SIMULATING DEMAND IN AN INDEPENDENT-ACROSS-FIRMS MANAGEMENT GAME

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ABSTRACT

This paper proposes that the demand function of business simulations, especially those of independent-across-firms design, should be composed of equations and algorithms simple enough for players to deduce the parameters from observing results. A set of tested equations and algorithms are discussed accounting for eight basic concepts: trend, stages, randomness, limits, seasonality, pricing, transient effects, and cumulative effects. Combining these simple equations and algorithms results in a demand function of apparent complexity, but because simplicity underlies the complexity, players are faced with the scientific challenge of discerning the underlying simplicity from apparent complexity.

INTRODUCTION

Since Goodsen (1981) expressed his concern that relatively little had been written that provided “enough information to help the novice designer develop business simulations in an efficient manner’ (p. 41), several papers have been presented discussing mathematical functions for simulating market demand. Pray and Gold (1982) have shown that the demand function for some published simulations are unstable, and Gold and Pray (1983, 1984) have proposed a set of equations that assure stability and have an easily-located inflection point. Frazer (1983) has proposed a simple linear model relating price to demand, and has shown how the complex concept of locating the optimal price through calculus can be taught with a simple model. Others, however, tended to favor complex models. Teach (1984) has proposed a gravity-flow model to account for differences in non-price attributes between products; Decker, LaBarre, and Adler (1987) have proposed two exponential logarithmic functions to account for price and non-price variables; and Golden (1987) has proposed an heuristic algorithm for simulations of service industries. Goodsen (1986), eschewing mathematical models, has shown that results similar to any mathematical model can be obtained by graphing the desired relationships, constructing a schedule of selected points from the graph, and interpolating for values between selected points.

The appropriate means of simulating demand necessarily depends largely on the nature of the simulation. Business simulations are of two kinds: dependent- across-firms and independent-across-firms. In simulations that are dependent-across-firms, the demand available to a firm is affected by the decisions of competitive firms. All of the total enterprise business games reviewed by Keys (1987a, 1987b) were of this kind, and Gold and Pray’s (1983, 1984) work on a robust demand system concerned simulations of this kind.

Simulations that are independent-across-firms have tended to be overlooked. Pray and Gold (1982) referred to these simulations as not interactive. Biggs (1987), noting the ambiguity in the term <u>interactive</u>, which could refer either to the relationship between player and machine or to the relationship among players, elected to use the term <u>noncompetitive</u>, but failed to note that this term is also ambiguous for it can refer to the nature of the market or to the nature of the relationship between players. This relationship depends on how the instructor chooses to grade the game rather than on the nature of the game itself, for a grading scheme based on comparative performance will tend to create competitive relationships, whereas a grading scheme based on aggregate performance will tend to create cooperative relationships. Chiesl (1985) referred to simulations that are independent across firms as being of the Monte-Carlo approach, a term that is unfortunately misleading because the randomness the term suggests can be present in, or absent from, both kinds of simulations. He proceeded, nevertheless, to discuss three advantages of independent-across-firms simulations, namely, they reduce time delays, they eliminate most hardware problems, and they raise the authenticity of the simulation.

In simulations that are independent across firms, the demand available to a firm is not dependent on the decisions of other firms. Thus, the results of a firm’s decisions can be calculated immediately. This quickens the pace so that whereas in a game of dependent-across-firms design, players might be expected to make one or two sets of decisions a week; in a game of independent-across-firms design, twelve to twenty-four decisions a week would be equivalently reasonable. At this rapid pace, players quickly accumulate data sufficient to deduce the parameters of the demand function, if the function had been designed simply enough to permit this deduction.

The possibility thus exists in a simulation of independent-across-firms design to teach students how to deduce simplicity from apparent complexity, how, in short, to be a scientist. This possibility for inducing a significant kind of learning depends, however, on the demand being modeled by simple functional forms put together such as to display apparently complex, but not bizarre, results. This paper puts forth a set of equations and algorithms to meet the requirement.

