CHAPTER 3

REVISIONS IN WAVE FORECASTING: DEEP AND SHALLOW WATER

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ABSTRACT

During the past six years since the latest revisions in wave forecasting (Bretschneider 1951) were made, much information has become available such that another revision is in order. An abundance of published (and unpublished) accounts of wave generation and decay in both deep and shallow water from various sources, as well as new ideas in the art of wave forecasting, are used in this revision. Deep water wave forecasting relationships, relationships for the generation of wind waves in shallow water of constant depth, and techniques for forecasting wind waves over the Continental Shelf are included in this paper. Forecasting hurricane waves is also discussed, from the engineering design point of view. The concept of significant wave is still retained as the most practical method in wave forecasting to date. The significant period has definite significance in that the wave energy is propagated forward at a speed approximately equal to the corresponding group velocity.

The graphical approach (Wilson 1955) for moving fetches and variable wind vectors is discussed, and is the best approach for forecasting waves. Without Wilson's graphical technique it is difficult for any two forecasters supplied with the same meteorological data to obtain the same degree of verification, or determine whether the forecaster or the forecasting relationships are in error. It is quite possible that by use of this technique further revisions in wave forecasting are possible.

The problem of wave variability is discussed, and the distributio functions are given. A short summary of the wave spectra (Bretschneide 1958) used in connection with the revisions is also given.

When the present forecasting relationships are applied to sections of the world, other than that from which the basic data were procured, it is recommended that atmospheric stability factors be taken into account. This essentially involves a slight modification or calibratic of the forecasting relationships and techniques, prior to general use in the area of interest.

INTRODUCTION

Ordinary gravity waves have been classified by Munk (1951), and are ocean waves having periods between 1.0 and 30 seconds. This range of periods is included in the spectrum of ocean waves, which at the lower limit are capillary waves having a period less than 0.1

second and the upper range transtidal waves having a period of 24 hours and greater. Waves having a period between 0.1 and 1.0 second are called ultragravity waves. Hence, gravity waves in general include both ordinary gravity and ultragravity, although the latter are affected by surface tension. When discussing ocean waves in this paper, the meaning is intended to be these gravity waves.

Deep water waves are defined as gravity waves unaffected by the depth of water. For all practical purposes deep water waves have wave lengths equal to or less than twice the water depth. All other waves are considered as waves in shallow or intermediate water depths.

When forecasting waves use is made of the term significant wave. The significant wave height is the average of the heights of the highest one-third waves in a wave train or wave record. The mean period of the significant wave is termed the significant period. Sometimes the significant wave is defined for the convenience of wave record analysis according to Wiegel (1953): A length of wave record is selected, usually of 20 minutes duration; the high groups of waves are selected to determine a mean period, called the significant period; the length of record in seconds is divided by the significant period to obtain a wave number; one-third of this number is the number of waves to consider in determining the significant height, beginning with the highest wave. Both of the above definitions give almost the same results, the latter being a time saver in the analysis of wave records.

Wave forecasting may be classified into three general groups, (a) ordinary deep water wind waves and swell; (b) wind waves and swell in shallow water; and (c) hurricane wind waves and swell, deep or shallow water. The above cases are discussed in this report, although none are completely understood at present.

By ordinary wind waves is meant waves generated by stationary or slowly moving storms, having more or less constant wind speed and direction. More knowledge is available on ordinary deep water wind waves than on any of the other above two topics. Wind waves in shallow water of constant depth is fairly well established, at least semiempirically. Hurricane wind waves in deep water are least understood, chiefly because of the lack of data, both on winds within hurricanes as well as the complex nature of the sea. The transformation of sea into swell and the decay of swell is partly understood, at least physically, although suitable wave theory and data are lacking for an accurate description of this phenomenon.

FORECASTING DEEP WATER WAVES

Little literature was available prior to World War II on wave generation and decay. During the war advance knowledge of wave activity was required for areas where amphibious landings were planned. This problem was first attacked by Sverdrup and Munk (1947), who combined classical wave theory with available data to obtain semiempirical or "semitheoretical" wave forecasting relationships.

This was the first great advance in the art of wave forecasting, and i known as the significant wave method of forecasting, sometimes referre to as the Sverdrup-Munk-Bretschneider or SMB method. Actually the B in the above method deserves little credit at the most, since it becam attached through a simple revision (Bretschneider 1952) of the origina work by Sverdrup and Munk (1947), which already had experienced its first revision by Arthur (1947). These revisions were based on additional wave data not available at the time of the original work of Sverdrup and Munk.

At present two main schools of thought exist for deep water wave forecasting, (a) the significant wave method, mentioned above, and discussed in more detail in the present paper, and (b) the wave spectr method, discussed by Pierson, Neumann and James (1955), referred to as PNJ method. Both the SMB and the PNJ methods have certain advantages and certain disadvantages, since neither method has realized the perfection desired in the art of wave forecasting. The desired perfection of either the SMB or PNJ method might be attained in the near future by use of the graphical approach to wave forecasting as given by Wilson (1955), supplemented with additional wave data for calibration purposes.

Two other methods of forecasting waves may also be mentioned, (a) the method of Darbyshire (1955), and (b) the method of Suthons (1945), both European methods. The Suthons method is the least familiar but similar in techniques and principle to the SMB method. Darbyshire method is based on the development of a wave spectrum, quite different than the Neumann (1953) wave spectrum. An important consideration is that each of the four methods is based essentially on wave data, and hence each must give forecasts as accurate as the data from which the particular method was derived. It is foolish for PNJ to evaluate the Darbyshire method using PNJ data, just as it is for the present author to evaluate the PNJ method using SMB data, and vice versa. That is to say each method has inherent characteristics associated with the procurement of data. A very objective verification study of the above four methods was made by Roll (1957), and the general conclusion was that each method works better for the particular region from which the principal data were obtained. Perhaps, even better verification might have been obtained provided the individual contributors made the forecast, each using their own methods and techniques.

GENERATION OF DEEP WATER WAVES BY WIND OF CONSTANT SPEED AND DIRECTION

The present revisions are based on the SMB technique. As shown by Johnson (1950) and perhaps others, one may arrive at the generating parameters by use of the PI-theorm (Buckingham 1914) and dimensional analysis. These parameters are:

$$\frac{gH}{U^2} = f_1 \left[\frac{gH}{U^2} \quad , \quad \frac{gt}{U} \right] \qquad (1)$$

$$\frac{C_o}{U} = \frac{gT}{2\pi U} = f_2\left[\frac{gF}{U^2}, \frac{gt}{U}\right], \quad \text{where} \tag{2}$$

$H = H_{1/3}$ = significant wave height, feet
C_o = wave speed in deep water, feet/second
$T = T_{1/3}$ = wave period, seconds
g = acceleration of gravity, 32.2 feet/second^2
U = wind speed in feet per second
t = duration of wind in seconds
F = fetch length in feet

Equations 1 and 2 are for constant wind speed and direction.

Figure 1-A is a revision of the Fetch Graph of Bretschneider (1952), originally revised from Sverdrup and Munk (1947). The upper limits of the generating parameters, corresponding to a fully developed sea are obtained by use of the wave spectrum proposed by Bretschneider (1958), a short abstract of which is given at the end of this paper. These upper limits are reached when

$$\frac{gT_{1/3}}{2\pi U} = \frac{g\bar{T}}{2\pi U} = 1.95 \tag{3}$$

$$\frac{gH_{1/3}}{U^2} = .282 \tag{4}$$

$$\frac{gF}{U^2} \approx 6 \times 10^5 \quad \text{where} \tag{5}$$

$T_{1/3}$ = significant wave period, $\bar{T}$ = mean wave period, and

$H_{1/3}$ = significant wave height.

The original curves of Sverdrup and Munk (1947) places the upper limit corresponding to $gF/U^2 = 10^5$, those revised by Bretschneider (1952) at $gF/U^2 = 4 \times 10^4$, and those utilized by Pierson, Neumann and James (1955) at $gF/U^2 = 10^5$. It can be seen that all previous investigations underpredict the parameters for a fully developed sea. A fully developed sea for moderate to large wind speeds can never be experienced on this world. However, about 90 percent of the fully developed sea is reached at $gF/U^2 = 10^5$, but the last 10 percent of the generation takes place over a very much longer fetch length. For low to moderate wind speeds, storms moving across the Pacific Ocean

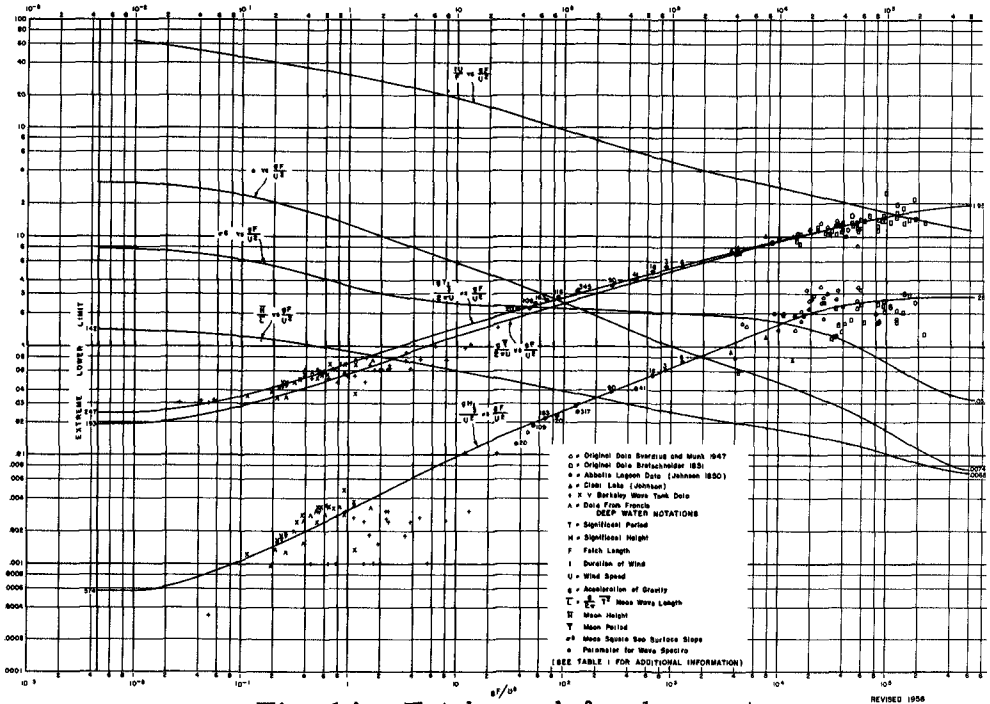

Fig. 1A. Fetch graph for deep water.

