Part 1

THEORETICAL AND OBSERVED WAVE CHARACTERISTICS

Chapter 1

SURFACE WAVES ON ENCLOSED BODIES OF WATER

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ABSTRACT

A series of observations was made on the surface waves on a reservoir half a mile wide and one mile in length in winds between 250 and 1300 cm/sec. A slight increase in wave period, and a considerable increase in wave height were observed when the air was colder than the surface water. The data obtained are compared with previous observations made on Abbott's Lagoon in California.

INTRODUCTION

For many engineering applications it is useful to know the relations which exist between the wind and the characteristics of waves on small lakes and reservoirs. This applies particularly in the design of earthwork constructions such as stop-banks near rivers or lakes, which may be seriously affected even by quite small waves.

In this paper the data obtained from observations on wind waves, for a wind range of 250 to 1300 cms/sec., and for fetches between 325 and 1300 metres, are described.

Such observations are a first stage in a complete investigation of the effects of wave action on constructions of different shapes and materials.

THE EXPERIMENTAL EQUIPMENT AND PROCEDURE

Waves were recorded simultaneously at two sites on a reservoir, using two capacitance-wire wave recorders similar to that described by M.J. Tucker and H. Charnock (1954) at the present Congress. A plan of the reservoir is shown in fig. 1; it has a uniform depth of 52 feet, and the concreted edges have a slope of one in three. All waves are deep-water waves and the energy reflected from the edges is small. For a distance of 3 miles the land is flat in all directions; the largest obstructions are trees and medium sized buildings. Within about one mile in the direction of the predominating winds (West to North) the ground is marshy with no trees except some small copses to the North. A few winds blew from the South or South-west, in which directions the land is clear for about half a mile with houses beyond. The reservoir banks slope down to the surrounding levels at a gradient of one in $2\frac{1}{2}$ from a height of 40 feet.

Wave profiles were recorded on photographic paper using a galvanometer in the output of each wave recorder. Frequent calibrations of the "lewmex" insulated wire units were made. Successive calibrations showed a trend which was almost linear with the log of time from first placing the unit in water, as the capacity per cm changes in the manner similar to that described by Tucker and Charnock. There is also apparent in an individual calibration a non-linearity, the wire becoming less sensitive with increasing wave height. This non-linearity was superimposed on the time trend, but apparently follows no regular law. The maximum deviation from the trend of the calibrations, in the range of wave heights considered in this paper, was 8%; this is less than the deviation of means of height values found in a single record. In a later series of observations a polythene insulated wire is to be used and the difficulties involved with water absorbtion in the dielectric should be overcome. A further correction is necessary to allow for the length of screened cable used to connect the wire unit to the electronic recorder. This was checked on several occasions when calibrating the units by placing a fixed condenser across the inner core and shield of the connecting cable at the unit end, and measuring the galvanometer deflection produced, with the units in the calibrating tank, and in position for recording. Calm conditions were chosen and the factors involved were constant to within the limit of measurement of the deflections, during the whole period over which observations were taken.

Wind velocity was recorded by flashing a lamp which was connected in series with the contacts of a standard Meteorological Office cup anemometer, to produce a pip on the photographic chart each time the contacts were made. The anemometer was mounted on a pole at the end of a pier, (see plan, fig. 1), 30 feet above the mean level of the reservoir, and 18 feet above a parapet on the pier.

The wind direction was visually observed during each wave observation by watching a vane placed above the anemometer. Fluctuations about a mean of the order of 20 degrees were not usually exceeded, and the mean direction was recorded to the nearest 15 degrees. The fetch was determined from the mean wind direction, as the distance to the upwind shore. The fetch is variable for a wind varying 15° on either side of North-west, but for Westerly and Northerly winds a good estimate of fetch can be made (see fig. 1).

Before and after each observation the air temperature on the downwind bank and the surface water temperature from a bucket sample were measured.

