CHAPTER 40 SHIP WAVES IN NAVIGATION CHANNELS

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INTRODUCTION

Ships moving through water generate surface waves which in many navigation channels cause severe wave-wash damage to the banks In some waterways, rather extensive protection works have been constructed to reduce this destructive action to levee faces (Hertzberg, 1954). This action, termed "foreshore erosion", has been described by Lewis (1956) for the lower Mississippi River as follows:

> "The attack, interestingly enough, takes place at lowwater. It is due largely to the waves created by passing ocean-going ships and is augmented by shallow draft traffic. It may be asked at this point why this attack is new, in view of the fact that ship and barge traffic has long existed on the river in very substantial volume? The answer is that the recent technological advances in ocean-going and river transportation have greatly increased maximum and average speeds, and accordingly, the wave making potentials."

Despite the importance of this problem, very little quantitative data are available on the characteristics of ship waves at a given distance from the sailing line in terms of type of ship, displacement, speed, and water depth. Naval architects have long been interested in the waves generated by ships, but primarily from the standpoint as to how the waves affect the resistance of the ship (Havelock, 1908, 1934, and 1951; Lunde, 1951; Birkhoff, et al, 1954). The pioneer work in the field of ship resistance was done by Rankine (1868) and Froude (1877); however, as discussed below under the section on theory, Lord Kelvin (1887a, 1887b) was perhaps the first to present a theory fu such waves by calling attention to the similarity of actual ship waves t the waves generated by a pressure point moving across the water surface. Standard text books on naval architecture (Taylor, 1943; Robb, summarize Lord Kelvin's theory which provides a method of plotting the wave patterns as well as the relative wave heights throughout the pattern. What was apparently the first comprehensive set of measure ments of actual ship waves were the observations of Hovgaard (1909) o the wayes generated by several different types of ships. He compared the actual angles of the diverging waves with the theoretical values of

Lord Kelvin for deep water conditions. For restricted waterways numerous investigations have been made to determine the form of canal cross section and system of bank protection to be adopted with a view to resisting the destructive wash of passing tows and motor boats. These studies consisted of model tests (Krey, 1913 and 1929) as well as tests and observations with boats in actual canals (De Bruyn and Maris, 1935; Rosik, 1935; Schijf, 1949; Sum, 1935; Vanderlinden, 1912; Van Loon, 1912; Wortman, 1894). As discussed below there is very little information in any of these various studies on the actual wave heights that occurred during the tests.

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THEORY

As originally discussed by Froude (1877), the main features of the entire disturbance behind a ship, which is confined between two straight lines, is that there are two distinct systems of waves. These are the transverse and the diverging waves (Figure 1). Curves of equal phase in the two systems meet on the straight line boundaries of the disturbed region in cusps. These lines often are designated as the "cusp locus". Wave heights are greatest and the crests the sharpest at the cusps, as well as at the point of the disturbance, so that breaking water will be found at these points, if anywhere.

There is a striking similarity between the waves generated by an actual ship moving across the water surface and the theoretical wave pattern resulting from a single moving point as developed by Lord Kelvin (1887a). His solution shows that for deep water the whole pattern of waves is comprised between two straight lines drawn from the bow of the ship and inclined to the wake on its two sides at equal angles of $\alpha = 19^{\circ} 28'$. Figure 2 shows a plot of a Kelvin wave group caused by a travelling disturbance, and Figure 3 shows a single crest of Kelvin's wave group with relative heights at various points. The height in this instance is the elevation of the wave crest above the undisturbed water level. The difference between the Kelvin wave groups and actual waves 1s that a Kelvin group 18 an ideal system resulting from forces applied at a single moving point; whereas an actual wave group is due to forces spread over a ship's hull. Actually, the application of the Kelvin theory perhaps is valid only at a distance of several ship lengths from the stern.

