CHAPTER 2

MODIFICATION OF WAVE SPECTRA ON THE CONTINENTAL SHELF AND IN THE SURF ZONE

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ABSTRACT

This paper discusses the problem pertaining to the modification of the wave spectrum over the continental shelf. Modification factors include bottom friction, percolation, refraction, breaking waves, ocean currents, and regeneration of wind waves in shallow water, among other factors. A formulation of the problem is presented but no general solutio is made, primarily because of lack of basic data. Several special solutions are presented based on reasonable assumptions. The case for a steep continental shelf with parallel bottom contours and wave crests parallel to the coast and for which bottom friction is neglected has been investigated. For this case it is found that the predominant period shifts toward longer periods. The implication is, for example, that the significant periods observed along the U. S. Pacific coast are longer than those which would be observed several miles westward over deep water.

The case for a gentle continental shelf with parallel bottom contou1 and wave crests parallel to the coast and for which bottom friction is important has also been investigated. For this case it is found that the predominant period shifts toward shorter periods as the water depth decreases. The implication is, for example, that the significant periods observed in the shallow water over the continental shelf are shorter than those which would be observed beyond the continental slope. In very shallow water, because shoaling becomes important, a secondary peak appears at higher periods.

The joint distribution of wave heights and wave periods is required in order to determine the most probable maximum breaking wave, which can be of lesser height than the most probable maximum non-breaking wav In very shallow water the most probable maximum breaking wave which fi occurs would be governed by the breaking depth criteria, whereas in deep water wave steepness can also be a governing factor. It can be expected that in very shallow water the period of the most probable maximum break ing wave should be longer than the significant period; and for deeper wate1 the period of the most probable maximum breaking wave can be less than the significant period.

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INTRODUCTION

During the last decade, a great number of studies has been carried out on wave forecasting and wave spectrum in deep water. Despite this fact, a great deal remains to be accomplished in this field.

One of the most important steps to be taken next is the study of the modification of the wave spectrum upon its arrival on the continental shelf and in the surf zone.

When the waves generated by a storm in deep water off the continental shelf are propagated as swell over the continental shelf, they are modified by bottom friction, percolation, refraction, shoaling and whitecaps As an overall effect, the waves composing the spectrum are damped due to the dissipation of wave energy. As a consequence, this damping involves a deformation of the wave spectrum with the result that the wave heights of the spectrum can no longer be defined by the special type Gamma function distribution such as the Rayleigh distribution.

Upon arrival in the surfing zone, each component of the spectrum breaks at a different water depth in the surfing zone. The distribution of breaking depth and the probability and distribution of the angle of breaking wave crest with the shoreline depend upon the wave spectrum characteristics arriving in the surfing zone and the change in the angle of the wave crest with the shoreline due to refraction effects.

GENERAL CONSIDERATIONS ON
THE MODIFICATION OF THE WAVE SPECTRUM

The significant wave method for dealing with the transformation of surface waves as they are propagated into shallow water, taking bottom friction, percolation and refraction into account, has been presented by Bretschneider and Reid[9]*. Instead of assuming a uniform wave train, it is assumed that the significant period of the wave is invariant. This implies essentially that there is no selective attenuation or selective amplification of the wave spectrum. The functions characterizing the dissipation of energy used in the above reference are the same as those introduced by Putnam and Johnson[1], and hence, all assumptions pertaining to these dissipation functions have been incorporated in the work by Bretschneider and Reid.

The significant wave method has been extended by Bretschneider[6] to forecasting wind waves in shallow water, taking into account wave energy loss due to bottom friction. This method is semi-empirical and has been correlated with wind and wave data from Lake Okeechobee, Florida.

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* Numbers indicate references listed at end of paper.

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These forecasting relationships for shallow water of constant depth are comparable to similar relationships obtained by Thijsse[8] by an entirely different method. The relationships given by Bretschneider[6] as well as those given by Thijsse[8], have certain practical applications.

The attenuation of energy by bottom friction over the continental shelf for waves of long period can be explained qualitatively as due to the fact that the long waves effectively "feel" bottom sooner than the short period waves and consequently are subjected to frictional dissipation over a greater distance. In a complex wave group this selective attenuation could produce, under certain conditions, a shift in the peak of the power spectrum towards lower periods as the waves travel towards shore. The is some evidence to support this in the shallow, flat Atchafalaya Bay regi of the Gulf of Mexico where a smaller significant period has been observe inshore compared with simultaneous measurements offshore, as reported by Bretschneider[6]. However, this is not conclusive since distortion of the power spectrum with a consequent change of significant period can als result when no energy is lost, as Pierson, et al, have shown [10] on the basis that each component of the spectrum has a different shoaling and refraction factor. However, shoaling and/or refraction can cause a shift either to higher or lower significant period depending on the actual conditions.