Most of the observations were made during the period April, May, June 1953; each record from this series containing two continuous wave observations from the two sites marked on the plan, the wind velocity pips, and time marks at two second intervals. The duration of each record varied between 20 and 75 minutes, and about 26 hours of observations were analysed for wave periods, wave heights and wind velocity. For the wave data described in this paper a selection was made as described in the next section, totalling about 15 hours of the observations. A few points from a previous series are also included; on these occasions waves were recorded at the site near the eastern shore only.

ANALYSIS OF THE RECORDS

Each record was divided into sections of one minute. The number of wave peaks and wave troughs were then counted during each minute, neglecting any obviously superimposed waves of much shorter period than the more or less regular run of waves, and making one count only for the 'double waves' typical of low amplitude regions in interference patterns. In this way a 'dominant' period was found. About fifty repeat counts were made on individual minute intervals selected at random at a later date, and in the worst case the discrepancy was 8% between separate counts; nearly all repetitions agreed to within ±3% with a standard deviation of less than 2%. The somewhat subjective operation of counting waves was always performed independently of wind and fetch analysis.

A representative wave height for each one minute interval was found by measuring the distance between two lines, one such that one third of the number of peaks of the dominant waves were above it and the other such that one third of the dominant troughs were below it. Applying the calibration factor appropriate to the record to this distance, the wave height H1/3 which is exceeded by one third of the dominant waves during each minute was found.

The wind velocity over intervals varying usually between 40 and 60 secs. was found by measuring the time between the appropriate number of anemometer pips.

Every record was then carefully examined and time intervals of five minutes or longer were selected, such that the wind velocity was steady to within ±15% (usually ±10%) and the mean wind direction was steady to within ±15 degrees. When wind conditions were steady, a steady state of wave periods normally exists, but with a time lag in most cases of three to five, and occasionally eight, minutes. The figures for both wave period and wave height were then averaged over those minutes for which the periods were apparently dependent on the approximately steady wind velocity. This method of selection supplied about half the data used in this paper.

At this stage it became clear that no significant differences from these figures could be noticed if longer time intervals were chosen during which the mean wind direction was constant to within 15 degrees, but the wind velocity fluctuations were as high as 30%. Particular care was taken to avoid intervals during which obvious trends could be seen. These intervals of more variable wind were always greater than ten minutes and usually more than 15 minutes. The root-mean-square deviation of the individual minute wave heights H1/3 over the interval selected for averaging was in the worst case 15%, but usually less than 10%.

