

EDDY VISCOSITY PROFILES FOR WAVE BOUNDARY LAYERS: VALIDATION AND CALIBRATION BY A k - ω MODEL

Rafik Absi¹; Hitoshi Tanaka²; Loreline Kerlidou³ and Alicia André⁴

Eddy viscosity in wave boundary layers is a key parameter in coastal engineering. Two analytical eddy viscosity profiles present a particular interest for practical applications: the *parabolic-uniform* profile (Myrhaug 1982, van Rijn 1993, Liu and Sato 2006) and the *exponential-linear* profile (Gelfenbaum and Smith 1986, Beach and Sternberg 1988, Hsu and Jan 1998, Absi 2010). The aim of our study is to assess and validate these two profiles by: (1) investigation of eddy viscosity in steady fully developed plane channel flow; (2) comparisons with numerical results of the two equation baseline (BSL) k - ω model (Menter 1994, Suntoyo and Tanaka 2009). Our study shows that these two profiles are able to describe the eddy viscosity distribution in the wave bottom boundary layer but for different wave conditions given by the parameter a_m/k_s , where a_m is the wave orbital amplitude and k_s the equivalent roughness. The *exponential-linear* profile is adequate for $a_m/k_s < 500$, while the *parabolic-uniform* profile is more appropriate for $a_m/k_s \geq 500$. We suggest empirical formulations for the different coefficients which appear in these two profiles based on numerical results of the BSL k - ω model.

Keywords: wave boundary layers; eddy viscosity profiles; two-equation k - ω model, validation, calibration

INTRODUCTION

The resolution of some coastal engineering problems such as coastal erosion, transport of sediments and pollutants needs the knowledge of the boundary layer flow in the vicinity of the sea bed. The turbulent boundary layer generated by a sinusoidal wave has been the subject of many researchers: Kajiuira (1968), Brevik (1981), Myrhaug (1982), Sleath (1990), Fredsoe and Deigaard (1992), Nielsen (1992), You *et al.* (1992), van Rijn (1993), Tanaka and Thu (1994), Madsen and Salles (1998) and others.

The distribution of eddy viscosity within the oscillatory boundary layer generated by waves in shallow waters is a key parameter in coastal engineering. Adequate description of eddy viscosity distribution in boundary layers is needed for accurate predictions of velocity profiles. Eddy viscosity is also related to sediment diffusivity which is involved in the calculation of sediment concentration profiles and therefore for the prediction of sediment transport by waves. For practical applications in coastal engineering, the eddy viscosity is obtained by mathematical models often based on statistical modeling of turbulence as two-equation models (k - ϵ , k - ω , ...), or by simple analytical or empirical models. Different assumptions were made about the variation of eddy viscosity with height in the bottom boundary layer: Kajiuira (1968), Lundgren (1972), Smith (1977), Grant and Madsen (1979), Brevik (1981), Myrhaug (1982). We will investigate particularly two analytical eddy viscosity profiles for wave boundary layers namely, the *parabolic-uniform* profile (Myrhaug 1982, van Rijn 1993, Liu and Sato 2006) and the *exponential-linear* profile (Gelfenbaum and Smith 1986, Beach and Sternberg 1988, Hsu and Jan 1998, Absi 2010).

In order to assess and validate these profiles, we will first investigate eddy viscosity profiles obtained: (1) for steady flows, as a test case we will consider the fully developed plane channel flow; (2) from a two-equation k - ω model for the case of oscillatory flows. The validation and calibration of analytical profiles need comparisons with the period-averaged eddy viscosity obtained from the BSL k - ω model for different wave conditions through the parameter a_m/k_s . The aim of this study is to provide engineers simple analytical tools for practical use.

TIME-INVARIANT EDDY VISCOSITY PROFILES FOR BOTTOM BOUNDARY LAYERS

Eddy viscosity in turbulent oscillatory boundary layers can be assumed to be a time-independent and real-valued parameter (You *et al.* 1992). Different time-invariant eddy viscosity profiles (figure 1) were proposed for oscillatory flows by Kajiuira (1968), Brevik (1981), Myrhaug (1982) and for combined steady and oscillatory flows by Lundgren (1972), Smith (1977), Grant and Madsen (1979).

