

CHAPTER 29
A STUDY OF DIFFUSION IN AN ESTUARY

W. E. Maloney and C. H. Cline
Division of Oceanography
U. S. Navy Hydrographic Office, Washington 25, D. C.

The problem of dispersion and flushing of contaminants from estuarine waters is of ever-increasing importance to engineers. To determine this reduction in concentration and removal of contaminants from estuarine water, several basic methods have been established. They are: (1) the classical tidal prism method, where the flushing is a function of the amount of water brought in and removed on each tide; (2) Ketchum's modified tidal prism method, where flushing is a function of tidal action and river discharge; and (3) a diffusion-advection method which may be based on a coefficient of eddy diffusivity. This third method, diffusion-advection, would seem to give the most realistic answer provided that confidence can be had in the coefficient of eddy diffusivity. This discussion will be confined to the third method. Furthermore, we will concern ourselves only with that portion of a contaminant that goes into solution and partakes of the motion of the water. Such factors as absorption onto particulate matter, settling out onto the bottom, and uptake of the contaminant by marine organisms will be neglected.

A convenient conservative property in an estuary from which a mean coefficient of diffusion may be computed is the distribution of river water. Stommel (1953) has described the finite difference form of the equation for the coefficient of eddy diffusivity as:

$$k = \frac{Q \ 2 \ a \ (1 - f_n)}{S_n (f_{n-1} - f_{n+1})}$$

where k = Coefficient of eddy diffusivity

Q = River discharge

a = Distance between segments

f_n = Concentration of fresh water at segment n

f_{n-1} = Concentration of fresh water at segment $n-1$ (upstream)

f_{n+1} = Concentration of fresh water at segment $n+1$ (downstream)

S_n = Cross sectional area at n .

This coefficient can then be used in a solution of the classical diffusion equation. Taylor (1954) derived the following equation as a solution for dispersion in turbulent flow with the conditions that the channel have uniform cross section, constant net velocity, and constant eddy diffusivity:

$$C = \frac{M}{2 \ S \ \sqrt{\pi k t}} \exp \frac{-(X - Vt)^2}{4kt} ,$$

A STUDY OF DIFFUSION IN AN ESTUARY

where C = Concentration of contaminant
S = Cross sectional area of channel
t = Time after introduction of contaminant
X = Distance from point of origin
V = Mean velocity of the stream
k = Coefficient of eddy diffusivity
M = Initial mass of contaminant.

Pritchard (1954) and Kent (1958) arrived at the same solution for estuaries having the same limiting conditions. The distribution with respect to X of the contaminant at fixed values of t is then symmetrical and Gaussian. Also the contaminant moves downstream with the mean speed of the stream flow. Experimental results have shown that the contaminant behaves as predicted by the equation. The important considerations of this equation are that it represents a one dimensional case in which at time t equal 0, the contaminant is considered to be uniformly distributed across the section S and subsequent diffusion takes place only in a direction normal to the section. A consequence of this is that for a normal point source release computations based on the equation will be too conservative for early time, that is, the equation will give concentrations which are too low for times less than a few tidal cycles. At some later time, probably after 2-3 tidal cycles, when the contaminant actually becomes uniformly distributed laterally, the results of the computation will be more nearly representative.

Assuming a mean value for the diffusion coefficient and a mean cross sectional area, a distribution of contaminant was computed along a longitudinal section of the James River Estuary (the embayment on which Norfolk and Newport News, Virginia are located), in the area shown in Figure 1. This computed distribution for 1, 2, 4, etc. tidal cycles after introduction is shown in Figure 2. The computed decrease of peak concentration as a function of time is shown in Figure 3.

In an effort to better determine the initial dispersion of a contaminant, a simple field test using sodium salt of fluorescein (uranine) was carried out in Hampton Roads during 5 and 6 August 1959. The results of this field test are reported here and comparisons made with the original computations.