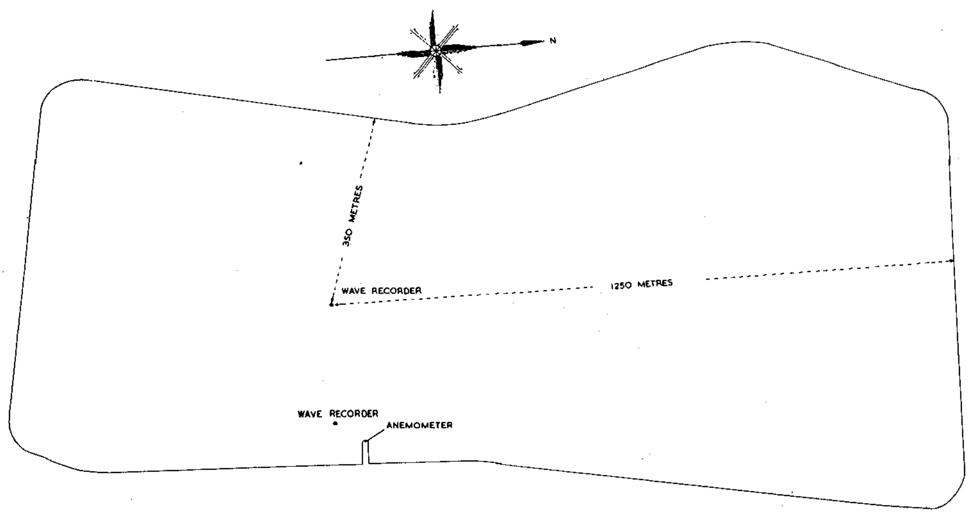

Fig. 1. Plan of reservoir

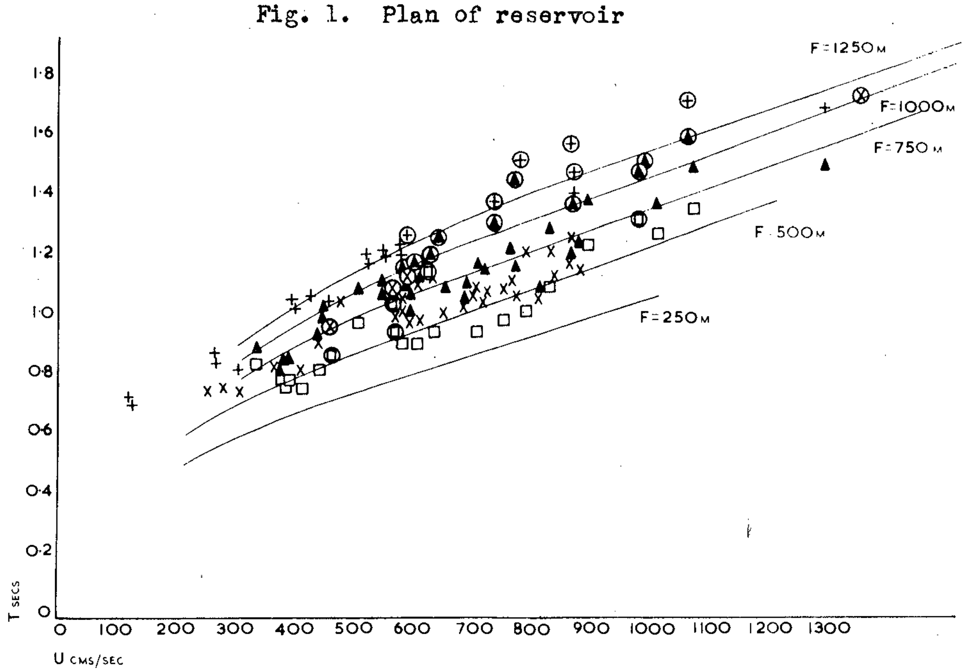

Fig. 2. Dominant wave period versus wind velocity.

□ fetch 350-500 meters
✕ fetch 500-750 meters
▲ fetch 750-1000 meters
+ fetch 1000-1350 meters

The circles denote occasions on which the air-water temperature structure was unstable. The lines drawn are computed from the mean line drawn in Figure 4 for neutral or stable conditions.

THE DISTRIBUTION OF WAVE PERIODS AND HEIGHTS

The data obtained by the procedure described in the previous section is shown in figs. 2 and 3. The symbols enclosed in a circle are those obtained under unstable conditions of the air relative to the water, i.e. when the air is colder (by 2°F or more) than the surface water.

A wave velocity C, and a wave length L were computed from the dominant wave period T using the classical formulae for small waves on deep water $C = \frac{gT}{2\pi}$, $L = \frac{gT^2}{2\pi}$ (where g = 981 cm/sec^{-2} is the acceleration due to gravity). The figures obtained were used to find the non-dimensional quantities (first plotted by Sverdrup and Munk (1947,); the wave age C/U, the non-dimensional height $gH_{1/3}/U^2$, and the wave steepness $H_{1/3}/L$. These are plotted in figures 4, 5 and 6 respectively, against the non-dimensional fetch parameter gF/U^2 (where F is the fetch distance, and all quantities are expressed in c.g.s. units). In these diagrams those points obtained under unstable air-water conditions are designated by a circle, and those under near-neutral or stable conditions by a cross. No significant differences were detected between the neutral state and increasing stability (i.e. the air warmer than the surface water), but the effect of instability is apparent.

The solid lines in figures 4 and 5 were drawn (by freehand) through the vertical means of the points obtained under neutral and stable conditions, and that in figure 6 from the values read from these graphs for given values of gF/U^2.

