



HARINGVLIET

PART 4
DYNAMIC ACTION OF WAVES

FLUSHING SEA WALL



CHAPTER 34
ON THE STABILITY OF RUBBLE-MOUND BREAKWATERS

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Some comments are presented on different formulas suggested for the design of rubble-mound breakwaters and results of laboratory tests concerning the design of these structures are mentioned. Iribarren's formula (the one, on the verification of which, the largest number of studies has been carried out) is then critically analyzed in the light of the results of laboratory tests. The applicability of laboratory studies to actual cases is discussed. Finally some suggestions are presented regarding questions to be taken into account in future research, due to the numerous points on which information is still lacking, in spite of the considerable volume of work already achieved.

I - INTRODUCTION

Until the beginning of the second quarter of the present century, characteristics of rubble-mound breakwaters were determined by entirely empirical methods, although harbour engineers had been dealing with this problem for many centuries. As a rule, designers merely compared the case under study with existing structures, prescribing sturdier breakwaters when those located in shores with a similar exposure had not withstood the most violent storms acting on them.

The first empirical formula for breakwater design did not appear before 1933, but this and other similar formulas did not go beyond ordering and reducing the use of arbitrary methods in the choice of the elements making up the breakwater slopes more directly subjected to wave action; no sensible progress resulting therefrom for the design methods of these structures. It can even be stated that, due to the use of Iribarren's formula - the most widely used in Europe - which leads to the utilization of too heavy blocks placed in steep slopes (about $4/3$), a tendency began to be observed in designers, towards a considerable reduction of these slopes.

Such a situation which, bearing in mind the knowledge available until about 10 years ago, was perfectly admissible, has been

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subjected to considerable changes thanks to: 1) the enormous advances achieved in the theoretical field, which placed our knowledge on the majority of Maritime Hydraulics subjects on a satisfactory level; 2) the invaluable help of small scale model tests, and 3) our improved knowledge on natural phenomena which makes possible a comparatively satisfactory estimate of the characteristics of the waves to be anticipated at any point of the coast.

We have merely to persevere along the route followed in the latter years in order to determine more accurate values for the coefficients of the available formulas, representing the results obtained by means of graphs and tables, resorting for that purpose both to model tests and to a careful observation of the behaviour of completed structures throughout the world, above all those which underwent damages. On the other hand efforts should not be spared in concentrated attempts to discover new formulas as phenomena are, no doubt, much too complex in the destruction of a breakwater to allow of a single satisfactory schematization.

It should be borne in mind that, in spite of the laboratory tests recently carried out, our knowledge is limited to the area directly affected by the wave breaking and so a total knowledge of the stability of rubble-mound breakwaters lies still a long way ahead.

II - EMPIRICAL FORMULAS

The first formula for the design of rubble-mound breakwaters was presented in 1933 by the Spanish engineer Eduardo Castro [1] :

$$W = 0.704 \frac{H^3 s}{(\cot \alpha + 1) (s-1)^3 \sqrt{\cot \alpha - \frac{2}{s}}} \quad (1)$$

where

W = weight of individual armor units in metric tons

H = wave height in meters

s = specific gravity of armor units

α = angle of breakwater slope measured from the horizontal

The preceding formula was based on the following theoretical assumptions: the destructive action of the wave is proportional to its energy, hence, the height of storm waves being proportional to their length, the energy of the waves is proportional to H^3 ; the weight of a unit required to resist the action of a given wave is directly proportional to its density in the air and inversely proportional to the cube of its density in water; the stability of the units under wave action is inversely proportional to a function of the angle of slope.

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This formula, yielding small values for W and making the angle of repose dependent on the specific gravity of the armor units, goes against what is known in Soil Mechanics. Harbour engineers rejected this formula which, as far as we know, remained without practical application.

The second formula, which is also due to a Spanish engineer - Prof. Iribarren Cavanilles -, is of particular interest, being in systematic use in Portugal since 1946. The formula was presented for the first time in 1938 [2] under the form:

$$W = K \frac{H_b^3 s}{(\cos \alpha - \sin \alpha)^3 (s-1)^3} \quad (2)$$

where, maintaining the above notations:

H_b = breaking-wave height, which can be determined by a method described in Iribarren's papers

$K = 0.015$ for quarry-stones

$K = 0.019$ for artificial blocks

According to the author [3], the following formula can be used when the water depth d at the toe of the structure does not exceed $0.06 L$, L being the wave length:

$$W = K \frac{H^3 s}{(\cos \alpha - \sin \alpha)^3 (s-1)^3} \quad (3)$$

where:

H = wave height in the absence of the structure

$K = 0.023$ for quarry-stone

$K = 0.029$ for artificial blocks

In 1950, Iribarren [4] generalized his formula so as to make possible its application in the design of underwater slopes.

According to this generalization, the design of breakwaters is also possible for slope elements at water depths exceeding H_b , by replacing H_b in formula (2), by

$$H' = H_z \frac{\frac{2\pi H_z}{L}}{\sinh \frac{2\pi z}{L}} \quad (4)$$

where:

Z = crown depth of the breakwater portion, the characteristics of which are to be determined

H_z = wave height at depth Z

Iribarren's original and modified formulas having aroused a deep interest in harbour engineering circles, they shall be present

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ed and discussed in detail below. We would emphasize, nevertheless, that this formula is similar to Castro's from which it only differs by the coefficient and by the function which takes into account the influence of the angle of slope. This is in fact the case with al most all the existing formulas. In addition, the application of this formula to steep slopes yields very high values for the weight of the armor units which, in the majority of cases, prevents the adoption of these slopes. This is the negative aspect of the formula which, as shall be seen below, disagrees most widely from nature in the ran ge of steep slopes (near $1/1$).

An analysis of the coefficients indicated by the author also shows that, all other factors being equal, the weight of the armor units required for a given breakwater is higher for artificial blocks than for quarry-stones which, as shall be seen below, is quite contrary to the facts observed in laboratory tests.

The coefficient $K = 0.015$ and $K = 0.019$ were determined by Iri barren from an analysis of the damages suffered by the breakwaters of Orio (quarry-stone) and San Juan de Luz (artificial blocks). The fact that the values of these coefficients were confirmed by a sole breakwater for each type and some peculiar conditions in both breakwaters (shallow depths at the toe as compared with the maximum wave heights attacking the structures and nature of the bottom), seems to indicate that the coefficients thus determined can at best apply to breakwaters in similar conditions. Consequently the author's generalization of his formula could only be confirmed by chance. In fact, K varies very widely with the different factors influencing the phenomenon.

Not before 10 years had elapsed after the presentation of Iri barren's formula did the problem begin to arise a wide interest in American engineers who, in a short time, proposed several formulas to solve the problem. In 1948, Mathews [5] of the Los Angeles District Corps of Engineers submitted a formula for discussion which, with the notations above, can be written thus:

$$W = 0.00149 \frac{H^2 T s}{(\cos \alpha - 0.75 \sin \alpha)^2 (s-1)^3} \quad (5)$$

where

T = wave period in seconds.

At the International Navigation Congress held in Lisbon in 1949, the American engineers Epstein and Tyrrel [6] presented the first results of their theoretical researches on rubble-mound breakwaters, which can be represented by the formula

$$W = K_t \frac{H^3 s}{(s-1)^3 (1 - \tan \alpha)^3} \quad (6)$$

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where

K_t = a function of α , μ and d/L including three additional coefficients defined as functions of the armor unit size
 μ = coefficient of friction stone on stone, practically equal to unity

The authors suggested laboratorial tests for determining K_t and its variation with the different parameters.