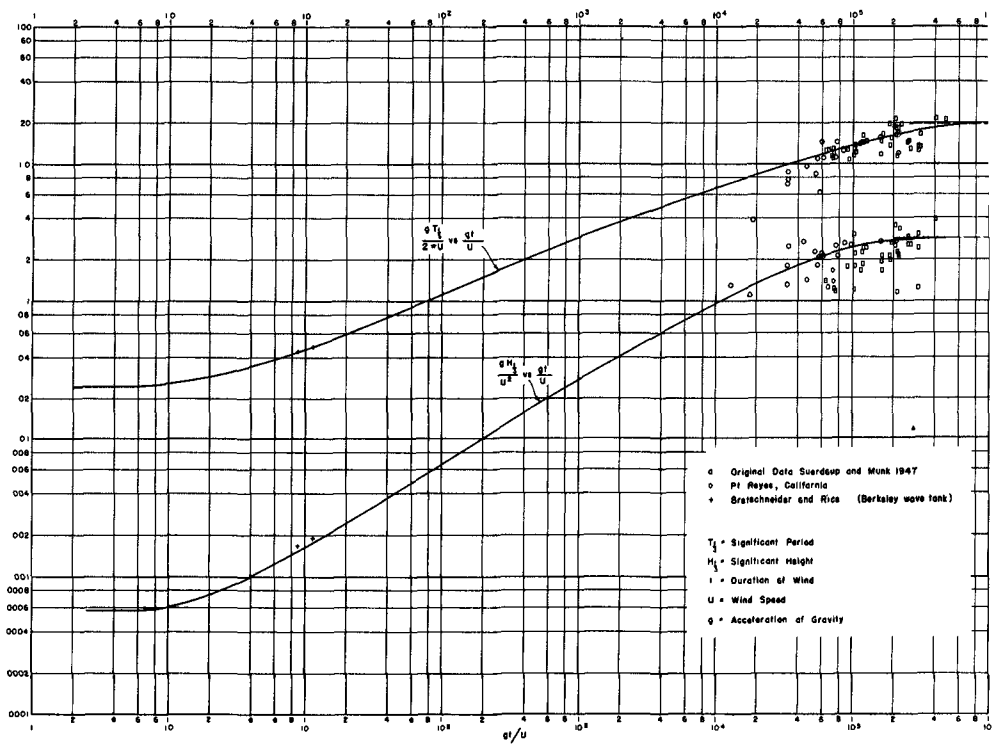

Fig. 1B. Duration graph.

might generate a fully developed sea.

The lower limits of the fetch graph are obtained from Bretschneider (1958):

$$\frac{g\bar{T}}{2\pi U} = .0193$$

$$\frac{gT_{1/3}}{2\pi U} = .0244 \qquad (6)$$

$$\bar{H}/\bar{L} = \frac{1}{7}$$

$$\bar{L} = \frac{g\overline{T^2}}{2\pi}$$

$$\overline{T^2} = 1.078(\bar{T})^2 \qquad (7)$$

$$\frac{g\bar{H}}{U^2} = .000357$$

$$\frac{gH_{1/3}}{U^2} = .000572$$

$$\frac{gF}{U^2} = .0046$$

Using the upper and lower limits given above, and the available wave data, Figure 1-A, the Fetch Graph, was constructed. The curve of $g\bar{T}/2\pi U$ is a first approximation for mean wave period, based on meager data and the asymtopic limits given above. This curve may need a slight revision when more data become available.

The Duration Graph, Figure 1-B, can be constructed from the Fetch Graph and the considerations following. The duration of time required for generation depends on the fetch distance traveled and the group velocity appropriate to the most energetic waves. It is shown by Bretschneider (1958) that the band of waves having a period very nearly

equal to the significant period are the most energetic. This is as should be expected. For very low values of gF/U^2 the significant period is equal to the period corresponding to the band of waves having maximum energy, and for a fully developed sea the significant period is 1.027 times the period corresponding to the band of waves having maximum energy. Thus it must be emphasized, that the signific period has definite significance, the significance being that it represents very closely the period appropriate to the band of waves having maximum energy, and hence may be used to determine the duratio time required for wave generation.

The general form of $F = C_g t$ (fetch distance is equal to group velocity times time) can be applied in differential form $dt = \frac{1}{C_g} dF$, where C_g, the group velocity is a variable and increases with time an distance. In parametric form the expression becomes:

$$\frac{gt}{U} = \int \frac{U}{C_g} \, d\,(gF/U^2), \text{ where} \qquad (8$$

$$\frac{C_g}{U} = \frac{1}{2} \left[\frac{gT_{1/3}}{2\,U}\right] \left[\frac{T_{op}}{T_{1/3}}\right], \text{ where } T_{op} \qquad (9$$

corresponds to the optimum period around which is concentrated maximum energy. $T_{op}/T_{1/3}$ varies from 1.0 at $gF/U^2 \rightarrow 0$ to 1.027 at $gF/U^2 = 6 \times 10^5$, with maximum of 1.0375 at $gF/U^2 \cong 200$.

Numerical means and Figure 1-A were used to establish the Duratic Graph, Figure 1-B. Figures 1-A and 1-B were used to construct the forecasting curves presented in Figure 2. Table I summarizes the fina selected values of generation parameters. The parameters α, σ^2 and ϵ a discussed later in the paper with respect to the wave spectra.

Short Fetches and High Wind Speeds. For short fetches and high wind speeds, it was shown by Bretschneider (1957) one might use the following formulae as first approximations:

$$H = .0555 \sqrt{U^2 F} \qquad (10$$

$$T = .50 \sqrt[4]{U^2 F}, \text{ and} \qquad (11$$

$$\frac{F_{min}}{t_{min}} = .57 \sqrt[4]{U^2 F} = 1.14\ T, \qquad (12$$

These equations will result in slightly different values of H and T than those obtained from Figure 2, and should therefore be used with caution. However, these equations become quite useful when discussing hurricane waves later in this paper. In the above equations:

TABLE I

SUMMARY OF DEEP WATER WAVE GENERATION PARAMETERS

$\frac{gF}{U^2}$	$\frac{gt}{U}$	$\frac{tU}{F}$	$\frac{gH_{1/3}}{U^2}$	$\frac{gT_{1/3}}{2\pi U}$	r	$\frac{T_{1/3}}{\bar{T}}$	$\frac{g\bar{H}}{U^2}$	$\frac{g\bar{T}}{2\pi U}$	$\frac{\bar{H}}{\bar{L}}$	$\frac{\bar{H}}{L}$	α	σ^2	ϵ	$\frac{H_{1/3}}{T_{1/3}^2}$
.01	.63	63	.000574	.0247	.998	1.2645	.000359	.0195	.1394	.1397	3.07	.766	.464	.767
.02	1.14	57	.000611	.0258	.995	1.2637	.000382	.0204	.1355	.1361	2.90	.728	.465	.748
.04	2.06	51.5	.000738	.0288	.992	1.2629	.000461	.0228	.1309	.1316	2.71	.685	.466	.725
.06	2.92	48.6	.000867	.0316	.985	1.2613	.000542	.0251	.1270	.1282	2.55	.650	.467	.708
.08	3.7	46.3	.000957	.0334	.981	1.2602	.000598	.0265	.1257	.1272	2.50	.640	.468	.699
.10	4.5	45.0	.00105	.0353	.979	1.2598	.000656	.0280	.1235	.1251	2.41	.620	.474	.687
.20	8.0	40.2	.00143	.0425	.965	1.2566	.000894	.0338	.1155	.1179	2.11	.553	.479	.645
.40	14.1	35.5	.00195	.0521	.949	1.2526	.001219	.0416	.1040	.1071	1.71	.460	.487	.585
.60	20.3	33.9	.00235	.0591	.936	1.2497	.001469	.0473	.0969	.1005	1.48	.408	.494	.548
.80	25.7	32.1	.00269	.0646	.927	1.2474	.001681	.0518	.0925	.0965	1.35	.377	.497	.525
1.0	31	31.0	.00301	.0695	.916	1.2450	.001881	.0558	.0885	.0928	1.24	.352	.510	.508
2.0	54	27.0	.00430	.0869	.878	1.2357	.002688	.0703	.0803	.0860	1.02	.311	.547	.464
4.0	94	23.5	.00610	.108	.827	1.2231	.003813	.0883	.0722	.0794	.824	.274	.572	.426
6.0	129	21.5	.00743	.124	.790	1.2141	.004644	.102	.0659	.0739	.686	.260	.596	.393
8.0	160	20.0	.00855	.137	.762	1.2071	.005344	.113	.0618	.0703	.603	.249	.016	.371
10.0	192	19.2	.00951	.147	.740	1.2017	.005944	.122	.0590	.0678	.550	.242	.632	.359
20.0	306	15.3	.0129	.179	.671	1.1845	.008063	.151	.0522	.0621	.431	.232	.677	.328
40.0	488	12.2	.0175	.215	.590	1.1636	.01094	.185	.0472	.0583	.352	.228	.713	.309
60.0	654	10.9	.0208	.240	.546	1.1524	.01300	.208	.0444	.0559	.311	.221	.736	.294
80.0	792	9.9	.0232	.261	.513	1.1437	.01450	.228	.0412	.0527	.268	.219	.750	.278
100	920	9.2	.0255	.279	.486	1.1367	.01594	.245	.0392	.0507	.243	.218	.766	.267
200	1520	7.60	.0337	.337	.400	1.1355	.02106	.297	.0352	.0469	.196	.212	.792	.242
400	2440	6.10	.0441	.403	.322	1.0918	.02756	.369	.0300	.0416	.142	.205	.819	.221
600	3300	5.50	.0522	.453	.275	1.0794	.03263	.420	.0273	.0386	.118	.202	.832	.208
800	4056	5.07	.0583	.486	.243	1.0705	.03644	.454	.0261	.0374	.108	.199	.840	.201
1000	4800	4.80	.0641	.519	.220	1.0640	.04006	.488	.0248	.0359	.0972	.198	.846	.194
2000	8000	4.0	.0841	.618	.160	1.0469	.05256	.590	.0223	.0330	.0786	.194	.863	.179
4000	13,800	3.45	.1110	.735	.102	1.03005	.06938	.714	.0201	.0294	.0638	.186	.875	.167
6000	18,960	3.16	.130	.816	.079	1.0232	.08125	.797	.0189	.0289	.0564	.178	.882	.159
8000	23,760	2.97	.145	.872	.062	1.0183	.09063	.856	.0183	.0281	.0529	.170	.885	.155
10,000	28,100	2.81	.157	.924	.052	1.0154	.09813	.910	.0175	.0270	.0484	.163	.888	.1550
20,000	48,200	2.41	.195	1.10	.027	1.0080	.1219	1.091	.0151	.0235	.0360	.142	.896	.131
40,000	82,000	2.05	.234	1.28	.010	1.003	.1463	1.276	.0133	.0208	.0279	.113	.901	.116
60,000	112,800	1.88	.253	1.39	.006	1.0018	.1581	1.388	.0121	.0190	.0231	.0962	.902	.107
80,000	140,000	1.75	.264	1.49	.002	1.0006	.1650	1.4891	.0110	.0173	.0191	.0815	.903	.0969
100,000	168,000	1.68	.270	1.54	.001	1.0	.1688	1.54	.0105	.0165	.0174	.074	.904	.0928
150,000	228,000	1.52	.277	1.67	0	1.0	.1731	1.67	.00916	.0143	.0133	.0583	.905	.0809
200,000	286,000	1.43	.279	1.74	0	1.0	.1744	1.74	.00850	.0134	.0114	.0500	.906	.0751
300,000	393,000	1.31	.281	1.84	0	1.0	.1756	1.84	.00766	.0119	.0093	.0410	.907	.0676
400,000	496,000	1.24	.282	1.90	0	1.0	.1763	1.90	.00721	.0113	.0082	.0370	.908	.0637
500,000	595,000	1.19	.282	1.93	0	1.0	.1763	1.93	.00699	.0110	.0077	.0345	.909	.0617
600,000	702,000	1.17	.282	1.95	0	1.0	.1763	1.95	.00685	.0107	.0074	.0337	.910	.0604

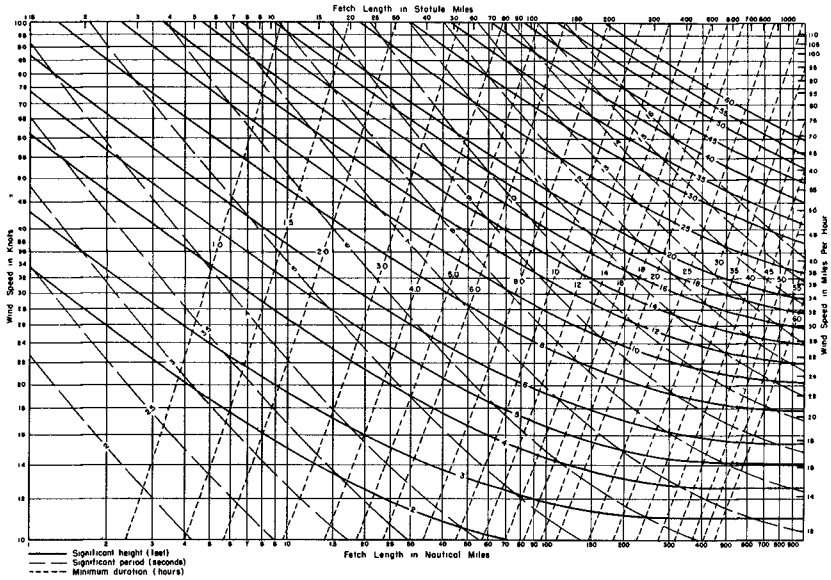

Fig. 2. Deep-water wave forecasting curves as a function of wind speed, fetch length, and wind duration.