The curves obtained by Bretschneider (1952), which are revised versions of the original curves published by Sverdrup and Munk, are also shown. To compare those curves involving wave heights, an adjustment must be made to the present data, since previous data has been based on the mean height of the highest one third waves, while in the present series the height exceeded by one third of the dominant waves is used.

A theoretical distribution of the wave amplitudes (the distribution for wave-heights being an approximation) in a system having a "narrow" waveband, is given by Longuet-Higgins (1952). Briefly this states that if σ_h is the root-mean-square value of the wave heights then the fraction p of wave heights which (in a large number of waves) exceeds a certain value r, is given by $p = e^{-r^2/\sigma_h^2}$. The mean height $a^{(p)}$ of the fraction p of highest waves (i.e. the waves which exceed r) may be calculated and is given by $\frac{a^{(p)}}{\sigma_h} = \left(\log\frac{1}{p}\right)^{1/2} + \frac{1}{p}\frac{\sqrt{\pi}}{2}\left[1 - H\left\{\left(\log 1/p\right)^{1/2}\right\}\right]$.* Where log refers to the natural base e, and where $H(\theta)$ is the probability function $\frac{2}{\sqrt{\pi}}\int_0^{\theta} e^{-x^2}\,dx$. Longuet-Higgins compares his theory with statistical data from various sources and despite the fact that wave

*See Reference (4)

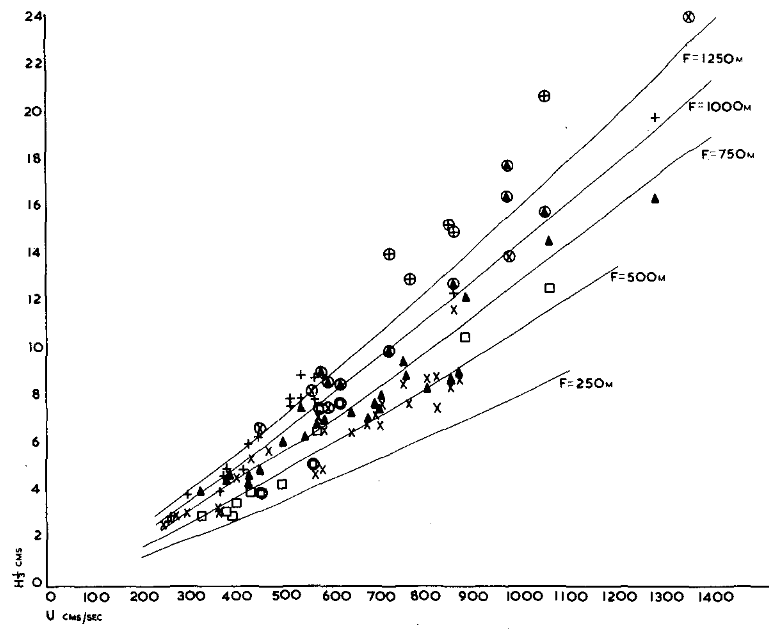

Fig. 3. The height H1/3 which is exceeded by one third of the dominant waves versus wind velocity. The symbols used correspond with those of Fig. 2, and the lines are computed from the mean line drawn through points obtained during neutral or stable conditions in Fig. 5.