At the Conference on Coastal Engineering, held in Long Beach in 1950, F.W. Rodolf [5] of the Portland District Corps of Engineers presented a formula, based on the observation of hydraulic operations carried out in gold mines, which can be written as follows using the previous notations:

$$W = 0.0162 \frac{H^2 T s}{\tan^3 (45 - \frac{\alpha}{2}) (s-1)^3} \quad (7)$$

According to the author, the formula has a small coefficient of safety in order to take into account any wave eventually higher than the highest wave considered.

Finally in 1952, another formula was developed by the French engineer Larras [7], based on the century-old experience of the breakwaters of Algiers. This formula has the following expression in the preceding notations:

$$W = \frac{H_o^3 s}{(\cos \alpha - \sin \alpha)^3 (s-1)^3} \times \left[\frac{\frac{2\pi H_o}{L}}{\sinh \frac{4\pi Z}{L}} \right]^3 \quad (8)$$

where:

H_o = deep water wave-height
 K_o = 0.0152 for quarry-stone
 K = 0.0191 for artificial blocks

For breakwaters directly subjected to the wave breaking, the author recommends to take $Z = H_o/2$.

As already pointed out in different articles on the subject [8], both the expression above and the coefficients indicated by the author coincide with Iribarren's formula and coefficients, with only a difference, namely that Larras considers the wave-height in deep water, thus leading to lighter armor units for the breakwater.

Before the results of model tests were available, attempts were undertaken to verify the reliability of the different formulas and especially their coefficients, by comparison with the break-

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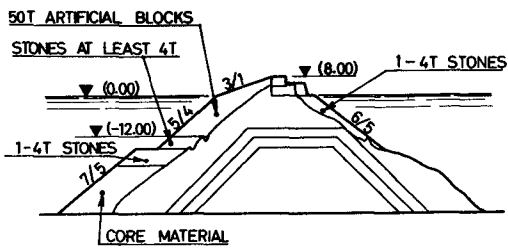


Fig. 1. Cross section of north break-water of Algier Harbour.

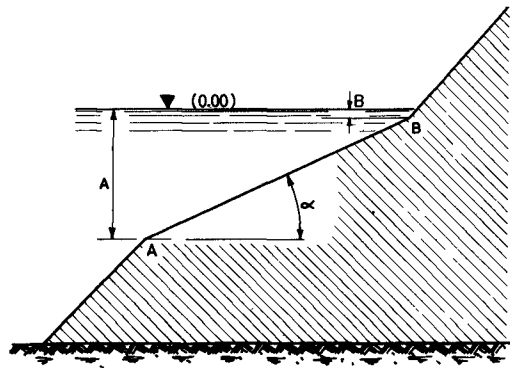


Fig. 2. Characteristic profile of equilibrium of a rubble-mound.

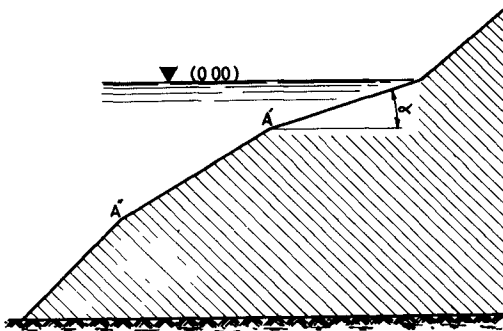


Fig. 3. Profile of equilibrium of a rubble-mound.

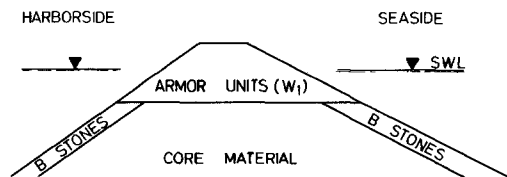


Fig. 4. Section of the rubble-mound breakwater tested in W.E.S.

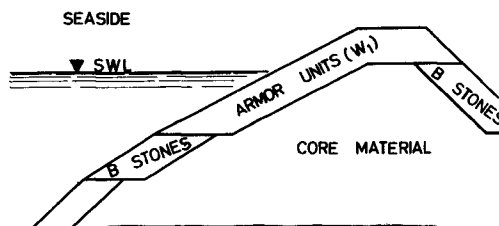


Fig. 5. Usual section of a rubble-mound breakwater.

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waters of Algiers harbour, chiefly the northern pier, the behaviour of which after being reinforced in 1933 (fig. 1) had been excellent notably during a storm in February 1934 in which it withstood, without serious damage, the attack of waves 9 m high.

From Hickson and Rodolf's comparison [5], between Iribarren's, Rodolf's, Mathews's and Castro's formulas and the slopes of the Algiers breakwaters deemed stable by Larras and Collin [9] and Iribarren [10], it was concluded that the values supplied by Iribarren's and Rodolf's formulas showed a perfect agreement with the stable slopes of Algiers breakwaters, whilst the values of Mathews's and Castro's formulas, although agreeing with one another, led to considerably steeper slopes (3.18/1 and 3.51/1 according to Rodolf and Iribarren, against 2.19/1 and 2.22/1 from Mathews's and Castro's formulas). As shall be seen later, this comparison with the slopes deemed stable of the Algiers breakwaters which had such a considerable influence on the wide acceptance of Iribarren's formula, had not the value then ascribed to it. In fact, subsequent model tests showed that comparatively reduced forces acted on the breakwater portion where the wave attack had been assumed to be strongest.

III - MODEL TESTS

The difficulties experienced by harbour engineers in the analysis of the different formulas based on the observed behaviour of breakwaters throughout the world; the impossibility of taking into account the influence of the different parameters which influence the stability of rubble-mound breakwaters; and above all the enormous advantages of model tests for improving our knowledge on the influence of each parameter, led to detailed laboratory tests on this problem, among which should be emphasized those carried out by the Waterways Experiment Station and by the Laboratoire de Neyrpic.

1) CHARACTERISTIC PROFILE OF EQUILIBRIUM

In the majority of cases studied at the laboratory, the profile of equilibrium of a homogeneous mound (the components of which undergo practically no displacements under the action of the waves) presented the shape indicated in fig. 2. This is a rather common shape called "characteristic profile of equilibrium" by Beaudevin [11]. As can be seen, the active zone of the breakwater, AB, extends practically from the still-water level down to a depth A ranging from 1.2 H and 1.6 H (mean value 1.3 H). It is in this zone that the influence of the different parameters has been studied in laboratory tests.

Below point A, the slope is approximately equal to the angle of repose in still water. Above point B the slope is often steeper

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than the angle of repose of the material [12]. This polygonal-line profile is always found, in its broad lines, whatever the material and the characteristics of the waves. Only angle α of zone AB with the horizontal, and the depth at point A (lower end of zone AB) are variable [11].

The shape of the "characteristic profile of equilibrium" seems to indicate that, whenever the depth at the base of the slope exceeds $1.3 H$, the influence of the depth is probably small.

This was indeed confirmed by the tests so far carried out in France [11,12] and U.S.A. [13]. This influence is not felt until the depth decreases beyond $1.3 H$ (that is the elevation at which the bottom intersects zone AB) but grows even together if the nature of the bottom allows under-toe sand scouring. It is noteworthy, as Miche observed in the discussion of Beau Devin's article, that $1.3 H$ is the breaker depth.

