.94-1.85	0.442-3.98
16.5	1.01-1.53	1.33 -2.28
17.0	0.80	4.04
20.0	0.87-2.09	0.497- 7.38
20.6	1.15-2.02	1.69 -3.69
23.1	1.02-2.00	2.15 -4.50
25.0	0.97-2.01	3.55 -7.55
30.0	1.00-2.00	1.35 -11.2
35.0	1.15-2.02	0.410-4.10
40.0	1.23-1.54	1.71 -6.67
45.0	1.30-2.28	2.35 -3.62

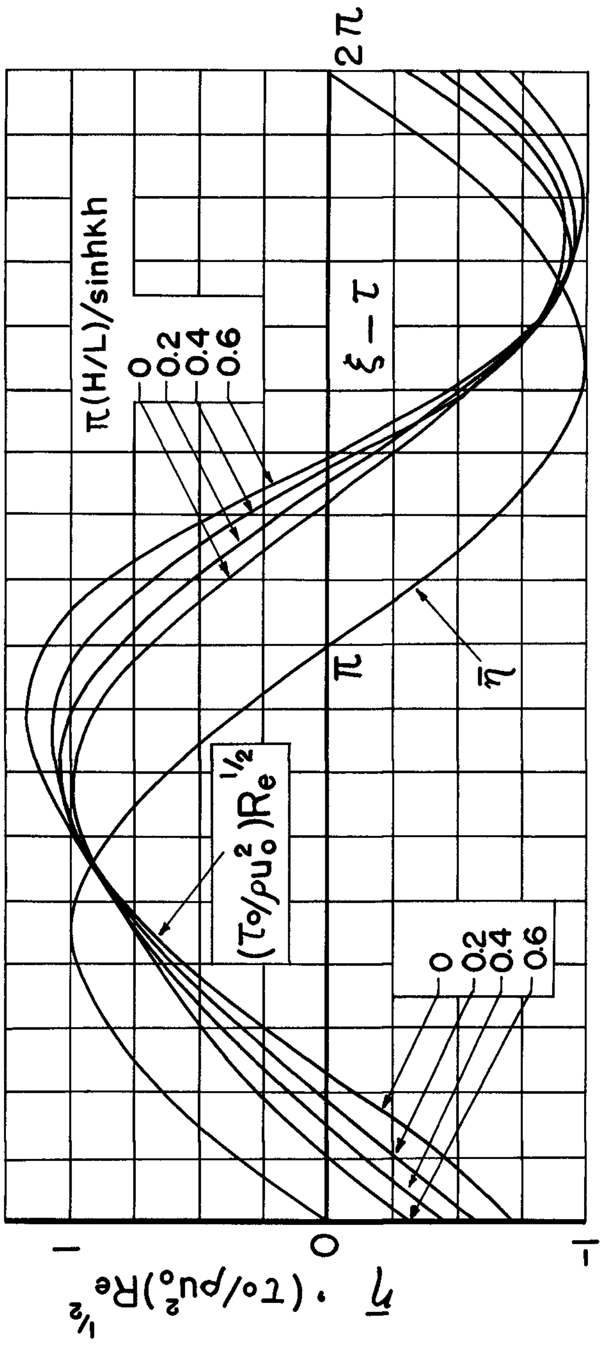

Fig. 1. Effect of convective terms in boundary layer equation on bottom shearing stress.

