CHAPTER 28

DESIGN OF PILING

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A theory for the forces exerted on piling by waves has been developed and confirmed by model studies (Morison, O'Brien, Johnson, and Schaaf, 1950). The results obtained by the application of this theory are of the correct order of magnitude; however, additional laboratory and field experiments are desirable to define more accurately the values and limits of certain coefficients which enter the theoretical relationships. It is the purpose of this paper to present the possible calculations and the assumptions made with the limitation involved.

In the development of the theoretical relationship for the force on a piling the wave profile is the trochodial form and the particle velocity and particle acceleration are sinusoidal in form. The assumptions made in connection with this theory are that the water depth is large compared to the wave length and that the wave height is small compared to the wave length. However, the calculations using this theory are accurate wherever the sinusoidal particle velocity distribution shows good agreement with the actual distribution. This situation includes steep waves in deep water and waves of low amplitude in shallow water where the wave height is small compared to the water depth. The obvious exclusion is steep waves in very shallow water. The horizontal particle velocity and acceleration are given by the expressions:

particle displacement

$$x = \frac{-H}{2} \frac{\operatorname{Cosh} \frac{2\pi(d+z)}{L}}{\operatorname{Sinh} \frac{2\pi d}{L}} \operatorname{Sin} \theta; \qquad (1)$$

particle velocity

$$u = \frac{\pi H}{T} \frac{\operatorname{Cosh} \frac{2\pi(d+z)}{L}}{\operatorname{Sinh} \frac{2\pi d}{L}} \operatorname{Cos} \theta \qquad (2)$$

particle acceleration

$$\frac{\partial u}{\partial t} = + \frac{2\pi^2 H}{T^2} \frac{\operatorname{Cosh} \frac{2\pi(d+z)}{L}}{\operatorname{Sinh} \frac{2\pi d}{L}} \operatorname{Sin} \theta \qquad (3)$$

where

H = wave height
L = wave length
T = wave period
d = still-water level
z = depth below still-water level (negative)
t = time
θ = angular particle position in its orbit = $2\pi t/T$

The physical significance of these symbols is shown in Fig. 1.

The force (F) on a differential section (dz) of a piling is

$$F = \frac{1}{2} \rho C_D D u^2 dz + C_M \rho \frac{\pi D^2}{4} \frac{\partial u}{\partial t} dz \qquad (4)$$

where

C_D = coefficient of drag

C_M = coefficient of mass

ρ = water density
D = pile diameter

The first part of the force equation represents the form drag caused by surface shear. The second part of the equation is the acceleration force on the displaced volume of fluid including the virtual mass effect. The term virtual mass may be explained as the increase in force caused by an apparent increase of the displaced mass of the fluid when an object is accelerated in a fluid as compared to in a vacuum. Thus, there is an apparent increase in displaced volume without an actual increase in the mass of the pile. The two effects of the force are shown illustrated in Fig. 2. The force (F) is a function of the depth below the still-water level and the position along the wave profile defined by θ. The position is given by the expression:

$$\frac{\bar{x}}{L} = \frac{\theta}{360} - \frac{H}{2L \tanh \frac{2\pi d}{L}} \sin\theta \quad (5)$$

where $\bar{x}$ is defined in Fig. 1.

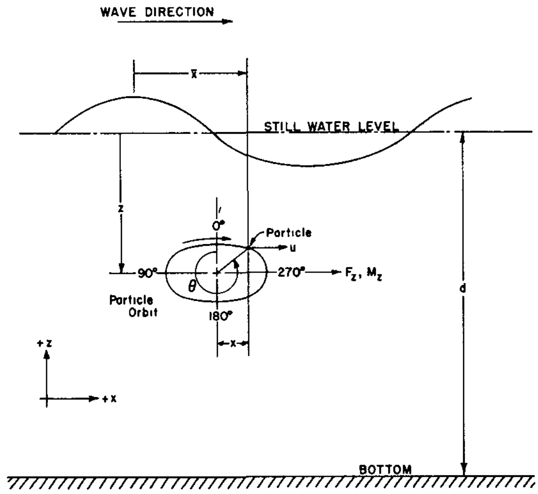

Fig. 1
Schematic diagram of particle motion

The moment at any point on the pile is given by the expression

$$M_z = \rho \frac{H^2 L^2 D}{T^2} \left\{ \pm C_D K_2 \cos^2\theta + \frac{\pi D}{4H} C_M K_1 \sin\theta - (d+z)\left[\pm C_D K_3 \cos^2\theta + \frac{\pi D}{4H} C_M K_4 \sin\theta\right] \right\} \quad (6)$$