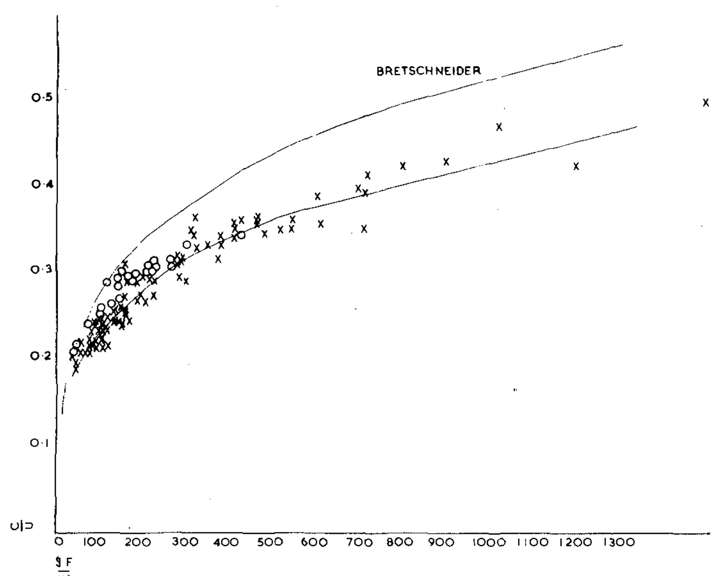

Fig. 4. Wave-age versus non-dimensional fetch. O denotes points found during unstable air-water temperature conditions. X denotes points found during neutral or stable conditions. The mean line drawn corresponds to this set of points.

systems in nature have not usually a very narrow wave-band there is agreement of observation with theory to within 8% in all cases.

If we put $p = 1/3$ in the above relations we find that the ratio of the mean height of the highest one third of the waves to the height exceeded by one third of the waves is 1.35. The adjusted curves, which in figures 5 and 6 are shown dashed, are those to be compared with Bretschneider's curves.

The curves in figures 2 and 3, were obtained from values read from the solid curves in figures 3 and 4, and are lines representing wave characteristics at different fetches when the waves are generated under neutral or stable conditions.

DISCUSSION

In figure 4 it is seen that the wave-age curve for the present work, is of the same form as that found by Bretschneider, but lies from 15 to 25% below it. In figure 5, although the curves are again of the same form, there is apparently an even greater discrepancy. But to compare the curves, it is first necessary to investigate the different methods of deriving the data through which the curves are drawn. Over the range of the parameter gF/U^2 considered in this paper, Bretschneider's evidence arises almost entirely from the observations of J.W. Johnson (1950) on waves generated on Abbott's Lagoon in California. Accordingly it is natural to compare the different techniques used in observing waves and in analysing the records used by Johnson and the present author.

At Abbott's Lagoon the reference height for wind velocity was 26 feet and for the present series 30 to 31 feet. However, the difference in wind speeds between these levels will be of an order less than 5%, according to turbulence theory, or for example the wind profiles shown in Johnson's (1950) paper. In fact a correction of about 35% would be necessary to bring the curves in figure 4 into coincidence. Even allowing for the different wind profiles possible in the two cases at the downwind shores, (at Abbott's Lagoon the prevailing wind came from the Ocean), or for the effects of the reservoir bank and the proximity of the anemometer to the pier, the wind value used in each case cannot be the main cause of the difference. This is more likely to be due to the methods of wave measurement and analysis. At Abbott's Lagoon wave pressure recorders were used, one at a depth of 6 ins. for smaller waves, and one at 18 ins. for larger waves. "For each five-minute interval the average period was obtained by counting a number of distinct waves and dividing into the elapsed time." For the present series the actual wave surface level was recorded, the galvanometers used responding linearly down to periods lower than 0.3 seconds. In each case the 'distinct' or 'dominating' waves were considered but the filtering effect of the pressure recorder will have a pronounced effect on the appearance of the record. For example the amplitude of pressure fluctuations produced by waves of one second at a depth of 18 ins. is less than 40% of that due to