An essential parameter which determines the wave pattern created by a ship on a straight course is according to the Kelvin theory

$$\lambda = \frac{C}{C_0} \tag{1}$$

In which C is the ship speed and C_0 is the velocity of a wave in shallwater; that is, $C_0 = \sqrt{gd}$, where g = acceleration of gravity, and d water depth. The wave amplitudes, and hence the wave-making resistance of the ship apparently depends on the character of the ship's hull. However, the general shapes of the curves formed by the wave crests and troughs and their spacing depend upon λ and the speed of the ship and not upon the shape of the hull. For $\lambda = 0$, i.e. infinite depth, the character of the wave pattern given by the Kelvin theory is as shown in Figures 2 and 3.

Relationships for wave patterns for a travelling disturbance water of any depth have been developed by Havelock (1908). A simpl fied method of plotting such wave patterns has been presented by Rol (1952). For a finite water depth d, the angle α between the ship's course and the cusp locus increases as λ increases and approaches : theoretical value of 90° as λ increases from zero to the critical value of 1.0. The value of the angle α increases, however, yary slowly with λ until λ is somewhat greater than 0.7 (Figure 12). For finite depth, the spacing of the waves continues to be proportional to the speed, but the constant of proportionality depends on the value of λ . The spacing increases for a given speed with diminishing depth, but this increase is small for λ values less than 0.7. At $\lambda = 1$, $\alpha = 90^{\circ}$, and the wave pattern consists essentially of a single wave with its crest at right angles to the ship. When the critical value $\lambda = 1$ has been passed, the character of the wave pattern behind the ship chan; completely, and there are no longer distinct systems of waves as th transverse system disappears, the disturbed region lies between tw straight lines with the angle α being expressed in terms of λ . The angle α theoretically approaches zero at high speeds (or low depth: and the straight line boundary is now a phase curve of the wave system and not a cusp locus of such curves as in the case where $\lambda < 1.0.$

OBSERVED SHIP WAVE CHARACTERISTICS

DEEP-WATER CONDITIONS

Observations on waves generated by actual ships have been made by naval architects in connection with studies on ship resistan Most of these investigations have been with models, and only a few systematic observations with full size ships appear to have been ma The most extensive observations were those of Hovgaard (1909) who made measurements on both wave height and wave pattern for variou sized ships and models operating in deep water. A typical example



Fig. 1. Froude's sketch of the characteristics of ship waves, Trans. Inst. Naval Arch., 1877.



Fig. 2. Crest of a Kelvin wave group in deep water caused by a travelling disturbance at 0 (Taylor, 1943).



Fig. 3. Single crest of Kelvin wave group in deep water with relative heights at various points given by numbers (Taylor, 1943).





Fig. 5. Relation between velocity travel and wave height at shore of canal under the influence of a sing tugboat (Franzius, 1936).

Fig. 4. Typical wave pattern as observed by Hovgaard (Trans. Inst. Naval Arch., 1909).

SS United States	10,000 tons	V= •/	(Knots)	ΥL	Locus (degrees)	Height of Bow Breaker
States		493	14 5	0 86	18	3 ft
11	10,000 tons		15	0 68	19	-
Danish Patrol	47 tons	84	45	0 48	16 5	3-6 inches
Boat						
1	0	11	6	0 68	17	4-6 "
D II	11	11	8	0 87	15-1/2	6-8 "
U U	n		10	1 09	17	9-12 "
0	n		12	1 31	-	9-12 "
Olfert Fisher	3,450 tons	272	14	0 85	17-1/2	7 5 ft
- H	11		9	0 55	19	1-2 ft
S S Dronning Maud	1,760 tons	271	15	0 91	18-1/2	4 ft
SS Kong Haakor	1.760 tons	271	16	0.97	16	-
Danish Mine	107 tons	68	7-1/2	0 91	17	12-15 inches
Destroyer Mod	el 505 lbs	20	3 01	0 67	10	-
Desit Of cr moor	**	n in	4.49	1 00	9-1/2	-
н	0	Ð	6 00	1 47	9	-
н	11		8 20	1 83	9	-
11	и	11	8 86	2 23	8	-
Battleship	2,820 lbs	20	3 16	71	17	-
100061	11		3 60	81	16-1/2	-
n	11	11	4 00	.88	15-1/2	-
11	11	11	4 50	1 01	14	-
h	n	0	4 90	1 10	13-1/2	-
Merchant Vesse	1 2,422 lbs	20.9	2 70	0 59	17	-
Moder	н		2 20	0 70	10 1/4	
	11	11	3 30	0.72	15-2/4	-
11	0	0	4 10	0 00	15-5/4	-
н	n	11	4 6 9	1 01	15	-
	11	11	- 03 5 06	1 11	14-9/4	-
n	11	n	5 45	1 10	14-3/4	-