TEST PROCEDURES

The general procedure consisted of releasing a quantity of dissolved fluorescein dye into the water and making longitudinal and/or transverse transits through the dye patch collecting water samples at 3 depths, 1 foot, 6 feet, and 15 feet. These water samples were examined on the boat for dye content using a Fisher Electrophotometer which has the capability of measuring fluorescein dye concentrations to about 1 part in 100 million. Fluorescein dye was used because it is commercially available in large quantities, is nontoxic and is relatively inexpensive. Although fluorescein will fade when exposed to sunlight, the rate of fading was not expected to appreciably affect the concentrations over a 6-hour period.

COASTAL ENGINEERING

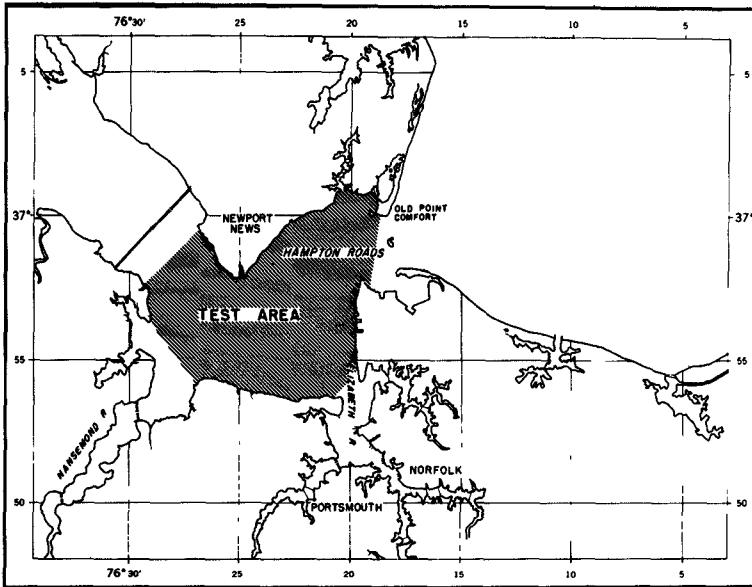
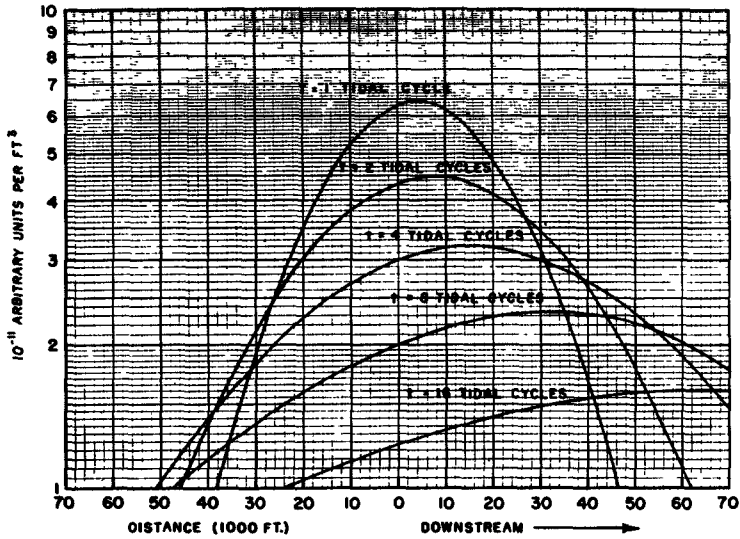


Fig. 1. James River Estuary.



COMPUTED FROM FOLLOWING EQUATION

$$C = \frac{M}{2S\sqrt{\pi kt}} \exp\left[-\frac{(X-Vt)^2}{4kt}\right]$$

C = CONCENTRATION

S = CROSS SECTIONAL AREA

t = TIME AFTER INTRODUCTION OF CONTAMINANT

M = INITIAL MASS OF CONTAMINANT

X = DISTANCE FROM POINT OF INTRODUCTION

k = COEFFICIENT OF DIFFUSION

V = MEAN VELOCITY

$$S = 288 \times 10^4 \text{ ft}^2$$

$$k = 1950 \times 10^4 \text{ ft}^2/\text{hr}$$

$$M = 1 \text{ ARBITRARY UNIT}$$

$$V = 323 \text{ ft/hr}$$

Fig. 2. Space distribution of contaminant for James River.