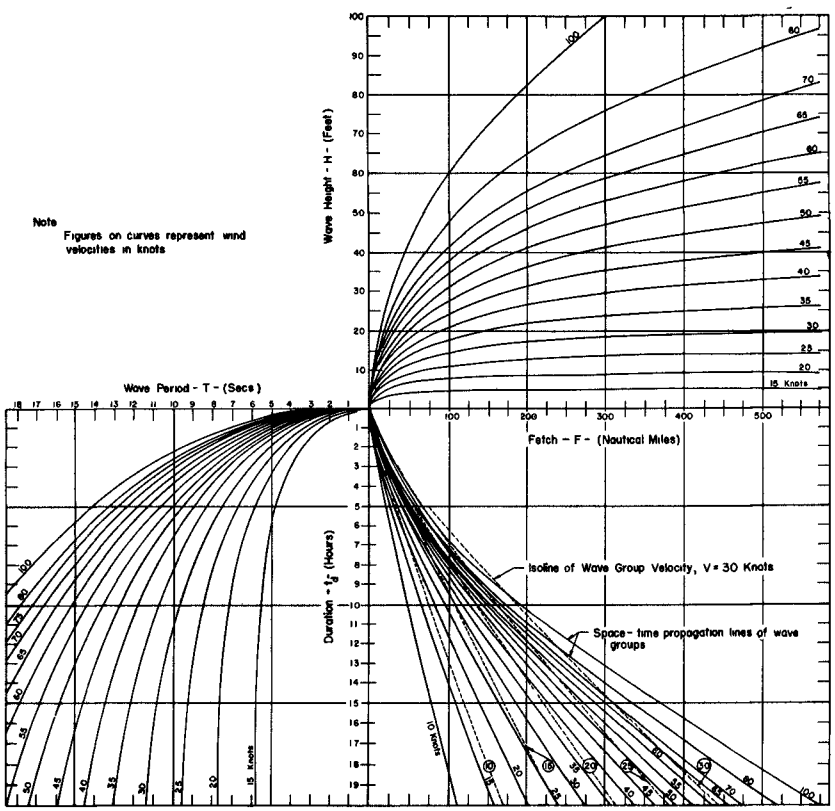

Fig. 3. Relationships for forecasting significant heights and periods of waves in deep water under wind of particular velocity, duration, and fetch.

H = significant height in feet
T = significant period in seconds
U = wind speed in knots
F = fetch length in nautical miles
F_{min} = minimum fetch length in nautical miles
t_{min} = minimum duration in hours

The value of F to use in equations 10 and 11 must be equal to or less than F_{min} obtained from equation 12.

GENERATION OF DEEP WATER WAVES BY VARIABLE WIND SPEED AND DIRECTION AND MOVING STORMS

If the wind field is not too irregular and the movement of the storm is fairly slow, then Figure 2 can be used to advantage. Methods of techniques used are given by Kaplan (1953). However, when the variables are ill-defined then the graphical method proposed by Wilson (1955) must be used, the method of which also applies for winds of constant speed and direction. Figure 3, wave forecasting curves used by Wilson (1955), is somewhat different than the revised curves of Figure 2. Whether Figure 3 or Figure 2 is more accurate under steady wind conditions is difficult to say. The important thing is that Figure 3 can be used readily for ill-defined storm situations whereas the use of Figure 2 becomes somewhat awkward, even with the additional techniques proposed by Kaplan (1953). Figure 4 is a typical profile of a variable wind field taken from Wilson (1955). Figure 5 is a typical example taken from Wilson (1955) in the application of the graphical technique. In explaining Figure 5 the following is quoted from Wilson (1955):

"Wind of Variable Velocity in a Variably Moving Wind System of Finite Fetch

"Dispensing now with the restriction of a uniform wind velocity, U, but retaining the concept of uniformity along closed contours of a space-time wind field, it becomes possible to represent a wind system that has both variable wind velocity and variable speed of forward (or rearward) progression by a wind-field of closed contour lines whose intervals apart represent equal increments of wind velocity U.

"Figure 5 shows such a wind-field with contours of wind velocity at 5-knot intervals from 20 to 40 knots. Superimposed thereon at an arbitrarily selected point O in space and time is the HtFt diagram with H(F), $F(t_d)$ and $T(t_d)$ curves drawn in for the same 5-knot intervals of wind velocity U from 20 to 40 knots.

"The problem now is one of determining the history of the height and period growth of the waves originating at the point O.

"In relation to the wind-field the origin O is seen to be at a point where the wind velocity would be of the order of 21 knots. Waves originating at O would be obliged to follow a space-time path somewhere along the belt of propagation lines forming the relationships $F_U(t_d)$. It is clear that the actual path of the waves must initially be along a line intermediate between the propagation lines for U = 20 and U = 25 knots as far as a, the intersection point with the 25-knot wind-field contour. Along the path Oa the waves would be under the influence of winds ranging from 21 to 25 knots so that, to all intents and purposes, Oa can be regarded as the propagation line for U = 23 knots.

"Over the same interval of time the growth in significant period of the waves will follow the line Ob (Figure 5), equivalent to the curve $T_U(t_d)$ for U = 23 knots.

"Having arrived at a, the waves pass into the next incremental wind zone over which wind velocity rises from 25 to 30 knots. Their further space-time path from a to e must be at a rate (or group velocity) appropriate to the average wind of U = 27½ knots, but the propagation rate must start off from a at the same slope as the line Oa has at a.

"To ensure that the group velocity shall remain the same at the transition, it becomes necessary to trace a line bc at constant period and locate a point c intermediate between $T_{25}(t_d)$ and $T_{30}(t_d)$. The condition of constant period ensures constant wave group velocity since group velocity is directly proportional to wave period under deep water conditions. By drawing the abscissa cd, the point d is found intermediate between the curves $F_{25}(t_d)$ and $F_{30}(t_d)$. An imaginary propagation line $F_{27\frac{1}{2}}(t_d)$, drawn through d would now have the same slope as the curve Oa at a. To find ae, therefore, it is only necessary to transcribe, as it were, a piece of the $F_{27\frac{1}{2}}(t_d)$ curve from d, parallel to itself, and add it to the curve Oa at a. By this means the point e is established.

"In the same sense, by transcribing a portion of the curve $T_{27\frac{1}{2}}(t_d)$ from c, parallel to itself, and adding it to the curve Ob at b, the point f can then be located (via ef), marking the further growth in period of the waves, bf, under the influence of the 25 to 30-knot wind.

"This procedure may be followed consistently to trace the actual space-time path of the waves, Oaekosw, through the wind-field and to give the history of the period growth of the waves, Obflptx. It will be noted

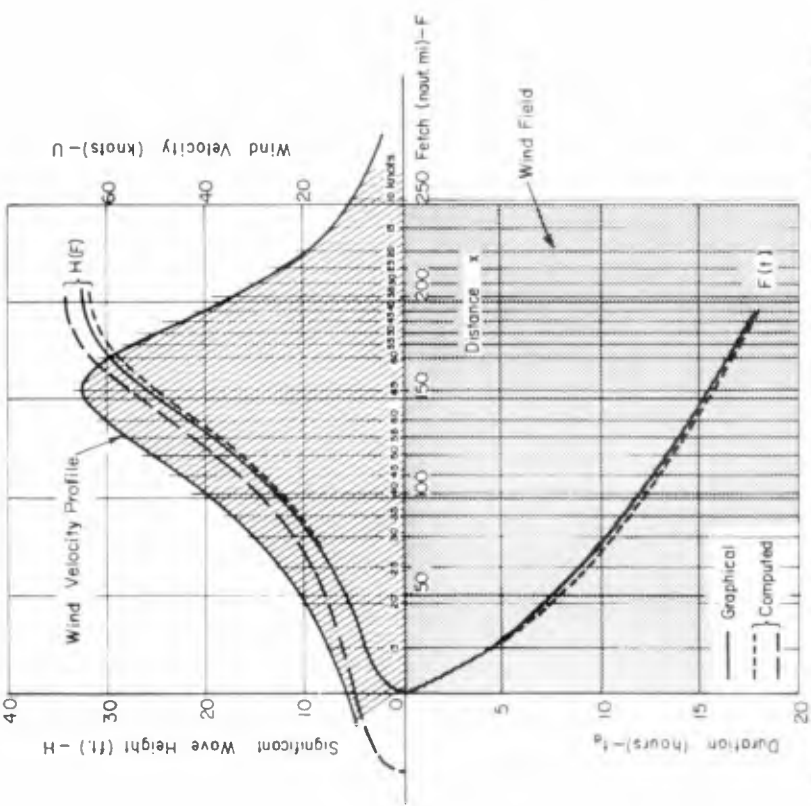

Fig. 4. Comparison of computed and graphically determined heights and travel distances of waves generated within a stationary hurricane.

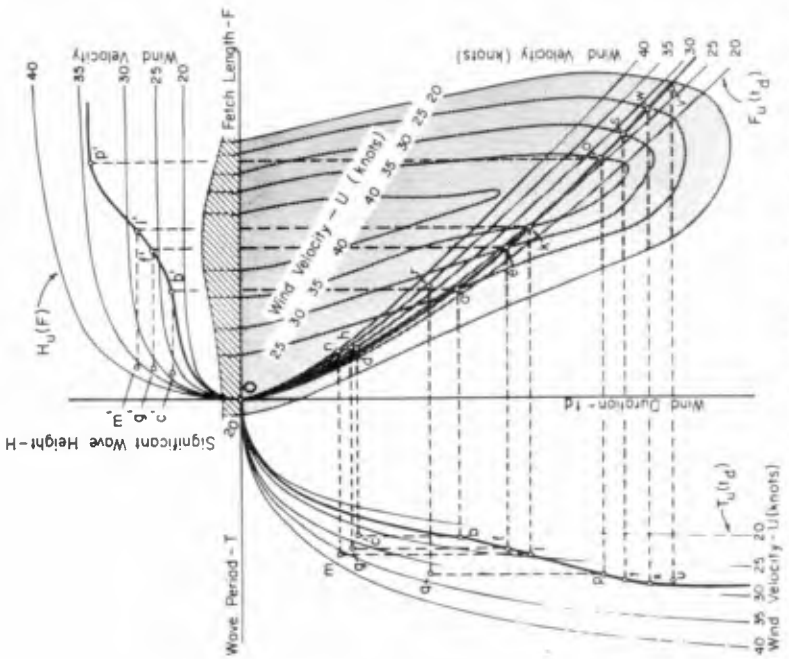

Fig. 5. Significant heights and periods of waves generated in a variably moving wind system of variable wind velocity.

that the same method applies in the zone of declining wind velocities as in the zone of increasing velocities. Thus the portion os of the wave propagation curve is drawn parallel to $F_{32\frac{1}{2}}(t_d)$ at r, the wave group velocity at 0 and r being different in the declining 35 to 30-knot wind zone from that at h and k in the increasing 30 to 35-knot wind zone.