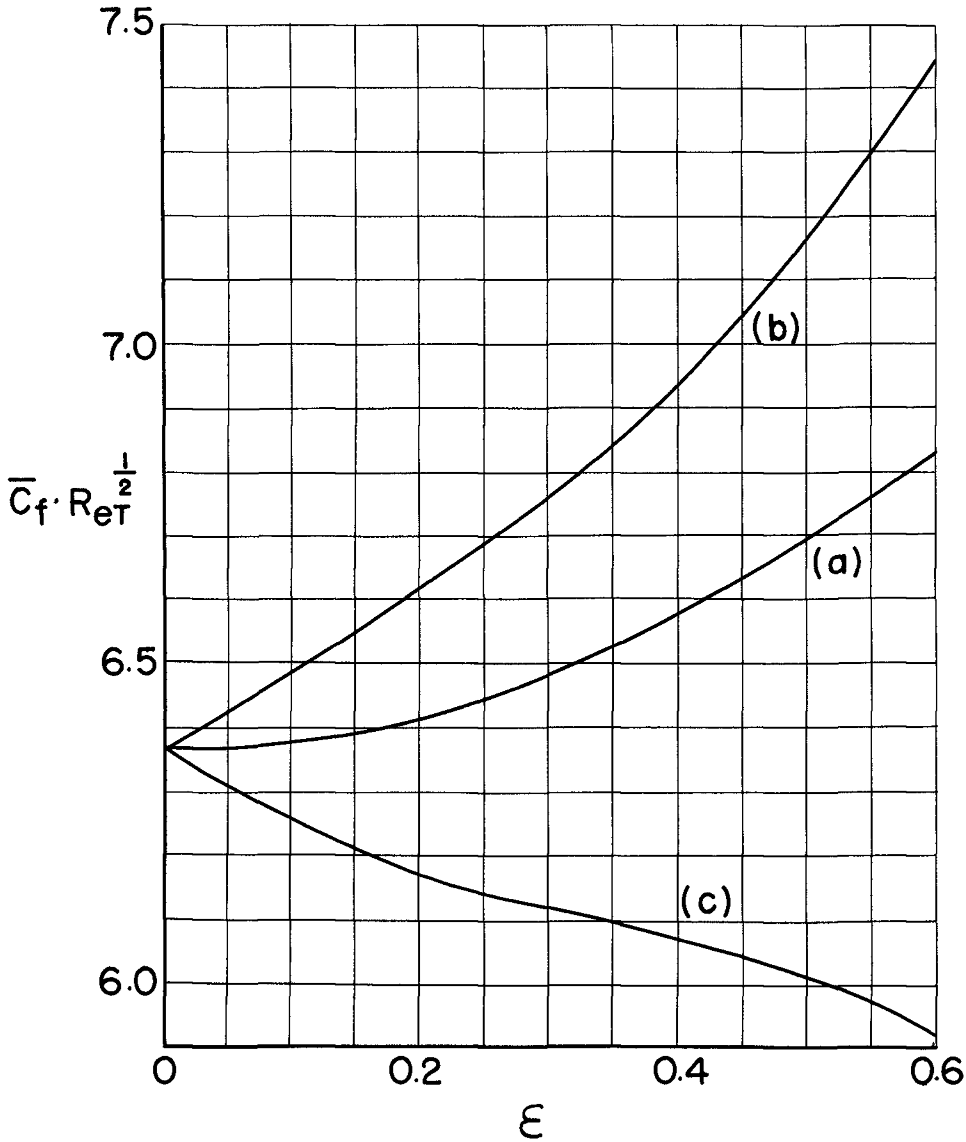

Fig. 2. Effect of convective terms in boundary layer equation on bottom friction coefficient.

