CHAPTER 3

A THEORY ON THE FETCH GRAPH, THE ROUGHNESS OF THE SEA AND THE ENERGY TRANSFER BETWEEN WIND AND WAVE

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ABSTRACT

It is the aim of this paper to give theoretical derivations of the wind-wave characteristics from a viewpoint of fundamental mechanism of wind-wave generation.

A hypothesis is proposed as a basic principle of the air-sea interaction, which asserts the maximum ratio of the energy transfer from wind to wave to the energy dissipation within wind. As the theoretical consequences of the hypothesis combined with the recent theories on wind-wave spectra by Miles and Phillips, wind-wave characteristics such as the fetch graph, the roughness of the sea, the spectral peak frequency, the ratio of pressure contribution to total drag and the energy transfer from wind to wave are derived with remarkable agreements with experimental data.

INTRODUCTION

In the latest decade, great progress in our knowledge on the mechanism of wind-wave has been made by the effort of physicists and oceanographers. In 1957, Phillips proposed a theory of wind-wave generation by the mechanism of resonance between the convective turbulent atmospheric pressure fluctuations and the water surface. While, in the same year, Miles presented the instability theory which yields the energy in each component to grow in the exponential manner characteristic of linearized instability model. Later on (1960), Miles has given the combined effect of the two mechanism in an analytical form. On the other hand, Phillips (1958) has given a negative five power law for the equilibrium subrange. These theories have been supported by many experimental data, especially by those of Longuet-Higgins et al (1963).

These theories are, the author believes, to be applied to obtain results of engineering importance. During a few years, the author has continued the investigations to derive from these fundamental theories characteristics of wind-wave such as the relation among the roughness of the sea, the wind speed and the fetch, the ratio of pressure contributions to total drag, the spectral peak frequency and the fetch graph, the non-dimensional relationship among wind speed, wave height, wave velocity and fetch. In 1963's paper, he obtained theoretically, with some degree of success, the roughness coefficient and the fetch graph using the Phillips-Katz assumption on the transitional frequency or the spectral peak frequency and the assumption that the roughness of the sea is proportional to the root-mean-square wave height. The results were only applicable in the case of small fetches because of the second assumption.

In the succeeding 1964 and 1965's papers, the above two assumptions are wholly rejected, a new hypothesis being proposed as an alternative. This paper is the final report on this problem.

THEORY

BASE OF THE THEORY

Basically, the theory to be presented here is composed of only two fundamental theories and a hypothesis on the mechanism of the air-sea interaction.

The wind-wave spectrum for the lower frequency side of the spectral peak is considered to be given by the Miles-Phillips theory. While, the higher frequency side of the spectrum is approximated by the Phillips' theory on the equilibrium subrange. The third principle, the maximum ratio of energy transfer from wind to wave, to energy dissipation within wind is introduced to determine the spectral form, because the Miles-Phillips' spectrum involves the shear velocity as an unknown value.

Although, the basic concept of the author's theory is very simple, several components of equations as given in the following sections are to be known for detailed numerical calculations.

GENERATION OF WIND-WAVE

The analytical form of the combined effect of the resonance-instability mechanism is expressed by eq. (1),

$$\Phi(\mathbf{k},t) \sim (2\rho_w^2 c^2)^{-1} t\, F(mt) \int_0^\infty \Pi(\mathbf{k},\tau) \times \cos[k(V\cos\varphi - c)\tau]\, d\tau$$
$$= (2\rho_w^2 c^2)^{-1} t\, F(mt)\, \Pi(\mathbf{k},o)\, \Theta(\mathbf{k}, c\sec\varphi - V) \quad \cdots \quad (1)$$

where ρ_w is the water density, $c(k) = (g/k)^{1/2}$ denotes the phase velocity of free surface wave of wave number k, φ is angles of travelling wave to wind blowing with the convection velocity V ; $\Pi(\mathbf{k},\tau)$ denotes the spectrum of the pressure fluctuation at the water surface at time τ in the moving frame of reference, Θ represents the integral time scale and

$$m = \frac{1}{2}\varsigma k c \quad (\ll 1), \qquad \cdots\cdots\cdots (2)$$

$$F(mt) = (2mt)^{-1}(e^{2mt} - 1) \qquad \cdots \quad \cdots\cdots (3)$$

ς, the fractional increase in wave energy per cycle is expressed as

$$\varsigma = (kc\overline{E})^{-1}(\partial\overline{E}/\partial t)$$
$$= \frac{\rho_a}{\rho_w}\beta\left(\mu\frac{U_1\cos\varphi}{c}\right)^2 - \frac{4g\nu_w}{c^3} - \frac{\rho_a}{\rho_w}\left(\frac{g\nu_a}{2c^3}\right)^{1/2}$$
$$\times \left[1 + 2(\alpha+\beta)\left(\frac{U_1\cos\varphi}{c}\right)^2 + (\alpha^2 - \beta^2 + 2\alpha\beta)\left(\frac{U_1\cos\varphi}{c}\right)^4\right]$$

where α and β are real, non-dimensional function of both c and k dependent on the solution of the Orr-Sommerfeld equation to the aerodynamic boundary value problem, $U_1 = U_* / \kappa$ and U_* and κ represent the shear velocity and the Karman constant, respectively. The second and third terms on the right hand side of eq. (4) (i.e. the viscous dissipation in water and air, respectively) are negligible compared to the first term, the positive energy-transfer from the shear flow. The numerical values of the non-negative damping factor β published in graphical forms by Miles are reduced to interpolation formulae for convenience of numerical calculation by means of a digital computer;

$$\left.\begin{aligned}
&\beta = 3.39 - 0.9406\,[\,\log(8.61\times10^{-3}/\xi_c)\,]^{1.860} \\
&\qquad\qquad (\xi_c \leq 8.61\times10^{-3}) \\
&\beta = 3.39 - 1.294\,[\,\log(\xi_c/8.61\times10^{-3})\,]^{2.323} \\
&\qquad\qquad (8.61\times10^{-3} < \xi_c \leq 5.48\times10^{-2}) \\
&\beta = -0.1402 - 2.181\,\log\xi_c \\
&\qquad\qquad (5.48\times10^{-2} < \xi_c \leq 3\times10^{-1}) \\
&\beta = \{\,\log(3/\xi_c)\,\}^{2.362} \\
&\qquad\qquad (3\times10^{-1} < \xi_c \leq 2) \\
&\beta = 0.017\,\exp\{\,2(2-\xi_c)\,\} \\
&\qquad\qquad (2 < \xi_c)
\end{aligned}\right\}\quad \cdots\cdots\cdots(5)$$

where $\xi_c = \Omega\left(\dfrac{U_1\cos\varphi}{c}\right)^2 \exp\{c/U_1\cos\varphi\}$, $\Omega = g z_0/(U_1\cos\varphi)^2$ and z_0 denotes the roughness parameter. The value of β becomes almost independent of Ω when $\xi_c < 2$.