EQUATIONS AND ALGORITHMS

The demand function discussed below accounts for a number of basic concepts: trend, stages, randomness, limits, seasonality, pricing, transient effects, and cumulative effects. Trend refers to the dependence of demand on time; stages, to its dependence on events; randomness, to its partial unpredictability; limits, to its maximum and minimum levels; seasonality, to its regular ups and downs; pricing, to its dependence on price; transient effects, to its temporary response to change; and cumulative effects, to its lasting response to presence. These concepts are captured by a set of simple equations and algorithms, and the results combined to form the demand function, one that has been tested in MANAGEMENT 500, an independent-across-firms game formerly called ANOTHER CHANCE (Thavikulwat, 1986).

Trend

A trend that may be increasing, decreasing, or level can be captured by the simple linear equation:

$$Q_0 = A + Bt \qquad (1)$$

where

Q_0 = quantity demanded

A = intercept parameter

B = slope parameter

t = time

This equation models a trend in a way that is simple and effective.

Stages

A simulation that will be played for many periods should provide for stages between which the parameters of the demand function may change. Such changes between stages may mimic a product life cycle or an exogenous alteration of the economic climate. Such changed might also be induced solely for pedagogical reasons, such as to challenge the ability of players to adapt to the change. Changes accompanying a new stage may be triggered either upon the game reaching a certain period, or upon a certain level of accumulated earnings, or upon a certain value of the firm's total assets, or upon some combination of these. But irrespective of the rule used to trigger the change, the demand function should be continuous across stages.

The continuity of the demand function is assured if a change in the slope of the trend in the demand is accompanied by a recalculation of the intercept such that the trend line prior to the change and the trend line subsequent to it intersect in the period of change. Thus, if the trend line at the earlier stage is represented by the equation:

$$Q_0 = A' + B't \qquad (2)$$

then, to preserve continuity, the trend line at the succeeding stage must be represented by the equation:

$$Q_0 = A' + (B' - B'')t^* + B''t \qquad (3)$$

where

Q_0 = quantity demanded

A' = intercept parameter, former stage

B' = slope parameter, former stage

B'' = slope parameter, latter stage

t = time

t^* = time of change

When the trend in demand is continuous, as assured by Equation 3, the change in stage will not be unnaturally aberrant.

Randomness

Randomness can be included in the trend by adding a random variable to the trend, as follows:

$$Q_1 = Q_0 + e \qquad (4)$$

where

Q_1 = quantity demanded, with randomness

e = a random variable of zero mean

A subtle problem arises, however, in programming a computer to supply this random variable. Moat pseudo-random number generators available with commonly-used compilers and interpreters will only supply numbers of uniform distribution bounded by zero and one. The bounding is a trivial problem for the numbers can be easily re-scaled to a mean of zero by subtracting the mean, 0.5, from each number. The uniformity of the distribution, however, is unnatural.

A number of sophisticated algorithms (Knuth, 1969) will convert a uniform distribution to a more natural normal distribution. Stubbs (1986), however, has suggested this simple method:

$$e = (s / 2^{\frac{1}{2}}) \ln(r_1 / r_2) \qquad (5)$$

where

s = standard deviation

r_1, r_2 = two pseudo-random numbers of a uniform distribution bounded by zero and one

Another simple method relies on the Central Limit Theorem, and is as follows:

$$e = s\,(12n)^{\frac{1}{2}}\,(1 / n)\,\Sigma\,(r_i - 0.5) \qquad (6)$$

where

n = number of pseudo-random numbers summed

Essentially, this second method invnives re-scaling the uniformly-distributed numbers to a mean of zero, averaging n of these re-scaled numbers, and adjusting for the standard deviation of the final distribution. Hanson (1985) gives results of this method implemented in a Basic program, and Latour (1986) compares the results with those of the more sophisticated polar method.

Although both methods discussed above are suitable, the second allows for more precise control over the shape of the distribution (the larger the n, the more normal the distribution) and is the method chosen for the game, MANAGEMENT 500.