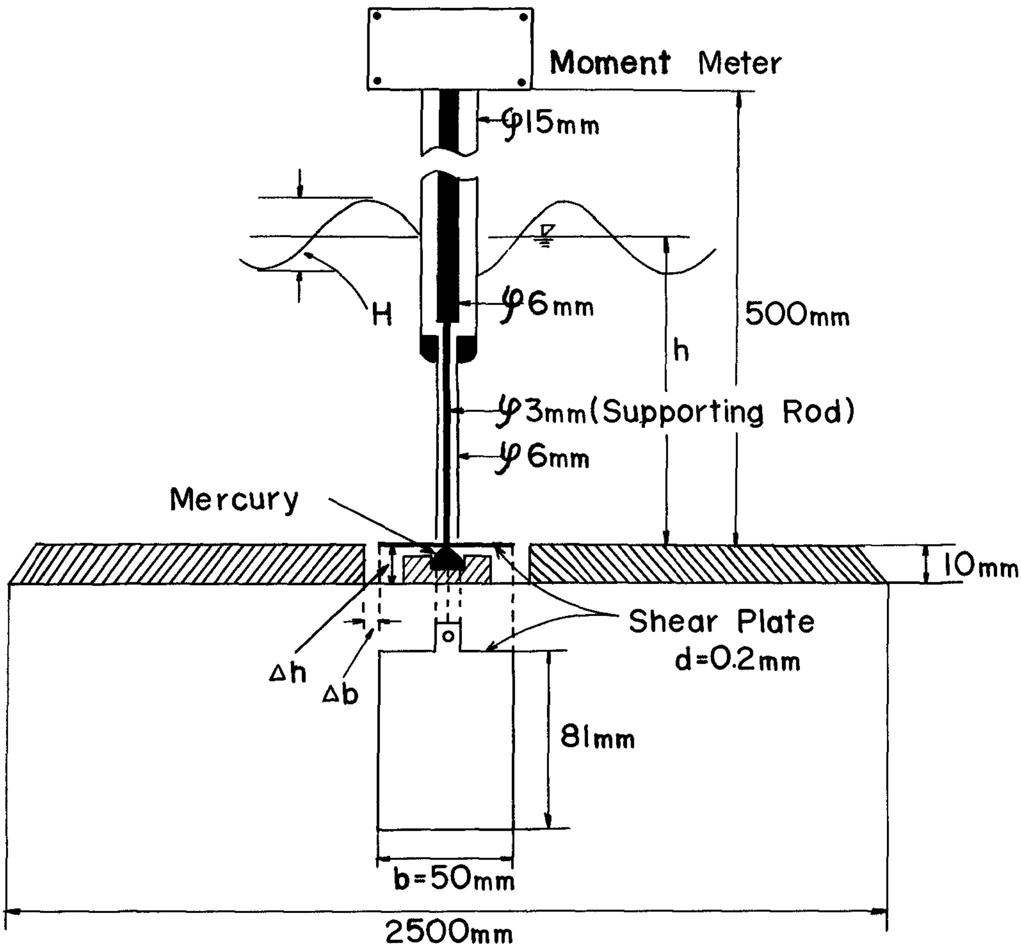

Fig. 3. Schematic diagram of shear meter.