In general, however, it is believed that the predominant period of the period spectrum will shift to lower periods because of wave energy loss due to bottom friction. This is an opposite effect to what is known about the increase in predominant period due to increase in fetch length and wind duration. On the other hand, in deep water the predominant wave period increases with distance from decaying swell, but decreases with time at any particular decay distance. Pierson and Marks[11] have also demonstrated a shift in the predominant period of the sub-surface pressure spectrum, in which case the predominant period increases with depth.

A SUMMARY OF EXISTING KNOWLEDGE
ON WAVE DAMPING EFFECTS

Most of the theoretical studies on wave damping have been carriec out for regular wave trains: Boussinesq[17], Lamb[18], Basset[19], Hough[20], Biesel[21], Keulegan[3], Putnam and Johnson[1], Miche[25], Reid and Bretschneider[9], among others.

From a theoretical point of view, two methods exist to attack the problem. The first method -- the analytical method -- consists of solving directly the basic differential equations -- momentum, continuity taking into account a friction term. This method presents the advantage of giving not only the damping but also the deformation of wave motion du to friction forces. This is more important for long waves in shallow

water and becomes essential at the limit for tidal waves in an estuary, for example.

For the problem under study, an approximate method -- the energy method -- is simpler and as accurate. It consists of determining the wave motion independent of the friction forces and neglecting the convective inertia. As a consequence the wave profile is symmetrical. This approximation is valid because the decay of wave amplitude over a wave length is very small.

The damping effect is simply defined as a decay in wave height calculated from energy consideration: the loss of energy over a given leng is equal to variation of the wave energy.

The theory for modification of wave height (assuming significant wave as a wave train) due to bottom friction, percolation and refraction has been presented in detail, particularly by Bretschneider and Reid[9]. Shoaling does not represent a loss in wave energy but this modification is included as a transformation process. It will suffice at present only to summarize the theory here. The theory is based on waves of small steepness. For steady state conditions, the rate at which the total power is altered per unit distance along one of the wave rays is given by

$$\frac{d\,(Pb)}{dx} \; = \; - (D_f + D_p) \, b \qquad\qquad . \, . \, . \, (1)$$

where x is the distance measured along the wave ray in the direction of propagation of waves (see Figure 1). P represents the energy propagated per unit time through a vertical area of depth d and unit width (normal to the wave ray) averaged over one wave length. The average power (or energy transfer per unit time) between two wave rays of horizontal spacing b is consequently Pb. Under steady conditions, this power would remain constant for given wave rays in the absence of dissipation, reflection, breaking and lateral dispersion of energy. However, in the presence of bottom friction and/or percolation, the value of Pb decreases slowly from one wave to the next in shallow water.

In equation (1) D_f and D_p are functions characterizing the dissipation of energy from bottom friction and percolation, respectively. The problem is to evaluate these functions. The bottom friction effect can be calculated theoretically when the flow is laminar. However, this flow is more often turbulent than laminar. This necessitates experiments or field observations. Laboratory studies on wave energy loss have been made by Keulegan[3], Savage[4], Ippen and Kulin[5], and others. (In fact, Keulegan, Ippen and Kulin considered the case of solitary waves, the results of which might be applicable to the surf zone.)

This leads to a more general discussion of the boundary layer problem and inception of turbulence in unsteady motion.