If a wind flows over the water surface of a finite fetch, transformation of the wind duration time, t, to the fetch, F, is given by (Phillips 1958a)

$$F = \frac{1}{2}\,c(k)\,t. \qquad \cdots\cdots(6)$$

The pressure fluctuations acting on the water surface play, according to the resonance theory, a role of trigger for the initiation of water-level perturbation which is further amplified by the instability mechanism. There is a scarcity of experimental data on the atmospheric pressure fluctuations, especially for those on water surface. While, theories as well as experimental data from aeronautical science are easily available (Kraichnam (1956), Lilley & Hodgson (1960), Lilley (1960), Serafini (1962), Corcos (1964)). Among them we choose the equation by Lilley (1960) for the composition of our wave theory. The pressure fluctuations along the wall are contributed from sources at the wall region of high shear-turbulence interaction as well as at the outer mixing region of free turbulence The equation by Lilley is

$$\Pi(k, o) = \frac{\rho_a^2\,\tau_1^2\,\overline{w'}^2\,k_1^2\,\exp(-k^2/4\sigma^2)}{4\pi\sigma^4\,(k+b+\ell_3^{-1})(k+b-\ell_3^{-1})} \qquad \cdots\cdots(7)$$

or

$$\Pi(k, o) = \frac{\rho_a^2\,(\overline{w'}^2/U_*^2)\,(\tau_1\,\delta_1/U_*)^2}{4\pi\,(\sigma\,\delta_1)^2} \times \frac{U_*^4\,\delta_1^4\,(k\cos\varphi)^2\,\exp(-k^2/4\sigma^2)}{\{(k\delta_1)^2 + 2(b\delta_1)(k\delta_1)\}} \qquad \cdots\cdots(8)$$

where τ means the mean shear given by dU/dz and is approximated by $\tau = \tau_1 e^{-by}$, $\sqrt{w'^2}$ denotes the root-mean-square value of velocity component normal to the wall, k_1 is the component of wave number k in the direction of flow axis x, σ defines the shape of the conventional longitudinal velocity correlation coefficient $f(r) = \exp(-\sigma^2 r^2)$ and ℓ_3 is the scale of the energy containing eddies in the direction normal to the surface. The values of constants in eqs. (7) and (8) are estimated from the results of either Laufer (1955) or Grant (1958) as follows; $\sqrt{w'^2}/U_* = 0\ 8$, $\tau_1 \delta_1 / U_* = 3\ 7$, $b\delta_1 = 0\ 31$, $\sigma\delta_1 = 1/2$ and $\delta_1 / \ell_3 = 1/2$.

Phillips (1957) assumed an expression of θ as

$$\theta \fallingdotseq 1/k\ [\ U(k) - c(k) \sec \varphi\ ] \qquad \ldots\ \ldots\ldots(9)$$

However, a more rational expression of integral time scale may be obtained. The spectrum of pressure fluctuations is described as, using the space-time correlation $R(\xi, \eta, \tau)$,

$$\Pi(\mathbf{k}, \tau) = (2\pi)^{-2} \iint R(\xi, \eta, \tau) \exp\{-i(k_1\xi + k_2\eta)\, d\xi\, d\eta \qquad \ldots\ \ldots(10)$$

which is converted into eq (11) with use of the Taylor's hypothesis of frozen turbulence —— $R(\xi, \eta, \tau) = R(\xi - U_c\tau, \eta, 0)$——,

$$\begin{aligned}\Pi(\mathbf{k}, \tau) &= (2\pi)^{-2} \iint R(\xi_1, \eta, 0) \exp\{-i(k_1\xi + k_2\eta)\} \cos(k_1 U_c \tau)\, d\xi_1\, d\eta \\ &= \Pi(\mathbf{k}, 0) \cos(k_1 U_c \tau)\,. \end{aligned} \qquad \ldots\ \ldots(11)$$

On the other hand, the cross-spectral density $\Gamma(\xi, \eta, \omega)$ defined by eq. (12)

$$\Gamma(\xi, \eta, \omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} R(\xi, \eta, \tau) \exp(-i\omega\tau)\, d\tau \qquad \ldots\ldots\ .(12)$$

has been verified experimentally to be expressed

$$\Gamma(\xi, 0, \omega) = \Gamma(0, 0, \omega) \cos(\omega\xi/U_c)\, A(\omega\xi/U_c) \qquad \ldots\ \ldots(13)$$

where $A(\omega\xi/U_c)$ is a decreasing function of $\omega\xi/U_c$ to be approximated by

$$A(\omega\xi/U_c) = \exp(-0\ 7\, \omega\xi / 2\pi\, U_c)\,. \qquad \ldots\ldots\ldots(14)$$

The right hand side of eq. (13) is eqivalent to the coversion of eq. (12), with use of the Taylor's hypothesis, multiplied by a function $A(\omega\xi/U_c)$.

Therefore, the integral time scale is finally rewritten as

$$\theta = \int_0^\infty \{ \Pi(\mathbf{k},\tau)/\Pi(\mathbf{k},0)\} \cos[k(U_c \cos\varphi - c)\tau]\, d\tau$$

$$= \frac{a_0 U_c \cos\varphi}{2k}\left[\frac{1}{(a_0 U_c \cos\varphi)^2 + \{2U_c \cos\varphi - c(k)\}^2} + \frac{1}{(a_0 U_c \cos\varphi)^2 + c^2(k)}\right], \qquad \ldots (15)$$

where $a_0 = 0.7/2\pi$. Phillips (1957) assumed the convection velocity of pressure fluctuations, U_c, as the mean wind speed at a height $2\pi/k$ from the water surface. Certainly, experiments by Serafini (1962) show a slight decrease in U_c with increasing wave number k; however, the degree of decrease in U_c is so small as to be treated almost constant, $U_c \doteqdot 0.8\, U_\infty$.

