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Simulation of wave propagation in a piecewise homogeneous elastic rod

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Abstract

This article deals with modeling and simulation of the wave propagation phenomenon in a heterogeneous linear elastic rod treated as piecewise homogeneous. Employing a reference configuration approach the piecewise homogeneous elastic rod is mathematically described by a linear hyperbolic system of partial differential equations. Its generalized solution is obtained by connecting intermediate states by contact shocks – thus allowing analytical solutions for some initial value problems even when boundary conditions are considered.

Keywords: Riemann problem, linear elasticity, piecewise homogeneous material.

1. Introduction

Stress waves propagation in solids is an important tool for studying the mechanical response of materials. Wave propagation provides information about the way solids behave when the forces acting on them are no longer in static equilibrium. Some phenomena such as scattering, dispersion and attenuation, which strongly influence wave propagation, affecting the thermomechanical response of the materials, may be attributed to nonlinearities like material heterogeneity, wave characteristics and loading conditions (Chen et al., 2004).

Zhuang et al. (2003) have performed a systematic experimental investigation of the influence of interface scattering on finite-amplitude shock waves – which affects shock waves dissipation and dispersion – by considering shock wave propagation in periodically layered composites and have observed that these materials can support steady structured shock waves. These authors noted that wave propagation through layered materials composed by isotropic layers allows investigating the effect of heterogeneous materials under shock loading.

Chen et al. (2004) presented an approximate analytical solution to one-dimensional wave propagation in layered heterogeneous materials subjected to high velocity plate impact loading conditions, based on Floquet's theory, using neither a sinusoidal wave loading nor unit step loading, which, according to these authors, have been used in previous analytical works.

Berezovsky et al. (2006) considered a piecewise homogeneous media, accounting for the wave distortion of nonlinear elastic waves. The authors performed numerical simulations of one-dimensional wave propagation in layered nonlinear heterogeneous solids, employing a finite volume approximation for hyperbolic problems, in which the Riemann problem is solved at each interface between discrete elements. They have considered finite amplitude nonlinear wave propagation to study scattering, dispersion and attenuation of shock waves, employing a wave propagation algorithm accounting for thermodynamic consistency, introduced by Berezovsky and Maugin (2001), for the two-dimensional problem in media with rapidly varying properties. The dispersion effects due to the microstructure in nonlinear deformation waves have been considered by Engelbrecht et al. (2007).

This article presents a discussion concerning the dynamical response of a specific class of heterogeneous linear elastic rods which is piecewise homogeneous. This elastic rod is left in a nonequilibrium state. Starting from a given

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initial data, the technique proposed in this paper allows obtaining analytical solutions for the strain, the stress and the velocity fields. Eventually this initial value problem may be subjected to some boundary conditions.

The piecewise homogeneous linear elastic rod considered in the present work gives rise to a one-dimensional problem in the reference configuration which is mathematically described by a linear hyperbolic system of partial differential equations with eigenvalues depending on the position. In fact, these eigenvalues are piecewise constant, since the rod is assumed to be piecewise homogeneous – being composed by *N* different materials.

Several problems in Mechanics are represented by hyperbolic systems, which permit very realistic descriptions, since the propagation of any quantity – or information – in real natural phenomena is characterized by a finite speed. However they may not admit a regular solution, requiring a larger space of admissible solutions. The hyperbolic system describing the piecewise homogeneous linear elastic rod considered in this article does not admit – in general – a solution in the classical sense, requiring an enlargement of the space of admissible solutions allowing working with the jump conditions associated with the set of equations in order to deal with discontinuous functions.

The generalized solutions of the problem are obtained by connecting intermediate states by contact shocks (Lax, 1971; Smoller, 1983). A contact shock may be viewed as the limiting case of a rarefaction in which the rarefaction fan is reduced to a single line; namely a discontinuity with associated eigenvalue corresponding exactly to the shock speed. Unlike ordinary shocks, the contact shock is reversible, without any associated entropy generation (Saldanha da Gama, 1990). The reference configuration approach employed to describe the piecewise homogeneous rod gives rise to a problem characterized by deformation jumps at any two distinct materials interface, represented by stationary shocks – in other words, the interface position is not modified. An adequate composition of these discontinuous functions gives rise to the complete analytical solution of the initial value problem even with boundary conditions.