"The graphical charting of the corresponding growth in significant height of the waves follows essentially the same procedure as described above. The curve Ob' follows the $H_{23}(F)$ isoline as far as b', which is the intersection point with the ordinate drawn through a. Further increase in height of the waves in the next incremental wind zone (U = 25 to 30 knots) must continue at a rate appropriate to $H_{27\frac{1}{2}}(F)$, starting, however, at the same height as at b'. Accordingly b'f' is drawn, parallel to $H_{27\frac{1}{2}}(F)$ at c', to give the intersection point f', with the ordinate drawn from e on the propagation line.

"The final curve of significant wave height follows the line Ob'f'l'p' and tapers off to a maximum value which is maintained to the end of the wind field. In the same way the curve of significant wave period, Obflptx, is found to taper off to a maximum value of wave period."

The above quote from Wilson (1955) should be sufficient to understand the practical applications of the graphical technique, but if more details are required the reader is referred to the paper by Wilson (1955). Experience and practice of course are necessary to perfect one's techniques in the above method.

DECAY OF WAVES IN DEEP WATER

When waves leave a generating area and travel through an area of calm or lighter winds a transformation takes place. The significant height decreases and the significant period shifts to the longer period waves, resulting in an increase in the mean period (significant period) of the significant height. To understand the meaning of the above statement one must refer to the spectrum of waves, and the original work of Barber and Ursell (1948). Although one forecasts the significant height and period, it must be remembered that the sea is actually made up of a spectrum of waves, with varying amplitudes and frequencies or periods. The work of Putz (1952) and that of Longuet-Higgins (1952) show the distribution of heights about the mean height, and Putz (1952) shows the distribution of periods about the mean period. The theoretical distribution of heights by Longuet-Higgins is known as the Rayleigh type distribution based on a narrow spectrum, and the agreement is surprisingly good with the distribution obtained by Putz (1952) based on the analysis of wave records. Thus the

significant height can be related to the distribution of all heights in the spectrum, and the significant period can be related to the distribution of all periods in the spectrum, when the significant and mean wave periods are nearly equal. The joint relationship between the individual heights and periods has not yet been established satisfactorily. An unsuccessful attempt to determine the joint distribution was made by Bretschneider (1956), except for the special case of zero correlation. A revision of the work on the joint distribution is presently under way. This, together with the wave spectrum derived therefrom, should be valuable in future studies in regards to the decay of waves. Remarks on this matter are discussed later.

However, the physical behavior of the decay of waves in the form of either joint distribution or wave spectrum can be visualized accordingly: first, the heights of long period waves are reduced proportionately less than the shorter period waves for the same decay distance. Furthermore, the long period waves travel faster than the shorter period waves and hence will be dominating. With respect to decay distance the height will decrease and the period increase for the significant waves. Consider now a fixed decay distance with respect to time. First will be noticed the arrival of long period swell, and some time later the shorter period waves begin to arrive, which in effect causes a decrease in significant period with respect to time. In case of deep water waves the decrease in significant period with respect to time will continue, but the lowest value of this significant period can never be less than the original (generated) significant period. Thus, with respect to distance, there will be a space time history of the joint distribution or the spectrum of waves, and a space time history of the significant waves. The work of Barber and Ursell (1948) shows this to be true. The same discussion applies to the mean height and the mean period, wherein certain low period waves are completely filtered out.

General expressions for the decay of waves can be written as follows:

$$\frac{H_D}{H_F} = f_1\left[\frac{D}{gT_D^2}, \quad \frac{gF}{U^2}, \quad \frac{F}{W}\right] \tag{13}$$

$$\frac{T_D}{T_F} = f_2\left[\frac{D}{gT_D^2}, \quad \frac{gF}{U^2}, \quad \frac{F}{W}\right] \tag{14}$$

where H_F = significant height at end of fetch
H_D = significant height at end of decay
gF/U^2 = generating parameter
D/gT_D^2 = decay parameter
F/W = ratio of fetch length to fetch width

Exact relationships for equations 13 and 14 cannot be obtained due to lack of proper wave data. In most cases F/W does not vary much, and its effect, if any, is lost in the scatter of wave data. By empirical means relationships were found (Bretschneider 1952) for wave decay as follows:

$$\frac{D}{H_D} = f_1 \left[\frac{D}{H_F} \quad , \quad \frac{D}{F} \right] , \text{ and} \qquad (15$$

$$\frac{D}{gT_D^2} = f_2 \left[\frac{D}{gT_F^2} \quad , \quad \frac{D}{F} \right] \qquad (16$$

The empirical relationships were transformed into practical curve for forecasting the decay of waves. This is shown in Figure 6. It must be emphasized that Figure 6 still requires revisions, based on more suitable wave data. In the development of Figure 6 wave data included that from the Pacific Ocean, both from the northern and southe hemispheres. Waves from the southern hemisphere had decay distances from 4,000 to 6,000 miles, and those from the northern hemisphere had decay distances from 50 to 3,000 nautical miles. The waves in the fetch were forecast by use of the generation graphs. Twenty-four hourly weather maps of the southern hemisphere and twelve hourly weather maps of the northern hemisphere were used to obtain wind speeds and fetch lengths. Twenty-four hourly maps are never very satisfactory.

Perhaps a properly calibrated wave spectrum method needs development for prediction of waves in the decay zone. In either case more reliable wind and wave data are required to obtain accurate decay relationships.

DECAY OF DEEP WATER SWELL OVER SHALLOW BOTTOM

Figure 6 presents curves for obtaining the decayed wave height and period for deep water. The swell may have advanced hundreds of miles and in some cases such as southern swell, several thousand miles The greatest rate of decay takes place over the first few hundred miles, after which the rate is not so great. However, if the swell advances into shallow water, the rate of decay may again increase due to dissipation of wave energy by bottom friction and percolation in the permeable sea bottom. Putman and Johnson (1949) have developed a dissipation function for bottom friction and Putman (1949) presents a dissipation function for percolation in a permeable sea bottom.

Using the above-mentioned dissipation functions, Bretschneider and Reid (1954) have obtained a number of solutions and present a number of nomographs for determining the change in wave height (or change in wave energy) due to bottom friction, percolation and refraction, for swell traveling over a shallow bottom. Since it is

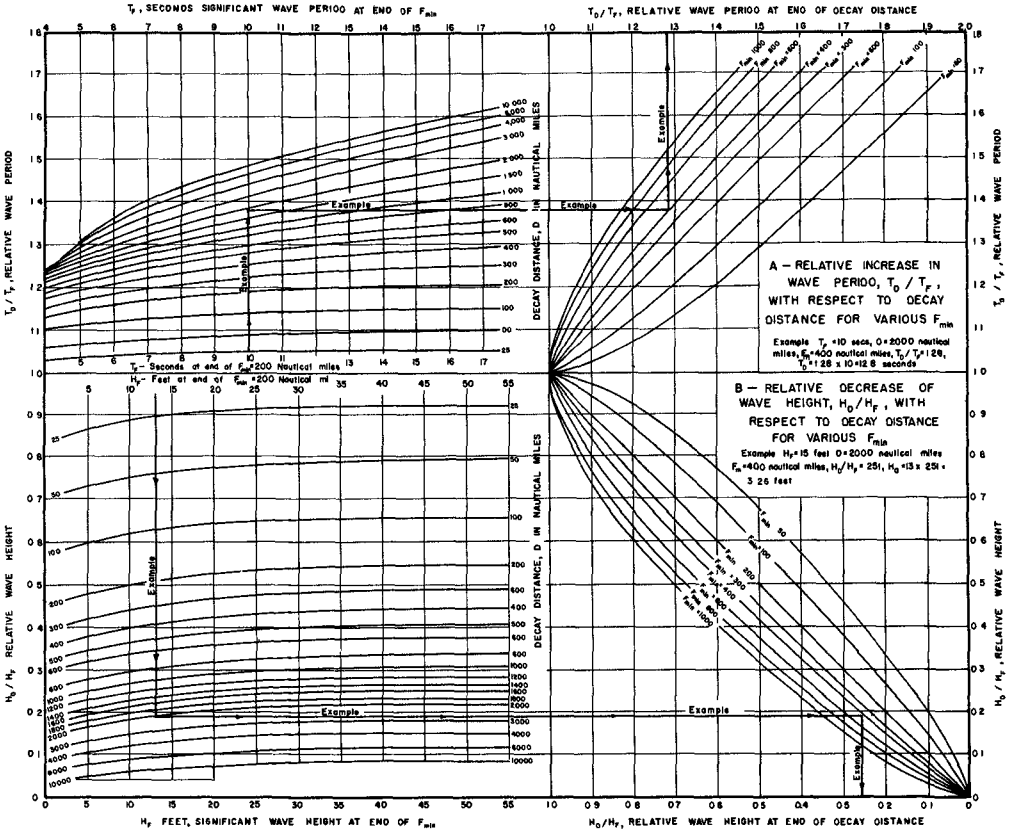

Fig. 6. Forecasting curves for wave decay.

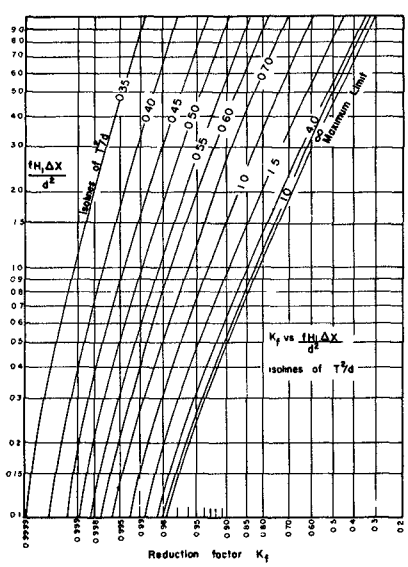

Fig. 7. Relationship for friction loss over a bottom of constant depth.

difficult to isolate the individual effects of percolation and bottom friction, an overall bottom friction factor can be used.

The actual height of swell traveling over an impermeable bottom, water of constant depth and no refraction is obtained from

$$H_{Ds} = K_f \, K_s \, H_{Do}, \text{ where}$$

H_{Do} is the deep water decayed height. K_f is the reduction factor to take into account wave energy loss due to bottom friction. K_f may be obtained by use of Figure 7. K_s is the shoaling coefficient and is given in terms of H_o/H_o', as a function of d/L_o in tables by Wiegel (1954). Figure 8 of the present paper gives K_s as a function of T^2/d.

At this point consider an example of a wave forecast, given:

U = 30 knots
F = 400 nautical miles
t_d = 36 hours, duration of wind
D_o = 600 miles in deep water
D_s = 50 miles decay in shallow water
d = 50 feet constant depth over the 50 miles in shallow water
f = .01 bottom friction factor

From Figure 2 for U = 30 knots, F = 400 nautical miles, t_d = 36 hours, read H_F = 17.0 feet and T_F = 12.1 seconds, F_{min} = 400 nautical miles. From Figure 6 read H_D/H_F = .43 and T_D/T_F = 1.22. Thus the decayed wave height and period at the end of the deep water section respectively, are

$$H_{Do} = 17.0 \times .43 = 7.3 \text{ feet}$$
$$T_D = 12.1 \times 1.22 = 14.8 \text{ seconds}$$

Over the last 50 miles further decay is possible by using D = 650 miles instead of 600 miles. However, this will be much smaller than the reduction due to bottom friction, and does not combine linearily. Now also, the significant period will shift back to the lower periods since waves having longer periods are first affected by bottom friction. At present this factor also will not be considered.