It also results from the shape of the "characteristic profile of equilibrium" that Iribarren's and Rodolf's comparisons of their formulas and coefficients with the slopes of Algiers breakwaters is not entirely correct since they took as active zone of the breakwater the portion above the hydrographic datum, which has a slope of $3/1$, instead of considering, as the shape of the "characteristic profile of equilibrium" indicates, the zone between the hydrographic datum and a depth of 12 m , where the slope is $5/4$ (fig. 1).

The comparison is further invalidated by the fact that during very violent storms the breakwater is often overtopped and stability conditions as the seaside face of a breakwater improve when this is overtopped.

But, even if this fact is neglected, a comparison of the slopes of the Algiers breakwaters (bearing in mind the results of the model tests) with the values supplied by Castro's, Iribarren's, Mathews's and Rodolf's formulas for the storm of February 1934, shows that the slopes obtained (respectively $2/1$, $3/1$, $1.95/1$ and $2.75/1$) are considerable gentler than the slope at the active zone of the breakwater. Hence the conclusion that the coefficients recommended by the different authors are much too high, at least for steep slopes; in other words, even if the formulas are reliable for certain values of the slope, the values yielded undergo very considerable changes with the variation of the angle of slope with the horizontal. For instance, whereas Iribarren indicates a value of $K = 0.019$ for breakwaters of artificial blocks, this coefficient for a $5/4$ slope should be, at most, $K = 0.00036$, i.e. 52 times smaller, according to the Algiers breakwater.

Besides the variation of Iribarren's coefficient K with the angle of slope, this disagreement seems to indicate a marked influence of an overtopping by the highest storm waves on the stability of the seaside face of the breakwater.

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2) INFLUENCE OF DIFFERENT PARAMETERS

a) Specific gravity of the armor units - The main purpose of the tests carried out at different laboratories was to study the validity of the existing formulas. Firstly the influence of different parameters on the weight of the armor units to be used at the active zone of a breakwater was investigated.

Practically all the authors admit that the influence of the specific gravity should be expressed by a term $s/(s-1)^3$. According to the first tests carried out in Grenoble [12], this law did not seem quite acceptable but no final conclusion could be reached due to the dispersion of the data points and the limited number of specific gravities studied. Nevertheless, subsequent tests carried out at the Waterways Experiment Station and also in Grenoble (although their primary purpose was different) showed that the law seemed quite valid.

Taking into account that the specific gravity of sea water, s_0 , is different from unity, the preceding expressions becomes $s.s_0/(s-s_0)^3$ which leads to an increase of about 10 to 15% in the weight of the armor units [12].

b) Wave-height - All the formulas, except Mathews's and Rodolf's, assume that the influence of the wave-height should be expressed by a law of the type $W = NH^3$.

Even the two exceptions noted above, in which a law of the type $W = NH^2T$ is assumed, yield for actual cases a variation of the type $W = NH^3$, since during storms, the wave-height and the period change, as a rule, in the same sense. Observations carried out for a period of 5 years in the Portuguese coast supplied for the ratio wave-height/wave period during storms an approximate value, $T = 2.5 H$ (in which T is expressed in seconds and H in meters) which is the same that was used in fig. 6, based on Mathews's and Rodolf's formulas.

Tests carried out at different laboratories have fully confirmed the law derived from the existing formulas.

As regards the influence of the wave-height on the "characteristic profile of equilibrium", tests carried out in Grenoble [11], showed that, for wave-heights below a certain value, the profile has the shape indicated in fig. 2, the lower point A lying at a depth proportional to H , as previously explained.

For wave-heights exceeding the value in reference, the profile presents the shape indicated in fig. 3.

c) Period - According to the tests so far carried out in order to investigate the influence of this parameter, a wave is all the more dangerous the smaller its period although, on the whole, this influence is never considerable [11]. This shows that Mathews's and Rodolf's formulas are incorrect as, for the same wave-height, they yield increasing weights for the armor units when the period

increases.

It was further concluded that, for characteristic profiles with the shape indicated in fig. 3, the depth A' depends on the wave period, whilst depth A'' depends on the wave-height only [11].

d) Depth in front of the structure - In the tests carried out at the different laboratories, the depth at the toe of the breakwater was always large as compared with the wave-height [12,13,14], hence rather larger than the depth at point A of the "characteristic profile of equilibrium". Thus no influence of the depth could be detected. Nevertheless, a few preparatory tests carried out at the Laboratório Nacional de Engenharia Civil with depths approaching A showed a marked influence of this parameter. Other tests are under way in order to study this influence in detail. Tests carried out at the Waterways Experiment Station concerning a breakwater for Narwiliwili harbour have also shown the enormous influence of the relative depth d/L and of the wave steepness H/L on the stability of the structure for waves breaking directly on the breakwater slope.

e) Shape of the armor units - Blocks of different shapes have been considered in the tests so far carried out: quarry-stones, cubes, tetrapods, tribars, besides other shapes less common in practical cases. The tests showed a very marked influence of the shape although the stability curves for the different types of blocks can be approximately derived from anyone of them by affinity. It is thus possible to characterize each shape means of a constant parameter [11].

One of the main conclusions drawn from the first tests carried out, was that cubic blocks were better than quarry-stones with respect to stability, contrary to Iribarren's and Larra's deductions for their formulas.

Tests carried out in Grenoble in 1953 showed that quarry-stone with a weight $3W$ would be required to supply the same stability as cubic blocks of weight W , but more numerous tests in 1955 corrected the above ratio to 2/1 only. This difference is due to the fact that cubic blocks with slightly rounded edges were considered in the second case. This assumption should be nearer to the actual phenomenon, as cavitation is observed in model tests near the edges of the blocks in the active zone of the breakwater.

The tests at the Waterways Experiment Station concerned, above all, the behaviour of quarry-stones, tetrapods and tribars [15], but very few tests were carried out regarding cubic blocks.

The ratio between the weights of quarry-stones and tetrapods required to ensure the same stability was verified to be about 2.6. A comparison of these tests with those carried out in Grenoble shows that tetrapods and sharp-edged cubes have a similar stability, which slightly exceeds the stability of cubes with rounded edges, as was confirmed by the tests of Funchal harbour [16].