¹ EBI-IPSL, Université Paris Grand Ouest (UPGO), 32 Bd du Port, Cergy-Pontoise, 95094, France

² Department of Civil Engineering, Tohoku University, 6-6-06 Aoba, Sendai 980-8579, Japan

³ EBI-IPSL, Université Paris Grand Ouest (UPGO), 32 Bd du Port, Cergy-Pontoise, 95094, France

⁴ EBI-IPSL, Université Paris Grand Ouest (UPGO), 32 Bd du Port, Cergy-Pontoise, 95094, France

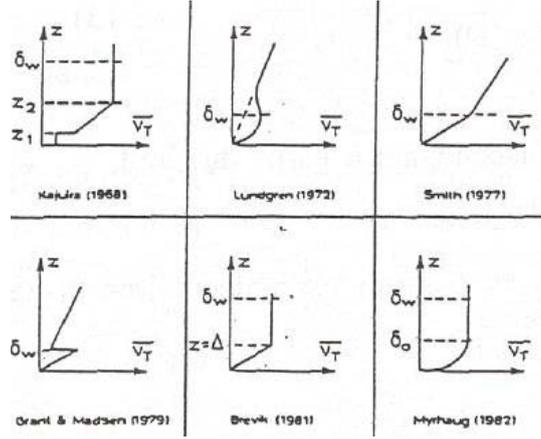


Figure 1. Assumptions for the variation of eddy viscosity with height in the bottom boundary layer (adapted from Fredsoe and Deigaard, 1992).

Among the eddy viscosity profiles for wave boundary layers, two seems to present a particular interest.

The first is given by a parabolic variation of eddy viscosity until the half of the boundary layer thickness then a uniform value (Myrhaug 1982, van Rijn 1993, Liu and Sato 2006, van Rijn 2007). We refer to this first profile as the parabolic-uniform profile. It is given by:

$$v_t = \begin{cases} \kappa u_* z \left(1 - \frac{z}{\delta}\right) & \text{if } z < \delta/2 \\ v_{t \max} & \text{if } z > \delta/2 \end{cases} \quad (1)$$

where κ is the von Karman constant ($=0.41$), u_* the friction velocity, z the distance from the bottom and δ the boundary layer thickness.

The second is given by an exponential law modulated by a linear function (Gelfenbaum and Smith 1986, Beach and Sternberg 1988, Hsu and Jan 1998, Absi 2010). We refer to this second profile as the exponential-linear profile. It is given by:

$$v_t = \alpha \kappa u_* z \exp\left(-C_A \frac{z}{\delta}\right) \quad (2)$$

where α and C_A are two coefficients.

We write equations (1) and (2) in dimensionless form respectively as:

$$v_{ta} = \begin{cases} \kappa C_L z_a (1 - z_a) & \text{if } z_a < 0.5 \\ v_{ta \max} & \text{if } z_a > 0.5 \end{cases} \quad (3)$$

and

$$v_{ta} = \alpha \kappa z_a \exp(-C_A z_a) \quad (4)$$

with $v_{ta} = v_t / (u_* \delta)$; $z_a = z / \delta$ and $v_{ta \max} = (\kappa C_L) / 4$. In equation (3), we introduced an adjustment coefficient C_L . The initial profile, given by equation (1), didn't contain this coefficient ($C_L = 1$).

In order to assess and validate these profiles, we will investigate eddy viscosity profiles obtained from:

- (1) Steady flows, we will consider as a test case the fully developed plane channel flow (figure 2) which is considered to be the simplest and most idealized boundary layer flow
- (2) A two-equation $k-\omega$ model applied to oscillatory flows



Figure 2. Test case: steady fully developed plane channel flow with smooth walls

ANALYSIS OF EDDY VISCOSITY IN BOUNDARY LAYERS

Analytical eddy viscosity for steady flows

In the equilibrium region of open-channel flows ($z^+ > 50$), the turbulent kinetic energy (TKE) is given by $k \approx u_* \exp(-C_k z / \delta)$ (Nezu and Nakagawa 1993). Since in the inner region the streamwise velocity profile is given by the log-law, it is possible to write a mixing length as $l_m = \kappa z \exp(-C_k z / \delta)$ and therefore Eq. (2) for eddy viscosity (Absi 2012).