Limits

Demand should not be allowed to increase or decrease without limit. This is especially important if the game will run for an indefinite number of periods, and if the slope of the trend is large. A ceiling should be placed on the demand, and also a floor, which might be zero or a number greater than zero but less than the ceiling. The following algorithm will direct demand to bounce off the ceiling and the floor:

```
IF (Q1 < floor) THEN
     Q2 = floor + (floor - Q1)
IF (Q2 > ceiling) THEN
     range = ceiling - floor
     gap = Q2 - floor
     multiple = TRUNCATE(gap / range)
     remainder = gap - multiple x range
     IF (multiple is even) THEN
          Q2 = floor + remainder
     ELSE
          Q2 = ceiling - remainder                    (7)
```

where

Q_2 = quantity demanded, bounced off the ceiling and floor

Seasonality

Seasonality can be included by multiplying seasonal relatives to the bounced trend, as follows:

$$Q_3 = S_j Q_2 \tag{8}$$

$$(1 / m) \Sigma S_j = 1 \tag{9}$$

where

Q_3 = quantity demanded, with seasonality

S_j = seasonal relative for the jth season

m = number of seasons in a season-cycle

To avoid bias, the seasonal relatives chosen must average to one. To fit the definition of seasons, they must cycle repetitively. They do not, however, have to cycle in sets of four. Two-season cycles would be simple and defensible, and in reality, many businesses experience only two: a high season and a low season.

Pricing

The economics of everyday life indicates that when price changes, demand will change in the inverse direction. Furthermore, the language of marketing suggests that a "right" price exists for every product. If this price is defined as the price that gives rise to the highest revenue, then a simple pricing function would be the following:

$$p^c + q^c = 2 \tag{10}$$

$$p = P / P^* \tag{11}$$

$$q = Q_4 / Q_3 \tag{12}$$

conditional upon

$$p^c \leq 2$$

where

P = actual price

P^* = price that maximizes revenue

Q_4 = quantity demanded, with pricing effect

c = demand curvature

p = relative price

q = relative quantity

Furthermore, since

$$R = PQ_4 = pP^*qQ_3 = pP^* (2 - p^c)^{(1/c)} Q_3 \tag{13}$$

where

R = revenue

It follows that

$$\frac{dR}{dp} = (1 - G)(2 - p^c)^{(1/c)} P^* Q_3 \tag{14a}$$

$$G = p^c / (2 - p^c) \tag{14b}$$

which, when set to zero, shows the maximum revenue is at the point where

$$p = q = 1$$

implying that

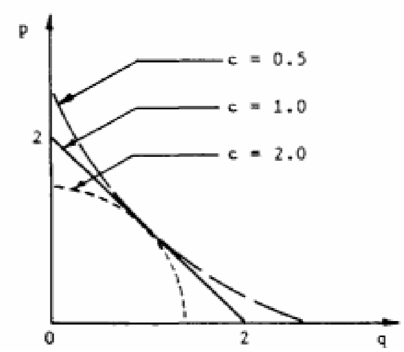
$$Q_4 = Q_3 \quad \text{and} \quad P = P^*$$

Thus, when the price is right (revenue is maximized), the demand will be at the seasonally-adjusted level irrespective of the demand curvature.

Figure 1 is a plot of Equation 10 for three values of the demand curvature: 0.5, 1.0, and 2.0. The plot shows that the equation is symmetrical on both axes, linear when the curvature equals 1, and constitute a quadrant of a circle when the curvature equals 2. Inasmuch as revenue is the area bounded by price and quantity, one can deduce from the plot that the greater the curvature, the more sensitive revenue will be to price. Thus, the parameter of curvature has a powerful and comprehensible meaning.

FIGURE 1

EFFECT OF PRICE ON QUANTITY DEMANDED



An alternative definition of the right price is the price that gives rise to the highest total contribution to income. Formally,

$$K = (P - V)\ Q_4 \qquad (15)$$

where

K = contribution to income

P = price

V = variable cost per unit

Q_4 = quantity demanded

By substitution and differentiation, it follows that

$$\frac{dK}{dp} = (1 - H)\ (2 - p^c)^{(1/c)}\ P^* Q_3 \qquad (16a)$$

$$H = [(pP^* - V)\ p^c]\ /\ [pP^*\ (2 - p^c)] \qquad (16b)$$

and thus, total contribution is maximized when

$$2pP^*\ [1 - (1\ /\ p^c)] = V \qquad (17)$$

Furthermore, in the case when

$$c = 1$$

which represents a linear relationship between price and quantity, it follows that

$$P = P^* + (V\ /\ 2) \qquad (18)$$

Thus, Equation 10 gives rise to simple relationships between prices and variable cost when contribution to income is maximized.