A STUDY OF DIFFUSION IN AN ESTUARY

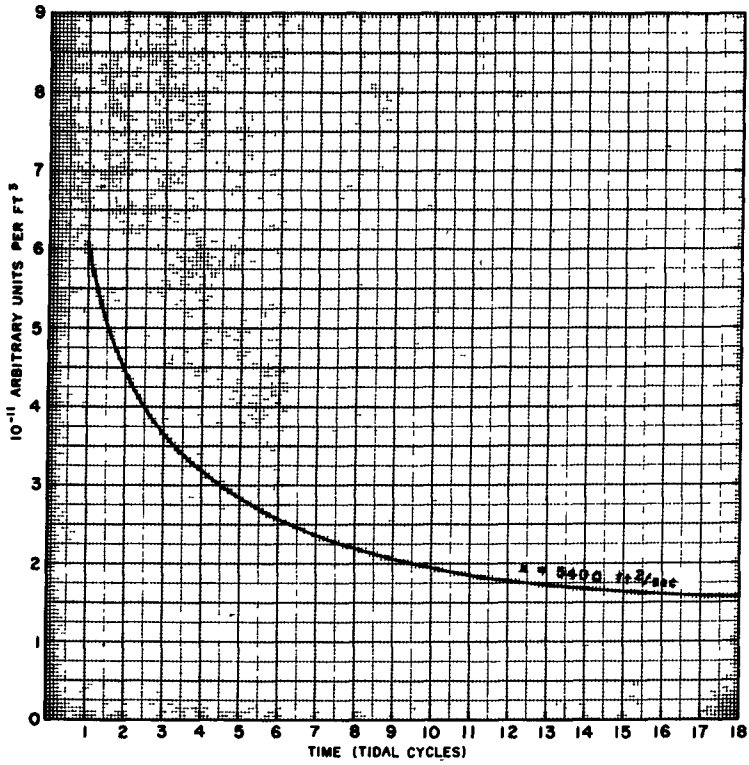


Fig. 3. Decrease of peak concentration with time.

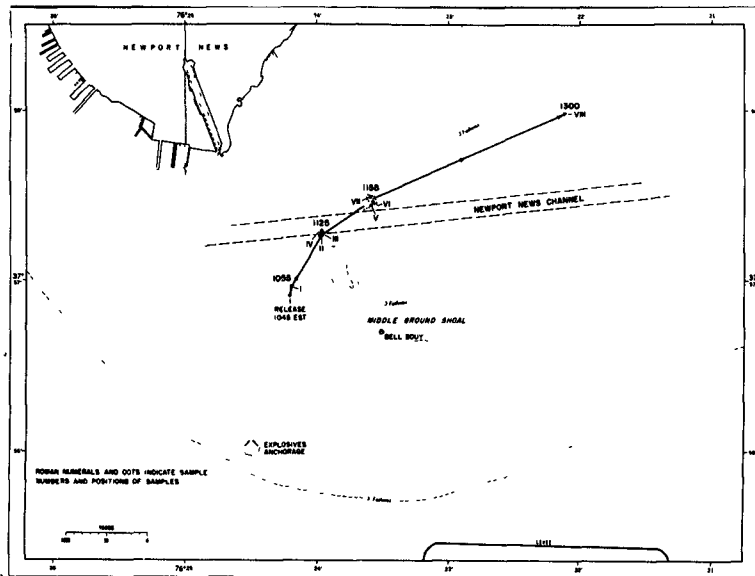


Fig. 4. Test location chart, 5 August.