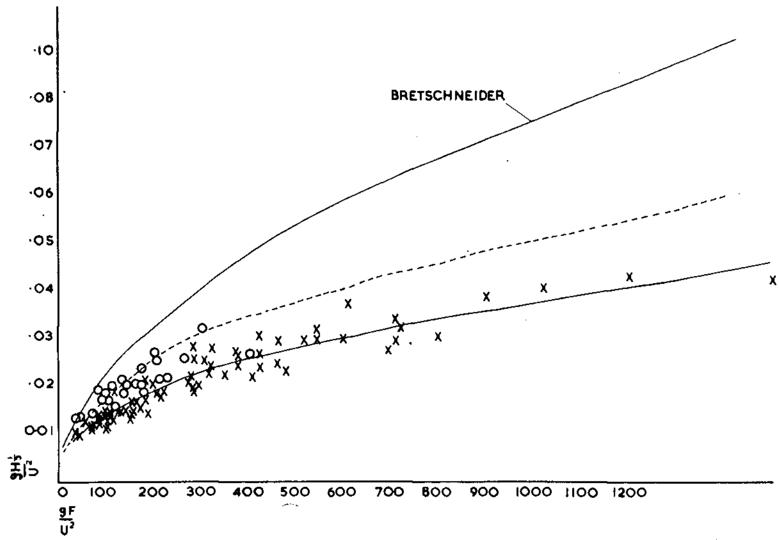

Fig. 5. Non-dimensional height versus non-dimensional fetch. The symbols used correspond to those in Fig. 4. The dotted line is the mean line adjusted such that the height used corresponds to the mean of the highest one third waves.

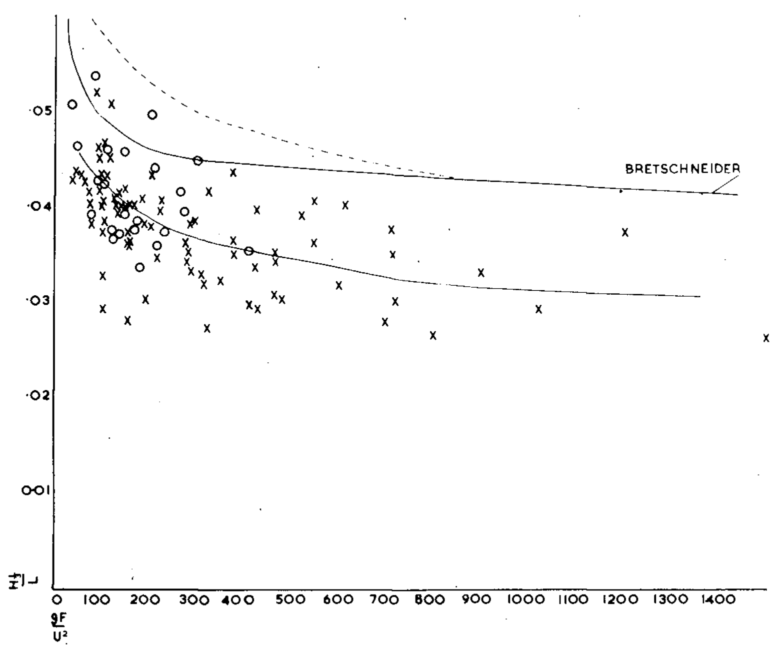

Fig. 6. Wave steepness versus non-dimensional fetch. The symbols used are those described for Fig. 4, and the mean lines are found from Figs. 5 and 6.

waves of the same height of period 1.4 secs. It may be seen that if the more obvious waves in a record are counted, then the resulting period will tend to be higher where a pressure recorder has been used than if the actual surface elevations at a point are considered. Some differences may also arise due to the observer's personal preference in choosing distinct waves.

If the statements regarding the measured period are correct, then we require a correction to the fraction of the number N of distinct "surface" waves used in measuring heights. If the period correction necessary is assumed to be 20% then the Abbott's Lagoon data uses not the fraction one third but the fraction $\frac{N/3}{N(1+\frac{1}{5})}$ of the total number N of distinct surface waves. The factor 1.35 found in the previous section to bring the measured height H1/3 into correspondence with the mean height of the highest one third waves on a pressure wave record, is increased by about 5% to 1.42. But by far the most significant feature in the non-dimensional height-fetch graph figure 5 is the increase in the non-dimensional height $gH_{1/3}/U^2$ during unstable conditions. This is of the order 25%, or more. Brown (1953) has found that waves at the Atlantic Ocean Weather stations are 25% higher when the surface water is $5\frac{1}{2}$°C warmer than the air, than during neutral stability conditions. An increase during unstable conditions has also been observed in a laboratory wave-tank by Francis (1954). This increase in wave heights could account for most of the discrepancy between Bretschneider's curve in figure 4 and the present data (for which the mean line is drawn for neutral and stable conditions), if the observations at Abbott's Lagoon were mostly made when the surface water was considerably warmer than the air. Indeed if the present data for figure 4 is corrected so that the wave heights are equivalent to those obtained at Abbott's Lagoon (i.e. present heights are multiplied by 1.42 as indicated above); then all points plotted for $gH_{1/3}/U^2$ lie symmetrically within the spread of Johnson's points up to $gF/U^2 = 500$. But the weighting of the latter series is in the upper region of values of $gH_{1/3}/U^2$, or that region corresponding to points in the present series obtained under unstable conditions. For values of gF/U^2 greater than 600 the points in both series are much less dense but the Abbott's Lagoon data lie always on the upper side. Bretschneider's curve in the non-dimensional height-fetch graph, figure 5, extends through a much wider range of the parameter gF/U^2, and in the range considered tends to lie fairly high in the distribution of Johnson's data.