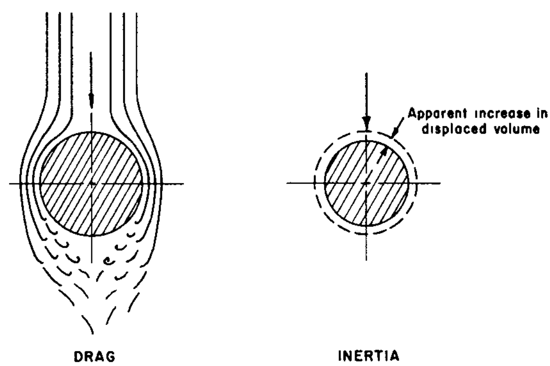

Fig. 2
Force components

where

$$K_1 = \frac{\frac{2\pi d}{L} \sinh \frac{2\pi d}{L} - \frac{2\pi (d+z)}{L} \sinh \frac{2\pi (d+z)}{L} - \cosh \frac{2\pi d}{L} + \cosh \frac{2\pi (d+z)}{L}}{2 \sinh \frac{2\pi d}{L}} \quad (7)$$

$$K_2 = \frac{\frac{1}{2}\left[\frac{4\pi d}{L}\right]^2 - \frac{1}{2}\left[\frac{4\pi (d+z)}{L}\right]^2 + \frac{4\pi d}{L} \sinh \frac{4\pi d}{L} - \frac{4\pi (d+z)}{L} \sinh \frac{4\pi (d+z)}{L} - \cosh \frac{4\pi d}{L} + \cosh \frac{4\pi (d+z)}{L}}{64 \left(\sinh \frac{2\pi d}{L}\right)^2} \quad (8)$$

$$K_3 = \frac{4\pi}{L} \left[\frac{\frac{4\pi d}{L} - \frac{4\pi (d+z)}{L} + \sinh \frac{4\pi d}{L} - \sinh \frac{4\pi (d+z)}{L}}{64 \left(\sinh \frac{2\pi d}{L}\right)^2} \right] \quad (9)$$

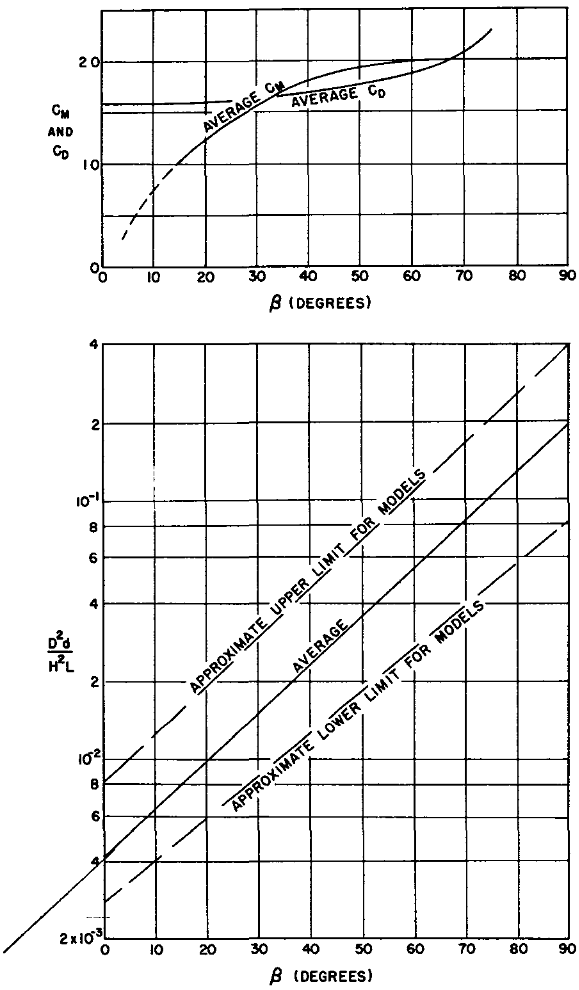

Fig. 3

Relationships of angular position of maximum total moment from model studies

$$K_4 = \frac{2\pi}{L}\left[\frac{\sinh\frac{2\pi d}{L} - \sinh\frac{2\pi(d+z)}{L}}{2\sinh\frac{2\pi d}{L}}\right] \tag{10}$$

Equation 6 does not include the variation of the leverarm due to the surface elevation. Thus, the equation applies only in the case where the wave height is small compared to the water depth.

The combination of forces indicates that a maximum moment would occur at an angular position, β, other than the position of the crest. The expression for β is given by

$$\beta = \sin^{-1}\left\{\frac{\pi}{8}\,\frac{C_M}{C_D}\,\frac{D}{H}\,\frac{(K_1-(d+z)K_4)}{(K_2-(d+z)K_3)}\right\} \tag{11}$$

The relationship of β to C_M and C_D is shown approximately by Fig. 3.

The angle, β, approaches zero for steep waves in shallow water when the pile diameter is relatively small compared to the wave height. It approaches 90^o when the wave length is long, the water is deep, and the pile diameter is of the same order of magnitude as the wave height.

As yet, no conclusive values of C_M and C_D for ocean condition have been obtained. These coefficients are empirical and can only be evaluated from measurements or by some relationship of oscillatory flow to steady flow, or by the extension of model study values.

The following sample calculations are presented to demonstrate the method and are based on results obtained by model studies.