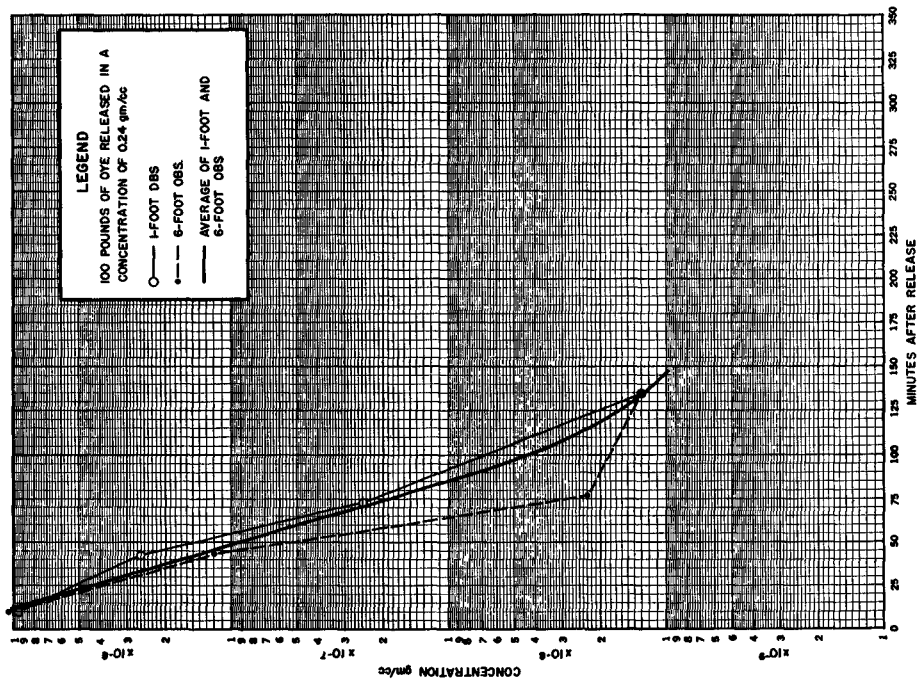


Fig. 6. Decrease of peak concentration with time - 5 August test.

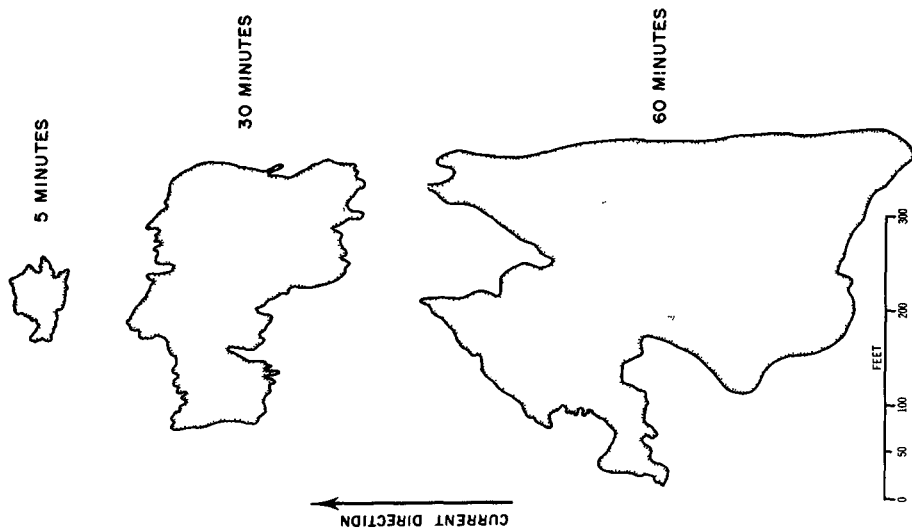


Fig. 5. Dye patch outlines at 5, 30, and 60 minutes after release, 5 August test.

A STUDY OF DIFFUSION IN AN ESTUARY

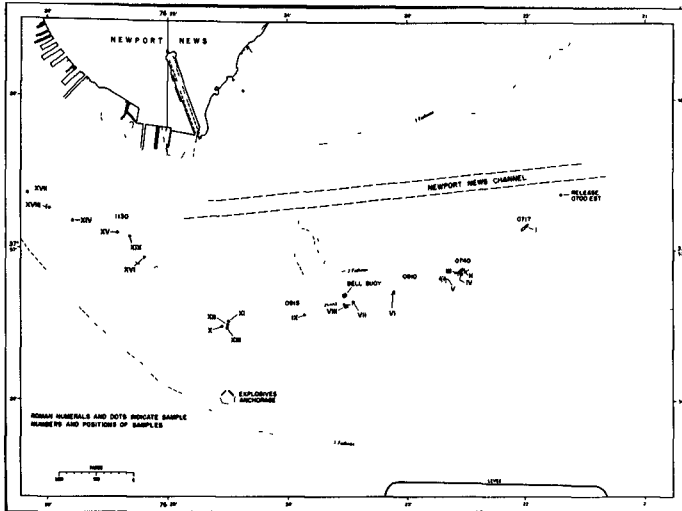


Fig. 7. Test location chart, 6 August.