First, it is assumed that a wave motion is well defined by a velocity potential function. Hence the flow is irrotational, i.e. without
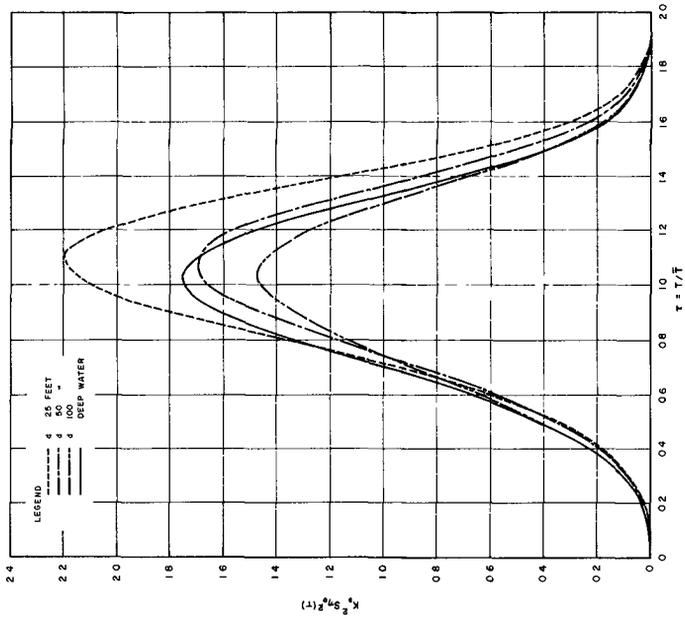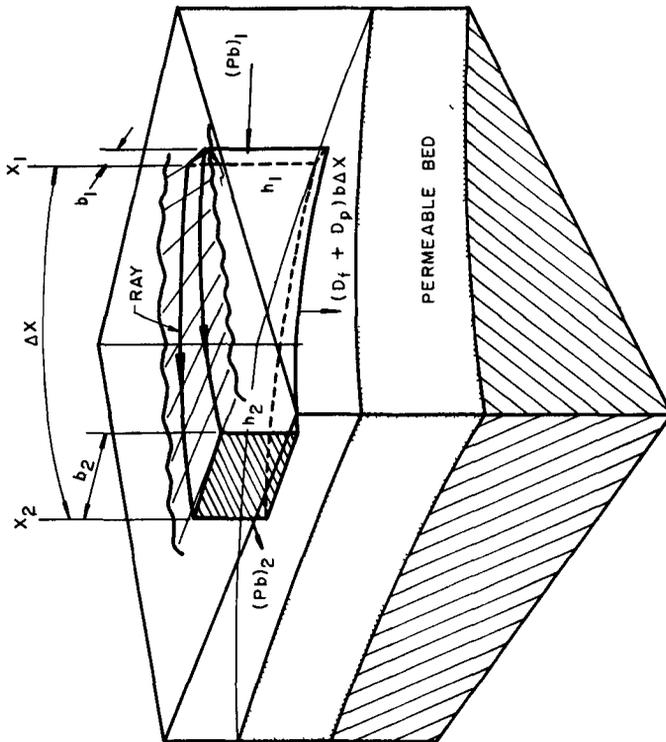
Fig. 2. Modification of wave spectrum due to shoaling.

Fig. 1. Schematic of energy transfer.

friction.

$$\mu \nabla^2 \vec{V} = \mu \nabla^2 \vec{grad}\, \varphi = \mu\, \vec{grad}\, \nabla^2 \varphi = 0 \qquad \ldots (2)$$

However, it often happens that a current, such as tidal current or long-shore current, is superimposed on the wave motion. This current may be the cause of instability and turbulence. By causing turbulence throughout the mass of water within the wave motion, there is an action on the internal damping mechanism of the wave spectrum, independent from the turbulence due to whitecaps.

Under this irrotational wave motion, there is the boundary layer where the flow is strongly rotational. Without whitecaps, it is in the boundary layer that the greatest part of wave energy is lost.

In the case of viscous flow, the periodic motion in the boundary layer on a smooth plane is relatively well known.

The thickness of the boundary layer δ is proportional to $[\nu T]^{1/2}$, that is, δ increases with the wave period. δ is of the order of magnitude of a few millimeters. The shearing stress τ is proportional to $\mu \cdot u_b / \delta$ where u_b is the bottom velocity. The damping effect due to the loss of energy in the boundary layer may also be calculated. It is found that the wave height decreases exponentially with distance. This decay is independent from the absolute value of H.

However, for some Reynolds' numbers the flow in the boundary layer becomes unstable and turbulent. The stability of such unsteady motion has not been investigated by theory. Some experiments (Collins[22]) show that the boundary layer becomes turbulent for a Reynolds' number

$$\frac{u_b \delta}{\nu} = \left(\frac{\pi}{\nu T}\right)^{1/2} \frac{H}{\sinh md} > 160 \qquad \ldots (3)$$

$m = \dfrac{2\pi}{L}$; H = wave height.

On a rough boundary the inception of turbulence and the damping depend also upon the relative roughness ε / δ. Hence, on a sandy bottom ripples have an effect on the wave damping. The thickness of the boundary layer is unknown but also grows as T increases. It often happens that the turbulence issued from the bottom is damped quickly throughout the mass of water. Hence, a turbulent boundary layer under a laminar wave motion is possible. A cloud of sand in suspension can be seen only a few inches above the ripples.