EQUILIBRIUM RANGE OF WAVE SPECTRUM

If the wind continues to blow over the water, the growth of waves is limited by the formation of white caps or white horses which dissipate a local excess of the energy supplied from the wind into turbulence in water. This critical or saturated state is the condition that the particle acceleration of water at the wave crest equals the gravitational acceleration. In the limiting equilibrium or saturated state, the form of the wave spectrum must be determined by the physical parameters which govern the form of wave crest. On dimensional grounds, Phillips (1958c) described the frequency spectrum as follows,

$$\psi_e(\omega) = \lambda g^2 \omega^{-5} \qquad [\omega_0 \ll \omega \ll (4\rho_w g^3 T^{-1})^{1/4}] \qquad \ldots\ldots (16)$$

where $\psi_e(\omega)$ is defined by

$$\psi_e(\omega) = (\pi)^{-1} \int_{-\infty}^{\infty} \overline{\eta(\mathbf{x},t)\,\eta(\mathbf{x},t+\tau)}\; e^{-i\omega\tau}\, d\tau \qquad \ldots\ldots (17)$$

and λ is a constant ($\lambda = 1.48\times10^{-2}$). The existance of such an equilibrium range even up to near spectral peak was supported by many investigators (Burling (Phillips 1958c), Kinsman (1961), Kitaigorotskii (1962), Kitaigorotskii & Strekalov (1962) and Longuet-Higgins (1962))

The Miles-Phillips spectrum, eq. (1), is represented in two-dimensional form in respect to wave number vector $\mathbf{k}$; while, the equilibrium spectrum eq. (16) is one-dimensional frequency spectrum. In order to obtain the intersection of the two spectra, we must transform the two-dimensional representation of eq. (1) into one-dimensional form $\psi(f)$ ($f = \omega/2\pi$),

$$\psi(f) = \int \Phi(\mathbf{k}) \frac{2\pi f}{c}\left[\frac{2\pi}{c + k\dfrac{\partial c}{\partial k}}\right] d\varphi$$

$$= \int \Phi(\mathbf{k},\varphi) \frac{16\pi^2 f}{\left[\dfrac{g}{k} + \dfrac{3T}{\rho_w} k\right]}\, d\varphi\,. \qquad \ldots\ldots (18)$$

On the other hand, two-dimensional form of eq. (16) is assumed to be

$$\Phi_e(\mathbf{k}) = \Phi(k_*, \varphi)(k/k_*)^{-4} \qquad \text{..........(19)}$$

where k_* denotes the transition frequency or the spectral peak frequency and $\Phi(k_*,\varphi)$ is the spectrum eq. (1) corresponding $k=k_*$.
Laboratory experiments by Hamada et al. (1963) support in the order of magnitude the values of β given by Miles, and the Phillips spectrum for equilibrium subrange has been proved to cover far wider range enough to the spectral peak However, the frequency of the intersection of eqs. (16) and (18) gives higher value than experimental value of the spectral peak frequency. Therefore, a constant factor μ (= 4) independent of other factors such as U_0, F, c and k is introduced in eq.(4), considering that the functional form F(mt) is to be modified by some method. Although Phillips and Katz (1961) considered that the transition from linear growth to an exponential one of an instability begins at wind duration such that mt is of order unity corresponding to the so-called steep forward face, a steep increase in the Miles-Phillips spectrum seems to occur at $mt = \mu^2$

INTRODUCTION OF ENERGY HYPOTHESIS

At first, the author considered that the above mentioned two spectra, the Miles-Phillips equation and the equilibrium spectrum, when they are connected determine an approximate shape of wave spectrum; i.e. the wave spectrum may be approximated both by eq. (1) for the lower frequency region of spectral peak frequency and by eq. (16) or (19) for the higher frequency region. However, it was soon found impossible because the value of U_* should be given by any method in order to fix eq. (1) under prescribed values of U and F.

It is to be prohibited to assume a ratio of U_* to U_0, since the ratio squared is the roughness coefficient of the sea, one of the very results to be derived theoretically in this paper. On the other hand, increasing an assumed value of U_* results at first in increase of the value U_{*p} ($U_{*p}=\sqrt{\tau_p/\rho}$; τ_p means the pressure contribution to total drag) and then in decrease of the value, the state of equality of U_* and U_{*p} being impossible to be found (Fig. 2).

$$\tau_p = \int_0^{k_*}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \rho_a (U_1 \cos\varphi)^2 \beta k^3\, \Phi(\mathbf{k}) \cos\varphi\, d\varphi\, dk$$

$$+ \int_{k_*}^{\infty}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \rho_a (U_1 \cos\varphi)^2 \beta k_*^4\, \Phi(k_*,\varphi)\, k^{-1} \cos\varphi\, d\varphi\, dk \qquad \text{..........(20)}$$

Here, as a determing principle, the author introduces a hypothesis. It asserts that the ratio of the energy transfer from wind to wave to the energy dissipation within the wind should be maximum Consequences of this hypothesis will be shown in the next section.

In relation to this hypothesis, let us consider the energy transport process within an inspection region enclosed by line ABCD in Fig. 3. Across the line AD, work, $U\tau_0$, is done by wind shear and a small amount of energy, D_f, is convected by turbulence. On the other hand, the energy of wind is dissipated mostly within wind by turbulence (ε_T) and partly by wake behind small wavelets and ripples (ε_K), the rest of wind energy being converted to the energy of wave generation (E_w). The energy thus supplied from wind in the form of wave generation is lost partly into heat and the momentum of drift current by white cap and wave breaking (E_B) and partly transferred to higher frequency components by the non-linear effect, finally being dissipated into heat by viscosity (E_v) (Fig. 4),

$$U\,\tau_0 - D_f = \varepsilon_T + \varepsilon_K + E_w$$

$$U\,\tau_0 - D_f = \varepsilon_T + \varepsilon_K + E_B + E_v\,.$$

The hypothesis presented above that the maximum ratio of energy transfer from wind to wave to energy dissipation within the wind above the sea surface is equivalent to say that Nature controls himself to the state that the energy transmitted outside of a system becomes maximum and the energy loss within the system be minimum. The proverb says that Nature prefers the principle of maximum or minimum.