The study of elastic wave equations accounting for their scattering in non-uniform media is important for acoustic problems. Approximate methods can lead to convenient simplifications, for weak inhomogeneities. (Tenenbaum and Zindeluk, 1992a) present an exact algebraic solution for the scattering of acoustic waves in one-dimensional elastic media, considering the wave propagation pattern in layered media, valid for strong inhomogeneities. In a subsequent article (Tenenbaum and Zindeluk, 1992b) the same authors propose a sequential algorithm with arbitrary inlet pulse to solve inverse acoustic scattering problems for plane waves in elastic nonhomogeneous media, consisting of a mathematical inversion of the previous direct problem.

It is worth noting that the methodology presented in this work could be extended to nonlinear homogeneous problems by employing the approximate Riemann solver developed by Roe (1997), considering “approximate solutions which are exact solutions to an approximate problem”. Essentially, this scheme allows each Riemann problem – which is to be solved for each two consecutive steps, in order to implement a difference scheme like Glimm’s one (see (Gama and Martins-Costa, 2009) and references therein) – to approximate an originally homogeneous nonlinear system by a heterogeneous linear one. The procedure for implementing this scheme is briefly described by an example in in which a nonlinear elastic rod is considered.

2. Mechanical model

The one-dimensional phenomenon considered is this work is mathematically described, from a continuum mechanics viewpoint (Billington and Tate, 1981) as

$$\begin{cases} \dfrac{\partial \varepsilon}{\partial t} - \dfrac{\partial v}{\partial X} = 0 \\ \rho \dfrac{\partial v}{\partial t} - \dfrac{\partial \sigma}{\partial X} = 0 \end{cases} \tag{1}$$

where ρ represents the mass density in the reference configuration (piecewise constant here), *v* the velocity, σ is the normal component of the Piola-Kirchhoff tensor and ε is the strain. The first equation above represents a geometrical compatibility while the second one represents the linear momentum balance in the reference configuration. In both the equations *t* represents the time while *X* represents the position (in the reference configuration).

The strain field ε is defined as

$$\varepsilon = \frac{\partial x}{\partial X} - 1 \tag{2}$$

in which *x* represents the position in the current configuration (spatial position).

Since a linear elasticity hypothesis is assumed in this work the Piola-Kirchhoff normal stress σ is a piecewise linear function of the strain ε. In other words,

$$\sigma = c_i \varepsilon, \text{ for } \quad X_i < X < X_{i+1} \tag{3}$$

where c_i is a positive constant. The mass density ρ is assumed constant in $X_i < X < X_{i+1}$.

$$\rho = \rho_i = \text{constant}, \quad \text{for} \quad X_i < X < X_{i+1} \tag{4}$$

3. The associated Riemann problem and its generalized solution

Associated with equations (1)-(4) there is an initial value problem – named associated Riemann problem, given by

$$\begin{cases} \dfrac{\partial \varepsilon}{\partial t} - \dfrac{\partial v}{\partial X} = 0 \\ \rho \dfrac{\partial v}{\partial t} - \dfrac{\partial \sigma}{\partial X} = 0 \end{cases} \qquad \begin{cases} (\varepsilon, v) = (\varepsilon_L, v_L) & \text{for} \quad X < X_0 \\ (\varepsilon, v) = (\varepsilon_R, v_R) & \text{for} \quad X > X_0 \end{cases} \tag{5}$$

in which ε_L, ε_R, v_L and v_R are known constants.