TABLE I		
Summary of Hovgaard's Observ	vations	on
Diverging Ship Waves		

of Hovgaard's observations is shown in Figure 4 and a summary of some of his observations are presented in Table 1.

The Kelvin theory predicts that the diverging wave system is the same for all hull forms and speeds; however, Hovgaard's measurements lead him to state that:

"The observations here recorded show that this is not the case, the obliquity of the waves being greatly influenced by speed and form of the ship, and being not ever the same for all waves in the same ship at a given speed."

SHALLOW-WATER CONDITIONS

As a ship moves through relatively shallow water the flow conditions around the hull, as well as the surface waves which are generated, will be different from deep-water conditions, and the ship resistance is increased. The change in flow conditions around the hull probably increases the skin friction resistance slightly, but the major increase in resistance apparently is due to the increased wave heights occurring with shoal-water conditions. Perhaps the best summary of the problem of ship resistance in shallow water is that by Taylor (1943), wherein previous investigations are reviewed and a tentative attempt is made to define the condition at which depth effects cause the resistance to begin to increase. Since the effect of the waves probably is the principal cause of increased resistance, the relationship presented by Taylor is of importance in evaluating the effect of depth on the wave characteristics. For moderate and slow speed vessels, that is, vessels capable of deep-water speeds in knots of $0.9 \sqrt{L}$ or less, Taylor shows that the relative minimum depth for no increase of resistance is given by the expression,

dmin	$= 10 D \frac{V}{T}$ (2)
d _{min}	= minimum depth for no increase
D	= draft - ft,
V L	= speed in knots = length of vessel - ft.

This relationship gives the critical point at which ship resistance starts to increase, but unfortunately no information is available 'rom the data on the actual change in the characteristics of the waves hemselves.

For a high speed vessel in water of depth less than the ship's length, Taylor (1943) states that "at a definite speed in a given depth the vessel will begin to experience appreciably increased resistance as compared with the resistance in deep water. The excess of resistance above the normal increases with speed until it reaches a maximum. This maximum appears to be at about a speed such that a trochoidal wave, travelling at this speed in water of the same depth, is about 1-1/4 times as long as the vessel."

From observations on the operation of canal boats in restricted waterways Rosik (1935) made some measurement of wave heights. He stated that:

> "The wave from the bow usually narrows suddenly about half the length of a vessel, whereas behind the stern it is suddenly thrust out. According to the speed at which the craft is travelling, the first wave behind it reaches a height of 2 meters measured from crest to hollow. This wave is followed by several others of an elongated form and two or three grouped waves having 1/2 to 2/3 the height of the first one. The effect of the wave is all the greater on the banks when a vessel runs close alongside them. On the Danube this distance does not drop below 15 to 20 meters. At a distance of 50 meters from the banks the effect is only half, and at a distance of 250 meters it scarcely makes itself felt. The influence of the waves makes itself felt if the ship speed craft exceeds 8 km an hour."

No mention is given by Rosik as to the type or displacement of the vessel or the water depth to cause such wave conditions.