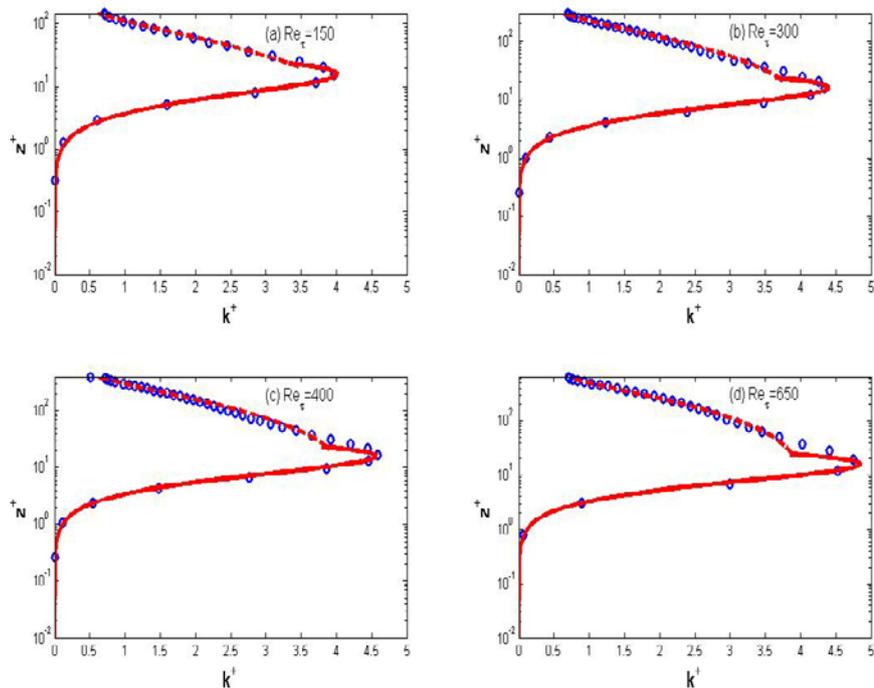


Figure 3. Analytical profiles for turbulent kinetic energy (TKE) for turbulent channel flow with smooth walls; dashed lines: TKE profile of Nezu and Nakagawa (1993); solid lines: TKE profile of Absi (2008); symbols: DNS data. (Absi *et al.* 2011, Absi 2012)

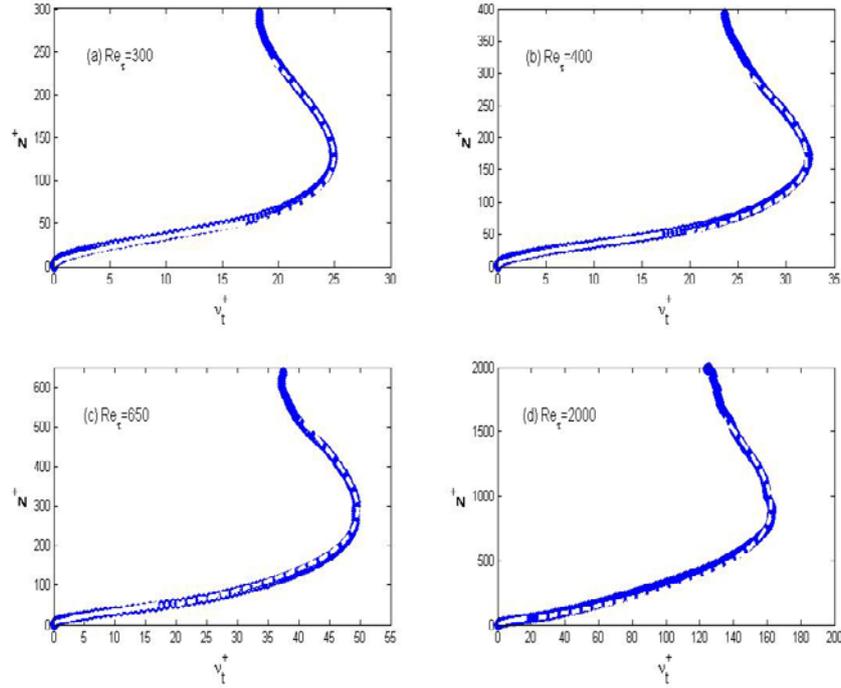


Figure 4. Analytical eddy viscosity profiles for turbulent channel flow with smooth walls; white dashed lines: Eq. (2); white solid lines: eddy viscosity based on van Driest mixing length equation; symbols (=blue bold solid line): DNS data. (Absi *et al.* 2011, Absi 2012).

Figure 3 shows TKE profiles given by Nezu and Nakagawa (1993) and Absi (2008), while figure (4) presents eddy viscosity profiles. In these figures, variables with the superscript of + are those nondimensionalized by the friction velocity and the kinematic viscosity as $z^+ = z u_* / \nu$; $k^+ = k / u_*^2$; $\nu_t^+ = \nu_t / \nu$. Comparisons with Direct Numerical Simulation (DNS) data (of Iwamoto *et al.* 2002 and Hoyas and Jiménez 2006) show that, for steady plane channel flow with smooth walls, Eq. (2) allows accurate description (white dashed lines, figure 4) of DNS in the equilibrium region (Absi *et al.* 2011, Absi 2012). For $z^+ < 50$, the eddy viscosity profile is based on van Driest mixing length equation (white solid lines, figure 4). Figure (4) shows that, for high friction Reynolds numbers, this layer ($z^+ < 50$) tends to be very small compared to the main profile given by Eq. 2 (figure 4.d). This could explain why in wave bottom boundary layers with large Reynolds numbers this layer is not observed and the profile is described mainly by Eq. (2). Eddy viscosity profiles of figure (4) are for a flat smooth wall, bottom roughness increases this effect and seems explain clearly why eddy viscosity is described only by Eq.(2) in wave boundary layers.