METHOD OF NUMERICAL CALCULATION

If we assume tentatively a value of shear velocity U_*, under prescribed wind speed at reference level U_0 and fetch F, the shape of wave spectrum is defined by eqs.(1) and (19) or by eqs. (16) and (18). Firstly, a tentative value of spectral peak frequency k_* is obtained from the intersection of eqs. (16) and (18). Then, using eqs. (1) and (19), the energy transfer from wind to wave E_w and the energy loss of wave by viscous dissipation may be calculated, respectively, by

$$E_w = 2\rho_a U_1^2 \int_0^\infty\int_0^{\pi/2} c\beta k^3\, \Phi(\mathbf{k}) \cos^2\varphi \, d\varphi\, dk \qquad \text{..} \quad \text{.....(21)}$$

$$E_v = 8\,\nu_w \rho_w \int_0^\infty\int_0^{\pi/2} k^4 c^2\, \Phi(k,\varphi)\, d\varphi\, dk \qquad \text{..........(22)}$$

The energy dissipation of wind below height $z = z_1$ is given by

$$D = \rho_a U_*^2 \left(U_0 - \frac{U_*}{\kappa} \ln \frac{10^3}{z_1} \right) \qquad \text{..........(23)}$$

where U_0 denotes wind speed at $z = 10^3$ cm. The relationship, eq.(23), assumes a logarithmic law of wind profile applicable for the neutral atmospheric stratification Although many experimental supports to the log-law have been reported, there exist some objections to the validity of the relationship. For instance, Stewart (1961) discussing the wave drag of wind concluded the gradual deviation from the log-law near the sea surface Sheppard (1951) and Takeda (1963) found experimentally a kink on the velocity profile above the sea surface. However, we shall proceed for the moment to the following discussions considering eq (23) to be valid. The spectra being treated as two-dimensional, whole calculations have been processed by means of a digital computer (IBM 7090). As the assumed value of U_* is increased the ratio E_w/D increases at first and then it turns to decrease. By interpolation we can determine U_* at which the ratio E_w/D takes a peak value According to the author's hypothesis, the value of U_* thus determined is the very shear velocity which actually occurs at the given wind speed and fetch. The spectral peak frequency k_* as well as the shape of wave spectrum are also determined definitly from the two spectra eqs. (1) and (19). The drag contributed by pressure τ_p is given by eq. (20). The drag coefficient γ^2 defined by

$$\tau_0 = \rho_a \gamma^2 U_0^2 \qquad \text{..........(24)}$$

is calculated from

$$\gamma^2 = (U_* / U_0)^2 \qquad \text{.... ....(25)}$$

Another definition of the roughness of the sea, the roughness parameter z_0, may easily be obtained from

$$\frac{U}{U_*} = \frac{1}{\kappa} \ln \frac{z}{z_0} \qquad \text{......... (26)}$$

by putting $U=U_0$ cm/s and $z=10^3$ cm. The significant wave height $H_{1/3}$ is given by (Longuet-Higgins (1952) and Cartwright & Longuet-Higgins (1956))

$$H_{1/3} = 2\sqrt{2}\,\sqrt{E} = 4\sqrt{\overline{\eta^2}} = 4\,[\int_0^\infty \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \Phi(\mathbf{k})\, d\varphi\, dk\,]^{\frac{1}{2}} \;. \qquad \text{..... ....(27)}$$

Thus, the important characteristics of wind-waves have been determined theoretically.

RESULTS AND DISCUSSIONS

PRESSURE CONTRIBUTION TO TOTAL DRAG

It is generally considered that a large propotion of the drag on a water surface is exerted by the pressure contribution, the wave drag (Steward 1961) The theory presented in this paper gives quantitatively the ratio as a function of the wind speed U_0 and the fetch. The energy hypothesis determines the shear velocity U_*, the spectral peak frequency k_* and consequently the shape of wave spectrum $\Phi(\mathbf{k})$. Then, the total drag $\tau_0=\rho U_*^2$ as well as the pressure drag $\tau_p=\rho U_{*p}^2$ may easily be calculated from eqs. (1), (19) and (20). It is shown in Fig. 5 that the pressure contribution to the total drag increases as fetches increase approaching the condition that the drag is wholly exerted in the form of wave drag. Stewart (1961) estimated a lower limit of τ_p/τ_0 to be 0.2 and considered that most of the momentum from the air enters in the form of wave motion, while in a recent paper Miles (1965) obtained the relationship $\tau_p/\tau_0=0\ 28\,(1+70\,U_0^{-1})^{-1}$. Considering the assumptions used in his paper, the Miles' estimate may be suitable only for small wind speeds and fetches, to be included as a part of the author's results.

It seems very natural that the drag on the water surface is due mainly to the frictional resistance for small fetches and to the wave drag for larger fetches, because for the former case the high frequency waves with small amplitude prevail acting as roughness elements, while for the latter the low frequency waves with high velocities comparable with wind speed predominate. The conclusion supports the results of calculation obtained in the first paper (Hino 1963) where the root-mean-square wave height is assumed to be proportional to the roughness parameter, although the assumption has been wholly abandoned in this paper.

RATIO OF ENERGY INPUT FROM WIND TO WAVE TO DISSIPATION WITHIN WIND

The determing principle of the energy hypothesis yields the ratio E_w/D as shown in Fig. 6. The fact that the ratio increases with increasing fetch is a counterpart of the above conclusion on the wave drag. The direct viscous dissipation into heat by waves of wave number less than $k = 4\ 0$ has been shown to be very small, while the most part of wave energy is dissipated by the process of wave breaking in equilibrium subrange, the energy transfer to high frequency region by non-linear effect and the viscous dissipation in high frequency waves. The process is explained schimatically in Fig. 4.

SPECTRAL PEAK FREQUENCY

The spectral peak frequency f_* is a feature characterizing the wind-wave The theoretically obtained results are plotted non-dimensionally in Fig. 7 which compares well with experimental results by Kitaigorotskii & Strekalov (1962). Neumann gives a relationship

$$2\pi f_* = \sqrt{2/3}\ \ g/U_0 \qquad \text{.... .....(28)}$$

which is further modified as

$$U_* f_*/g = 1\ 3 \times 10^{-1}\ U_*/U_0$$
$$= (1\ 6 \sim 2\ 1) \times 10^{-2}\ . \qquad \text{...... ...(29)}$$

This non-dimensional relation between the fetch F and the spectral peak frequency f_* also agrees with the author's results.