Figure 7 is used to obtain the reduction factor K_f .

Compute $$\frac{fH_1\Delta X}{d^2} = \frac{.01 \times 7.3 \times 50(6080)}{50^2} = 8.9$$

and $$\frac{T^2}{d} = \frac{(14.8)^2}{50} = 4.38$$

From Figure 7 read $K_f = .38$
From Figure 8 read $K_s = 1.05$

Thus, the 7.3-foot wave height, after traveling 50 miles over a shallow depth of 50 feet, will be

$$H_{Ds} = 7.3\ (.38)(1.05) = 2.9 \text{ feet.}$$

The general procedure, however, since the Continental Shelf is not a flat bottom, is to segment the traverse between two orthogonals, each segment assuming a mean water depth. In this manner refraction can also be taken into account. If more detailed information is required one might decay elements of the joint distribution or spectrum of waves and thereby obtain also the change in significant wave period. The primary purpose of the above discussion is to show that a numerical process is possible to obtain wave height reduction in shallow water due to bottom friction, percolation, and refraction.

FORECASTING WIND WAVES IN SHALLOW WATER

Less information is available on wind waves in shallow water than for deep water. This is true in regards to both theory and available data. The first information on this subject is given by Thijsse (1949), based on laboratory data. Additional data and relationships were brought forth by Dr. Garbis Keulegan of the National Bureau of Standards, although never published to the knowledge of the present author. The U. S. Corps of Engineers, Jacksonville District (1955), performed an extensive field investigation on wind, waves, and tides in Lake Okeechobee, Florida. Based on the hurricane wind and wave data from Lake Okeechobee, and some ordinary wind wave data from the shallow regions of the Gulf of Mexico, Bretschneider (1954) was able to establish a numerical procedure for computing wind waves in shallow water taking bottom friction into account. A friction factor of $f = .01$ appears satisfactory. Presently these techniques are used for the Continental Shelf, but may require further calibration when more wind and wave data are available.

GENERATION OF WIND WAVES OVER A BOTTOM OF CONSTANT DEPTH

If $d/T^2 < 2.5$ feet/sec^2, then the waves effectively "feel bottom" and the depth and bottom conditions enter as additional factors with respect to the heights and periods of waves which can be generated. The effect of frictional dissipation of energy at the bottom for such waves limits the rate of wave generation and also places an upper limit on the wave heights which can be generated by a given wind speed and fetch length.

The following expression for the reduction in height of waves traveling over an impermeable bottom of constant depth without refracti is obtained from Bretschneider and Reid (1954).

$$H = H_1 \left[\frac{f H_1 \phi_f \Delta X}{K_s T^4} + 1 \right]^{-1} \qquad (1$$

where H = the final height at X
H_1 = original height at $X = X_1$
$\Delta X = X - X_1$, the horizontal distance of wave travel in feet
f = friction factor (dimensionless), a characteristic of the bottom
T = wave period in seconds
K_s = shoaling factor, given as H/H_o' in tables by Wiegel (1954), also given in Figure 8

$$\phi_f = \frac{64\pi^3}{3g^2} \left[\frac{K_s}{\sinh 2\pi\, d/L} \right]^3 , \ \sec^4 \text{ ft}^{-2} \qquad (1$$

d = depth of water in feet

$L = g/2\pi T^2 \left(\tanh \frac{2\pi d}{L}\right)$, wave length in feet

g = acceleration of gravity in feet/sec^2

Equations 18 and 19 are based on consideration of waves of small steepness and therefore represent only an approximation for waves near the maximum steepness.

The solution of Equations 18 and 19 is given in Figure 7, where $K_f = H/H_1$.

Figure 1, the deep water wave forecasting relationships, in effect represents the generation of wave energy in deep water as a function of F, U, and t, since the energy is proportional to H^2; whereas Figure 7 represents the dissipation of wave energy due to bottom friction. Figure 1 and equation 18 were combined by a numerical method of successive approximation to obtain relationships for the generation of waves over an impermeable bottom of constant depth. Best agreement between wave data and the numerical method was obtained when a bottom friction factor f = .01 was selected. Perhaps a "calibration friction factor" is a more appropriate term, since it would take into account other influential factors not normally included in the friction factor term. Figures 9 and 10 are the results of these computations. Actually the curve of gT/U versus gd/U^2 is based on the wave data, whereas the curves of gH/U^2 versus gd/U^2 and gF/U^2 are based on the numerical computations. The curves of these figures are not too much different from those presented by Thijsse and Schijf (1949). Figure 11, based on Figure 9, gives wave forecasting curves for shallow water of constant

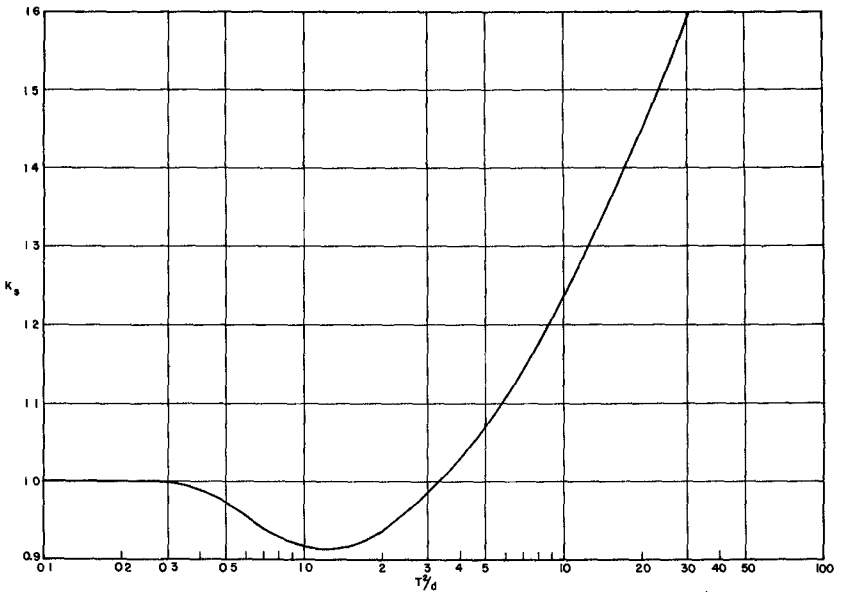

Fig. 8. Shoaling coefficient K_s vs T^2/d.

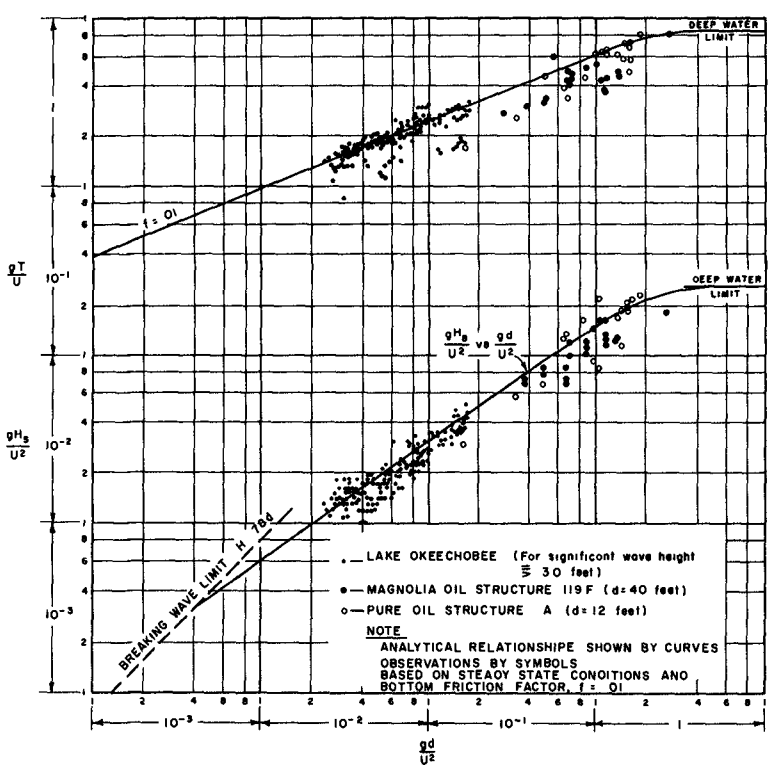

Fig. 9. Significant wave height and period as functions of constant water depth and wind speed.

depth, and unlimited wind duration and fetch length. Figure 10 may be used when both the fetch length as well as the depth are restricted.

The important fact from the above material, however, is the establishment of a numerical procedure for computing wind waves in shallow water of constant depth which can be verified by use of wave data. This procedure can be extended to a bottom of constant slope, wherein the bottom is segmented into elements, each element having a mean depth assumed to be constant.

FORECASTING WAVES OVER THE CONTINENTAL SHELF

In general, for any locations on the Continental Shelf, three special cases for wind-wave generation exist: (a) winds blowing parallel to the coast, (b) winds blowing from land to sea, and (c) winds blowing from sea to land. These are discussed below.

(a) Case I - Winds Blowing Parallel to the Coast - In this case, except where very irregular bottom topography exists, the best approach is to use the flat bottom relationships, Figures 9, 10, or 11, as the case may be. Wherever wave data are available, however, it is recommended that a calibration be made of the forecasting curves. When refraction becomes important, the numerical method must be used.

(b) Case II - Winds Blowing From Land to Sea - In a report by Bretschneider and Thompson (1955) it was shown that for most offshore winds, waves are generated which do not feel the bottom, at least for the Gulf of Mexico. This is probably true for other Continental Shelf areas, and the reason is that the fetch length, increasing seaward, is generally limiting. As the fetch length gets longer the wave period gets longer, but the water depth becomes greater. This would indicate that for offshore winds, one may use the deep-water forecasting curves, Figure 2. However, for the cases where the Continental Shelf is long and relatively flat, Figure 10 might be used, or perhaps the numerical method.

(c) Case III - Winds Blowing From Sea to Land - This is perhaps the most complex situation for wave generation, and no one set of generalized curves can be developed similar to those for Cases I and II. However, forecasting curves have been worked out by Bretschneider (1956) for various sections of the Gulf of Mexico. In general, each section or location has a different bottom profile leading shoreward from various directions. In some cases refraction must also be considered, and hence the numerical method must be used. The numerical method is calibrated by use of hurricane wind wave data from Lake Okeechobee, Florida and a limited amount of ordinary wind wave data from the Gulf of Mexico, and it appears that a bottom friction factor of $f = .01$ is also applicable to the Continental Shelf. It must be emphasized, however, that when more wave data become available for the Continental Shelf, a refined calibration of the method should be made.