Other wave data of limited extent are given by Franzius (193) in Figure 5. Information is lacking on the size, shape, and displacement of the tugboat and the depth and width of the canal under which these observations by Franzius were made.

As mentioned previously, observations on the movement of tow boats in restricted waterways have been made both by model tes and with actual canal boats. Most of these latter studies were concerned primarily with the maximum speeds that were allowed in various canals. This maximum speed was not based on scientific stuc but provisional speeds were adopted and the extent of bank erosion c served. The maximum speeds usually were expressed as a functior of the ratio of the mid-ship cross sectional area of the boat to the c

sectional area of the canal. For example, Vanderlinden (1912) states that where the canal cross-sectional area is eight times that of the boat a maximum speed of 10 to 12 km per hour is possible. Allowable speeds in various canals also have been given by Wortman (1894) and Van Loon (1912). In a discussion of the effect of ship waves on the banks and beds of canals Sum (1935) observes that the greatest damage occurs about one meter above and below normal water level. In narrow canals he recommends a maximum speed of 5 km per hour. Where a river's width is 70-150 meters Sum (1935) states that the waves are from 30 to 80 cm in height. No mention is made, however, of the size or type of boat that produced such waves.

Perhaps the most comprehensive investigation on boats operating in comparatively narrow waterways was that of Schijf (1949) who used both models and actual power boats in his studies. His tests show that in a canal with erodible bed and banks the maximum velocity is dependent on the ratio of the midship cross-section of the boat to the cross-section of the canal, the shape of the hull, and the method of propulsion.

SHIP WAVE MEASUREMENTS

MODEL STUDIES

Equipment and Procedure - As evidenced by the above discussion, relatively few data are available on the wave heights to be expected at a given distance from the sailing line of a ship with a given hull shape and displacement moving through shallow water at a given speed. To obtain such data and better define the importance of the variables involved, a series of model experiments were made at the University of California model basin. In these studies ship models were towed at various speeds in water of various constant depths, and the wave characteristics measured at various distances along a perpendicular to the sailing line. The general arrangement of the test facilities is shown in Figure 6, and the characteristics of the models tested are shown in Figure 7 and Table II. In addition to the geometric characteristics of the various models, Table II also shows values of the "block coefficient" and the "displacement-length ratio." These coefficients commonly used by naval architects are defined as follows:

Block Coefficient = $b = \frac{v}{L \times D \times B}$

where v = volume of displacement

L = waterline length

B = waterline beam at midship

D = draft at midship

Displacement-Length Coefficient =
$$\frac{\Delta}{(L)^3}$$
 -

where Δ = displacement in tons

L = waterline length in feet

TABLE II

Model Characteristics and Test Conditions (Fresh Water Tests)

Model	Displacement	Water I	line Dim	nens10ns		Λ	Wa
		L	В	D		<u> </u>	deŗ
	lbs.	Length	Beam	Draft	b	$\left(\begin{array}{c} L \end{array} \right)$	d
		(ft)	(ft)	(ft)		,100	(ft)
AL	9.27	3.32	0.94	0.13	0.37	127	0.
А _Н	27.63	3.40	0.98	0.23	0.58	352	$\left\{\begin{array}{c}1\\1\end{array}\right.$
в	27.63	3.44	0.82	0.19	0.82	340	0.
С	27.63	2.92	1.00	0.27	0.56	553	0.
D	51.5	4.0	0.67	0.33	0.93	402	1.
Е	20.94	3.0	0.50	0.25	0.93	398	1.;
F	5.89	2.0	0.33	0.167	0.93	36 8	0.



Fig. 6. Equipment used for investigating ship waves by models.



Fig. 7. Photographs of Models A, B, and C used in ship wave studies.



Fig. 8. Typical chart record of ship waves showing the various terms determined in the tests.