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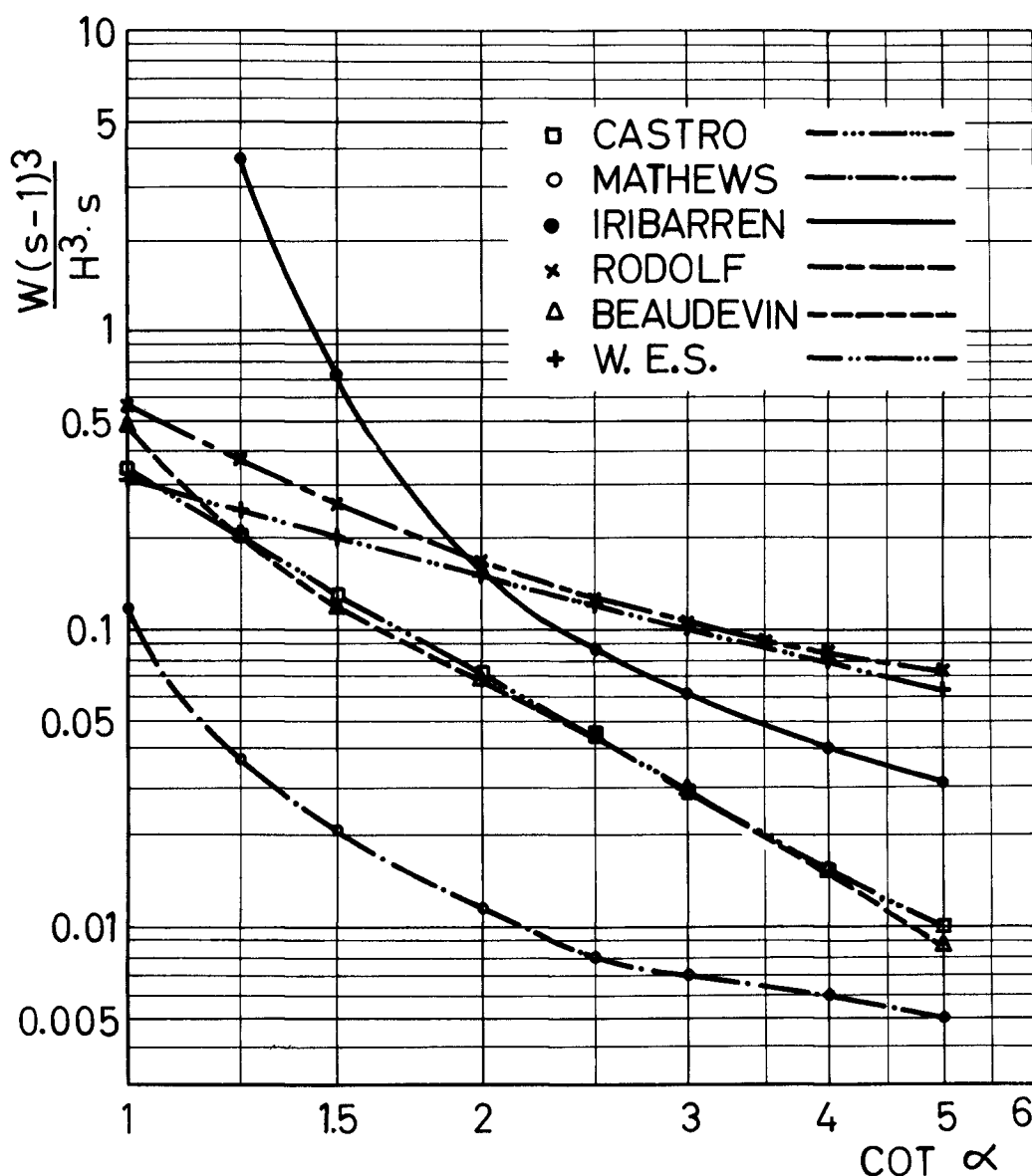


Fig. 6. Variation law of the parameter $\frac{W(s-1)^3}{H^3 \cdot s}$ with α , based on the following assumptions:

- In Iribarren's formula $\mu = 1$ and $K = 0.015$;
- In Castro's formula $s = 2.5$;
- In Rodolf's and Mathew's formula $T = 2.5 H$, T being in seconds;
- In Beaudevin's formula $K = 0.10$;
- In all formulas H is expressed in meters and W in metric tons.

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Quarry-stone cover layers being made up, in actual cases, of units with different weights ranging between comparatively narrow limits, tests were carried out in Grenoble in order to study the stability of these structures. The results showed that "the characteristic weight" of the quarry-stones is equal to or greater than the mean weight.

It was even observed that, in some cases, the stability of the mixed quarry-stone exceeded the stability of the heavier component blocks. Nevertheless, in practical cases it is recommended to take as characteristic weight of a given type of quarry-stone the mean weight of the type in reference. In the great majority of cases, a comparatively small safety factor is thus secured [11] .

3) FORMULAS OBTAINED FROM LABORATORY TESTS

When attempts began to study the stability of breakwaters by means of laboratory tests, the first problem to be solved consisted in determining up to what extent the conclusions drawn from the model tests could be applied to actual structures. For that purpose, an extensive program of basic research was undertaken at the Waterways Experiment Station which showed that Froude's law applied to all the phenomena which take place when waves attack a rubble-mound breakwater. Besides a comparison of the results obtained on models built at different scales, the program included a comparison of the damages undergone by completed breakwaters with the damages observed in models of those structures when subjected to the waves responsible for the actual damages. This comparison, the results of which were decisive for the acceptance of model test studies on the stability of rubble-mound breakwaters, showed a remarkable agreement between the behaviour of models and prototypes [13] . Analogous tests with the same purpose carried out in Grenoble yielded the same results [17] , what proves that this practical and serviceable tool - small scale models - is quite reliable in the study of these complex problems. Nevertheless, tests are under way at the Beach Erosion Board with a view to studying the influence of Reynolds's number, i.e. the scale effect. The results seem to show that there is indeed a certain scale effect which, at the usual scales, yields rather conservative results, a certain margin of safety being thus secured.

The research program of the Waterways Experiment Station was then extended so as to include the determination of the most adequate design-formula. In the first place, the validity of the existing formulas, in special Iribarren's and Epstein-Tyrrel's was investigated. These and, to a smaller extent, the preceding tests, imply the consideration of a "criterion of stability" which is a very important factor in the conclusions of tests of this type. In the first series of tests [13] , the design-wave height considered was slightly less than that required to move any of the armor stones of

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stones of the breakwater. Based on this so-called "no-damage" criterion, the conclusion was reached that Iribarren's formula is the most suited to the test results, although the coefficient K varied to a considerable extent with the angle of the breakwater slope measured from the horizontal. This comparison was preceded by a slightly theoretical study which had the purpose of making Iribarren's formula dimensionally homogeneous. The formula then becomes

$$W = K \frac{H^3 \gamma_r \gamma_f \mu^3}{(\mu \cos \alpha - \sin \alpha)^3 (\gamma_r - \gamma_f)^3} \quad (9)$$

where:

H = wave height at the breakwater toe
 μ = tangent of the angle of repose of armor units
 γ_r = specific weight of armor units
 γ_f = specific weight of water

Other tests were carried out with a different criterion. The wave was allowed to move some of the units but not, however, to induce sensible changes in the breakwater.

By means of this so-called "slight-damage" criterion, it was concluded that breakwaters designed according to the preceding criterion could withstand the attack of waves 50% higher than the design-wave, without undergoing serious damage.

This led the Waterways Experiment Station to modify his "no-damage" criterion which was too severe, as the fall of a few units is not due, as a rule, to deficient stability of the breakwater but to the fact that these units were placed in a peculiarly unstable position during construction.

In the subsequently adopted "no-damage" criterion, the design-wave height has a value which can induce some damage but the number of armor stones moved shall not exceed 1%.

Based on the modified criterion, a new series of tests was carried out at the Waterways Experiment Station. From the conclusions obtained, presented by Hudson [14], a new design formula resulted for this type of breakwater:

$$W = \frac{H^3 \gamma_r}{K_D (s_r - 1)^3 \cot \alpha} \quad (10)$$

where:

K_D = coefficient depending on the percentage of damage with values:
 $K = 3.2$ for quarry-stones (no damage)
 $K = 8.3$ for tetrapods (no damage)
 $s_r = \gamma_r / \gamma_f$

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The introduction of this objective numerical criterion was one of the major improvements achieved in model tests studies of stability of rubble-mound breakwaters. Nevertheless, the test section adopted in the W.E.S. for quarry-stone breakwaters (fig. 4) is not the most usual in structures of this type. In fact (fig. 5) armor units are normally placed in two or three layers instead of making up a mound. So, it is likely that, even if the results applying to the "no-damage" case may be the same, they are not correct as regards to the tests for determining the safety factor in "damage" cases. It is in fact possible that in many actual cases the structure would collapse for values of $H/H_{D=0}$ which in the tests carried out at the W.E.S. induced damages of only 15% to 40%.