In regards to wave generation by onshore winds, there are two conditions to consider. First, the initial deep-water waves generated may be propagated shoreward as swell under the continued influence of the generating winds; and second, regeneration of wind waves is

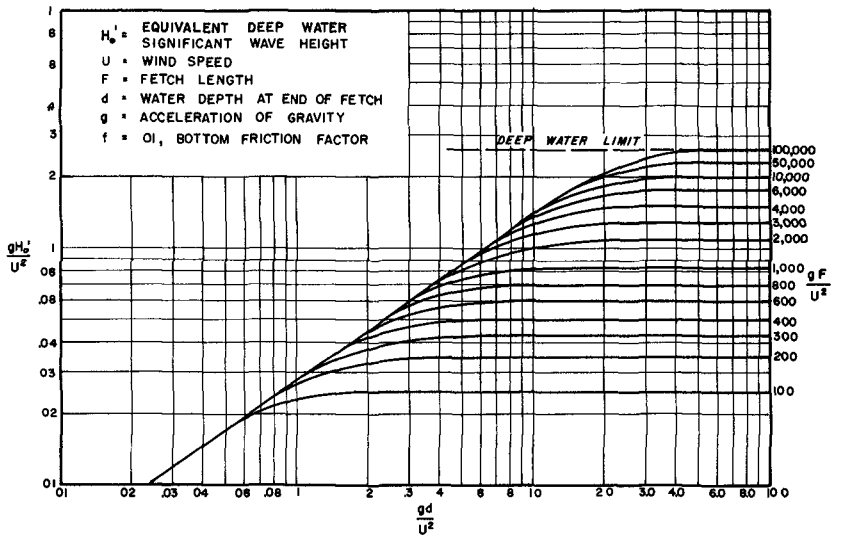

Fig. 10. Generation of wind waves over a bottom of constant depth for unlimited wind duration represented as dimensionless parameters.

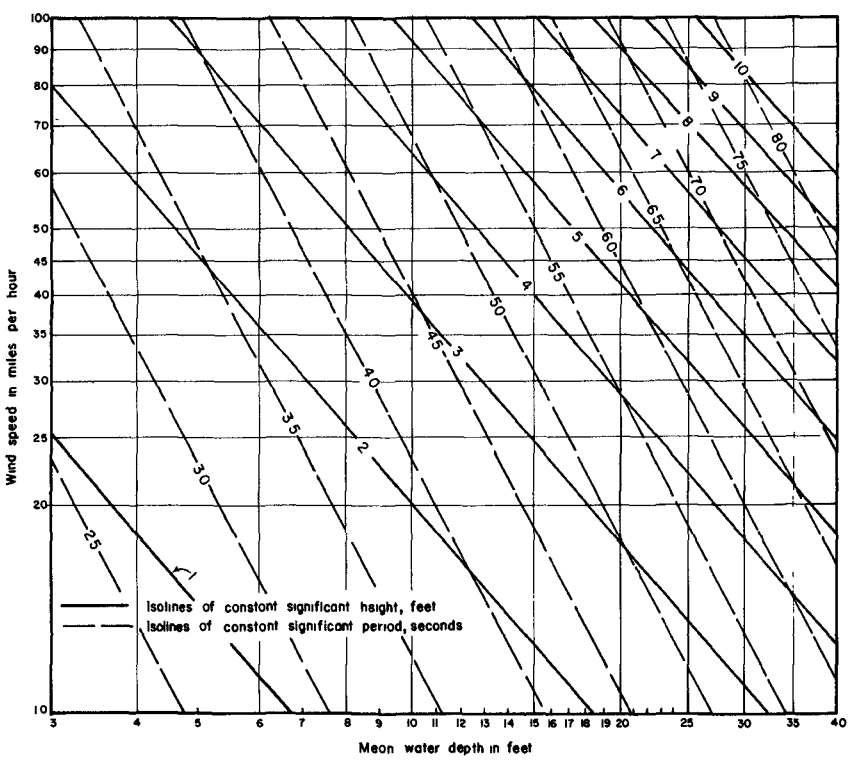

Fig. 11. Wave forecasting relationships for shallow water of constant depth.

constantly taking place all along the fetch over the Continental Shel Swell will feel bottom far from shore and commence losing energy at a early stage, whereas wind waves with shorter periods continue to grow and do not feel bottom until they are sufficiently large and are near ing the coast. In deep water one would observe the largest wave heig and longest periods, and in the shallow water both the significant wi wave heights and periods will have decreased, although a presence of swell might be noted. In the breaker zone one would observe both swe and wind waves. Hence, the shift of significant waves in the spectru will be toward the lower periods, opposite to that for deep water dec where the shift was toward the longer periods.

Steps in numerical procedure for computing wind waves generated up the Continental Shelf from deep water shoreward are as follows:

(a) A wind speed is selected, and a graph of H_o and T versus fetch length is computed from Figure 2.

(b) The minimum fetch length F_{min} is selected correspondin to the wind speed, actual fetch length and minimum duration. H_o and ' are determined at $F = F_{min}$. The deep water wave length is computed fro $L_o = 5.12\ T^2$ and the waves will begin to feel bottom at a depth $d = L_o/2$. This is the initial point from which to begin computations.

(c) The bottom profile along the fetch toward the location of interest is determined. The traverse is segmented into at least 10 to 15 equal increments ΔF, of about 10 to 5 miles or less in leng each, depending on the bottom slope and width of the Continental Shelf. Figure 12 is a schematic diagram illustrating the procedure.

(d) An average depth, d_{ave}*, is determined over each increment (Figure 12).

(e) A deep water wave height, H_o, and wave period, T, is determined at the beginning of the first increment of ΔF using deep water relationships.

(f) This value of H_o is then assumed to travel over the increments ΔF as swell, taking bottom friction into account. This is done by use of Figure 7. The quantity $\frac{fH\Delta X}{d^2}$ is determined, using

*It is assumed that wind set-up has been computed and is included in the depth. For high winds and shallow water, the wind set-up or storm surge must first be computed before wave computations begin.

bottom friction factor ($f = .01$); $H_1 = K_r K_s H_o$, where K_s is the shoaling coefficient obtained from Figure 8; K_r is the refraction coefficient over the increment ΔF; $\Delta X = 6080 \Delta F$, where ΔF is in nautical miles; $d = d_{ave}$, average total water depth over the increment ΔF; and T^2/d is computed using the average significant period over the increment and the average depth, d_{ave}. K_f is read from Figure 7 and the actual significant height at the end of the increment ΔF is equal to $H_s = H_o K_f K_s K_r$.

(g) An equivalent deep-water wave height H_o' is obtained from

$$H_o' = K_f H_o \, . \tag{20}$$

(h) Using H_o' and Figure 2 an equivalent deep water fetch length F_e' is obtained. For the case of regeneration of wind waves one also obtains an equivalent deep-water period, T_o.

(i) An equivalent deep-water wave height is determined at the end of the second increment for $F = F_e' + \Delta F \leq F_{min}$, using Figure 2. For the case of regeneration of wind waves one also obtains an equivalent deep-water period.

(j) With the average wave height $1/2(H_o' + H_{o2}')$ steps f, g, h, and i are repeated. (This gives the swell height when the wave period T is held constant; T is given at the beginning of the first increment for U at $F = F_{min}$). Using the average of the periods $1/2(T_{o1} + T_{o2})$ steps f, g, h, and i are repeated. This gives the regenerated wind-wave height. The above procedure is used for all except the last increment or until the waves break, whichever occurs first. The last increment of ΔF cannot be treated by the above method since here the bottom slope increases too rapidly and the surf zone is experienced. The procedure can be used for depths from deep water to about 20 feet, but has been applied up to depths of 12 feet, when the winds are not too great. Figure 13 is a typical example of wind-wave forecast for a 26-knot wind. Note, the last increment must be treated as surf.

The above procedure can also be set up on a high speed computer, and for a particular area one could determine a family (or families) of forecasting curves similar to Figure 13. This would be desirable, once sufficient data are available for a refined calibration.

If the wind is variable in speed and direction, the graphical method of forecasting waves in deep water by Wilson (1955) might be extended to include shallow water computations. This can become quite involved, but could be programmed on a high speed computer.

FORECASTING WAVES GENERATED BY HURRICANES

The problem of forecasting hurricane waves in deep water is somewhat handicapped by lack of adequate hurricane wind and wave data. In this respect the Japanese (Orakuwa and Suda 1953 and Unoki and Nakano 1955), and others have been doing a great deal of work on the study of

winds, waves and swell in typhoons, which are somewhat similar to the hurricanes.

The most satisfactory tool for predicting hurricane waves is the graphical method by Wilson (1955). This method has been used by Wilso (1957) to compile hurricane statistics in deep water for the Gulf of Mexico.

Three of the most important fundamental differences between gener tion of wind waves under hurricane conditions and that of normal wind waves are as follows: (a) winds within a hurricane are not constant in speed; (b) winds within a hurricane are circular in direction as oppos to straight line; and (c) the hurricane moves over waves generated at various angles of direction to the path of the storm. This fact may cause pyramidal waves formed by two trains approaching at a wide angle each other.

Figure 14, a typical wind field for a standard project hurricane off the Texas Gulf coast, was constructed by the Hydrometeorological Se the U. S. Weather Bureau (1957). The wind field will be slightly different for a similar hurricane off the east coast of the United Stat The fact that the wind speed varies in direction and speed poses no difficult problem, since in equations 10 and 11 one may replace U^2F by

$$U^2F = \int_{x_1}^{x_2} U_x^2 \, dx \qquad ($$

U_x is the component of wind in some arbitrary straight line direction. In case of a moving hurricane the space-time distributions $(U_x^2)t$ can be used. The integral can be evaluated numerically. The graphical method of Wilson (1955) can also be used, in which case a space-time wind field is determined. Figure 4 from Wilson (1955) shows a comparison between the numerical and the graphical methods.

It is interesting to note that the maximum value of the significant height is slightly afront the peak wind. If the hurricane moves at a moderate speed, steady state may not necessarily exist, and the peak height and peak wind speed may coincide.

There are occasions when a general knowledge of hurricane waves is important, and little time is available to perform the work involved in the graphical approach by Wilson (1955). The following material is presented to obtain significant waves within a hurricane for a slowly moving model or standard project hurricane in the Gulf of Mexico, such as might be used for design purposes.

By taking various cross sections of a hurricane wind field one may obtain wind distributions, similar to that given in Figure 4. Applying the numerical formula 20 and equation 10, one obtains the significant height distribution. This could also have been done by Wilson's (1955) graphical method. Based on a few theoretical model or standard project hurricanes moving at a slow to moderate speed in the Gulf of Mexico, wave distributions were computed similar to that shown in Figure 4. It was found that if these distributions were expressed in the dimensionless form $H\ 1/3/(H\ 1/3)_{max}$ versus r/R, the

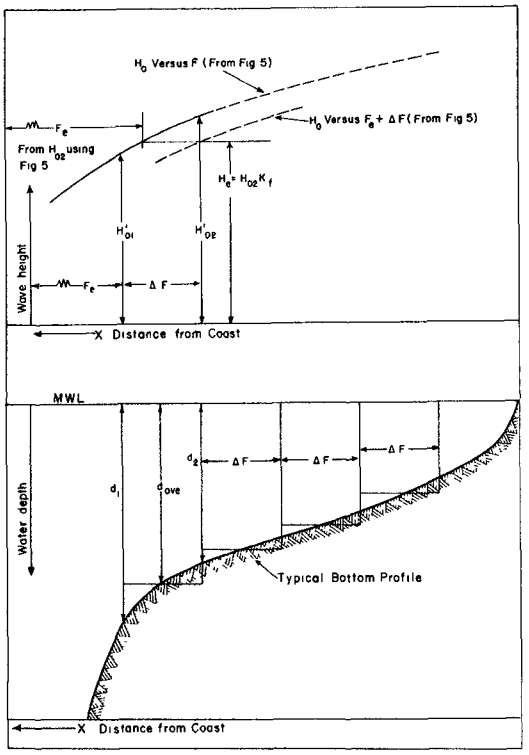

Fig. 12. Schematic diagrams illustrating procedure for computing wind waves in shallow water.