SAMPLE CALCULATION I

TOTAL MOMENT ABOUT THE BOTTOM OF A PILE

Conditions:

$H = 10$ ft. $\quad T = 10$ sec.

$d = 100$ ft. $\quad D = 1.5$ ft.

Calculations ($z = -100$ ft.)

$L_o = 5.12\,T^2 = 512$ ft. (subscript o designates deepwater values)

Therefore

$d/L_o = 0.196$

From the Bulletin of the Beach Erosion Board, Special Issue No. 1, July 1, 1948,

$$d/L = 0.221 \quad (=0.2218)$$

and

$$L = \frac{100}{0.2218} = 452 \text{ ft}$$

$$L = 452 \text{ ft.}$$

$$H/L = 0.02$$

$$D/H = 0.15$$

$$\frac{D^2 d}{H^2 L} = \frac{1.5^2 \times 100}{10^2 \times 452} = 4.97 \times 10^{-3}$$

corrections from page 370, IV th proc.

Hence from Fig. 3

$$\beta \approx 0 \text{ to } 15^\circ \text{ (assume average value of } 5^\circ)$$

Therefore

$$C_D \approx 1.6 \qquad C_M \cong 2.0 \text{ (use theoretical value of 2.0)}$$

~~$C_M \approx 1.0$ (a C_M less than 1.0 should not be used)~~

From equations 7 and 8 and the Bulletin of the Beach Erosion Board, Special Issue No. 1, July 1, 1948

$$K_1 = \frac{1.389\ (1.880) - 2.129 + 1.000}{2(1.880)} = 0.395$$

$$K_2 = \frac{1/2[2.777]^2 + 2.777(8.006) - 8.068 + 1}{64\ (1.880)^2} = 0.0837$$

From equation 6 the terms,

$$(d + z)\ K_3 = 0$$

$$(d + z)\ K_4 = 0$$

From equation 11

$$\beta = \sin^{-1} \frac{\pi(1.5)\ 1.0\ (0.395)}{8\ (10)\ 1.6\ (0.0837)} \ \text{~~= 0.1737~~} = 10^\circ$$

2.0 ? = arcsin 0.349 = 20° ?

Equation 6 reduces to,

$$M_z = \rho \frac{H^2 L^2 D}{T^2} \left\{ \pm C_D K_2 \cos^2\theta + \frac{\pi}{4} \frac{D}{H} C_{M_i} \sin\theta \right\}$$

$$= \frac{2.0(10)^2(452)^2(1.5)}{(10)^2} \left\{ +1.6(0.0837)(0.9848)^2 + \frac{\pi}{4} \frac{1.5}{10} \text{~~(1.0)~~ (0.395)(2.0)} (0.1737) \right\}$$

$$= 612{,}000 \left\{ 0.1294 + \text{~~0.0205~~ } 0.0162 \right\} = \text{~~91,600~~ } 99{,}000 \text{ ft.lbs.}$$

SAMPLE CALCULATION II

TOTAL MOMENT ABOUT THE BOTTOM OF A PILE

Conditions

$H = 10$ ft. $\qquad T = 10$ sec.

$d = 100$ ft. $\qquad D = 6$ ft.

Calculations ($z = -100$ ft.)

$$d/L = 0.221$$

$$H/L = 0.0221$$

$$D/H = 0.6$$

L = 452 ft. (same as for Sample Calculation I)

$$\frac{D^2 d}{H^2 L} = \frac{6^2 \times 100}{10^2 \times 452} = 7.97 \times 10^{-2}$$

Hence from Fig. 3

$$\beta \approx 54^\circ \text{ to } 90^\circ \text{ (assume average value of } 70^\circ\text{)}$$

Therefore

$$C_D \approx 2.0$$

$$C_M \approx 2.0$$

Similarly, as in Sample Calculation I,

$$K_1 = 0.395; \qquad (d + z)\, K_3 = 0$$

$$K_2 = 0.0837; \qquad (d + z)\, K_4 = 0$$

$$\beta = \sin^{-1} \frac{\pi(6)\ 2.0\ (0.395)}{8\ (10)\ 2.0\ (0.0837)} = \sin^{-1}(1.11), \therefore \text{ use } 90^\circ$$

Equation 6 reduces to

$$M = \rho \frac{H^2 L^2 D}{T^2} \left\{ \frac{\pi}{4} \frac{D}{H} K_1 C_M \ (1) \right\}$$

$$= \frac{2.0\ (10^2)(452)^2(6)}{10^2} \left\{ \frac{\pi}{4} \frac{6}{10} (0.395) (2.0) \right\} = \text{~~2,310,000 ft.lbs.~~ } 916,600 \text{ ft lbs}$$

REFERENCE

Morison, J.R., O'Brien, M.P., Johnson, J.W., and Schaaf, S.A. (1950). The force exerted by surface waves on piles: Petroleum Transactions, Amer. Inst. Mining Engineers, vol. 189, pp. 149-154.