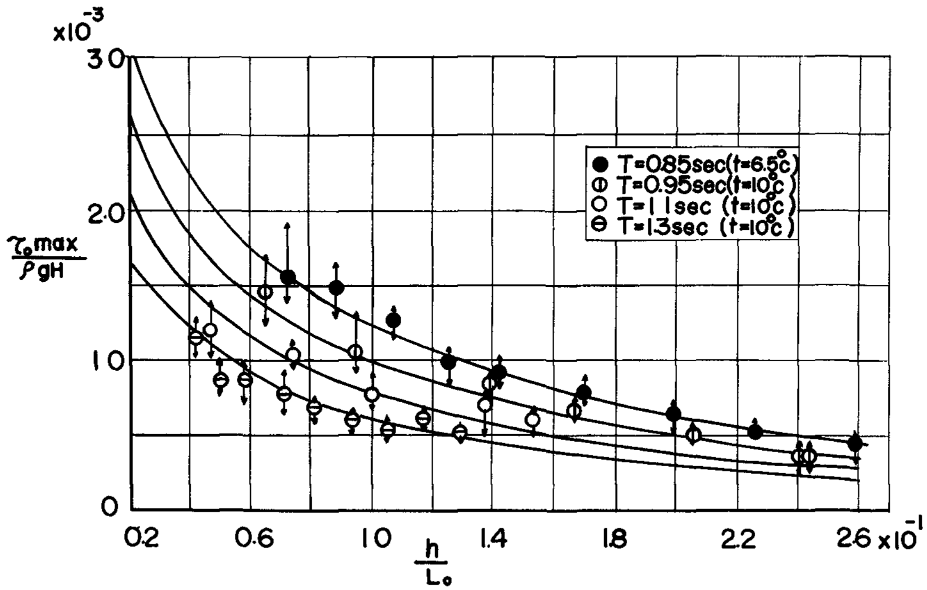

Fig. 4. Comparisons between theoretical curves and experimental results of maximum bottom shearing stress.