MODEL	DIMEN	ISION	vs,	in Inc	hes	
Model	L	в	D	е	f	h
D	48	8	4	7	41	8
Е	36	6	3	5.25	30.25	8
F	24	4	2	3.5	20.5	5





ELEVATION

Fig. 9. Idealized models used in determining scale effects in ship wave studies.

The towing system shown in Figure 6 was driven by falling weights with the model being attached fore and aft by strings to a taut line which passed over sheaves at two ends of the model basin. Craft speeds were varied by varying the weights on the driving system. All tests were made with fresh water in the model basin. Wave characteristics were measured in a locality where the model had attained a constant speed. These measurements were made at five positions by parallel-wire resistance elements (Wiegel, 1956), located as shown in Figure 6, and recording on a six channel Brush recorder. The sixth channel of the recorder was used to determine the speed of the craft from timing marks made on the recorder chart by a contact actuated during each revolution of one of the sheaves on the drive system. Measurement of the waves was made at various constant speeds from the lowest speed that waves could be recorded accurately to the highest speed that the driving system could be operated. For a model with "good" lines (Model A_{I} , Figure 7) the resistance to motion was small enough that sufficient weights could be placed on the driving mechanism to attain a craft speed well beyond the critical speed where the wave height was a maximum. A typical example of a wave height record is shown in Figure 8. Such chart records were analyzed for the maximum crest elevation above still-water level and for the maximum wave height. This maximum wave height usually was the vertical distance of the maximum crest height above the preceding trough (Figure 8). The angle between the cusp locus and the sailing line was computed for each test from the time lag between maximum wave height occurrence at two wave gages, the distance between gages, and the model speed.

To give a measure of the effect of scale on the results from model tests a series of runs were made with an idealized ship form constructed to three different sizes as shown in Figure 9. These three models were tested under conditions in which certain variables were held constant to determine the effect of absolute size of the craft on the wave characteristics.

<u>Test Results</u> - A typical example of the type of wave height data obtained from the tests on a particular model is shown in Figure 10, where for Model A_L the values of maximum wave height, H, and maximum crest elevation, S, as determined at various distances x from the sailing line, are plotted as a function of ship speed. The water depth was held constant during the tests. Also indicated in Figure 10 are scales showing values of V/\sqrt{L} and λ (as defined by equation 1). In the term, V/\sqrt{L} , which is commonly used by naval architects in comparing the performance of ships, V is the ship speed in knots, and L is the waterline length in feet. It is of importance to note in Figure 10 how rapidly the wave height increases with speed up to a critical point.

Beyond this critical speed the wave heights decrease and approach a con stant value for relatively high speeds. The rapid increase in wave height with ship speed for speeds below the critical speed confirms the observation of Lewis (1956) as quoted in the Introduction--namely, that increased ship speeds has resulted in greatly increased wave heights and consequently serious wave-wash erosion problems. An item of interest in connection with Figure 10, and similar plots for other models is that the maximum wave height, H, is approximately twice the value of S, the maximum crest elevation above the still-water level.

In addition to the data on wave heights as measured from the recorder charts the time between the occurrence of the maximum crest height and the preceding trough (see Figure 8) was determined and has been termed the "half-period." Figure 11 shows such data plotted again λ . These data were obtained from the same series of tests for which th height data in Figure 10b is presented. Within the accuracy of measure ment the half-period was independent of the distance, x, from the sailing course.

For the tests from which the wave height data shown in Figure 10 were obtained the angle, α , between the cusp locus, or point of maximum wave height, and the sailing line was computed and is plotted against λ in Figure 12. Also shown on this plot is the theoretical curve of Lord Kelvin. There is fair agreement between theory and experiment.

Wave height data similar to that shown in Figure 10 were obtained for the other models and test conditions listed in Table II. To permit a comparison of the wave generating capacity of the various hull forms represented by the models, dimensionless plots were prepared using the following parameters:

$$\frac{H}{D} = f\left(\frac{V}{\sqrt{L}}, \frac{d}{D}, \frac{x}{L}\right)$$
(5)

where

- H = maximum wave height, ft.
- D = ship draft at mid-section, ft.
- L = ship length at water line, ft.
- V = ship speed in knots
- d = water depth, ft.
- x = perpendicular distance from sailing line to point of wave measurement.