Iribarren's formula was abandoned in the W.E.S. tests due to the fact that the influence of the angle of slope, according to this formula, disagreed very sharply with the experimental results. Accurate values of the friction coefficient of the different materials were also extremely hard to obtain in the laboratory, which led to a wide variation of coefficient K [14]. For all these reasons, it was decided to adopt another formula, presented in (10). As regards to form, this formula is apparently not altogether correct, as it omits the angle of repose of the armor units but, on the other hand, it presents the enormous advantages of containing a coefficient K_D depending exclusively on the type of armor unit, and of being very easy to handle since the function expressing the influence of the angle of slope is very simple.

Another research program on the stability of rubble-mound breakwaters by means of model tests was also carried out in Grenoble, at the Laboratoire Dauphinois d'Hydraulique. Another stability criterion was used: the wave was allowed to model a profile of equilibrium in a homogeneous mound of the armor units to be studied. This was the "characteristic profile of equilibrium" since, according to the tests, it was stable for wave-heights not exceeding the height of the wave that had shaped it but unstable for wave-heights above that value [11] under the action of which, fall of blocks were observed. Some units can be unstable on the breakwater, undergoing alternate movements with the same period as the wave [12].

From these tests the following practical formula was obtained

$$W = K \frac{H^3 s}{(s-1)^3} \left(\frac{1}{\cot \alpha - 0.8} - 0.15 \right) \quad (11)$$

The tests supplied the following values for K

$K = 0.10$ for slightly rounded quarry-stones

$K = 0.05$ for cubes with slightly rounded edges

Grenoble recommends a safety factor of 2.5, the values of K

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being then

$K = 0.25$ for quarry-stones

$K = 0.12$ for cubes

The same tests also showed that, in order to avoid the blocks being moved along by oblique waves, an additional condition must be introduced, with which "absolute stability" is achieved. This condition is given by the expression:

$$W > K' \frac{H^3 s}{(s-1)^3} \quad (12)$$

where

$K' = 0.03$ for quarry-stones

Making $K = 0.25$ in formula (11), this condition is fulfilled for breakwater slopes steeper than $9/2$, that is practically for all breakwaters used in actual cases.

IV - COMMENTS ON THE EXISTING FORMULAS

A comparison of the different formulas presented shows that the main difference between them lies in the type of function used to express the influence of the breakwater slope. In fact the term $\frac{W (s-1)^3}{H^3 s}$, can be said to be common to all the formulas. Even in

Mathews's and Rodolf's expressions, the influence of the characteristics of the wave can be expressed by H^3 by taking, as explained above, $T = 2.5 H$ which seems to agree with observed phenomena.

The variation of this term in the function of the breakwater slope, as obtained from the different formulas, is shown in fig. 6.

For Beaudevin's formula, a value $K' = 0.10$ was taken which corresponds to the value directly obtained from the tests, that is without safety factor.

The differences observed between the test results obtained at the W.E.S. and in Grenoble are due to the different stability criteria used and to the design-wave height adopted at the W.E.S. In the way the tests were carried out at the W.E.S., the breakwaters were not subjected to the action of a uniform-height train of waves as in Grenoble, but to a succession of waves, the first and the last of which were higher than the others due to the starting and stopping of the wave generator. Because these waves have an obvious influence on the breakwater stability, the "significant height" of the train of waves attacking the breakwater was the wave height

selected for the final calculation of the results.

Nevertheless, the differences between the parameters of wave trains in nature, on the one hand, and the same parameters as analysed in the laboratory are enormous [14] and this fact should be borne in mind in the choice of the design-wave height and of the safety factor.

For almost every type of breakwater, the W.E.S. formula yields values exceeding those obtained in similar conditions by means of Beaudevin's formula, although its variation with the angle of slope is not so marked. In fig. 6, these curves intersect for an angle of slope between $1/1$ and $5/4$; the value supplied by Beaudevin's formula for a slope of 45° exceeding the value obtained from the W.E.S. formula.

This is due to the fact that the functions expressing the influence of the angle of slope were obtained from tests on slopes not steeper than $5/4$, at the American laboratory, and of about $4/3$, in Grenoble. The extrapolation for steeper slopes and even the results obtained for limit values of the angle of slope may not entirely agree with the actual behaviour of very steep slopes. Nevertheless, taking into account that the stability criterion adopted at the W.E.S. is better suited to actual conditions, it seems preferable, in practical cases, to use the American formula.

As shown in fig. 6, Mathews's formula is obviously deficient yielding weights W for the armor units much below the values obtained in the Grenoble and W.E.S. tests and so its use, even with a high safety factor, should be discontinued. On the other hand, as already pointed out by Barbe and Beaudevin [12], the values determined from Castro's formula for $s = 2.5$ entirely agree with the results obtained in the tests on rubble-mound breakwaters carried out in Grenoble. This thus means that by the way the tests were carried out in Grenoble, the values supplied correspond to rather peculiar limit conditions of stability and so, as indicated by Barbe and Beaudevin, Castro's formula may be used with a safety factor of no less than 2.5.

As for Rodolf's formula, provided that, as previously indicated, a value $T = 2.5 H$ is taken, the figure indicates that, for slopes gentler than $2/1$ the values supplied slightly exceed those of W.E.S. formula; whilst for steeper slopes the values disagree more strongly with the results of the American tests. This means that, for gentle slopes, the safety factor can be slightly above unity, its value increasing as the slope becomes steeper, reaching 1.5 for a $5/4$ slope and about 1.6 for an angle of 45° , which agrees with what the author had in mind. An increase of the safety factor seems admissible since damages are much more dangerous in very steep than in gentle slope breakwaters.

All these comments above refer to quarry-stone rubble-mound breakwaters as, apart from the formulas obtained from laboratory tests, only Iribarren and Larras indicated coefficients for arti

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ficial blocks which, however, by no means express the differences observed between stability conditions in quarry-stone and in artificial block breakwaters.

Comments on Iribarren's formula are presented below in greater detail, due the wide acceptance of this formula in harbour engineering circles. These comments apply, to a certain extent, to Larra's formula as this and its coefficients expressing the influence of the shape of armor units coincide with Iribarren's formula and coefficients.

V - COMMENTS ON IRIBARREN'S FORMULA

Iribarren's formula presented in II is the most widely known and used both in designs (notably in Europe) and in test verifications. In fact, although the formula

$$W = \frac{K \gamma_r \gamma_f^3 H^3 \mu^3}{(\mu \cos \alpha - \sin \alpha)^3 (\gamma_r - \gamma_f)^3} \quad (13)$$

has been in use for all values, both steep and gentle, of the breakwater slope, there is another formula by the same author

$$W' = \frac{K' \gamma_r \gamma_f^3 H^3}{(\mu \cos \alpha - \sin \alpha)^3 (\gamma_r - \gamma_f)^3} \quad (14)$$

recommended for steep slopes for which, however, the value of K' is unknown. In the present chapter only the first formula will be discussed, because, as shall be seen below, it is possible to adopt this formula to any breakwater slope gentle or steep, and so it is unnecessary to take the second formula separately into account since, for $K' = K \mu^3$, it becomes equal to the first.

Before analysing the formula more in detail, let us summarily pass in review the conditions in which Iribarren adopted the value of K , which he assumed constant for any breakwater slope.

