ABSTRACT

The effects of recruitment, benefits, training, and labor cost on the human component of production can be modeled plausibly by the concave exponential, convex exponential, logistic, and cusp catastrophic functions, respectively. When continuous mathematical functions such as these are used in computerized business simulations, the student who researches the algorithms of the simulation will develop a deep understanding of the mathematics. This deep understanding may be more valuable than the surface understanding of business that such simulations nominally teach.

INTRODUCTION

Since Pray and Gold’s (1982) analysis of the demand functions of published computerized business simulations, a lively discussion has ensued on the problem of modeling demand (Decker, LaBarre, & Adler, 1987; Frazer, 1983; Gold & Pray, 1983, 1984; Golden, 1987; Goodsen, 1986; Lambert & Lambert, 1988; Thavikulwat, 1988). The problem of modeling supply, however, has tended to be neglected. Yet, the supply side of modeling presents issues that are at least as involved as those of the demand side.

In particular, how best to incorporate the human component of the supply side is an interesting problem in the design of such simulations. This problem can be approached in four ways: First, the human aspect of the human resources can be ignored. Thus, human resources would be modeled linearly, as if they were another kind of material resource. Second, the human aspect could be inserted by the administrator. Thus, the administrator would change worker productivity, cause workers to quit, or call strikes (Dickson & Kinney, 1982). Third, ad hoc algorithms based on “research findings, conventional wisdom, common sense, and supposition” (Estes, 1986) could be incorporated. Fourth, continuous nonlinear functions could be utilized.

The first approach of using Linear models is expedient, but it gives rise to implausible results. The second approach of administrative intervention is flexible, but it makes the simulation difficult to administer. The third approach of using ad hoc algorithms suffers because ad hoc rules have no generality. Thus, the fourth approach would be ideal, provided suitable functions can be found. This paper proposes that the concave exponential, convex exponential, logistic, and cusp catastrophic (Thom, 1975) functions constitute a set of continuous nonlinear functions that can plausibly model the effects, respectively, of recruitment, benefits, training, and labor cost on the human component of production.

THE MODELS

Recruitment

Recruitment serves primarily to raise the productivity of new hires. As recruitment cost rises, the productivity of new hires should rise also, but at a diminishing rate until it finally levels off. These characteristics can be captured by a concave exponential function moderated by three parameters: base productivity, recruitment asymptote, and recruitment reaction. The function is graphed in Figure 1, and is defined as follows:

\[
\hat{p}_n = p_b \left[ 1 + \left( r - 1 \right) \left( 1 - \frac{1}{e^{AR}} \right) \right]
\]

where

- \( \hat{p}_n \) = productivity of new hires
- \( p_b \) = base productivity
- \( r \) = recruitment asymptote
- \( A \) = recruitment reaction
- \( R \) = recruitment cost per new hire

FIGURE 1
CONCAVE EXPONENTIAL MODEL OF RECRUITMENT ON THE PRODUCTIVITY OF NEW HIRES

Benefits

Benefits serve primarily to lower attrition. As benefits cost rises, the rate of attrition should decrease, but at a diminishing rate until it finally levels off. These characteristics can be captured by a convex exponential function moderated by three parameters: attrition asymptote, attrition reaction, and attrition base. The function is graphed in Figure 2, and is defined as follows:

\[
q = q_b + \frac{q_a - q_b}{1 + \frac{e^{AR}}{e^{AR}}}
\]
Training

Training serves primarily to raise the productivity of trained employees who remain with the firm following the period of training. As training cost rises, the productivity of trained employees should rise also. Because training cost directly affects training quality and because fatigue limits people's ability to benefit from training, the effectiveness of training should be most sensitive to training cost when the training cost is moderate, and it should eventually level off at higher levels of training cost. These characteristics can be captured by a logistic function moderated by four parameters: training asymptote, training base, training reaction, and training inflection. This S-shaped function is graphed in Figure 3, and is defined as follows:

\[ q = \frac{G_b - C_b}{1 + e^{-(G_a - C_b)A_s(T - T')}} \]  

where

- \( q \) = rate of attrition
- \( q_a \) = attrition asymptote
- \( q_b \) = attrition base
- \( A_q \) = attrition reaction
- \( B \) = benefits cost per employee

**Figure 2**  
**CONVEX EXPONENTIAL MODEL OF BENEFITS ON THE RATE OF ATTRITION**

**Figure 3**  
**LOGISTIC MODEL OF TRAINING ON THE PRODUCTIVITY OF TRAINED EMPLOYEES**

Labor Cost

Labor cost, combining salaries and wages, directly affect the size of the labor pool. The effect of labor cost, however, is contingent upon the productivity of labor. The more productive the labor, the higher the labor cost that should be required to maintain a labor pool of a given size. Because people are capable of collective action, the labor pool should be subject to discontinuities occasioned by strikes and settlements of strikes. These characteristics can be captured by a cusp catastrophic function moderated by eight parameters: asymmetry intercept, asymmetry labor cost, asymmetry productivity, bifurcation intercept, bifurcation labor cost, bifurcation productivity, labor pool midpoint, and labor pool deviation. The cusp catastrophic function is a three-dimensional cubic function, as illustrated in Figure 4. A two-dimensional cross section is shown in Figure 5, and the function is defined as follows:

\[ z^3 - bz - a = 0 \]  

\[ a = a_0 + a_1I_1 + a_2I_2 \]  

\[ b = b_0 + b_1I_1 + b_2I_2 \]  

\[ z = \frac{b - n}{s} \]  

where

- \( z \) = intermediate state variable
- \( a \) = asymmetry control variable
- \( b \) = bifurcation control variable
- \( n \) = intermediate state variable
- \( a_0 \) = asymmetry intercept
- \( a_1 \) = asymmetry labor cost
- \( a_2 \) = asymmetry productivity
- \( b_0 \) = bifurcation intercept
- \( s \) = labor pool deviation
The cusp catastrophe is an element of catastrophe theory, which is a relatively recent method of applying topographical mathematics to the modeling of divergent and discontinuous phenomena by the use of continuous functions (Stewart & Pereyoy, 1983; Zeeman, 1976). In catastrophe theory, discontinuities are represented by folds. Thus, in the cross-section of Figure 5, the edges of the two folds represent the strike and settlement points. The backward sloping region between the two edges represents a region of instability. Movement along the curve follows Path 1 in the forward direction and Path 2 in the reverse direction. Movement does not take place in the backward sloping region.

**DISCUSSION**

The concave exponential, convex exponential, logistic, and cusp catastrophic functions can model the effects of recruitment, benefits, training, and labor cost on the human component of production. Thus, a simulation incorporating these functions should be a useful device for teaching students about them. With the easy availability of sophisticated equation-solving programs, such as MathCAD (Mathsoft, 1987), solving nonlinear functions has become an exercise simple enough for undergraduate college students.

Students who work with business simulations often show great interest in the computational algorithms of the simulation, for a very sensible reason—to win. Success in simulation depends on decisions that fit the algorithms. Thus, if the algorithms are ad hoc, the intelligent student will learn ad hoc rules that have no real-world application. On the other hand, if the algorithms are continuous mathematical functions, the intelligent student will learn about the functions, and will leave the simulation with an understanding of the mathematics. In time, this deep understanding of mathematics, the science of patterns (Steen, 1988), may be more valuable to the student than the surface understanding of business that the simulation nominally taught. Thus, the algorithms of business simulations ought to consist of continuous mathematical functions such as those proposed here.

**REFERENCES**