The shearing stress is then proportional to the square of the bottom velocity.

$$\tau = \rho \int u_b \, | u_b |$$

. . . (4)

where f is a friction coefficient which varies with the relative roughness and a Reynolds' number. u_b is the bottom velocity at the top of the boundary layer. u_b can be defined from the velocity potential defining the wave motion.

In relatively deep water δ is very small by comparison with the depth d . Hence,

$$u_b = u \quad \cong u_d = \frac{\partial \varphi}{\partial x}\Big|_{z=-d}$$

. . . (5)

Using such assumptions, Putnam and Johnson have found that the wave energy D_f dissipated per unit area at the bottom per unit time (averaged over a wave length) is given by:

$$D_f = \frac{4}{3}\pi^2 \frac{\rho f \, H^3}{L^3 \left(\sinh \frac{2\pi d}{L} \right)^3}$$

. . . (6)

and the variation of wave height varies hyperbolically with distance and depends upon the absolute value of H .

Biesel[23] and Putnam[2] have also examined the oscillatory percolation of water through a permeable sea bed, associated with sinusoidal waves of small amplitude in the overlying water. According to Putnam, the amount of energy D_p dissipated by viscous forces in the permeable bed per unit area of the bottom per unit time (averaged over a wave length) is given by:

$$D_f = \frac{\pi g^2}{\nu} \frac{\rho K \, H^2}{L \left(\cosh \frac{2\pi d}{L} \right)^2}$$

. . . (7)

for a permeable bed whose depth is greater than 0.3 L. In the above equation, K equals the permeability coefficient of Darcy's law. The other symbols have previously been explained. (It should be noted that Reid and Kajura[12] have shown that equation (7) is in error by a factor of 4 ; i.e. equation (7) should be preceded by 1/4.)

Using D_f and D_p as given by the above expression, the differential equation (1) has been formulated by Bretschneider and Reid[9] and certain special solutions have been obtained. The general solution is obtained in part by numerical integration of the functions characterizing the dissipation of energy.

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One of the most important requirements is a knowledge of the friction coefficient   f   and this presents a number of difficulties.

First, the difference between an exponential damping in laminar flow or a hyperbolic damping in turbulent flow is so small that both solutions could be considered as possible. However, for long waves in very shallow water, the boundary layer becomes thicker and the velocity distribution in the vertical plane is influenced very much by the bottom friction effect. Moreover, the theoretical value for u_b equals

$$u = \frac{\partial \psi}{\partial x} = f(x, t) \text{ whatever the value of } z.$$

From this point of view, long waves in shallow water can be considered almost as a succession of steady flows. The velocity distribution in a vertical plane is close to the velocity distribution for steady flow. At the limit, the tidal motion in an estuary is defined on such an assumption.

The friction coefficient   f   can then be expressed as a function of the Chezy coefficient C_h or the Manning coefficient   n :

$$f = \frac{g}{C_h^2} \quad , \quad C_h = \frac{1.486 \, d^{1/6}}{n} \qquad \ldots (8)$$

From laboratory data on short waves, Bagnold[14] has obtained the following formula for the friction coefficient:

$$f = .074 (R/p)^{-0.75} \qquad \ldots (9)$$

where   R   is half the total horizontal displacement of particle velocity at the bottom, and   p   is the horizontal distance between the sand ripple crests at the bottom.

SOME CONSIDERATIONS ON THE MODIFICATION OF WAVE SPECTRUM

It is of interest to anticipate the influence on the wave spectrum due to various modification factors. In this respect an example is given assuming that in deep water the root mean square wave height and the mean wave period are given respectively by $H_r = 30$ feet and T = 12 seconds. In the following example the deep water wave spectrum equation given by Bretschneider[13] will be used although similar manipulations could be applied to the spectra equations given by Neumann[30] and Darbyshire[31 & 32] among others.