Fig. 10. Typical data on wave height as a function of ship speed and distance of wave recorder from sailing course. Model A_L with a water depth of 0.52 ft. used in tests.





Fig. 11. Relationship between pe- I riod for maximum wave and λ . Model AL, with a water depth of 0.52 ft. used in tests.

Fig. 12. Relationship between an \propto , between the cusp locus and sailing line and λ for Model A₁.



Fig. 13. Relationship between the wave height-ship draft ratio and the relative ship speed, $V/\sqrt{-L}$.

Figure 13 shows a plot of H/D as a function of V/ \sqrt{L} for a constant water depth and position of wave measurement for models A_{I} , A_{H} , B, and C. Due to the slight differences in the length, L, and draft, \overline{D} , for the various models, there is a slight difference in the values of x/L and d/D between models. The curve for model A_T in Figure 13 was derived from the basic data presented in Figure 10. This model was the only one in which the resistance to motion was low enough that the model could be towed at speeds in excess of the critical condition for maximum wave height. Comparison of the curves for models AL and AH gives an indication of the effect of varying the displacement with a particular hull shape. The actual wave heights were greater for model A_H than for model A_L at a particular speed; however, because the draft is less for model A_L than for A_H , the H/D versus speed curve for the lighter \cdot model $(A_{I_{i}})$ plots above the curve for the heavier model (A_{H}) . Comparison of the curves for models A_H , B and C in Figure 13, where the displacement was a constant, shows the general effect of hull form on the relative wave making capacity of the various models; that is, the better the ship lines the lower are the wave heights for a particular speed. To provide information on the variation of wave height with distance from the sailing course Figure 14 has been prepared for one of the models (Model A_H). This figure shows the ratio H/D plotted against the ratio x/L for a constant speed, V/\sqrt{L} , with d/D as a parameter. The curves shown are cross plots from such curves as shown in Figure 13. The curves in Figure 14 show that when the water depth 1s relatively small compared with the draft the wave height decreases rapidly with distance from the sailing line; whereas, for the larger water depth, the rate of decrease in wave height with distance, is considerably less.

Further information on the effect of water depth on the height of ship waves is shown in Figure 15 which is a cross plot from Figure 14. In this figure the wave height-draft ratio, H/D, is plotted against the depth-draft ratio, d/D, for a constant speed of $V/\sqrt{L} = 1.1$ at a constant distance from the sailing line of x/L = 1. In addition to the three values of d/D shown in Figure 15 (namely, 2.26, 4.35, and 7.95) some data were obtained in a relatively narrow towing tank with a d/D value of 22.5. These data were rather limited; hence the point on the curve in Figure 15 at the d/D value of 22.5 is not accurately defined; consequently the curve beyond a value of d/D = 7.95 is shown as a dotted line. Also shown in Figure 15 is the limit for no increase in ship resistance as computed from equation 2. This plot indicates that when the ratio of depth to draft (d/D) is less than about 8, the wave heights increase rapidly with a decrease in depth, that is, when the depth is greater than about 8 to 10 times the draft the waves are essentially those for deepwater conditions.



Scale Effects - As mentioned above, a series of tests were made with the idealized ship form shown in Figure 9. The three geometrically similar models were towed at various speeds in water of various depths such that a constant value of depth-draft ratio of 5.3 existed in the tests. The wave heights were measured at the distances from the sailing line as shown in Figure 6. The wave height data for each model were plotted on diagrams similar to Figure 10. From these basic data the dimensionless parameters presented in equation 5 were computed and plotted as shown in Figure 16, which is similar to Figure 14 for Model A_{H} . Figure 16 shows for each model the ratio H/D plotted as a function of x/L for a constant speed ratio, V/\sqrt{L} , of 1.0 and a constant depthdraft ratio of 5.3. Inspection of this graph shows that the scaling law for the larger models (models D and E) agree almost exactly, but the curve for the smallest model (model F) plots considerably below the other curves, thus indicating that there is an effect of absolute size of model. In other words, if models are too small in absolute size the prediction of prototype values from model data is of questionable value. Obviously more experimental data are required to better define the limit of model size for reliable prototype predictions.