This coefficient was obtained from the damages observed in a sole breakwater, which is obviously insufficient. On the other hand, the wave-heights were not observed but merely calculated from theoretical considerations of the water depths near the structure, which also correspond to very particular conditions. In fact, for a sandy bottom, the water depth is practically zero for the lower low water and about 4.5 m for the higher high water. These conditions are indeed extremely peculiar both as regards the depth and the nature of the bottom on account of not only the influence of the relative depth on the form assumed by the wave breaking but also of the considerable increase in the specific gravity of sea water due to the bottom sand in suspension.

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Another doubtful point Iribarren's formula is the coefficient μ . According to Iribarren this parameter, defined as the coefficient of friction stone on stone, is very nearly equal to the slope of the angle of repose for quarry-stones (i.e. unity) and, by taking $\mu = 1$, Iribarren believes that an adequate safety factor is introduced in the formula. Admitting Iribarren's assumption that μ is practically the slope of the angle of repose for quarry-stones, the value of this angle remains to be determined. Attempts to measure it in laboratory tests yield widely differing values. Thus, laboratory tests carried out at the Waterways Experiment Station have furnished values ranging between 1.06 and 1.18. Nevertheless even assuming that a real value of μ , say $\mu_r > 1$, could be defined and determined, it would be necessary to know the value of the safety factor which, according to Iribarren, is secured by taking $\mu = 1$. Some comments are presented below regarding this subject.

1) COMMENTS ON COEFFICIENTS K AND μ OF THE FORMULA

$$W = \frac{K \gamma_r \gamma_f^3 H^3 \mu^3}{(\mu \cos \alpha - \sin \alpha)^3 (\gamma_r - \gamma_f)^3}$$

In the first place let us accept Iribarren's assumption in which K is constant for any value of the breakwater slope. Let K_1 be the value of the constant for $\mu = 1$ and K_r for $\mu = \mu_r$. Bearing in mind the conditions α_0 , γ_0 , H_0 and $\bar{P}_0$, for which Iribarren calculated K_1 , by taking $\mu = 1$, it is obvious that no safety factor is introduced as regards the use of the formula with $K = K_r$, by taking $\mu = \mu_r$.

Let us consider an angle of slope approximately equal to the value used by Iribarren in the determination of the coefficient for quarry-stones breakwaters, i.e. $\cot \alpha_0 = 3.0$. For $\mu = 1$, the formula becomes

$$W_1 = \frac{K \gamma_r \gamma_f^3 H^3}{(\cos \alpha - \sin \alpha)^3 (\gamma_r - \gamma_f)^3}$$

and for $\mu = \mu_r$

$$W = \frac{K_r \gamma_r \gamma_f^3 H^3 \mu^3}{(\mu_r \cos \alpha - \sin \alpha)^3 (\gamma_r - \gamma_f)^3}$$

hence

$$\frac{W_1}{W_r} = \frac{K_1}{K_r} \frac{(\mu_r \cos \alpha - \sin \alpha)^3}{\mu_r^3 (\cos \alpha - \sin \alpha)^3}$$

As $W_1 = W_r$ for $\alpha = \alpha_0$, it follows that

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$$\frac{K_1}{K_r} = \frac{\mu_r^3 (\cos \alpha_0 - \sin \alpha_0)^3}{(\mu_r \cos \alpha_0 - \sin \alpha_0)^3}$$

Consequently, the ratio W_1/W_r for any value α is

$$m = \frac{W_1}{W_r} = \frac{\mu_r^3 (\cos \alpha_0 - \sin \alpha_0)^3}{(\mu_r^3 \cos \alpha_0 - \sin \alpha_0)^3} \cdot \frac{(\mu_r \cos \alpha - \sin \alpha)^3}{\mu_r^3 (\cos \alpha - \sin \alpha)^3}$$

The ratio m would be the safety factor introduced by taking $\mu = 1$ instead of $\mu = \mu_r$.

In order to give an idea of the variation of m with α , let us take $\cot \alpha_0 = 3$, making μ_r equal to 1.2 and 2.4. The computations carried out, plotted in the graph of fig. 7, are presented in Table I.

TABLE I

	m	
$\cot \alpha$	$\mu_r=1,2$	$\mu_r=2,4$
10,0	0,82	0,55
5,0	0,88	0,68
3,0	1,00	1,00
2,0	1,21	1,77
1,5	1,87	4,67
1,33	2,60	9,49
1,25	3,57	16,92
1,00	∞	∞

TABLE II

	m	
$\cot \alpha$	$\mu_r=1,2$	$\mu_r=2,4$
10,0	0,48	0,32
5,0	0,52	0,40
3,0	0,59	0,59
2,0	0,71	1,04
1,5	1,10	2,74
1,33	1,52	5,56
1,25	2,09	9,92
1,00	∞	∞

An analysis of the table shows that:

- 1) Iribarren's formula with $\mu = 1.0$ does not necessarily contain a safety factor;
- 2) This safety factor applies exclusively to slopes steeper

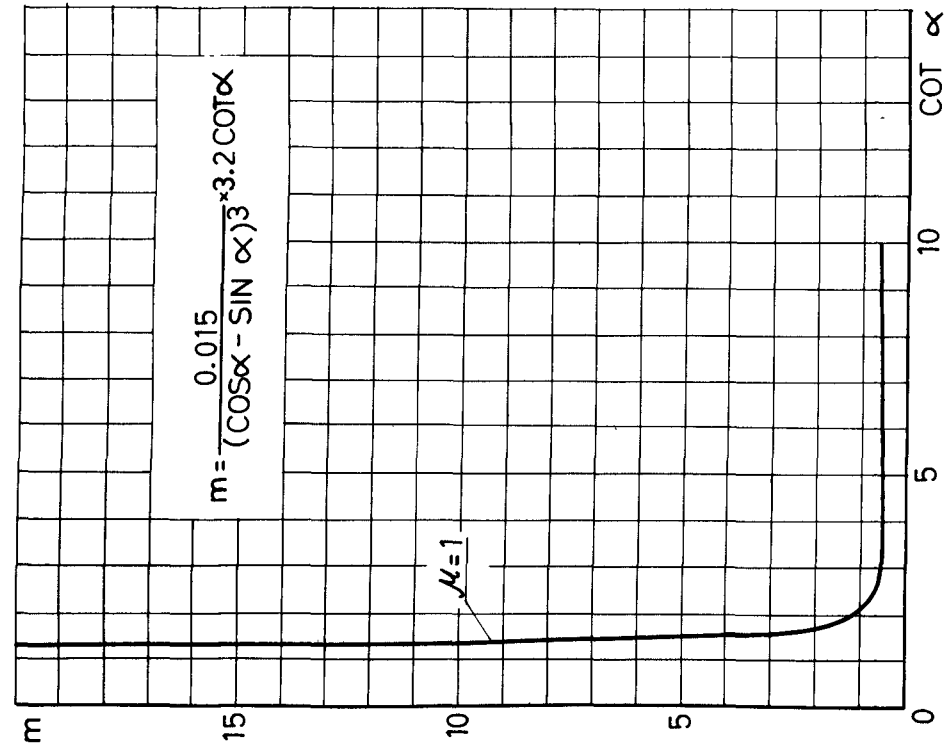


Fig. 8. Safety coefficients introduced by Iribarren's formula ($\mu = 1; K = 0.015$) when compared with Hudson's formula.