For the case of deep water, the wave period spectrum given by Bretschneider[1] is:

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$$S\eta_o^2(\tau) = K_1 \tau^3 e^{-B_1 \tau^{-4}}$$

$$\ldots (10)$$

in which case the wave frequency spectrum is

$$S\eta_o^2(\nu) = K_2 \nu^{-5} e^{-B_2 \nu^{-4}}$$

$$\ldots (11)$$

In the above equations $S\eta_o^2(\tau)$ and $S\eta_o^2(\nu)$ respectively represent the sum of the squares of the wave height as a function of wave period and wave frequency. The symbols used are:

$$\eta_\bullet^2 = H_o^2/\overline{H_o^2} \quad , \quad \tau = T/\overline{T} \quad , \quad \nu = \omega/\overline{\omega}$$

where $\omega = 2\pi/T$, $\overline{H^2}$ is the average of the squared wave heights, $\overline{T}$ is the average of the wave periods, $\overline{\omega}$ is the average of the wave frequencies, and K_1, K_2, B_1 and B_2 are constants. The subscript o is used to denote deep water wave spectrum.

As the spectrum is propagated across the continental shelf, unaffected by winds, the modifications that can take place are: (1) wave energy loss due to bottom friction, (2) wave energy loss due to percolation in a permeable sea bed, (3) shoaling, and (4) refraction. In addition, wave energy can be lost due to whitecaps, ocean and tidal currents, longshore currents, etc.

It will be convenient to define a modification factor K^2, which includes the above considerations, and which naturally will vary with wave period, water depth, bottom conditions, etc., such that the wave spectrum in shallow water becomes

$$S\eta^2(\tau) = K^2 S\eta_o^2(\tau)$$

$$\ldots (12)$$

or in terms of frequency

$$S\eta^2(\nu) = K^2 S\eta_o^2(\nu)$$

$$\ldots (13)$$

If the continental shelf is steep, such as off the Pacific Coast of the United States, wave energy loss due to bottom friction and percolation will be negligible. If the wave crests are parallel to parallel bottom contours, refraction will be absent, and if no waves break, then the modification will be due entirely to shoaling, whence $K = K_s$. K_s is the

shoaling factor and is a function only of h/L where h is the water depth and L the wave length, corresponding to the appropriate wave period (or wave frequency) of the spectrum. Based on T = 12 seconds the shoaling coefficients were determined for various wave periods and water depths, and applied to equation (10). The results are shown in Figure 2. I is interesting to note the shift in the predominant wave period. In deep water the predominant period is equal to 1.027 x 12 = 12.3 seconds and at a depth of 25 feet becomes 1.1 x 12 = 13.2 seconds. The same computations could have been performed for the frequency spectrum, but the period spectrum seems to illustrate the shift in period more clearly.

If the wave spectrum is propagated across the continental shelf off the East Coast or Gulf Coast, then bottom friction becomes important. Considering only bottom friction (without shoaling) $K = K_f$, and equation (10) or (11) becomes an equivalent deep water wave spectrum. Considerinį a bottom of constant slope and assuming a constant friction factor, K_f was determined for various water depths for f/m = 7.6 , f being the bottom friction factor and m the bottom slope. The results of these computations are shown in Figure 3. It is interesting to note now that the predominant wave period has shifted to the lower wave periods. That is, the predominant wave period is 12.3 seconds in deep water and about 9.0 seconds in a depth of 25 feet. The same sort of shift to lower wave periods can also occur by the breaking of the longer period waves, which also can have an effect on the high frequency components of the spectrum.

A wave recorder would naturally record the shoaled waves, the deduced spectrum of which would not be in accordance with Figure 3. Applying the shoaling coefficients to Figure 3 results in the squared wave height spectrum such as would be obtained from a wave recorder. The results of these computations are shown in Figure 4. It can be seen from Figure 4 that the shift in the predominant wave period is also toward lower wave periods, but not so pronounced as in Figure 3. For the 25-foot water depth, the combined effect of shoaling and bottom friction introduces an additional predominant period.

Corresponding frequency spectrum for the combined effect of shoaling and bottom friction is given in Figure 5.

THE MAXIMUM POSSIBLE WAVE AND THE MAXIMUM PROBABLE WAVE

The maximum possible wave is governed by the breaking wave criteria, whereas the maximum probable wave represents a statistical probability.