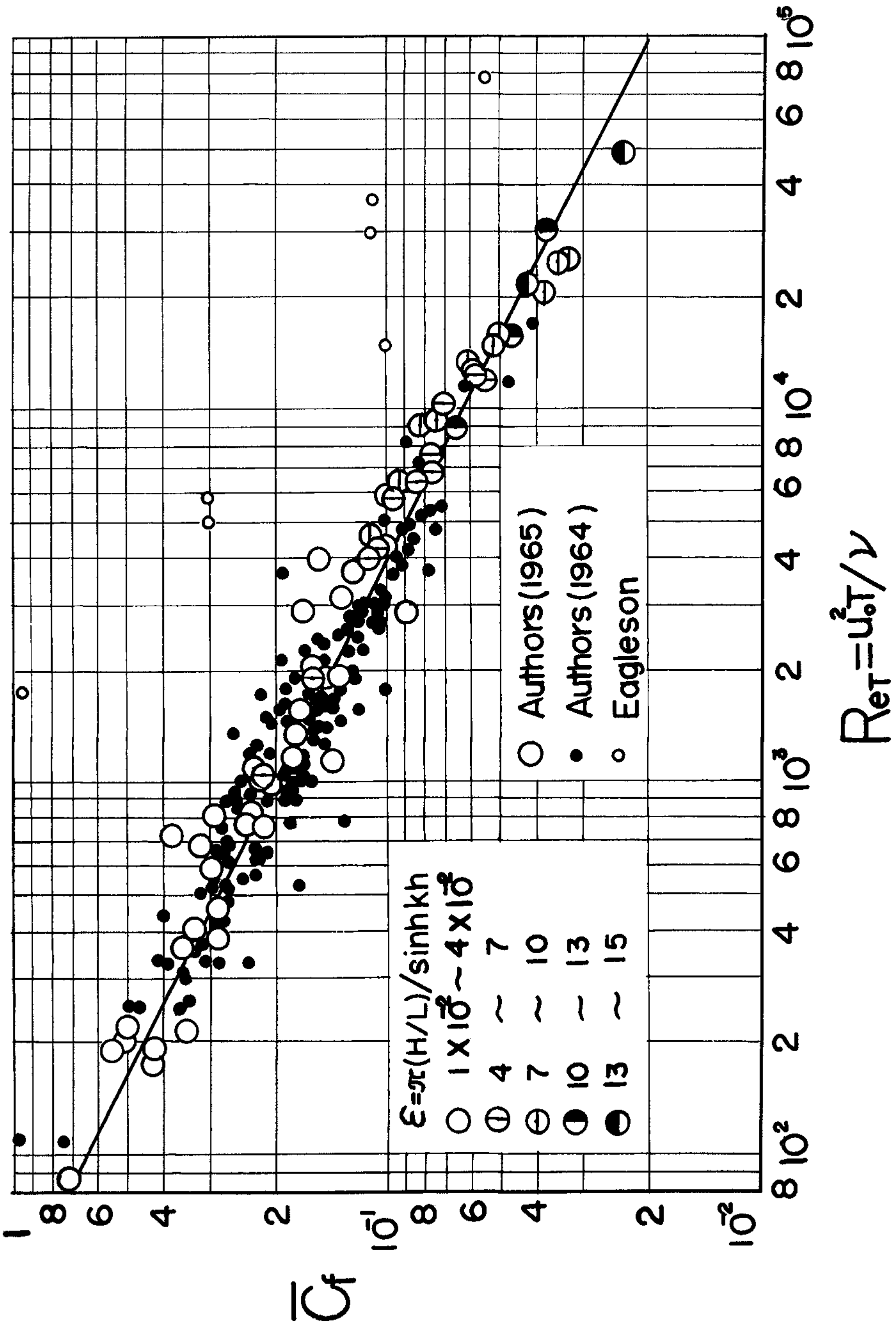

Fig. 5. Relation between average friction coefficient and wave Reynolds numbers.

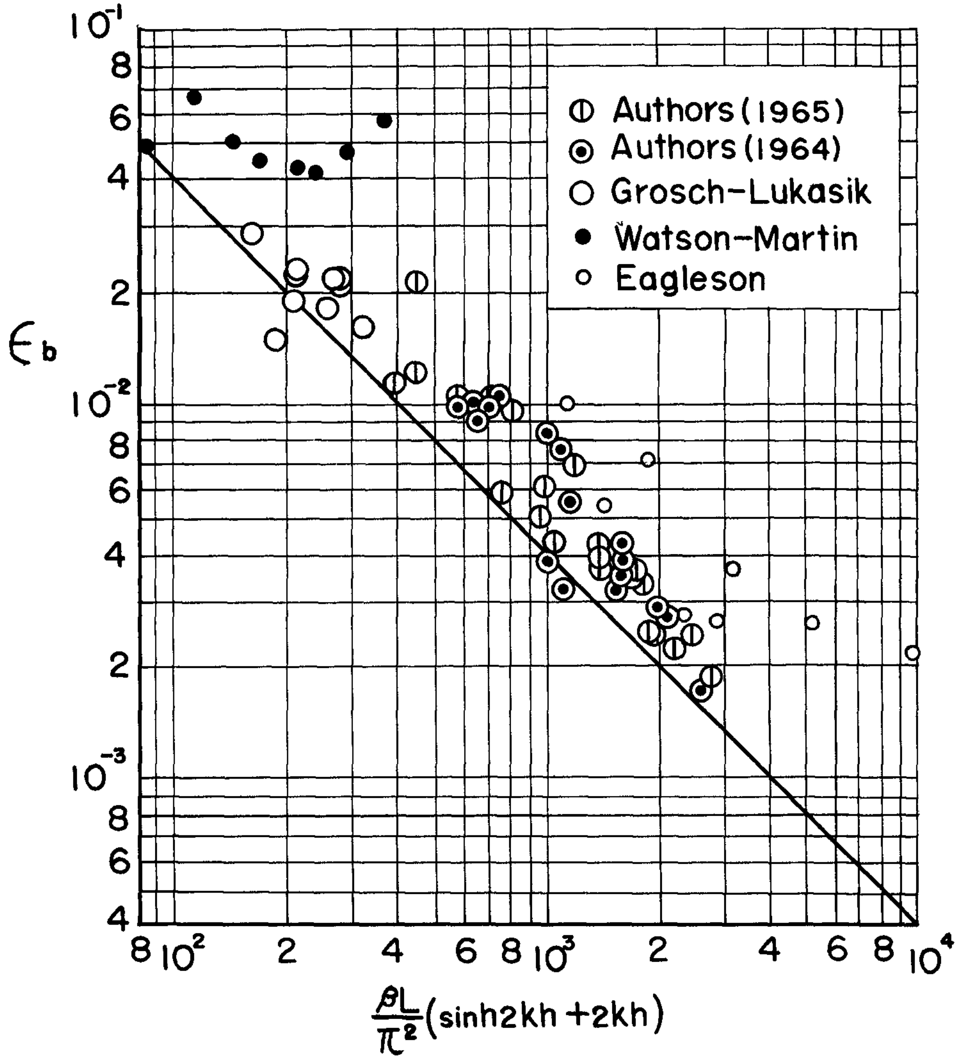

Fig. 6(a). Comparison between theoretical relationship obtained by the linearized theory and experimental results of dimensionless decay modulus.