ROUGHNESS OF THE SEA

The roughness of the sea is one of the most important characteristics of wind-waves as a basis of calculations of oceanographical phenomena, such as storm surges, drift currents and so on.

Neither well-defined empirical formula nor theories have been found in the present stage (1965). In spite of agglomerations of a lot of experimental data, scatters between them are so remarkable that derivation of any reliable empirical formula seems to be impossible. However, the author's theory developed above yields the shear velocity U_* as a function of U_0 and F. Thus, the roughness of the sea is calculated from eqs. (25) or (26) (Figs. 8 and 9). The roughness coefficient increases with increasing wind speed and fetch. In the range of these calculations, the roughness coefficient seems not to approach a constant value with increasing wind speed but to continue to increase The tendancy contradicts the widely believed concepts; however, the recent laboratory experiments by Kunishi (1966) also gives the same results.

FETCH GRAPH

The non-dimensional representation of the relation among wind speed, wave height (significant wave height), wave velocity and fetch is called the fetch graph. The theory of Sverdrup-Munk-Bretschneider (Ishihara & Hom-ma 1958) is composed of a complexity of assumptions including the empirical relationship between the roughness of the sea and the wind speed. While, the theory developed above yields, without such assumptions, $\sqrt{\overline{\eta^2}}$ and c for given values of U_0 and F, to give the so-called fetch graph (Fig. 10). The theoretical curves fit well the experimental data collected by Wiegel (1963). The curve for $gH_{1/3}/U^2$ versus gF/U^2 coincides remarkably well with the recently publsined empirical formula by Wilson (1966), Formula IV eq. (12 ii), and the one for c/U versus gF/U^2 fits reasonably well with the empirical curve, Formula IV eq. (12 i), over a middle range of gF/U^2 values, but tends to deviate at high and low values.

As already given in the previous paper (Hino 1963), the fetch graph shows the effect of fetch which is a fact supported by the experiments by Kunishi (1962) However, detailed calculations on this effect was not performed in this paper because the effect becomes predominant only for the case of very low wind speed, having negligible importance from the viewpoint of engineering.

CONCLUSION

In conclusion, it is believed that the theory presents the first theoretical results ever published on the first four problems, the ratio of pressure contribution to total drag on wave, the ratio of energy transfer from wind to wave to energy dissipation above the water surface, the spectral peak frequency and the roughness of the sea, each as a function of wind speed and fetch.

Moreover, the fetch graph is derived with as less assumptions as possible from a quite different point of view from the theory of Sverdrup and others.

It must be emphasized that the above theoretical results are deduced from a higher altitude of basic theories of wind-wave mechanism and they are independent conclusions each other.

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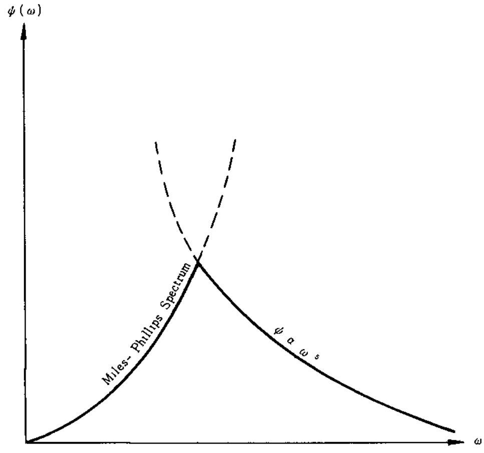

Fig. 1. Model of wind-wave spectrum.

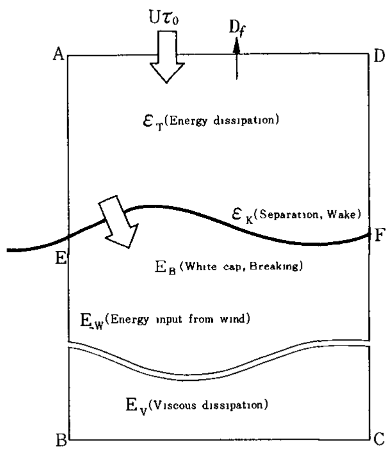

Fig. 3. Energy process between wind and wave.

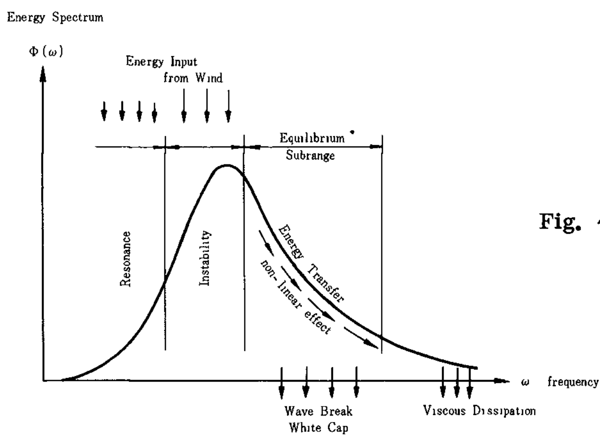

Fig. 4. Energy transfer process in wind-wave spectrum.