The solution of the hyperbolic system represented by Eq. (5) is reached by connecting the left state (ε_L, v_L) to the right state (ε_R, v_R) by means of rarefactions (continuous solutions) and/or shocks (discontinuities satisfying the entropy conditions). Two states are connected by a rarefaction if, and only if, between these states, the corresponding eigenvalue is an increasing function of the ratio $(X - X_0)/t$ (Smoller, 1983; Lax, 1971; John, 1974).

The eigenvalues associated to the hyperbolic system described by Eq. (5) are given, in increasing order, by

$$\lambda_1 = -\left[\frac{\sigma'}{\rho_i}\right]^{1/2} = -\left[\frac{c_i}{\rho_i}\right]^{1/2} \text{ and } \lambda_2 = \left[\frac{\sigma'}{\rho_i}\right]^{1/2} = \left[\frac{c_i}{\rho_i}\right]^{1/2} \text{ for } \quad X_i < X < X_{i+1} \tag{6}$$

in which σ' represents the first derivative of the normal component of the Piola-Kirchhoff tensor with respect to the deformation ε.

When $X_i \to -\infty$ and $X_{i+1} \to +\infty$, the solution of Eq. (5) depends on the ratio $(X - X_0)/t$ only and, since the eigenvalues are constant, the generalized solution is discontinuous. In other words, the left state (ε_L, v_L) is connected to an intermediate state (ε^*, v^*) by a discontinuity (called 1-shock or back shock) while the right state (ε_R, v_R) is connected to an intermediate state (ε^*, v^*) by another discontinuity (called 2-shock or front shock). Since $\lambda_1 < 0 < \lambda_2$, the entropy conditions (Keyfitz and Kranzer 1978; Callen 1960) ensure that the shock speed s_1 (back shock speed) is always negative while s_2 (front shock speed) is always positive.

The intermediate state (ε^*, v^*) is obtained from the Rankine-Hugoniot jump conditions, which for the hyperbolic system (5) are given by (Smoller 1983)

$$\frac{[v]}{[\varepsilon]} = \frac{[\sigma]}{\rho[v]} = -s \tag{7}$$

where *s* denotes the shock speed and the brackets denote the jump.

Equation (7) allows concluding that

$$\begin{aligned} \frac{v_L - v^*}{\varepsilon_L - \varepsilon^*} &= \frac{\sigma_L - \sigma^*}{\rho(v_L - v^*)} = -s_1 \\ \frac{v^* - v_R}{\varepsilon^* - \varepsilon_R} &= \frac{\sigma^* - \sigma_R}{\rho(v^* - v_R)} = -s_2 \end{aligned} \tag{8}$$

The set of equations (5)-(8) gives rise to the following (generalized) solution for the strain and the velocity

$$\varepsilon = \begin{cases} \varepsilon_L & \text{for} \quad -\infty < (X - X_0)/t < s_1 \\ \varepsilon^* & \text{for} \quad s_1 < (X - X_0)/t < s_2 \\ \varepsilon_R & \text{for} \quad s_2 < (X - X_0)/t < \infty \end{cases} \qquad v = \begin{cases} v_L & \text{for} \quad -\infty < (X - X_0)/t < s_1 \\ v^* & \text{for} \quad s_1 < (X - X_0)/t < s_2 \\ v_R & \text{for} \quad s_2 < (X - X_0)/t < \infty \end{cases} \tag{9}$$

in which

$$\varepsilon^* = \frac{v_R - v_L}{2\sqrt{c_i/\rho_i}} + \frac{\varepsilon_R + \varepsilon_L}{2}, \qquad v^* = \frac{v_R + v_L}{2} + \frac{\varepsilon_R - \varepsilon_L}{2}\sqrt{c_i/\rho_i},$$
$$s_1 = -\sqrt{\frac{c_i}{\rho_i}} \quad \text{and} \quad s_2 = \sqrt{\frac{c_i}{\rho_i}} \tag{10}$$

It is important to note that, since $s_1 = \lambda_1$ and $s_2 = \lambda_2$, both the 1-shock and the 2-shock are called contact discontinuities and no entropy generation is associated with these shocks (Lax 1971).