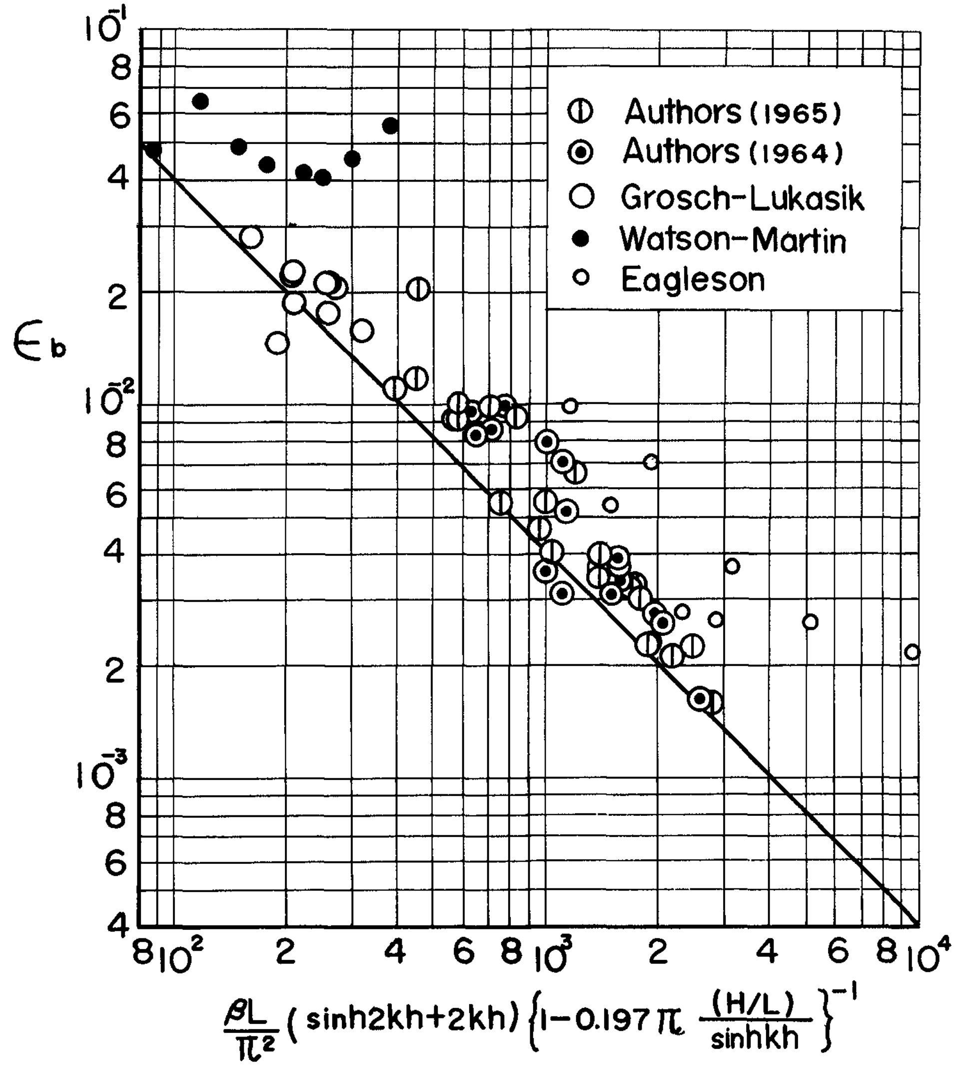

Fig. 6(b). Comparison between theoretical relationship obtained by the non-linear theory and experimental results of dimensionless decay modulus.