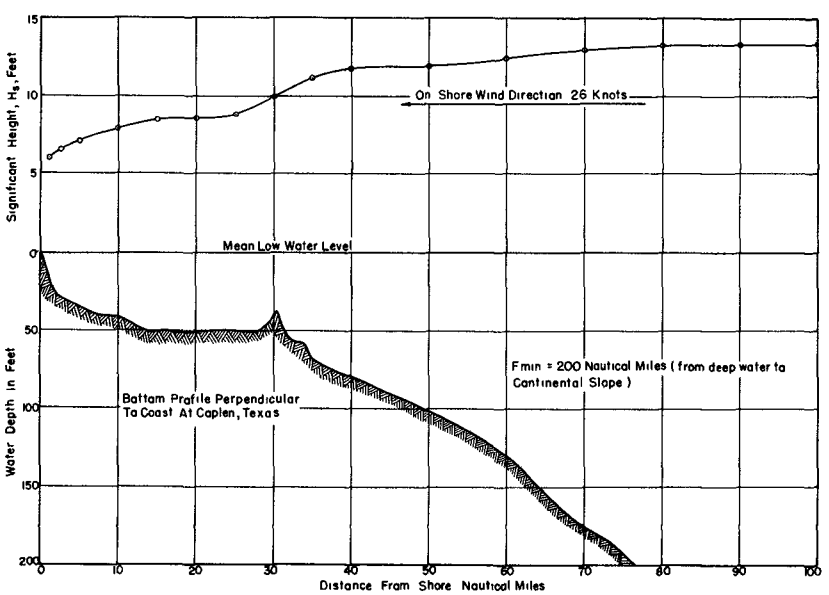

Fig. 13. Wind-wave forecast for a 26-knot wind blowing perpendicular to coast at Caplen, Texas.

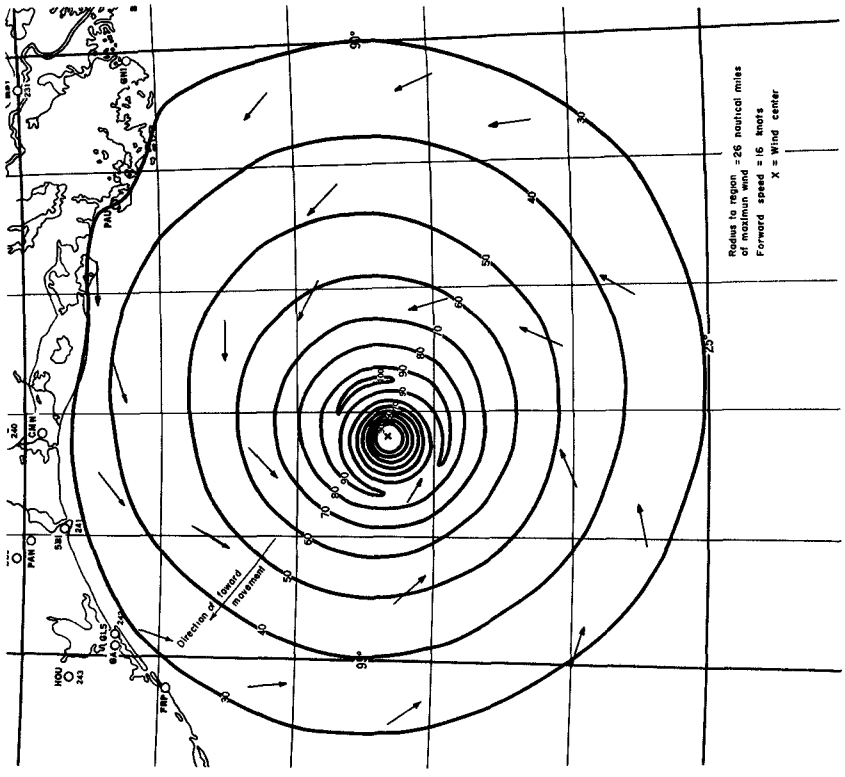

Fig. 14. Wind speed (MPH) at 30 feet over water for standard project hurricane, Galveston, Texas area.

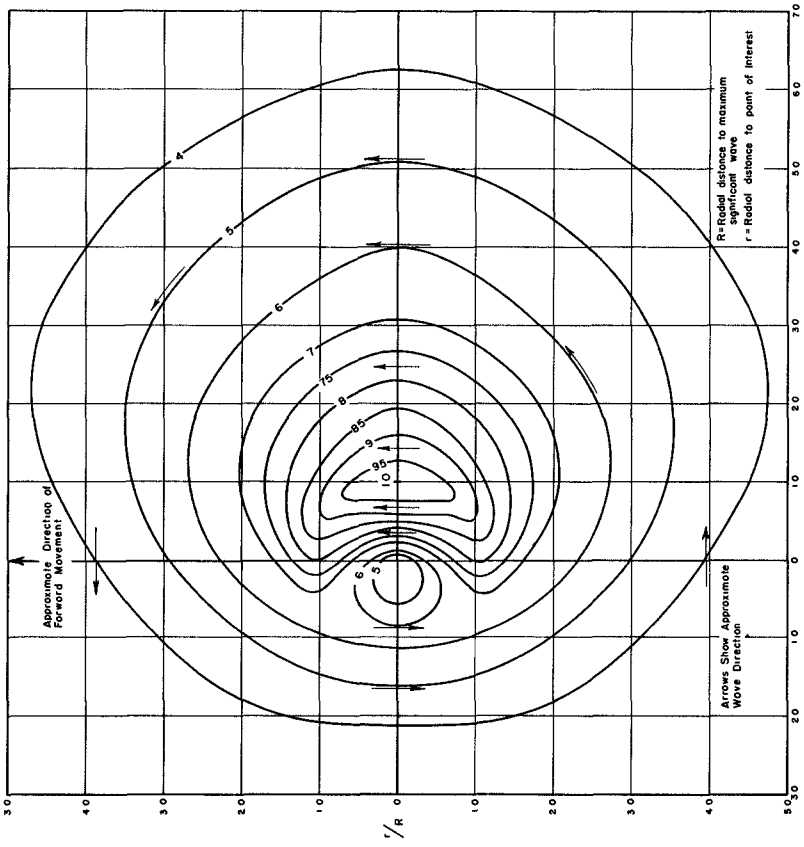

Fig. 15. Isolines of relative significant wave height for slowly moving hurricane.

results of the hurricanes investigated could be represented by a single family of curves. $H_{1/3}$ is the significant height at any position in the hurricane and $(H_{1/3})_{max}$ is the maximum value of the significant height in the hurricane. R is the radial distance to maximum wind and r the radial distance to any coordinate in the wave field. The results of these computations are given in Figure 15. Deviations from heights obtained by use of Figure 15 are less than ±5 percent obtained by the numerical method, and hence are within the same degree of accuracy as might be obtained by the graphical method of Wilson (1955).

In the use of Figure 15 it is only necessary to predict the maximum value of the significant height which might be generated by a design hurricane. A reasonable estimate of the significant period may be obtained from

$$T = \sqrt{H/.22} \tag{22}$$

Based on the analysis of thirteen east coast hurricanes, Bretschneider (1957) obtained a simple formula for obtaining the maximum value of the deep water significant height and period that might be generated by a hurricane under steady state conditions. These formulae are as follows:

$$H_o = 16.5\, e^{R\Delta P/100} \left[1 + \frac{\alpha .208\, V_F}{\sqrt{U_R}} \right] \tag{23}$$

$$T_s = 8.6\, e^{R\Delta P/200} \left[1 + \frac{\alpha .104\, V_F}{\sqrt{U_R}} \right], \text{ where} \tag{24}$$

$H_o = (H_{1/3})_{max}$ = maximum value of the significant height in feet

T_s = period of significant wave, seconds

R = radius of maximum wind, nautical miles

ΔP = atmospheric pressure reduction from center of hurricane in inches of mercury

V_F = forward speed of hurricane

α = percent effectiveness of V_F to be added to the wind field of a stationary hurricane to obtain the wind field of the moving hurricane. For slowly moving hurricanes, $\alpha = 1.0$

U_R = maximum wind in knots at R for stationary hurricane

TABLE II

JOINT DISTRIBUTION OF H AND T FOR ZERO CORRELATION
Number of waves per 1,000 consecutive waves for various ranges in hei and period.

Range in Relative Height	RANGE IN RELATIVE PERIOD $T/\bar{T}$											
$H/\bar{H}$	0-.2	.2-.4	.4-.6	.6-.8	.8-1.0	1.0-1.2	1.2-1.4	1.4-1.6	1.6-1.8	1.8-2.0	0-2.0	Accumulat
0- .2	.03	.50	2.05	4.86	7.68	8.09	5.31	1.92	.34	.03	30.81	30.81
.2- .4	.10	1.41	5.81	13.78	21.76	23.92	15.05	5.14	.98	.07	88.32	119.13
.4- .6	.14	2.06	8.54	20.23	31.95	33.65	22.10	7.99	1.44	.11	128.21	247.34
.6- .8	.16	2.40	9.91	23.48	37.08	39.06	25.65	9.27	1.67	.12	148.80	396.14
.8-1.0	.16	2.40	9.92	23.51	37.13	39.11	25.69	9.28	1.67	.12	148.99	545.13
1.1-1.2	.15	2.14	8.87	21.02	33.19	34.97	22.96	8.30	1.49	.11	133.20	678.33
1.2-1.4	.12	1.74	7.21	17.07	26.96	28.40	18.65	6.74	1.21	.09	108.19	786.52
1.4-1.6	.09	1.30	5.37	12.72	20.09	21.16	13.90	5.02	.90	.07	80.62	867.14
1.6-1.8	.06	.90	3.72	8.82	13.93	14.67	9.64	3.48	.63	.05	55.90	923.04
1.8-2.0	.03	.48	1.99	4.72	7.45	7.85	5.15	1.86	.33	.03	29.89	952.93
2.0-2.2	.03	.42	1.72	4.09	6.45	6.80	4.47	1.61	.29	.02	25.90	978.83
2.2-2.4	.01	.18	.76	1.80	2.84	2.99	1.97	.71	.13	.01	11.40	990.23
2.4-2.6	.01	.09	.39	.93	1.47	1.55	1.02	.37	.07		5.90	996.13
2.6-2.8		.04	.18	.43	.67	.71	.47	.17	.03		2.70	998.83
2.8-3.0												
0-3.0	1.09	16.06	66.44	157.46	248.65	262.93	172.03	62.16	11.18	.83		
Accumulative	1.09	17.15	83.59	241.05	489.70	752.63	924.66	986.82	998.00	998.83		

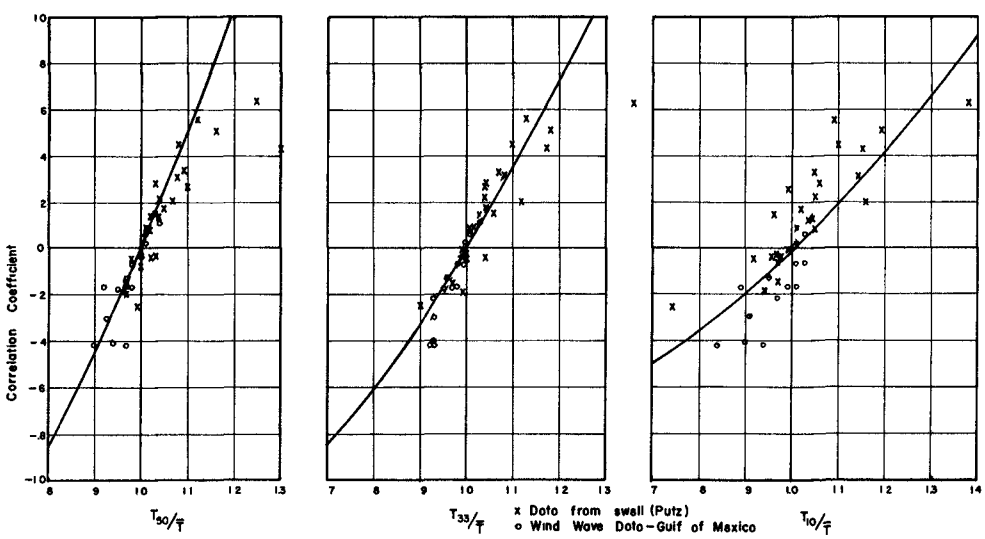

Fig. 16. Period ratio of highest 50 percent, 33 percent, and 10 perce wave height versus correlation coefficient.