Thus, the right price for maximum contribution is always greater than or equal to the right price for maximum revenue. Furthermore, because

$$V \geq 0$$

it follows from Equation 17 that

$$p \geq 1$$

Transient Effects

Transient effects may be caused by changes in price or promotion, among others. In the game, MANAGEMENT 500, changes in price cause a transient effect in addition to the permanent pricing effect discussed earlier. A simple way to account for transient effects is to relate them to the concept of sensitivity, and to restrict their effects to a single period. Thus, the transient effect of a variable on demand can be modeled as follows:

$$Q_5 = Q_4\ (1 + Sx) \qquad (19a)$$

$$x = (X_t - X_{t-1})\ /\ X_{t-1} \qquad (19b)$$

conditional upon

$$(1 + Sx) \geq 0$$

and

$$Q_5 = Q_4 \quad \text{when} \quad X_{t-1} = 0$$

where

Q_5 = quantity demanded, with transient effect

S = sensitivity

X_t = value of causal variable at current period

X_{t-1} = value of causal variable at previous period

In this usage, sensitivity relates a relative change in the causal variable between two periods to a relative difference in the quantity demanded with respect to what it would have been without the transient effect. Thus, the concept of sensitivity as used here resembles the concept of elasticity, which relates a relative change in the causal variable between two periods to a relative change in the quantity demanded between the same two periods.

Cumulative Effects

Some variables, such as advertising and product service, have cumulative effects on product demand. In the game MANAGEMENT 500, advertising demonstrates a cumulative effect. These are effects that appear gradually, and become strengthened with time. A cumulative effect can be modeled by exponential smoothing, as follows:

$$Q_6 = Q_5\ (Z_t\ /\ Z^*) \qquad (20)$$

$$Z_t = aY_t + (1 - a)Z_{t-1} \qquad (21)$$

conditional upon

$$Z_t\ /\ Z^* \leq 1$$

and

$$Q_6 = Q_5 \quad \text{when} \quad Z^* = 0$$

where

Q_6 = quantity demanded, with cumulative effect

Z^* = saturation level of cumulative effect

Z_t = efficacy of cumulative effect at current period

Z_{t-1} = efficacy of cumulative effect at previous period

Y_t = value of causal variable at current period

a = smoothing constant

The rationale underlying this model of cumulative effect is that the base demand presumes a saturation level of the causal variable, and furthermore, that the efficacy of the cumulative effect cannot exceed the saturation level. The model assures that the cumulative causal variable cannot induce an unstable movement in demand, an issue about which Pray and Gold (1982) has shown reasons for concern. Furthermore, the final result can be tested with the preestablished limits and confined between the limits with the following algorithm:

```
IF (Q6 > ceiling) THEN
      Q6 = ceiling
IF (Q6 < floor) THEN
      Q6 = floor                                    (22)
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CONCLUSION

The basic concepts that a comprehensive demand function must model in a business simulation are trend, stages, randomness, limits, seasonality, pricing, transient effects, and cumulative effects. Because business simulations are primarily designed to assist learning, and because simplicity facilitates learning, computerized models should be fundamentally simple, although they may be combined to demonstrate apparently complex phenomena. This paper discussed simple models for the basic concepts, and showed how a complex demand function can result from combining the simple models. The resulting demand function has been tested in the simulation game, MANAGEMENT 500.

When models are fundamentally simple, the structure of the models can be deduced from a straightforward analysis of the results without resorting to black-box mathematical analyses, such as those described by Zernik (1987). Because students have a natural curiosity to understand the models of the simulations they play, simple models amenable to deductive investigation rewards, and therefore reinforces this scientific curiosity.

The essential mission of science is to discover simplicity in apparent complexity. To the scientist, the world is composed of simple mechanisms that often combine in such a way as to demonstrate a complex outcome, A scientist tries to discover the underlying simplicities. To the extent that students playing simulations can be encouraged and rewarded for similar efforts, to that extent, the students have received encouragement to think scientifically.

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