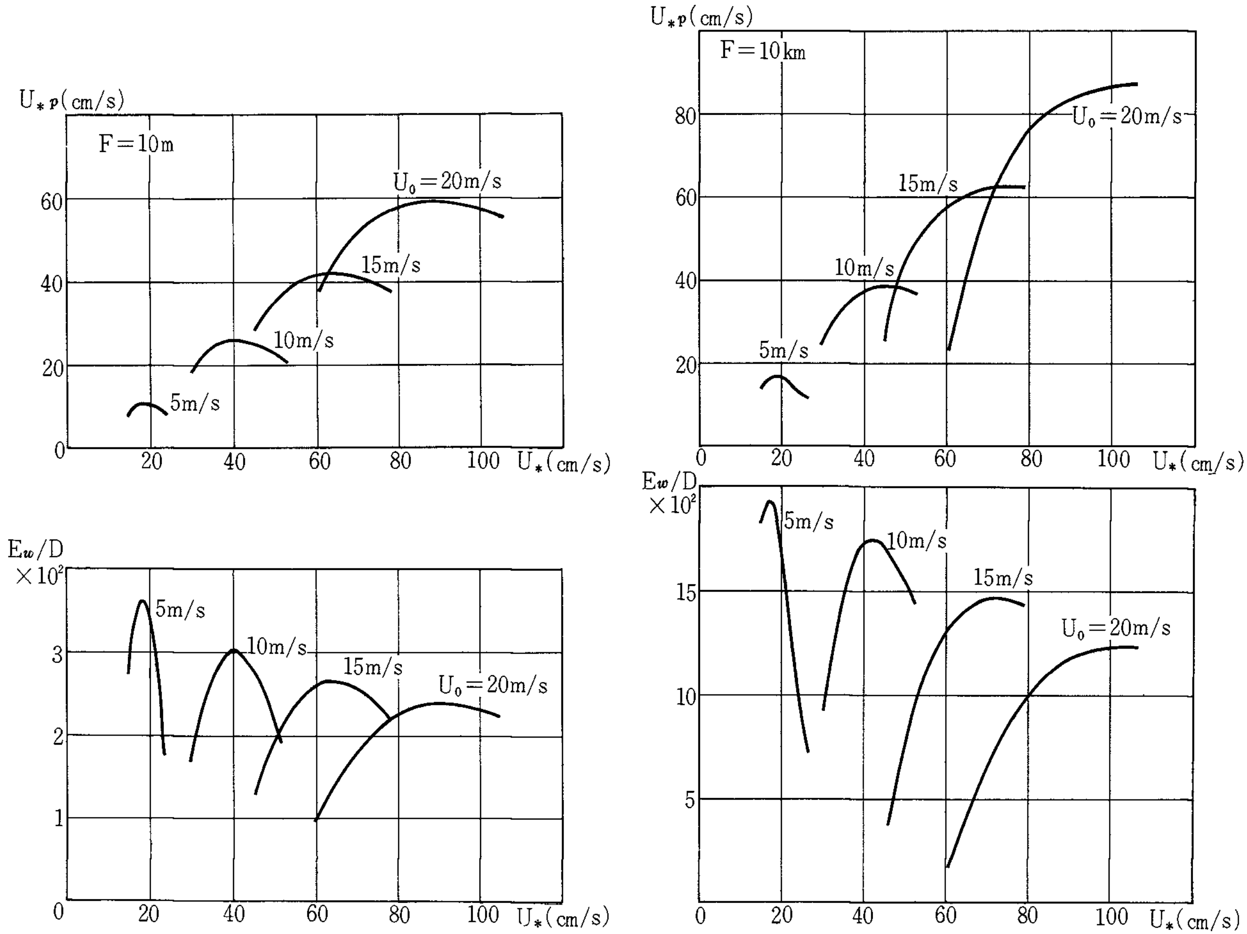

Fig. 2. The behavior of the pressure drag ($\tau_p = \rho U_{*p}^2$) and the ratio of energy input to wave to dissipation (E_w/D) due to increase in the assumed shear velocity ($U_* = \sqrt{\tau_0/\rho}$)

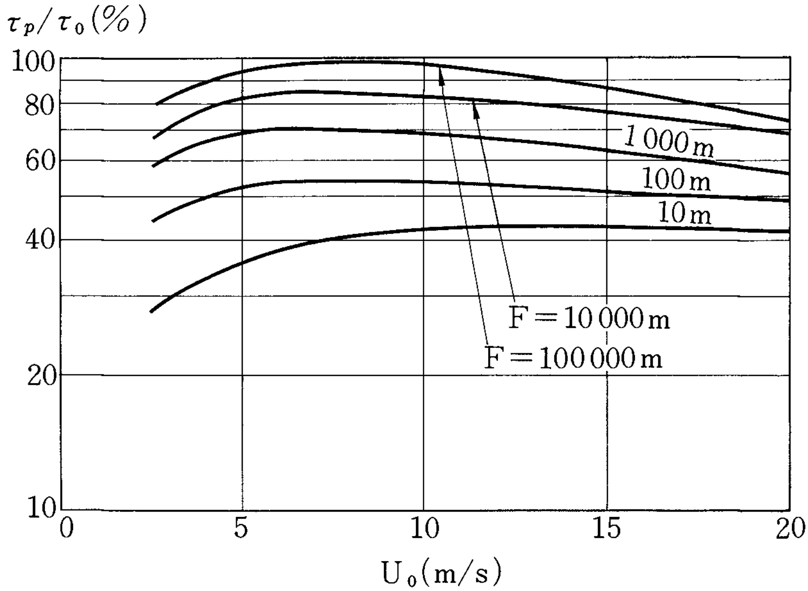

Fig. 5. The relationship between the ratio of pressure drag to total drag and wind velocity, with fetches as a parameter.

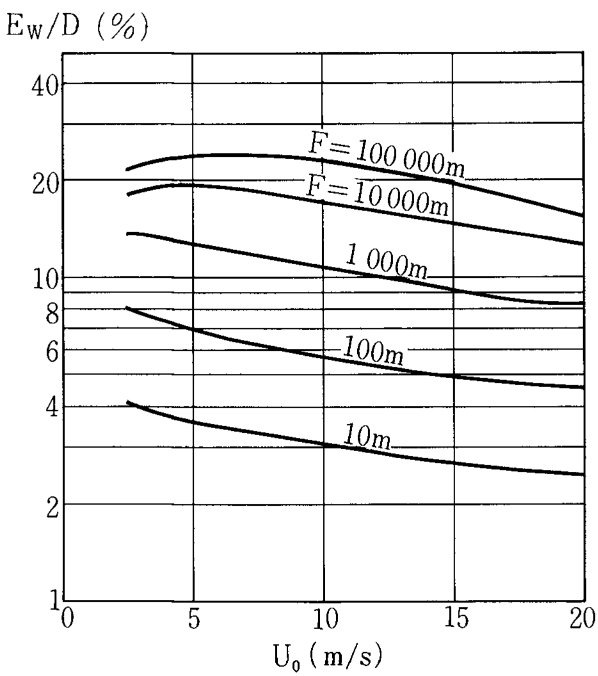

Fig. 6. The relationship between the ratio of energy of wave supplied from wind to energy dissipation within wind and the wind velocity, with fetch as a parameter.