WAVE VARIABILITY

Although one forecasts the significant height and period, it may be desirable to predict the distribution of heights and also the periods. The work of Putz (1952) and that of Longuet-Higgins (1952) are quite useful in this respect. It was shown by Bretschneider (1957) that the distribution of wave periods squarred as well as the wave heights can be represented very closely by the Rayleigh type distribution, as utilized by Longuet-Higgins (1952) for wave heights. The accumulative form of the distribution function for heights is given by

$$P\left[H\right] = 1 - e^{-\pi/4\left[\frac{H}{\bar{H}}\right]^2}, \text{ where} \tag{25}$$

$\bar{H} = .625\ H_{1/3}$, and may be obtained from forecasting relationships presented before.

The accumulative form of the distribution of periods is given by

$$P\left[T\right] = 1 - e^{-.675\left[\frac{T}{\bar{T}}\right]^4}, \text{ where} \tag{26}$$

$\bar{T}$ may be obtained from Figure 1-A.

Equations 25 and 26 were found to apply approximately for the swell as well as wind waves in deep or shallow water. In case of very long swell, however, agreement is not always satisfactory, except when a long record is used.

When $g\bar{T}/2\pi U = gT_{1/3}/2\pi U$, according to Figure 1-A, the stage of generation is in zero correlation with respect to H and T^2. In this case the joint distribution function is the direct product of the marginal distributions, whence

$$P\left[H,T\right] = P\left[H\right] \cdot P\left[T\right]. \tag{27}$$

Table II presents the number of waves per 1,000 consecutive waves that may be expected to fall within various ranges of heights and periods. This is for zero correlation only.

It can be seen from Figure 1-A, if the relationship of $g\bar{T}/2\pi U$ is approximately correct, that zero or near zero correlation begins at a moderate stage of generation and persists to the limit of the fully developed sea, and equation 27 is quite applicable. However, when $\bar{T} \neq T_{1/3}$ then correlation exists and equation 27 does not apply,

except perhaps approximately for low correlations. As mentioned earlier, the equation given by Bretschneider (1957) for the joint distribution is not correct, except for the special case of zero correlation, and therefore should be used accordingly.

For decayed waves, the marginal distribution functions, equations 25 and 26, still apply, at least approximately, since they are in close agreement with the relationships given by Putz (1952) based on the analysis of 25 records of ocean swell. As soon as waves begin to decay the correlation rotates to positive values of the correlation coefficients, the larger the decay distance the larger the positive correlation. Hence, equation 27 does not apply, and thi phase of the problem has not been established to date.

WAVE SPECTRA

Although the wave spectra is not intended to be part of this paper, it seems appropriate to include results of some recent studies on which certain revisions in this paper are based. When, and only when $\bar{T} = T_{1/3}$, the period spectrum is given by Bretschneider (1958) according to

$$S_{H^2(T)} = \frac{\alpha\, g^2 T^3}{(2\pi)^4}\, e^{-.675\left[\frac{T}{\bar{T}}\right]^4} \tag{28}$$

The corresponding frequency spectrum is given by

$$S_{H^2(\omega)} = \alpha g^2 \omega^{-5}\, e^{-.675\left[\frac{g}{\bar{T}\omega}\right]^4} \quad \text{, where} \tag{29}$$

$$\omega = \frac{2\pi}{T}\,,\ \frac{g\bar{T}}{2\pi U} = f\left[\frac{gF}{U^2}\,,\ \frac{gt}{U}\right] \quad \text{, and}$$

$$\alpha = 3.437\left[\frac{g\bar{H}}{U^2}\right]^2\left[\frac{2\pi U}{g\bar{T}}\right]^4 = 16\pi^2\left[\frac{\bar{H}}{\bar{L}}\right]^2 \tag{30}$$

The above relationships evolve directly from the joint distribution function for wind generated waves when zero correlation exists. For large ω equation 29 becomes

$S_{H^2(\omega)} = \alpha\, g^2 \omega^{-5}$, which is exactly that form given by Burling (1955) based on very accurate measurements. The form of equation 31 has also been proven by Phillips (1957) from an entirely different approach by use of the definition of the energy spectrum and dimensional analysis, a priori reasoning. (31)

For the case of a fully developed sea

$$\frac{g\bar{T}}{2\pi U} = 1.95 \text{ and } \frac{g\bar{H}}{U^2} = .625 \left[.282\right] = .172, \text{ according to}$$

Figure 1-A, from which one obtains the minimum value of

$$\alpha_{min} = 7.4 \times 10^{-3}, \text{ which is in agreement} \tag{32}$$

with the value reported by Burling (1955). Actually, the value of 7.4×10^{-3} was used together with

$g\bar{T}/_{2\pi U} = 1.95$ obtained from the period spectrum to arrive at the value of $g\bar{H}/_{U^2} = .172$, corresponding to $gH_{1/3}/_{U^2} = .282$.

The mean wave steepness for zero correlation evolves from the joint distribution function according to

$$\overline{\left[\frac{H}{L}\right]} = \frac{\pi}{2}\,\frac{\bar{H}}{\bar{L}} \tag{33}$$

The mean square sea surface slope σ^2 for zero correlation evolves from the period spectrum according to

$$\sigma^2 = \alpha\left[\ln\frac{g\bar{T}}{2\pi U} - \ln\frac{g\,T_{min}}{2\pi U} - .0671\right] \tag{34}$$

If the spectrum is composed of waves generated from the lowest value of $gF/_{U^2}$ to the value of $gF/_{U^2}$ as imposed by F, t, and U, one might assume that $\frac{g\,T_{min}}{2\pi U} = .0193$ and $g\bar{T}/_{2\pi U}$ a function of $gF/_{U^2}$. This would result in a maximized value of σ^2 according to

$$\sigma^2 = \alpha\left[\ln\frac{g\bar{T}}{2\pi U} + 3.88\right] \tag{35}$$

The value of $\frac{g\,T_{min}}{2\pi U} = .0193$ corresponds to the lowest possible period in the spectrum that might be generated by a wind speed equal to the critical wind speed of 6 meters per second, assuming this period is equal to .074 seconds as governed by the capillary limit. From actual measurement, however, using measuring instruments which

attenuate a portion of the high frequency components, $\frac{g\,T_{min}}{2\pi\,U}$ may be considerably larger, in which case σ^2 measured will be lower than tha given by the above equation.

The non-dimensional spectral width parameter, ϵ , as defined by Williams and Cartwright (1957) evolves from the spectrum according to:

$$\epsilon = \sqrt{1 - \frac{\pi}{4}\frac{\alpha}{\sigma^2}} \qquad (3$$

Since α and $\frac{g\bar{T}}{2\pi U}$ are both functions of $gF/_{U^2}$, the mean square sea surface slope σ^2 and the spectral width parameter ϵ are functions of $gF/_{U^2}$. From actual measurements if the high frequency components are attenuated ϵ will be measured less than that predicted by use of equation 36.

Equation 28 for the period spectrum or equation 29 for the frequency spectrum can be used together with the forecasting relationsh to obtain the corresponding wave spectra for all cases of generation where $\frac{g\bar{T}}{2\pi U} = \frac{gT_{1/3}}{2\pi U}$.

When $T_{1/3} \neq \bar{T}$ then the correlation parameter enters the problem. Where r is the correlation coefficient between H and T^2, the following equations are presented by Bretschneider (1958):

Period Spectrum

$$S_{H^2}(T) = \frac{\alpha\, g^2 T^3}{(2\pi)^4} \frac{\left[1 - r + .925r^2\left(\frac{T}{\bar{T}}\right)^2\right]^2}{1 + .273r^2}\, e^{-.675\left[\frac{T}{\bar{T}}\right]^4} \qquad (37$$

Frequency spectrum

$$S_{H^2}(\omega) = \alpha\, g^2 \omega^{-5} \frac{\left[1 - r + .925r^2\left(\frac{2\pi}{\bar{T}\omega}\right)^2\right]^2}{1 + .273r^2}\, e^{-.675\left[\frac{2\pi}{\bar{T}\omega}\right]^4} \qquad (38$$

Mean sea steepness

$$\overline{\left[\frac{H}{L}\right]} = \frac{\pi}{2}\,\frac{\bar{H}}{\bar{L}}\left[1 - r\left(1 - \frac{2}{\pi}\right)\right] \qquad (39$$

Mean square sea surface slope

$$\sigma^2 = \frac{a}{1 + .273r^2}\left[(1 - r)^2\left(\ln \frac{g\bar{T}}{2\pi U} - \ln \frac{gT_{min}}{2\pi U} - .0671\right) + r - \left(\frac{\pi - 1}{\pi}\right)r^2\right] \quad (40)$$

Spectral width parameter

$$\epsilon = \sqrt{1 - \frac{a}{\pi\,\sigma^2}\left[\frac{.5708\,(1 - r)^2 + 1}{1 + .273r^2}\right]^2} \quad (41)$$

Ratio of significant period to mean wave period

$$T_{1/3}/\bar{T} = \sqrt{1 + .6r} \quad (42)$$

Ratio of mean period of the highest p- percent waves to the mean wave period

$$T_p/\bar{T} = \sqrt{(1 - r) + r\,\frac{\eta e^{-\pi/4\,\eta^2} + 1 - \Phi_p}{e^{-\pi/4\,\eta^2}}} \quad (43)$$

where $\eta = \frac{H}{\bar{H}}$

$$\Phi = \frac{2}{\sqrt{\pi}}\int_0^x e^{-u^2}\,du \quad (44)$$

$$u^2 = \pi/4\,\eta^2$$

Figure 16 shows the comparison between theory and wave data for the ratio of the significant period (T_{33}) to the mean wave period $\bar{T}$. Ratio of the mean period of the highest 50 percent waves to the mean wave period and the highest 10 percent waves to the mean wave period are also given.

All of the above equations will result in answers to the same degree of accuracy as the forecasting curves if used for predictions. However, if measured values of $\bar{H}$, $\bar{T}$, and r between H and T^2 are used, quite satisfactory results should be obtained, assuming the record is sufficiently long and representative.

SUMMARY AND CONCLUSIONS

The present paper presents the latest revisions in wave forecasting based on the significant wave method. It is emphasized that the significant period as well as the significant height has definite significance. Three special classes of wave forecasting are discussed: (a) forecasting deep water wind waves and swell, (b) forecasting shallow water wind waves, and (c) forecasting hurricane waves. It is recommended that in using the techniques discussed, that wind and wave data, where available, be used to improve one's technique as well as a possible refinement in calibration. There are certain conditions under which one might use the wave spectra, and a summary of useful formulae are presented. The greatest hope for future revisions in either the significant wave method or the spectrum method rests with the procurement of more and better data and the utilization of the graphical method for forecasting waves.

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