A great number of studies has been carried out by defining the breaking conditions of a regular wave. The most significant formulas are those proposed by:
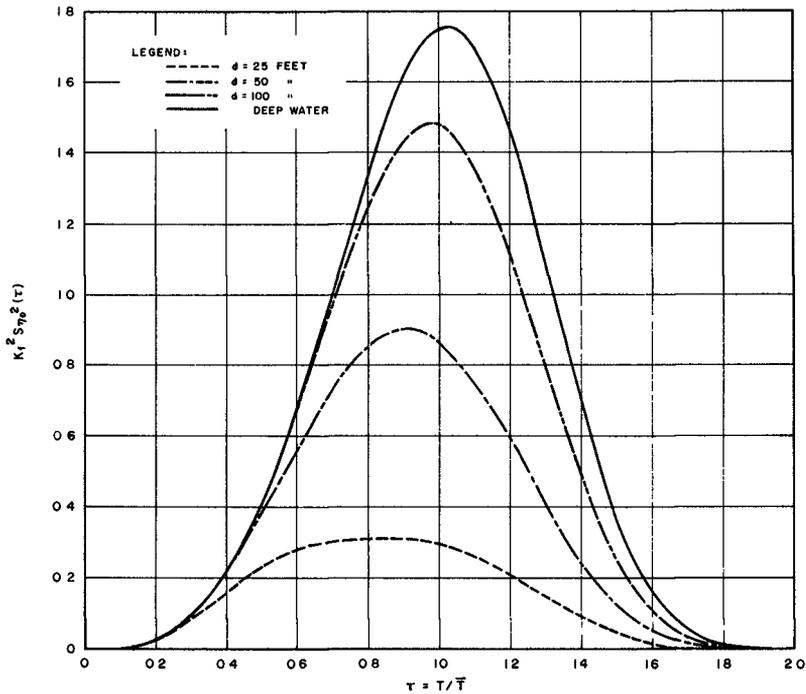
Fig. 3. Modification of period spectrum due to bottom friction
(f/m = 7.6).
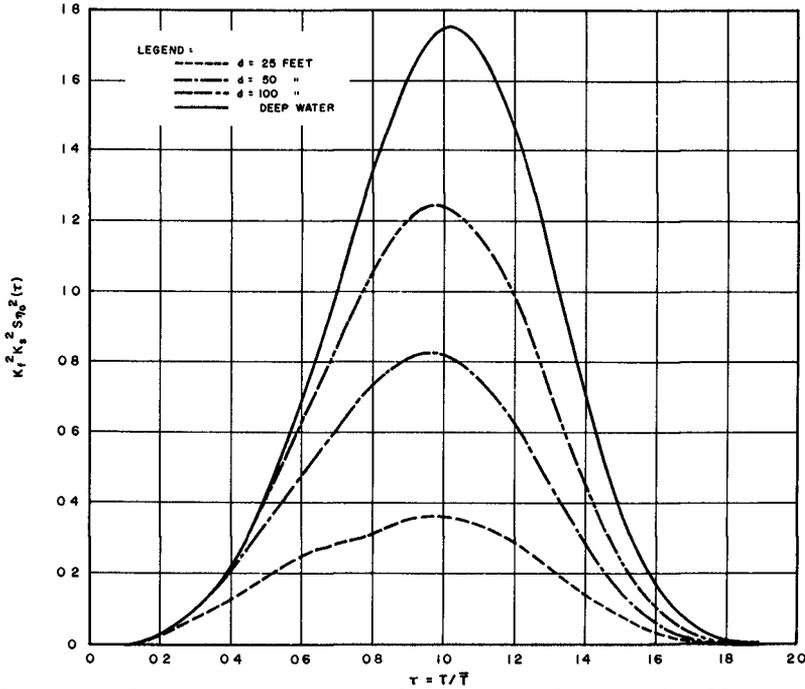

Fig. 4. Modification of period spectrum due to shoaling and
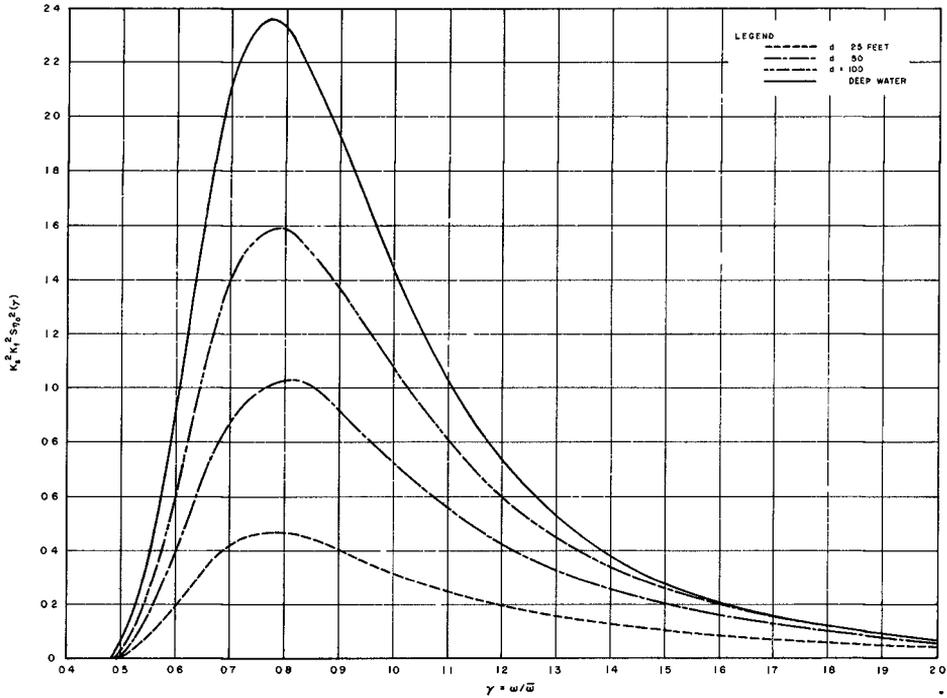bottom friction (f/m = 7.6).