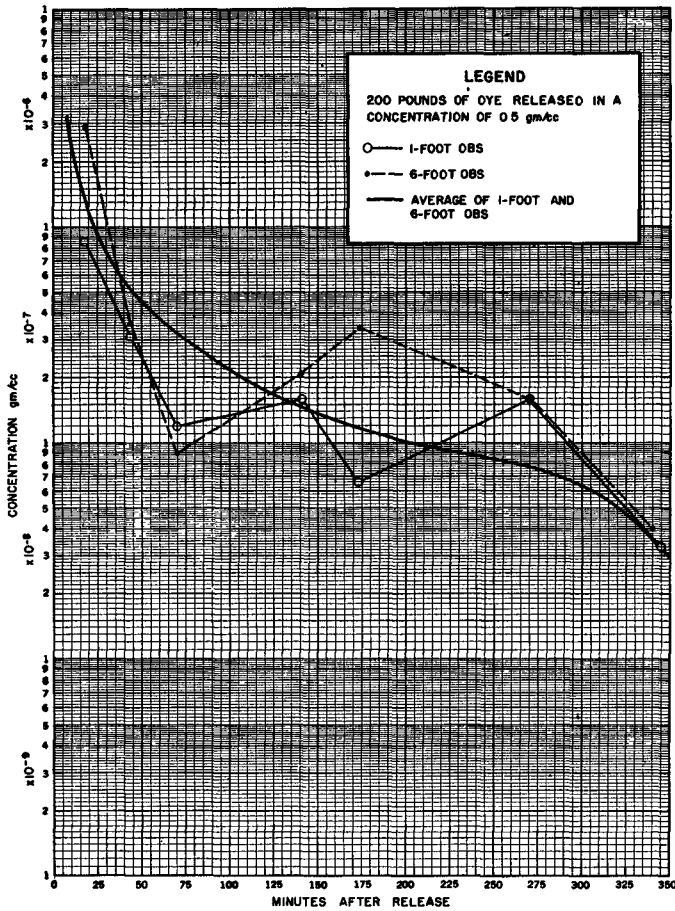


Fig. 8. Decrease of peak concentration with time - 6 August test.

COASTAL ENGINEERING

The first test was held on 5 August 1959; the dye release point was at the west end of Middle Ground Shoal. The positions within the dye patch at which the samples were taken, the general outlines of the dye patch at each sampling time, and the direction of movement are shown in Figure 4. Winds were light; seas were less than 2 feet. At the time of release the water temperature was 77°F. and vertically isothermal. On this test, 100 pounds of fluorescein dye were used making an initial concentration of 0.3 gm/cc. At slack before ebb this solution was released beneath the surface at a depth of 6 feet by gravity feed through a 1½ inch hose. Release took only 1 to 2 minutes so that the dye was released essentially as a point source.

Aerial photographs of the dye patch were taken from a helicopter at 5, 30, 60, and 120 minutes after release. Figure 5 shows the outlines of dye patches at +5, +30, and +60 minutes drawn to the same scale and with the same orientation with respect to the current direction. The patch at +120 minutes was too diffused to attempt to determine an outline. A vertical temperature trace taken at this time showed a decrease in water temperature from 79.4°F. at the surface to 76.8°F. at 10 feet and was isothermal from there to the bottom at 20 feet.

The decrease of peak dye concentration with time is shown in Figure 6. The smooth curve is a mean of the 1- and 6-foot peak concentrations found at each sampling time. It shows a dilution factor of approximately 10^5 in 35 minutes and 10^6 in 70 minutes.

