Chapter 11

MODEL TESTS ON THE RELATIONSHIP BETWEEN DEEP-WATER WAVE CHARACTERISTICS AND LONGSHORE CURRENTS

Arthur Brebner
Chairman, Department of Civil Engineering
Queen's University at Kingston, Ontario

and

J.W. Kamphuis
Research Fellow, Department of Civil Engineering
Queen's University at Kingston, Ontario

INTRODUCTION

It has long been recognized that the movement of littoral material takes place, in the main, in the onshore regions of a beach where breaking of waves occurs. Waves whose crests in deep water make an angle \( \alpha_0 \) with the shoreline, and which break at an angle \( \alpha_B \), are the main source of energy for the generation of the forces which manifest themselves in long-shore currents and the resulting littoral transport. This littoral material is put into motion before, during and after breaking but it is extremely difficult to separate the effects of the forces and currents in these three zones. In what follows the authors have attempted to measure the intensity of the current around the breaking zone in a highly idealized beach model in which the shoreline is straight, has a constant beach slope, \( \theta \), and is attacked by waves of constant deep-water wave-height, \( H_0 \), and period, \( T \).

During refraction and shoaling the angle of the wave-crests with the shore-line is reduced from \( \alpha_0 \) to \( \alpha_B \) and during this process some of the deep-water energy being transmitted shorewards may be dissipated by friction. The exact value of \( \alpha_B \) is a function of \( \alpha_0 \), \( H_0/L_0 \), and the friction loss, but will increase, both theoretically and experimentally, with increasing \( H_0/L_0 \), as shown by Brebner and Kamphuis (1963).

Based on the angle of breaking, \( \alpha_B \), the wave-steepness at breaking, \( H_B/L_B \), the depth of breaking \( d_B \), and the beach slope, \( \theta \), it is possible to formulate relationships for the long-shore current, \( v_L \), using the principle of conservation of energy and momentum and the principle of continuity.

Using energy considerations,

\[
V_L = K_1 \left[ \frac{H_B^2}{T} \cdot \sin 2 \alpha_B \right]^{1/3}
\]

and using momentum considerations,

\[
V_L = K_2 \left[ \frac{H_B^{3/2}}{T} \cdot \sin 2 \alpha_B \right]^{1/2}
\]

where \( K_1 \) and \( K_2 \) are empirical "constants" depending on the friction energy offered to the longshore current, the amount of energy dissipated in the breaking process, and the amount used in maintaining on-shore

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off-shore motion at right angles to the longshore current.

Using continuity considerations and a random-walk distribution of wave-heights Chiu and Per Bruun (1964) arrive at a value of $v_L$ which is apparently in reasonable agreement with field observations. However, since wave-forecasting methods give $H_0$, $T$, and $\alpha_0$ the authors deem it preferable in this instance to express $v_L$ in terms of the deep-water characteristics instead of breaking characteristics, and also to deal with regular waves since these are normally used in laboratory models simulating prototype installations.

MODEL TESTS OF LONGSHORE CURRENTS

a) Procedure:— The tests were carried out in a basin approximately 2 ft. deep with a plan area of 100 ft. x 50 ft. Sixteen differing wave periods ranging from 0.78 to 1.13 seconds, five differing wave-heights from 0.075 to 0.258 feet, six differing values of $\alpha_0$ from 10° to 60°, and two differing impermeable fixed-bed beach slopes, 1:10 and 1:20 were used. The longshore velocities in and around the breaking zone were measured by timing coloured neutral density bubbles of trichloroethylene and benzene over known distances. About 50 such readings were taken per incident wave and a mean value of $v_L$ established. In all about 500 values of $v_L$ were obtained.

b) Presentation of Results:— The variables in the tests were $\alpha$, $\alpha_0$, $H_0$ and $\alpha_0$ (or $T$). In view of the form of the theoretical relationship for $v_L$ it was decided to put $v_L$ in the form

$$v_L = K \alpha^a H_0^b T^c \left( \text{Function } \alpha_0 \right)$$

For given values of $\alpha_0$ it was possible, using a 1620 IBM computer, to perform regression analyses using values of $a$ varying from 0.33 to 1, $b$ from 0.5 to 1 and $c$ from 0.33 to 1. Further, since $T$ remains constant during refraction and the breaking height $H_B$ is of the same order as $H_0$, it is possible to carry out a Fourier analysis on $\alpha_0$ to give (Function $\alpha_0$) as a sine series.