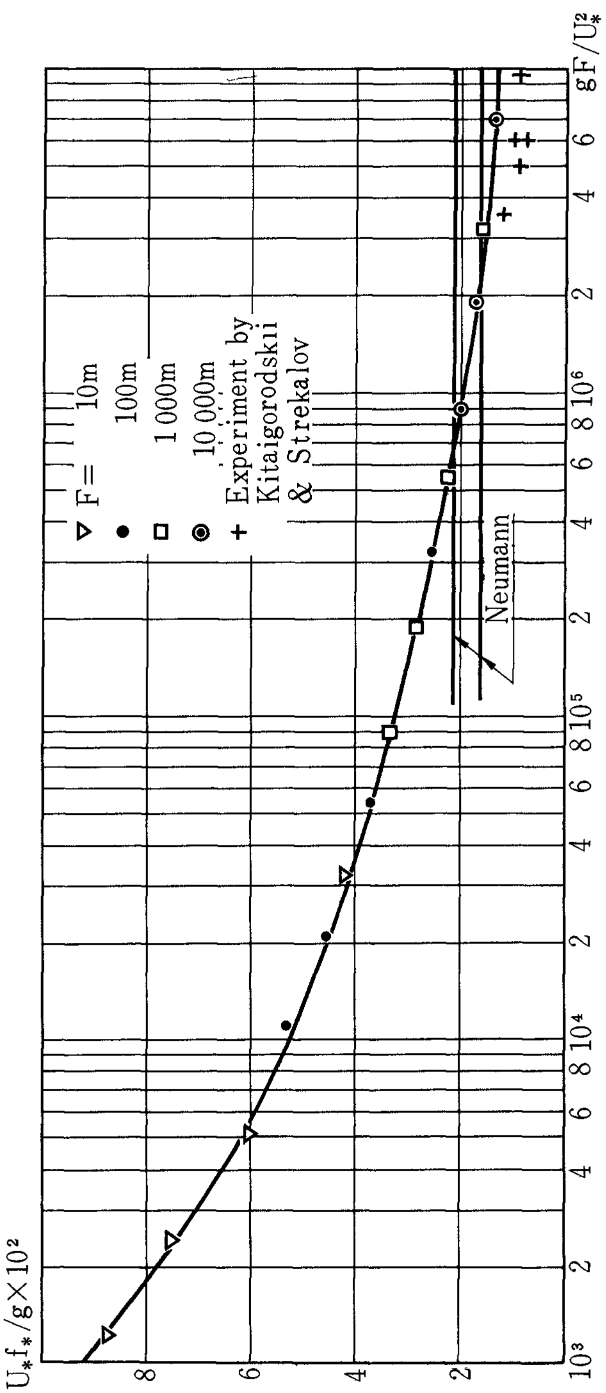

Fig. 7. Non-dimensional representation of the relation between the spectral peak frequency and the fetch.

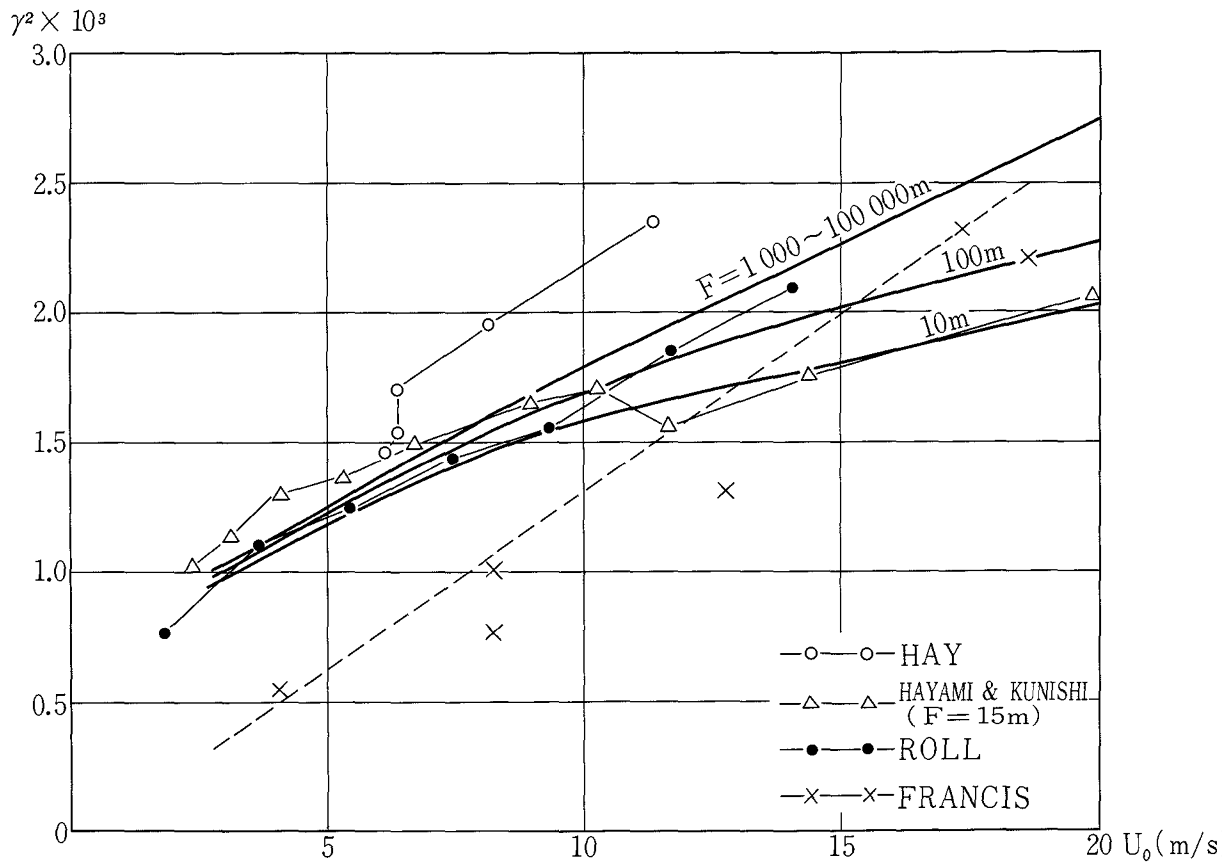

Fig. 8. The relationship between the roughness coefficient r^2 and the wind speed at 10 m from the mean sea surface, with fetch as a parameter.