Fig. 5. Modification of frequency spectrum due to shoaling and bottom friction (f/m = 7.6).
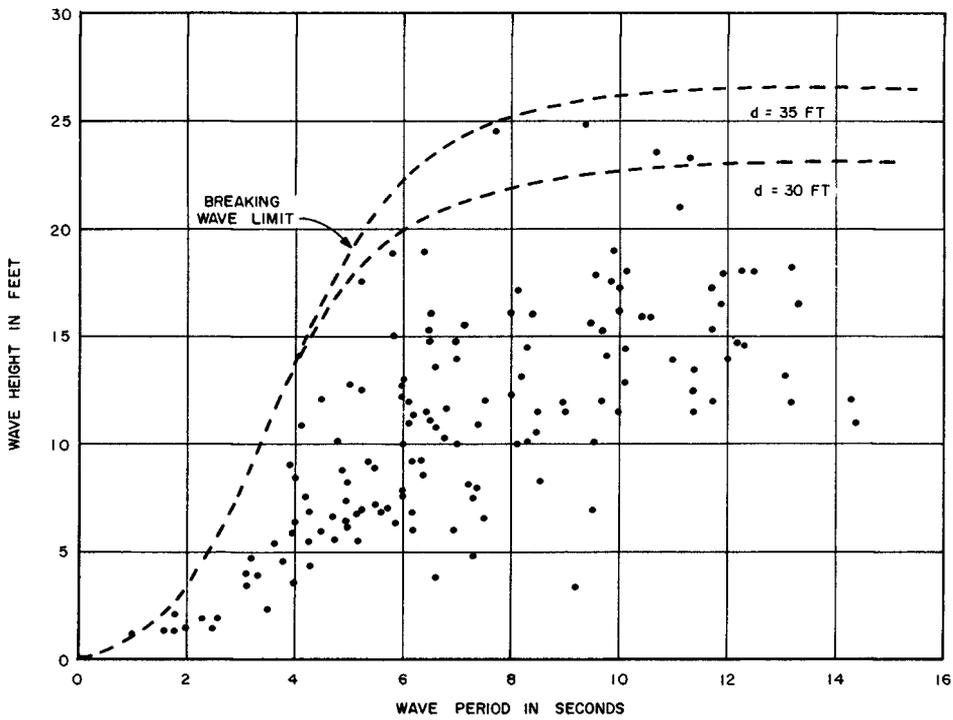

Fig. 6. Wave heights and periods, Hurricane Audrey, June 27, 1957.

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Stokes[27] in deep water: $\dfrac{H}{L} = 0.14$ . . . (14

Miche[26] for any depth: $\dfrac{H}{L} = 0.14 \; \text{Tanh} \dfrac{2\pi d}{L}$ . . . (15

Munk[28] and others for
solitary waves, valid for
long waves in very shallow $\dfrac{H}{L} = 0.78$ . . . (1(
water:

Experiments by Danel[29] have shown the limit of validity of the
Miche theory. Bretschneider[24] has presented, by empirical interpola-
tion, practical graphs valid for any case. Le Méhauté[15] has analyzed
the case of a wave breaking at an angle with the shoreline at a first order
of approximation, in which case the angle of wave crest in deep water is
related to the angle of breaking wave crest in shallow water.

Based on the assumptions of a narrow (linear) wave spectrum,
Longuet-Higgins[16] derived the Rayleigh distribution for ocean wave
heights. For a long record the asymptotic solution for the most probable
maximum wave height is given by:

$$H_{max} = .707 \; H_S \left(\ln N \right)^{1/2}$$ . . . (1(

where H_{max} is the most probable maximum wave height,

H_S is the significant wave height, and

N is the total number of waves.