In the wave-age graph, figure 4, there is an increase of about 10% in the ratio C/U, and thus of the wave period, during unstable conditions. Since the wave length L depends on the square of the period, the increase in wave-steepness $H_{1/3}/L$, figure 6, for these cases is small.

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RESUME

ONDES DE SURFACE SUR DES VOLUMES D'EAU EN ENCEINTE FERMEE

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Une série d'observations a été faite sur les ondes de surface dans un réservoir d'une largeur de 1 mile sous un vent compris entre 250 et 1300 cm/s. L'exposition du réservoir au vent est bonne ; il n'y a pas d'obstacles considérables dans une limite de 3 miles, mais les berges s'élèvent suivant une pente assez raide jusqu'à une hauteur de 40 pieds au-dessus du terrain environnant.

Les ondes furent enregistrées simultanément en deux postes à l'aide d'un enregistreur d'onde à fil capacitif répondant presque linéairement aux variations de hauteur des ondes de surface. La vitesse du vent et sa direction à une hauteur de 30 pieds furent enregistrées durant chaque observation. Les températures de l'air et de la surface de l'eau furent mesurées avant et après chaque observation.

Les enregistrement d'ondes furent analysés en comptant le nombre plus ou moins régulier d'ondes dominantes par minute pour trouver la période ; la hauteur dépassée par 1/3 du nombre des ondes dominantes fut mesurée. Les intervalles de vent comparativement stable et des périodes d'ondes stables correspondantes furent choisis et les moyennes des caractéristiques d'ondes pendant les minutes comprises dans ces intervalles furent calculées. Les résultats sont traduits suivant la façon non dimensionnelle habituelle employée pour la première fois par Sverdrup et Munk et sont comparés avec les observations faites par J.W. Johnson sur la lagune d'Abbott en Californie.

Un accroissement de hauteur d'onde de l'ordre de 25 % et pour des périodes de 10 % environ est apparu quand les ondes sont engendrées sous des conditions de température air-eau instables. Une courbe moyenne tracée d'après les présents résultats obtenus dans des conditions stables et neutres se tient considérablement au-dessous des résultats moyens de la lagune d'Abbott à la fois dans les graphiques ondes-temps et dans les graphiques hauteur non dimensionnelle $\frac{gH}{U^2}$ fonction de la longueur non dimensionnelle $\frac{gF}{U^2}$.

Il est suggéré que l'effet d'amortissement des enregistreurs de pression employés sur la lagune d'Abbott est le principal responsable des différences dans le graphique onde-temps. Peut-être une partie de ces écarts est-elle dûe au procédé quelque peu subjectif de comptage des ondes. Dans le graphique non dimensionnel hauteur-longueur, la courbe obtenue (corrigée pour que les hauteurs soient équivalentes à la moyenne de trois des ondes les plus hautes) est encore au-dessous de la courbe moyenne d'après les résultats de la lagune d'Abbott.

Si les observations à la lagune d'Abbott ont été faites pour la plupart dans des conditions instables, la différence entre les deux séries d'observations est faible.