In keeping with the theoretical results it was possible to produce two expressions for $v_L$, namely,

$$v_L = 2.5 \left[ \frac{g H_0^2}{T^2} \right]^{1/3} \left[ \sin (1.65 \alpha_0) + 0.1 \sin (3.30 \alpha_0) \right]$$  \hspace{1cm} (1)

and

$$v_L = 6 \left[ \frac{g H_0^2}{T^2} \right]^{1/4} \left[ \sin (1.65 \alpha_0) + 0.1 \sin (3.30 \alpha_0) \right]$$  \hspace{1cm} (2)

In both these expressions the constant is dimensionless. The experimental values are shown on Figures 1 and 2. Of the total energy available prior to breaking only about 8% is used in maintaining the long-
$V_L (\text{Calculated}) = 2.5 \left[ \frac{g \theta H^2}{T} \right]^{1/3} \left[ \sin 1.65 \alpha + 0.1 \sin 3.30 \alpha \right]$

FIGURE 1
\[ V_L \text{ (Calculated)} = 6 \left( \frac{g \theta^2 H^3}{T^2} \right)^{1/4} \left[ \sin(1.65 \theta) + 0.1 \sin(3.30 \theta) \right] \]

**FIGURE 2**
shore current at the intensity given by either equations 1 or 2, indicating the high energy loss in the breaking phenomenon itself and in maintaining on-shore off-shore movement of water, normal to the longshore current.

The \( \phi \) term reveals that the maximum value of \( v_L \) occurs when \( \phi \) is about 55°, which is in good agreement with Bruun's value of 54° and Sauvage and Vincent's value of 53°.

**DISCUSSION OF RESULTS**

The model may be considered distorted or undistorted. If undistorted the Froude scale is about 1:100 for linear dimensions and at this scale the "Reynolds Number" damping effect, using Eagleson's (1962) theory of damping of oscillatory waves, is not marked.

If \( n_d \) is the ratio \( \frac{\text{prototype depth}}{\text{model depth}} \) and if the suffix \( L \) refers to wave-length, \( H \) to wave-heights, and \( c \) to wave-speed, then for such a refraction model as this which demands homologous angles of breaking, homologous shoaling, and homologous breaking depths,

\[
\frac{n_c}{n_T} = \frac{n_L}{n_H} = \frac{n_d}{n_x} \quad \frac{1}{2} \quad \frac{1}{2}
\]

Referring to equations 1 and 2, if \( n_x \) is the plan length scale,

**Longshore current scale**

\[
v_L = \frac{v_{Lp}}{v_{Lm}} = \left[ \frac{n_d}{n_x} \right]^{1/3} \frac{n_H}{n_T} \]

or

\[
v_L = \frac{v_{Lp}}{v_{Lm}} = \left[ \frac{n_d}{n_x} \right]^{1/4} \frac{n_H}{n_T}^{3/4}
\]

If the model is considered undistorted, \( n_d = n_x \) and \( n_vL = n_x^{1/2} \), as in a simple Froude model.

In effect equations 1 and 2 should be universally applicable if one assumes that

a) the model beach slopes are not so steep as to excessively deform the orbital paths compared with the prototype.

b) the prototype slopes are neither so flat, rough, nor permeable as to cause excessive attenuation due to friction compared with the model.

c) the exponent of the slope \( \theta \) is to be trusted in view of the fact only 2 differing slopes were used in the tests.
In conclusion, equations 1 and 2 probably give a maximum attainable envelope value of $v_L$ when extrapolated to a prototype situation since, in nature,

a) beaches are not straight and uniform,

b) rip currents are set up which limit the longshore current,

c) due to the presence of longshore bars, energy and momentum are transferred to the longshore process at various locations,

d) waves are not regular and thus the breaking zone is not too well defined.

For use on prototype situations with non-regular waves the authors' suggest that $H_o$ be replaced by $H_s$ on the argument that the mean height of all waves in a random-walk sample is the significant wave-height divided by 1.6. Thus, on a one-bar beach having a slope $\Theta$ of 0.017 radians, wave-period $8 \text{ secs.}$, $\alpha_o = 45^\circ$, and $H_s = 5.5$ metres, the longshore current is $1.4 \text{ m/sec.}$ by equation 1 and $1.2 \text{ m/sec.}$ by equation 2. A typical prototype value is about $1 \text{ m/sec.}$ under such conditions, indicating as suggested previously that the authors' relationships probably give limiting values of the maximum longshore current for any particular situation.

REFERENCES


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