The above formula is intended to apply when N is large (say 100 to 1000)
and for near-steady state conditions. This formula is based on linear
assumptions and should therefore be used with caution for very steep
waves or for waves in very shallow water, where breaking occurs.

It is desirable to define a most probable maximum breaking wave,
which would be given by the intersection of the breaking index criteria
and the joint probability distribution of wave heights and periods. Figure
6 is an example of the joint distribution of wave heights and periods ob-
tained in the Gulf of Mexico during Hurricane Audrey in 1957. The data
were obtained by the California Company and analyzed at the Beach
Erosion Board. The breaking criteria is shown by the dashed lines for
two water depths covering the possible range of total water depth, in-
cluding storm surge and tide.

It is apparent (but not too clearly demonstrated in figure 6) that
there can be a most probable maximum breaking wave height which is
smaller than the most probable maximum wave height.

The data given in figure 6 have been analyzed by a very quick
method for determining the distributions shown in figure 7. The upper
graph (6a) represents the histogram or distribution of periods. The
other two graphs (6b and 6c) are related to the period spectrum. The
solid blocks are for periods grouped 1-3, 3-5, etc. and the dashed blocks
are for periods grouped 0-2, 2-4, 4-6, etc. A careful look at both
figures 6 and 7 seems to indicate that a detailed spectrum analysis
would result in a spectrum having at least two well-defined peaks, one
at about six seconds and the other at about ten seconds. The ten-second
peak probably results from the fact that the shoaling coefficient for water
depths 30 to 35 feet is greater for the ten-second wave than for the six-
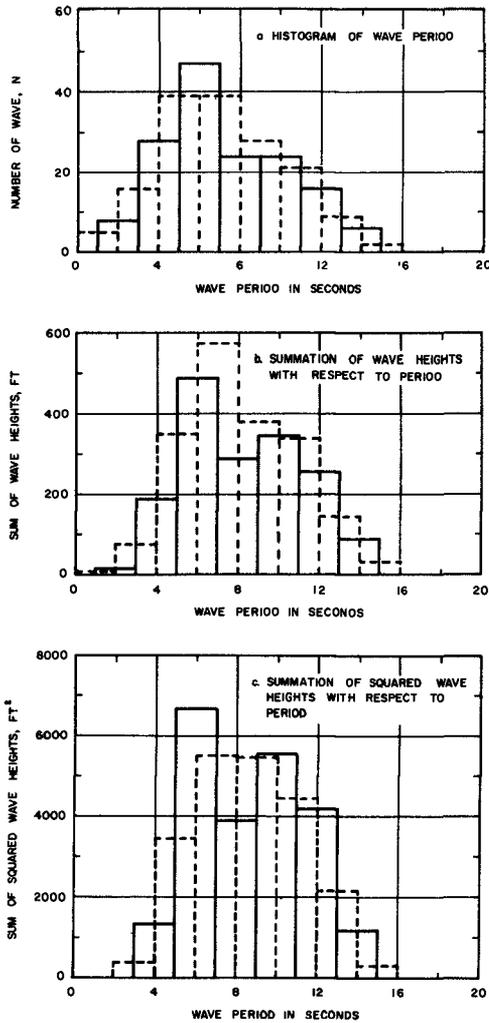second wave.

Fig. 7. Results of wave data, Hurri-
cane Audrey, 1957.

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The peak at six seconds probably occurs because bottom friction has not been important for the short period waves, and also the regeneration of waves is more rapid for the shorter periods than for the longer period waves. This phenomenon of a double peak is expected from the considerations given in figure 4. In fact, the peak at τ = 0.6 (figure 4) would be much higher if the wind effect had been included.

SUMMARY AND CONCLUSIONS

There can be a most probable maximum breaking wave which is equal to or less than the height of the most probable maximum wave. This should be considered in selecting the design wave. For example, for a particular design storm, the most probable maximum wave might be H = 22 feet and T = 9 seconds for d = 30 feet. This is a non-breaking wave. For the same spectrum there can be a 20-foot, 6-second wave which is a breaking wave. As another example, for another particular design storm, the most probable maximum wave might be H = 23 feet and T = 12 seconds for d = 30 feet. This is a breaking wave. However, for the same spectrum there can also be a 23-foot, 14-second wave which is also a breaker.

ACKNOWLEDGEMENTS

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