The 6 August tests were also conducted in the James River but with the dye release being near the east end of the Newport News Channel as shown in Figure 7. In contrast to the previous days test, these were conducted during the flood tide. During this test, winds and seas were light. Surface water temperature was 76.9°F. and nearly vertically isothermal. The current was flooding at 1.1 knots in a direction 220°F. In an effort to increase the time duration of the tests, 200 pounds of dye were used on this day.

The current had begun to flood by the time the dye solution was released so that the dye patch at completion of release was a plume about 10 feet wide by 70 to 80-feet long. At +5 minutes the patch had lengthened to approximately 125 feet. Because of the rapid movement of the patch with the currents, some of the sample locations are not shown within the limits of the dye patch on Figure 7, which shows the general outline of the dye patch at each sampling time. The limits of the patch are drawn for the time of the first sample taken at each sampling time. By the time the boat arrived in position to sample the west end of the patch, the whole water mass had moved 300 to 400 yards. Similarly, when samples 11, 12, and 13 were taken, the patch had moved westward enough so that these samples were taken in about the longitudinal middle of the patch. The outline of the visible dye patch at +270 minutes was very vague, and only an estimated size is shown on Figure 7.

A STUDY OF DIFFUSION IN AN ESTUARY

At slack before ebb, the dye patch had reached its greatest intrusion up river. At this time a definite tide rip was observed with a line of debris and seaweed in the clear water on the north and the dye water on the south. The rip line seemed to extend in a northeast-southwest direction. As the rip moved through the dye patch, several rather strong local concentrations were noted along the rip line. Elsewhere, dye color was not visible or only vaguely visible. After the rip passed, several runs were made across the river but no additional dye was visible. A vertical temperature trace taken at 1300 showed isothermal water conditions with a surface temperature 77.1°F.

The decrease of peak dye concentration with time is shown in Figure 8. The smooth curve is a mean of the 1- and 6-foot peak concentrations found at each sampling time. The general leveling off of the rate of change of peak concentrations after 1 to 2 hours was not apparent on the first test and is quite possibly a result of the sampling technique. However, the concentrations vary so much after 1 to 1½ hours that the mean curve could be drawn considerably different. The curve shows a dilution factor of 10^5 in about 10 minutes and 10^6 in about 40 minutes. The method of release and the faster current speeds probably account for the initial faster dilutions noted in this test than on the 5 August test.

DISCUSSION OF RESULTS

One of the original purposes of the tests was to determine a value for the coefficient of eddy diffusion to compare to that obtained from the salinity gradient. To do this, some estimate of the extent of vertical and horizontal dispersion was necessary. In the formula assumed for the decrease of peak concentration of any contaminant,

$$C = \frac{M}{2 S \sqrt{\pi k t}},$$

the cross sectional area through which the pollutant has spread should be known. Obviously, in the first few hours the contaminant will not extend completely and uniformly across the channel, nor completely from top to bottom as assumed in Figures 2 and 3. However, to use the same formula for the first few hours as well as for later times, some estimate of this cross sectional area should be made.

The method of sampling used in this test does not permit any precise indication of the vertical extent of mixing. However, mixing from top to bottom apparently took place rather rapidly. All releases took place at the surface or 6 feet, and within two hours or less, concentrations at 15 feet were as high as the 1- or 6-foot samples. The extent of horizontal dispersion, at least at the surface, can be readily determined from visual observations and aerial photographs. Using the 5 August test, because of the better aerial photographs, a measure of the vertical and surface lateral spreading with time was determined. From these two dimensions of the dye patch, the cross sectional area was determined as a function of time.