Now supposing that $X_i \to -\infty$, $X_{i+1} = X_0$ and $X_{i+2} \to \infty$, an infinite rod composed by two homogeneous parts is represented.

In such a case the solution presents a stationary shock at the (reference) position X_{i+1} and the generalized solution of Eq. (5) also depends only on the ratio $(X - X_0)/t$. Nevertheless the 1-shock (left) and the 2-shock (right) speeds have different absolute values.

Since there exists a stationary shock at $X = X_{i+1} = X_0$, it may be concluded, from the jump conditions across this shock, that velocity and stress do not jump at this point. So, only the strain ε jumps across the stationary shock and, since $[\sigma] = 0$, it comes that (Keyfitz and Kranzer, 1978)

$$c_i\varepsilon_-^* = c_{i+1}\varepsilon_+^* \quad \text{with} \quad \varepsilon_-^* = \lim_{X \to X_{i+1}, X < X_{i+1}} \varepsilon \quad \text{and} \quad \varepsilon_+^* = \lim_{X \to X_{i+1}, X > X_{i+1}} \varepsilon \tag{11}$$

In this case, the jump conditions give rise to the following set of equations

$$\frac{v_L - v^*}{\varepsilon_L - \varepsilon_-^*} = \frac{\sigma_L - \sigma^*}{\rho_i(v_L - v^*)} = -s_1$$
$$\frac{v^* - v_R}{\varepsilon_+^* - \varepsilon_R} = \frac{\sigma^* - \sigma_R}{\rho_{i+1}(v^* - v_R)} = -s_2 \tag{12}$$

and the complete solution is given by

$$\varepsilon = \begin{cases} \varepsilon_L & \text{for} \quad -\infty < \xi < s_1 \\ \varepsilon_-^* & \text{for} \quad s_1 < \xi < 0 \\ \varepsilon_+^* & \text{for} \quad 0 < \xi < s_2 \\ \varepsilon_R & \text{for} \quad s_2 < \xi < \infty \end{cases} \qquad v = \begin{cases} v_L & \text{for} \quad -\infty < \xi < s_1 \\ v^* & \text{for} \quad s_1 < \xi < s_2 \\ v_R & \text{for} \quad s_2 < \xi < \infty \end{cases} \tag{13}$$

where $\xi = (X - X_{i+1})/t$ and

$$\varepsilon_-^* = \frac{c_{i+1}(v_R - v_L)}{c_{i+1}\sqrt{c_i/\rho_i} + c_i\sqrt{c_{i+1}/\rho_{i+1}}} + \frac{c_{i+1}(\varepsilon_L\sqrt{c_i/\rho_i} + \varepsilon_R\sqrt{c_{i+1}/\rho_{i+1}})}{c_{i+1}\sqrt{c_i/\rho_i} + c_i\sqrt{c_{i+1}/\rho_{i+1}}}$$
$$\varepsilon_+^* = \frac{c_i(v_R - v_L)}{c_{i+1}\sqrt{c_i/\rho_i} + c_i\sqrt{c_{i+1}/\rho_{i+1}}} + \frac{c_i(\varepsilon_L\sqrt{c_i/\rho_i} + \varepsilon_R\sqrt{c_{i+1}/\rho_{i+1}})}{c_{i+1}\sqrt{c_i/\rho_i} + c_i\sqrt{c_{i+1}/\rho_{i+1}}} \tag{14}$$
$$v^* = \frac{c_{i+1}\varepsilon_R - c_i\varepsilon_L}{\sqrt{c_i\rho_i} + \sqrt{c_{i+1}\rho_{i+1}}} + \frac{v_L\sqrt{c_i\rho_i} + v_R\sqrt{c_{i+1}\rho_{i+1}}}{\sqrt{c_i\rho_i} + \sqrt{c_{i+1}\rho_{i+1}}}$$
$$s_1 = -\sqrt{c_i/\rho_i} \quad \text{and} \quad s_2 = \sqrt{c_{i+1}/\rho_{i+1}}$$

Remark: Equation (10) consists of a particular case of Eq. (14), obtained when $c_i = c_{i+1}$ and $\rho_i = \rho_{i+1}$. In this case there is no stationary jump at X_{i+1} and $\varepsilon_-^* = \varepsilon_+^* = \varepsilon^*$ even for $(X - X_{i+1})/t = 0$.