POWER BOAT STUDIES

A limited number of observations were made on the waves generated by a 42 ft. power boat of 241 cu. ft. displacement operating at various speeds in water about 7 ft. deep. Measurements were made of wave heights in the tests. In a few instances vertical aerial photographs were made to determine wave patterns. Photographs of wave patterns for values of λ from 0.70 to 1.32 are shown in Figures 17 and 18. A summary of the pertinent data for each of the tests shown in the photographs is presented in Table III. The angle α shown in this table is the angle between the sailing line and a line drawn from the bow through what appears to be the point of maximum wave height.











Fig. 19. Relationship between the ratio of wave height and draft and relative speed for a 42 ft. power boat with relative distanc x/L and depth-draft ratio as parameters.

TABLE III

Summary of Ship Wave Patterns

Boat length = 42 ft. Boat displacement = 241 cu. ft. Boat draft (midship) = 1.9 ft. Block coefficient = 0.37 Displacement-length coefficient = 101

	С	d	,	Co		α
Figure	ft/sec	ft.		ft/sec	λ	degrees
17a	10.6	7.2	3.8	15.2	0.70	15° 30'
17b	13.5	6.8	3.6	14.8	0.91	17°
17c	14.0	6.8	3.6	14.8	0.95	16° 30'
18a	14.8	7.0	3.7	15.0	0.99	15°
18b	16.6	6.8	3.6	14.8	1.12	1 3°
18c	19.0	7.1	3.7	15.1	1.32	10°

Examination of the photographs in Figures 17 and 18, and in particular Figure 18a, indicates a discrepancy between the character of the actual ship waves and those expected by theory near the critical value of $\lambda = 1$. The model test data summarized in Figure 12 indicates that the angle α approximates that expected by the Kelvin theory. In Figure 18a, however, where λ has a value close to 1.0, the value of the angle α is only 15° (Table III) instead of about 90° as would be expected by theory. It is possible that the short steep waves which appear to form the cusp locus are in reality lower in height than are the long low steepness waves which are outside of what has been assumed to be the cusp locus.

The measurement of wave height during the tests were made by water-level recorders installed at two distances, x, from the sailing line. From the water-level recorder charts the maximum wave height (distance from maximum crest elevation to preceding trough) was determined. The ratio of wave height to draft was then computed and plotted as a function of the relative speed of the craft, V/\sqrt{L} , as shown in Figure 19 with the ratio, x/L, as a parameter. A scale of λ is also shown in this figure. All runs were made with a value of the ratio to depth, d/D, equal to 3.3 except in one run where d/D equaled 3.7. Although the data are limited and some scatter occurs, examinaion of Figure 19 shows results similar to those determined from the nodel studies--namely, that for a particular ratio of depth to draft here is a certain speed at which the wave height is a maximum. In 'elatively shallow water this point occurs in the vicinity of $\lambda = 1.0$.

For values of $\lambda > 1$ the wave heights reduce with increase in speed and appear to approach a constant value at high speeds. As to be expected the wave heights decrease with increasing relative distances, $\frac{x}{L}$, from the sailing line.

CONCLUSION

The data from a series of model tests define the variables involved and their relative importance in determining the characteristics of waves generated by passing ships. It appears that with models of sufficient size reliable prototype prediction can be made of the waves generated by a particular hull shape; however, additional model studie and prototype observations are required to permit such predictions to be made with confidence.

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