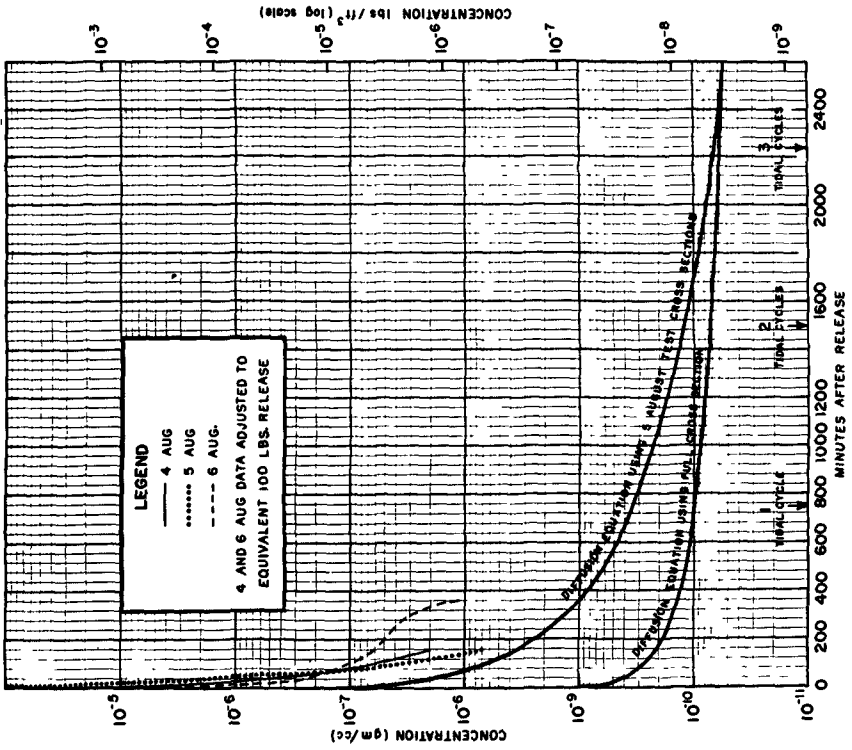


Fig. 10. Comparison of test and computed peak concentration curves.

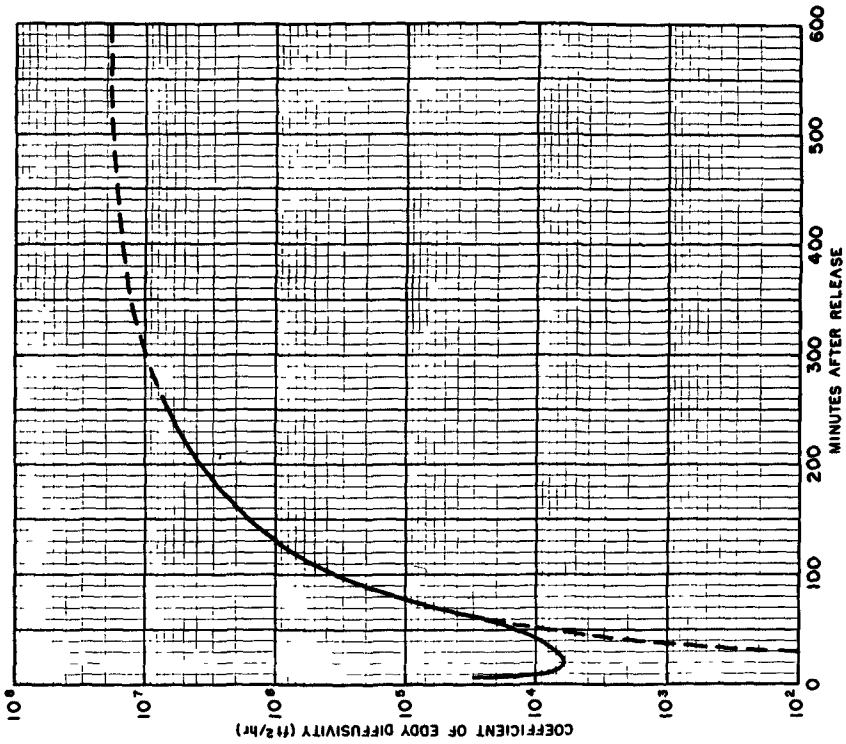


Fig. 9. Coefficient of eddy diffusivity with time - 5 August test.

A STUDY OF DIFFUSION IN AN ESTUARY

Using these values of cross section, and the peak concentration values from Figure 6, the original formula for peak concentration was solved for k , the coefficient of eddy diffusion. These values of k , as a function of time, are shown in Figure 9.

This curve shows that the value of the coefficient was not a constant, at least for the first few tidal cycles. Instead k was an increasing function of t for small dispersion times, eventually approaching a constant for large dispersion times. In the case of the James River in the Hampton Roads area, k presumably should have approached $1950 \times 10^4 \text{ ft}^2/\text{hr}$, the mean value derived from the salinity data.