Figure (1) presents the solution, obtained by employing Eq. (13) in the plane $X - t$, for a case in which $X_i \to -\infty$, $X_{i+1} = X_0$ and $X_{i+2} \to \infty$. It is worth noting that the representation in the plane $X - t$ presented in Figure 1 does not depend on the initial data (ε_L, v_L) and (ε_R, v_R), once the propagation speeds do not depend on the states (ε, v).

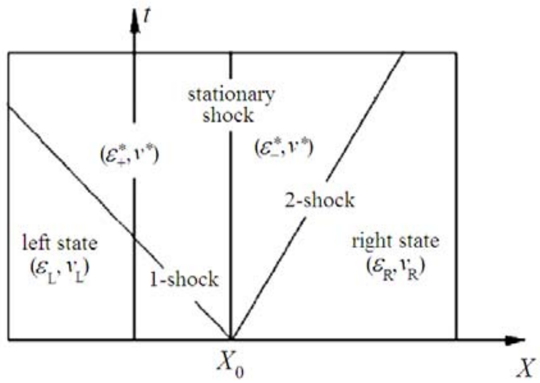

Figure 1: Riemann problem (Eq. (5)) in the plane $X-t$ for $X_i \to -\infty$, $X_{i+1} = X_0$ and $X_{i+2} \to \infty$.

4. The associated Riemann problem when $X_0 \neq X_i$ for any i

This section studies problems in which the interface between two different materials (placed at any position X_i) is not coincident with the jump in the initial data (placed at X_0). In this case the solution of the Riemann problem no longer depends on $(X - X_0)/t$. In fact, the solution depends on $(X - X_0)/t$ only until a shock (either front or back) reaches a stationary shock, characterizing a shock interaction. At this point, a new Riemann problem arises, centered at the position of the stationary shock, having as "initial time" the time in which the shock interaction has taken place.

Now, considering the problem defined by Eq. (5) and assuming $X_i < X_0 < X_{i+1}$, since X_0 is different from any X_i, both the 1-shock and the 2-shock are centered at X_0, while a stationary shock is present at each X_i. It is important to note that while there is no shock interaction between shocks coming from distinct points, the solution depends on the ratio $(X - X_0)/t$ only. When the 1-shock reaches the stationary shock at $X = X_i$, the solution behaviour is changed. In any case, the intermediate state becomes new initial data (with respect to the time in which the shock interaction occurred) giving rise to a new Riemann problem. The solution of this new Riemann problem has always the same structure of Eq. (13) enabling Eq. (5) to be solved for any piecewise constant initial data.

In order to illustrate the solution procedure, a particular case is now considered: an infinite rod composed by three different homogeneous parts such that: $X_1 \to -\infty$, $X_2 = -0.7L$, $X_0 = 0$, $X_3 = 0.3L$ and $X_4 \to \infty$ with $\sqrt{c_1/\rho_1} = 3\sqrt{c_2/\rho_2}$ and $\sqrt{c_3/\rho_3} = 0.3\sqrt{c_2/\rho_2}$.

Starting from the initial data $(\varepsilon, v) = (\varepsilon_L, v_L)$, for $X < X_0$ and $(\varepsilon, v) = (\varepsilon_R, v_R)$, for $X > X_0$, the intermediate state (ε^*, v^*) – which will be treated as a left state (ε_L, v_L) – is given by Eq. (10). At the point "a" the front shock (with speed $\sqrt{c_2/\rho_2}$) reaches the stationary shock placed at X_3, giving rise to a new Riemann problem, centered at "a", characterized by the left state (ε_1, v_1) and the right state (ε_R, v_R). The intermediate states (ε_-^*, v^*) and (ε_+^*, v^*) for this "new" Riemann problem, are given by Eq. (14). Repeating this procedure, a solution in the plane $X-t$ may be constructed, as depicted in Figure 2. Table (1) relates Eq. (14) and each of the states presented in Figure 2.