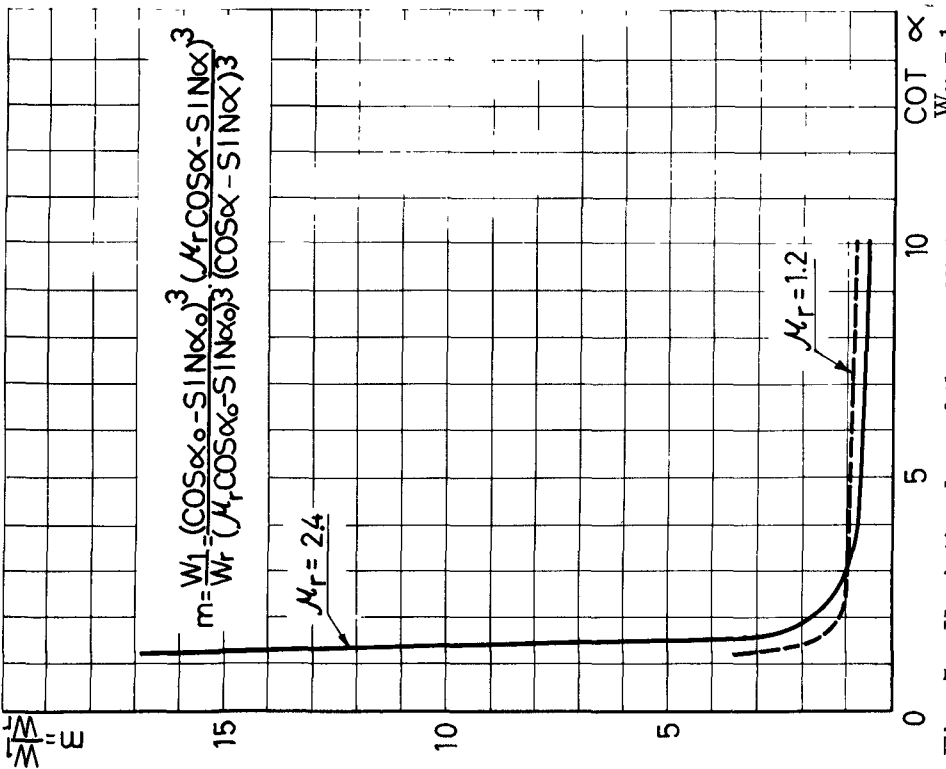


Fig. 7. Variation law of the coefficient $m = \frac{W_{\mu=1}}{W_{\mu}} = \frac{1}{\mu_r}$ when using Iribarren's formula with $\mu = 1$ and $\mu_r = 1.2$ and 2.4 .

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than the one for which the constant was determined;

3) The same safety factor increases with the angle of slope measured from the horizontal.

4) For angles of slope less than the value for which Iribarren determined his coefficient, the above-mentioned ratio $\underline{m}$ is an unsafety factor.

2) VARIATION OF THE COEFFICIENT OF IRIBARREN'S FORMULA WITH α

Hudson's formula for quarry-stone rubble-mound breakwaters has the expression

$$\frac{\gamma_r^{1/3} H}{W^{1/3} (S_r - 1)} = (K_D \cot \alpha)^{1/3}, \quad K_D \text{ being constant}$$

for all the values of $\underline{\alpha}$.

Iribarren's formula can be written in like manner

$$\frac{\gamma_r^{1/3} H}{W^{1/3} (S_r - 1)} = \frac{(\mu \cos \alpha - \sin \alpha)}{K^{1/3} \mu}$$

From these two equations the following is obtained

$$K = \frac{(\cos \alpha - \frac{1}{\mu} \sin \alpha)^3}{K_D \cot \alpha}$$

By making $\mu = \underline{\mu}_r$ (real unknown value of $\underline{\mu}$) the expression

$$K_r = \frac{(\cos \alpha - \frac{1}{\underline{\mu}_r} \sin \alpha)^3}{K_D \cot \alpha} \quad \text{is obtained which shows}$$

that K_r varies with $\underline{\alpha}$.

Consequently the coefficient $\underline{K}$ of Iribarren's formula should vary with $\underline{\alpha}$, whatever the value selected for $\underline{\mu}$.

Let us now determine the stability factor yielded by Iribarren's formula with the coefficient recommended by him, by comparison with the observed behaviour of a breakwater which reached its profile of equilibrium for $\cot \alpha = 3$, assuming that Hudson's experimental formula is exact and that Iribarren's coefficient would not vary with α .

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A comparison of the values $\frac{W (s_r - 1)^3}{H^3 s_r}$ for Iribarren's and Hudson's formulas shows that, for $\cot \alpha = 3$ Iribarren's formula with $\mu = 1$ and the coefficient recommended by him, yields weights $\frac{0.104}{0.061} = 1.7$ smaller.

Therefore the stability factors indicated in Table I become those of Table II.

As explained these coefficients imply that K_r is independent of α . Considering now that K_r varies with α , an expression is obtained which represents the general stability factor of a quarry-stone rubble-mound breakwater obtained from Iribarren's formula as usually applied:

$$m = \frac{0.015}{(\cos \alpha - \sin \alpha)^3} \times 3.2 \cot \alpha$$

This expression, plotted in the graph of fig. 8, shows that m varies with α and can be greater or less than 1. From the brief analysis above it results that:

a) If $\cot \alpha > \mu_r$ and $\mu < \mu_r$, Iribarren's formula,

$$W = \frac{K \gamma_r \gamma_f^3 H^3 \mu^3}{(\mu \cos \alpha - \sin \alpha)^3 (\gamma_r - \gamma_f)^3}$$

becomes Hudson's formula,

$$W = \frac{H^3 \gamma_r \gamma_f^3}{(\gamma_r - \gamma_f)^3 K_D \cot \alpha}$$

whatever the value selected for μ , provided that K varies according to the expression

$$K = \frac{(\cos \alpha - \frac{1}{\mu} \sin \alpha)^3}{K \cot \alpha}.$$

b) Accepting Iribarren's formula with the coefficient recommended by him, the safety (or unsafety factor) varies with α according to the expression

$$m = \frac{0.015}{(\cos \alpha - \sin \alpha)^3} \times 3.2 \cot \alpha$$

VI - FINAL REMARKS

During the preceding comparative analysis of empirical and semi-empirical breakwater-design formulas (the latter including experimental expressions) nothing was said regarding the actual