A comparison of the peak concentration curves with time from the two tests to those that were computed from the diffusion equations are shown in Figure 10. In an attempt to make the test results compatible, the 6 August concentrations were reduced by one-half. This should closely approximate the results that would have been attained if 100 pounds of dye had been used on each day.

The different rates of dilution on each day must have been largely a result of the speed of the tidal currents. On the 5th the dye was injected at slack before ebb with the currents gradually increasing from near slack to more than 1 knot. On the other hand, on 6 August even though the dye was released very near slack before flood, the currents at the beginning of the test were more than 1 knot, increasing to almost 1.5 knots within one-half hour and gradually decreasing toward the slack before ebb some 5 hours later. The spreading of dye in a very long plume in the direction of the current probably was due not only to the method of release, but also to the higher current speeds with their greater longitudinal turbulent fluctuations.

The test concentration curves when compared with those computed from the original equation using the full cross section and assuming a 100 pound release, show concentrations as much as 3 to 4 orders of magnitude higher. When the test cross sections are used in the diffusion equation, this difference becomes about 1 to 2 orders of magnitude in the first hour with differences decreasing with increasing time. After perhaps 1 or 2 tidal cycles, when near the mean value of eddy diffusion is actually attained, this difference should disappear; in fact, Figure 10 indicates such a trend.

A rough check on the concentrations computed for any area can be made by determining the lowest limits of possible concentrations. That is: (1) any dissolved pollutant released will be diluted by the volume of water contained in one tidal prism (or volume delineated by one tidal excursion) within one tidal cycle, and (2) the pollutant will be diluted by all the water available within several days.

In the lower James River Estuary, an average tidal prism or volume of a tidal excursion is something more than $3 \times 10^{14} \text{ cc}$. The entire volume of

COASTAL ENGINEERING

water available for dilution is assumably that volume lying between Old Point Comfort and the James River Bridge, approximately 12×10^{14} cc.

Using these approximate volumes, it can be estimated that 45,400 grams (100 pounds) of contaminant would be diluted by 3×10^{14} cc. of water in about one tidal cycle. This gives a mean concentration of 1.5×10^{-10} gm/cc and approximates the peak value of 3.5×10^{-10} gm/cc indicated on Figure 10 for one tidal cycle. After several days, the 100 pounds should be diluted by 12×10^{14} cc of water; its mean concentration should be 3.8×10^{-11} gm/cc, which is very close to that estimated by the diffusion equation.

To summarize and to simplify using the results of the test and the theory on the release of other amounts of dissolved contaminants in this area, a set of empirical dilution factors have been computed. They were derived from the curves of Figure 10 and could be used to estimate the peak concentrations that would occur in the vicinity of Hampton Roads if a known mass of contaminant was introduced. These dilution factors are shown in Table 1 and, when multiplied by the amount of introduced contaminant, will give the peak concentration of the substance at the indicated time.

TABLE 1

DILUTION FACTORS FOR HAMPTON ROADS, VIRGINIA AREA.

Time after release t	Dilution Factor	
	/cc	/ft ³
10 min.	2.2×10^{-10}	6.3×10^{-6}
30 min.	7.1×10^{-11}	2.0×10^{-6}
60 min.	1.1×10^{-11}	3.0×10^{-7}
3 hours	1.3×10^{-13}	3.8×10^{-9}
6 hours	2.4×10^{-14}	8.5×10^{-10}
1 tidal cycle	7.7×10^{-15}	2.2×10^{-10}
2 tidal cycles	2.6×10^{-15}	7.5×10^{-11}
3 tidal cycles	1.5×10^{-15}	4.2×10^{-11}

1. The total amount of contaminant released multiplied by the appropriate dilution factor gives the concentration at that time.

2. Two dilution factors are given so that computations may be made either in terms of cubic centimeters or cubic feet.

3. The rapid leveling off of the dilution factors after about 1 tidal cycle assumes that none of the contaminant is advected out of the area.

A STUDY OF DIFFUSION IN AN ESTUARY

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