Since all the propagation speeds are previously known and constant, the time associated with each shock interaction (a, b, c, d, e, f and g) is easily determined. For instance, point "a" is reached when $t = 0.3L\sqrt{\rho_2/c_2}$; point "e" when $t = 1.7L\sqrt{\rho_2/c_2}$ and point "c" when $t = L\sqrt{\rho_2/c_2}$.

Table 1: States 1 to 13 and their relation with Eq. (14).

Riemann problem centered at	LEFT STATE	INTERMEDIATE STATE "−" Eq.(14)	INTERMEDIATE STATE "+" Eq.(14)	RIGHT STATE
a	1	2	3	R
b	L	5	4	1
c	4	6	6	2
d	5	8	7	6
e	6	9	10	3
f	7	11	11	9
g	11	12	13	10

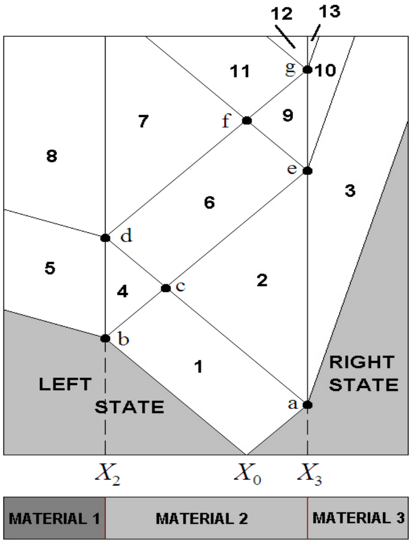

Figure 2. Riemann problem solution for $X_1 \to -\infty$, $X_2 = -0.7L$, $X_0 = 0$, $X_3 = 0.3L$ and $X_4 \to \infty$; with $\sqrt{c_1/\rho_1} = 3\sqrt{c_2/\rho_2}$ and $\sqrt{c_3/\rho_3} = 0.3\sqrt{c_2/\rho_2}$.

5. Finite rods – problems involving boundary conditions

The tools presented up to this point are sufficient for describing wave propagation in rods in which one edge is assumed to be fixed $(v=0)$ and the other is either fixed $(v=0)$ or free $(\sigma = 0$ and $\varepsilon = 0)$. Such boundary conditions are automatically satisfied by introducing artificial states beyond the actual rod. In other words, for imposing a fixed edge at X_1, it suffices to assume the existence of a rod at the left ($X < X_1$), with a state such that $v^* = 0$ while a fixed edge at X_{N+1} is imposed by assuming the existence of a rod at the right-side ($X_{N+1} > X$), with a state such that $v^* = 0$. On the other hand, for imposing a free edge boundary condition, it suffices to consider an artificial rod with a state such that $\varepsilon^* = 0$. This can be done in an easy way too. The choice of the state in the artificial extension of the rod is done based on Eq. (10), assuming the same material for both the artificial extension and the actual rod.

For instance, in this work a problem in which $X_1 = 0$, $X_2 = 4L$, $X_3 = 12L$, $X_0 = 7L$ and $\sqrt{c_1/\rho_1} = 2\sqrt{c_2/\rho_2}$ is considered for two distinct simulated situations – namely situation (***i***) representing a rod fixed at both edges and situation (***ii***) representing a rod fixed at the left edge, with the right edge free.

Some selected results associated with the cases defined above as (***i***) and (***ii***) are presented in Table (2), assuming the rod at rest for $t = 0$ and defining $w = v\sqrt{\rho_2/c_2}$. The solution is reached by employing Eq. (14) after each shock interaction. The quantitative results are presented for specified left and right states as well as given boundary conditions.