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value of experimental results. Without discussing the important detail of the stability criteria used in model tests, let us pose the following question. Up to what extent do model tests reproduce actual conditions in the prototype? Let us recall, in the first place, the general similitude conditions prevailing in the model study of a breakwater: similitude obeys Froude's law, the wave train being characterized in each test by the expressions $T = \text{const.}$ and $H = \text{const.}$ Taking into account the scales usually employed in these tests (normally $\lambda \geq 1/50$), viscosity effects are negligible and, only short-period waves being of interest in these studies, the use of Froude's similitude law seems justified. The same is not true, however, concerning the usually considered simplification which consists in taking $H = \text{const.}$ and $T = \text{const.}$ for the wave trains. Thus assuming, as it seems reasonable to do, that the term $\frac{\partial V}{\partial t}$ expressing the effect of the inertia forces resulting from the dissipation of the kinetic energy of the waves, plays an important rôle in the stability of the breakwater, such a simplification may seem doubtful. In fact, whilst for $T = \text{const.}$ and $H = \text{const.}$ the term $\frac{\partial V}{\partial t}$ is a periodic function with a period T , this is no longer true if T and H are any time functions whatsoever. That $\frac{\partial V}{\partial t} = f(t)$ is an important function to be considered in the study, may be a deficiency to be pointed out in model tests in which it is assumed that $T = \text{const.}$ and $H = \text{const.}$ Unfortunately no sure knowledge is available about the influence of $\frac{\partial V}{\partial t}$ on the stability of breakwaters and until this influence is carefully investigated, designers will have no choice but to observe the behaviour of breakwaters designed by means of model tests. In fact, a considerable number of breakwaters having been designed at the laboratory (those with tetrapod or tribar cover-layers for instance), the observation of the behaviour of the completed structures can give practical indications on the value of model test design of breakwaters. Such an observation, if carefully carried out, may even yield valuable experience likely to improve to a great extent the experimental design of breakwaters. Bearing in mind that several breakwaters designed by means of model tests and completed, some of them, years ago, present satisfactory behaviours, it seems natural to believe that, so far, there are no indications regarding improvements or changes required in the technique used, up to the present, in model tests of the stability of breakwaters. We believe, nevertheless, that an attempt should be made to generalize the use of wave generators able to reproduce actual wave trains with an adequate accuracy. A comparison of test results obtained by means of actual wave trains with results obtained by replacing these by

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uniform wave trains ($T = \text{const.}$ and $H = \text{const.}$) could be a great help for improving laboratory studies.

It is altogether different, nevertheless, to recommend as reliable, results of laboratory tests carried out on specific models, and also to recommend the indiscriminate use of the formulas so far presented based on reduced model tests. Thus both Beaudevin's (Neyrpio) and Hudson's (Waterways Experiment Station) formulas result from test conditions which can be regarded as being too particular. In fact these formulas contain as parameters, the weight and the specific gravity of the blocks, the angle of slope and the wave-height. Effects of parameters deemed important are omitted, such as the ratio H/d near the breakwater toe, the nature of the bottom and its relief near the structure. The latter showed itself particularly important in the study carried out at the Laboratório Nacional de Engenharia Civil, in which H/d was very large (about 0.8) [18]. As for the nature of the bottom, it is admitted that its influence is considerable, whenever d has the same order of magnitude as the depth at which waves break.

Nevertheless until all the parameters with a marked influence on the stability of breakwaters are known, it is advisable that a structure be individually tested, whenever it will have to withstand actions other than those for which the formulas were determined in the laboratory. For similar conditions, these formulas can be used in the design of the protection zone subjected to the most violent action of the waves (i.e. down to a depth of about $1.5 H$ below still-water level). As none of the formulas tentatively presented for the design of the underwater layers of breakwaters has yet been experimentally confirmed, there is no alternative but to test the whole breakwater. This means that the formulas at present available must still be augmented with the experimental design of breakwaters. It is recommended to carry out experimental studies for designing the cover layer at any elevation below water level. Another important detail to be dealt with in laboratory tests concerns the stability of singular points such as the breakwater head. This zone, in fact, has to be carefully studied in every breakwater design, and for lack of experimental data, some designers recommend a considerable increase of the resistance in this zone.

VIII - CONCLUSIONS

A comparison of empirical and semi-empirical breakwater-design formulas (both experimental or not), shows that:

a) Castro's formula presents a shape rather similar to that of the experimental formula suggested by Beaudevin;

b) For slopes such that $\cot \alpha > 2$, Iribarren's formula is intermediate between the Waterways Experiment Station and Beaudevin's experimental formulas, but for $\cot \alpha < 2$ it is different

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from both;

c) The two experimental formulas present different shapes, what may be due to either the different stability criteria or the different wave-heights adopted at each laboratory;

d) Any of the two experimental formulas can serve as a basis for the preliminary design of a breakwater, provided that the conditions in which the tests were carried out and the stability criteria followed are duly borne in mind so that a suitable safety factor is adopted according to the stability criteria followed in each case;

e) In view of the innumerable parameters, many more still yet unknown, which play a rôle in the behaviour of breakwaters, the design formulas presently available do not dispense with experimental tests in each actual case. Nevertheless, as a first approximation, these formulas can give indications on the technical and economical feasibility of rubble-mound breakwaters;

f) The design of the underwater portions and of the singular points, notably the head, of rubble-mound breakwaters should be included in laboratory research programs on breakwaters.

REFERENCES

- [1]- Castro, Eduardo - "Diques de escollera" - Revista de Obras Públicas, Abril de 1933.
- [2]- Iribarren Cavanilles, R. - "Una formula par el calculo de diques de escollera" - Revista de Obras Públicas - 1938.
- [3]- Iribarren Cavanilles, R. - "Obras Marítimas. Oleaje y diques". Editorial Dossat, Madrid 1954.
- [4]- Iribarren Cavanilles, R. and Nogales, C. - "Generalizacion de la formula para el calculo de los diques de escollera y comprobacion de sus coefficients" - Revista de Obras Públicas, Maio 1950.
- [5]- Hickson, R. and Rodolf, F. - "Design and Construction of jetties" - Proceedings of First Conference on Coastal Engineering - 1951.
- [6]- Epstein, H. and Tyrrel, F. - "Design of rubble-mound break water" - XVII^e Congrès International de Navigation, Section II, Communication 4.
- [7]- Larras, J. - "L'équilibre sous-marin d'un massif de matériaux soumis à la houle" - Le Genie Civil, 15 Septembre 1952.

COASTAL ENGINEERING

- [8] - Abecasis, Carlos K. - "Données d'expérience sur les cotes du continent et des îles Portugaises de l'Atlantique du Nord" - XVIII^e Congrès International de Navigation - Section II, Question I.
- [9] - Larras, J. and Colin, H. - "Les ouvrages de protection du port d'Alger à talus inclinés" - Travaux, vol. 31, pp.603-609; vol. 52, pp.163-168.
- [10] - Iribarren Cavanilles, R. and Nogales, C. - "Nueva Confirmacion de la Formula para el Calculo de los Diques de Escollera" - Revista de Obras Públicas - Janeiro 1953.
- [11] - Beaudevin, C. - "Stabilité des diques à talus à carapace en vrac". La Houille Blanche - Mai-Juin 1955.
- [12] - Barbe, R. and Beaudevin, C. - "Recherches expérimentales sur la stabilité d'une jettée à talus incliné soumise à la houle". La Houille Blanche - Juin-Juillet 1953.
- [13] - Hudson, R. Y., and Jackson, R.A. - "Stability of rubble-mound breakwater" - W.E.S. Technical Memorandum 2-365, Vicksburg 1953.
- [14] - Hudson, R. Y. - "Design of quarry-stone cover layers for rubble-mound breakwaters" - W.E.S. Research Report 2-2, Vicksburg 1958.
- [15] - Hudson, R. Y. and Jackson, R.A. - "Design of tribar and tetrapod cover layers for rubble-mound breakwaters" - W.E.S. Miscellaneous Paper 2-296, Vicksburg 1959.
- [16] - "Estudo experimental do porto do Funchal - Ensaio de estabilidade. Relatório complementar" - Laboratório Nacional de Engenharia Civil, Lisboa 1957.
- [17] - Schaer, G. - "Stabilité des ouvrages maritimes. Comparaison modèle-réalité" - Compte Rendu des Quatrièmes Journées de l'Hydraulique. Paris 1956.
- [18] - "Nota sobre os ensaios de estabilidade dos molhes da Figueira da Foz" - Laboratório Nacional de Engenharia Civil, Lisboa 1959.