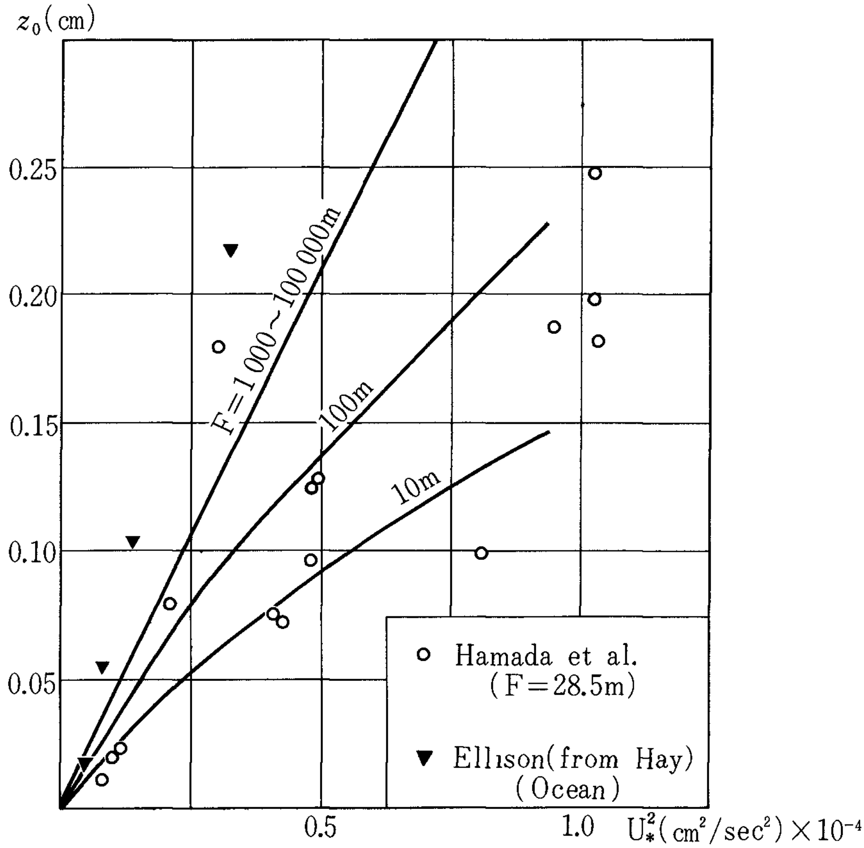

Fig. 9. The relationship between the roughness parameter z_0 and the wind speed at 10 m from the mean sea surface, with fetch as a parameter.

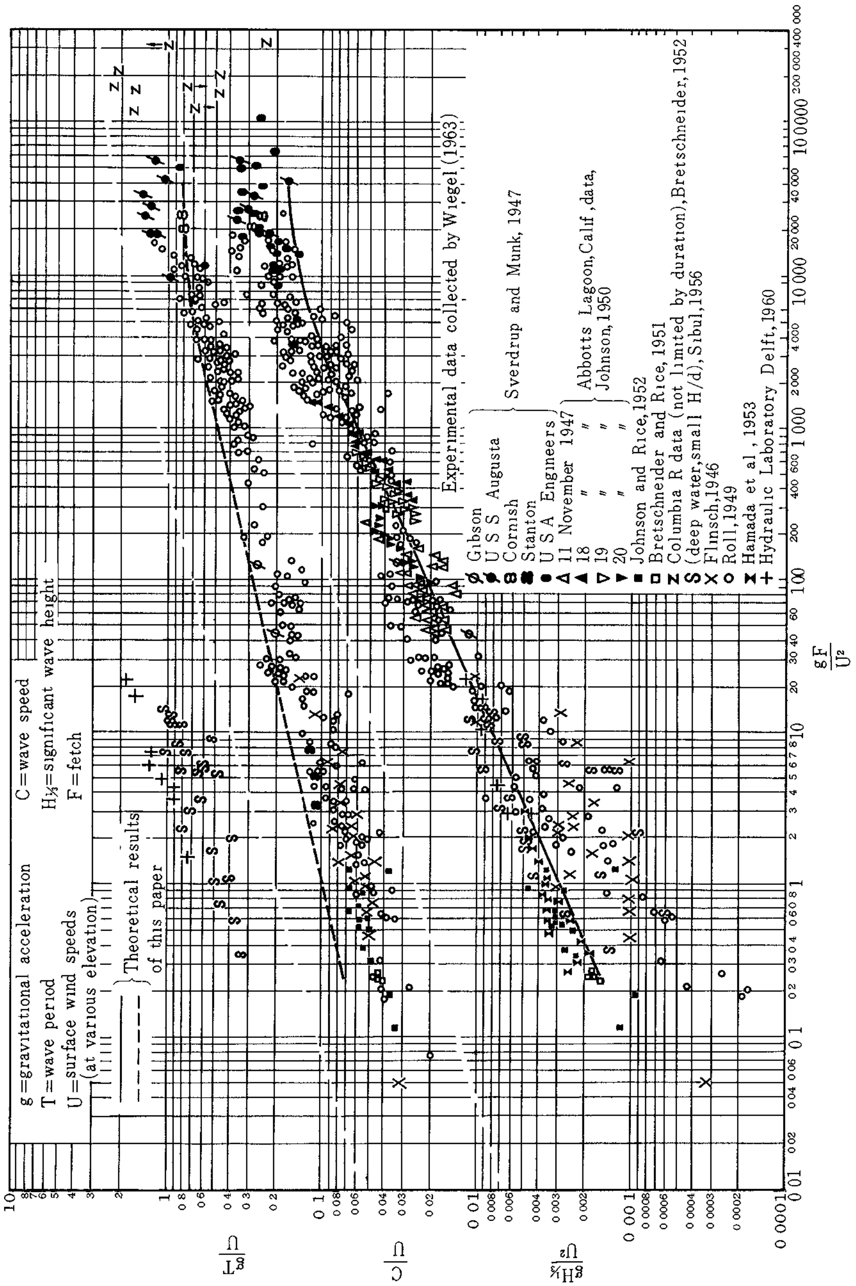

Fig. 10. Comparison of the fetch graph obtained from the present theory with experimental data collected by Wiegel (1963).