Table 2: Some results for cases (*i*) and (*ii*).

case	ε_L	ε_R	ε_1	w_1	ε_2	w_2	ε_3	w_3	ε_4	w_4	ε_5	w_5	ε_6	w_6
i	0.20	0.20	0.20	0.00	0.10	-0.20	0.40	-0.20	0.00	0.00	0.20	0.00	0.03	0.07
ii	0.20	0.20	0.20	0.00	0.10	-0.20	0.40	-0.20	0.00	0.00	0.00	-0.20	0.03	0.07
i	-0.40	-0.40	-0.40	-0.40	-0.20	0.40	-0.80	0.40	0.00	0.00	-0.40	0.00	-0.07	-0.13
ii	-0.40	-0.40	-0.40	-0.40	-0.20	0.40	-0.80	0.40	0.00	0.00	0.00	0.40	-0.07	-0.13
i	0.00	0.50	0.25	0.25	0.08	0.17	0.33	0.17	0.17	0.00	0.00	0.00	0.14	-0.06
ii	0.00	0.50	0.25	0.25	0.08	0.17	0.33	0.17	0.17	0.00	0.00	0.00	0.14	-0.06
i	0.00	-0.50	-0.25	-0.25	-0.08	-0.17	-0.33	-0.17	-0.17	0.00	0.00	0.00	-0.14	0.06
i	0.80	0.40	0.60	-0.20	0.33	-0.93	1.33	-0.93	-0.13	0.00	0.80	0.00	0.02	0.09
ii	0.40	0.60	0.50	0.10	0.23	-0.33	0.93	-0.33	0.07	0.00	0.00	-0.40	0.12	0.11
ii	-0.50	-0.10	-0.30	0.20	-0.18	0.63	-0.73	0.63	0.13	0.00	0.00	0.50	-0.21	-0.21

6. An application to nonlinear elasticity

In this section a nonlinear elastic rod is considered. In this case, the Piola-Kirchhoff normal stress σ may assume, for instance, the following nonlinear constitutive equation

$$\sigma = f\left(\varepsilon\right)\varepsilon, \text{ for } X_i < X < X_{i+1} \tag{15}$$

A convenient redefinition is now considered for the constant c_i, before each advance in time

$$c_i = f\left(\bar{\varepsilon}\right), \quad \text{where} \quad \bar{\varepsilon} = \frac{1}{2}\left(\varepsilon\big|_{X=X_i} + \varepsilon\big|_{X=X_{i+1}}\right) \tag{16}$$

giving rise to the following approximation for the stress

$$\sigma \cong c_i\varepsilon, \text{ for } X_i < X < X_{i+1} \tag{17}$$

It is important to note that, is this case, the problem lies within the range of the procedure proposed in this work, being reduced to a piecewise linear function of the strain ε.

The associated Riemann problem, given by equation (5) is now obtained from equations (1), (2), (17) and (4) and all the previously described steps after equation (5) remain unchanged.

7. Final Remarks

Although this article presents a discussion concerning piecewise homogeneous linear elastic rods, the results can be extended to any linear heterogeneous rod. This extension is performed by approximating the heterogeneous rod by a piecewise homogeneous one. This latter could be composed by any number of different materials, for instance, 10, 100 or 1000 materials could be considered, according to the required accuracy. Also, taking advantage of the scheme proposed by Roe (1997), which approximates nonlinear homogeneous problems by linear heterogeneous ones, it could be directly extended to piecewise nonlinear elastic rods, as briefly stated in section 6.

In addition, Glimm's scheme allows building an approximation for hyperbolic problems subjected to any arbitrary initial data. It suffices to approximate the arbitrary initial condition by piecewise constant initial data. In the sequence, a Riemann problem – an initial value problem characterized by a step function initial condition – is to be solved for each two consecutive steps. The main idea behind the method is to appropriately gather the solution of as many Riemann problems as desired to successively march from a given time instant $t = t_n$ to the successive time instant $t_{n+1} = t_n + \Delta t$.

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