Eddy viscosity distribution obtained by a two-equation k- ω model

Eq. (2) for eddy viscosity is validated for the case of steady plane channel flow. However for use in wave boundary layers, we need to assess this equation for the case of oscillatory flows. Eq. (2) is therefore analyzed by the baseline (BSL) k- ω model proposed by Menter (1994). This model allows accurate prediction of velocity profiles in oscillatory boundary layers (Suntoyo and Tanaka 2009). The BSL k- ω model is a two-equation model that gives results similar to the k- ω model of Wilcox (1988) in the inner boundary layer but changes gradually to the k- ϵ model of Jones and Launder (1972) towards the outer boundary layer and the free stream velocity. The blending between the two regions is done by a blending function F1 changing gradually from one to zero in the desired region. Equations of turbulent kinetic energy k , specific dissipation ω and eddy viscosity are given respectively by:

$$\frac{\partial k}{\partial t} = \frac{\partial}{\partial z} \left\{ (v + \nu_t \sigma_{k\omega}) \frac{\partial k}{\partial z} \right\} + \nu_t \left(\frac{\partial u}{\partial z} \right)^2 - \beta^* \omega k \quad (5)$$

$$\frac{\partial \omega}{\partial t} = \frac{\partial}{\partial z} \left\{ (v + v_t \sigma_\omega) \frac{\partial \omega}{\partial z} \right\} + \gamma \left(\frac{\partial u}{\partial z} \right)^2 - \beta \omega^2 + 2(1 - F_1) \sigma_\omega \frac{1}{\omega} \frac{\partial k}{\partial z} \frac{\partial \omega}{\partial z} \quad (6)$$

$$v_t = \frac{k}{\omega} \quad (7)$$

The constants of the model are given by $\sigma_{k\omega} = 0.5$, $\beta^* = 0.09$, $\sigma_\omega = 0.5$, $\gamma = 0.553$ et $\beta = 0.075$ (Suntoyo and Tanaka 2009).

In order to allow a validation of analytical eddy viscosity profiles $v_{ta}(z_a)$ by the two-equation BSL k- ω model, we will apply the following method (figure 7):

- (1) Obtain time-dependent eddy viscosity $v_{ta}(z_a, t)$ from BSL k- ω model
- (2) Calculate the eddy viscosity averaged over one period (time-independent) $\overline{v_{ta}}(z_a)$
- (3) Compare the period-averaged eddy viscosity obtained from the BSL k- ω model $\overline{v_{ta}}(z_a)$ with the analytical profiles $v_{ta}(z_a)$

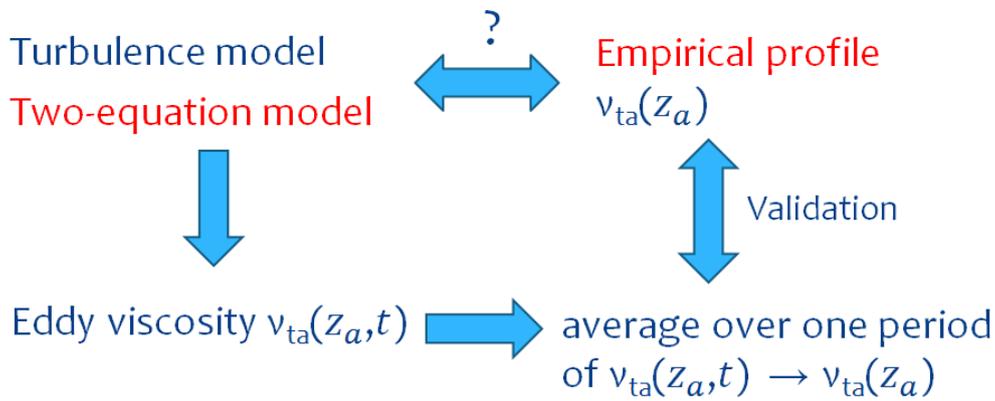


Figure 5. Summary of the approach used for the present study.

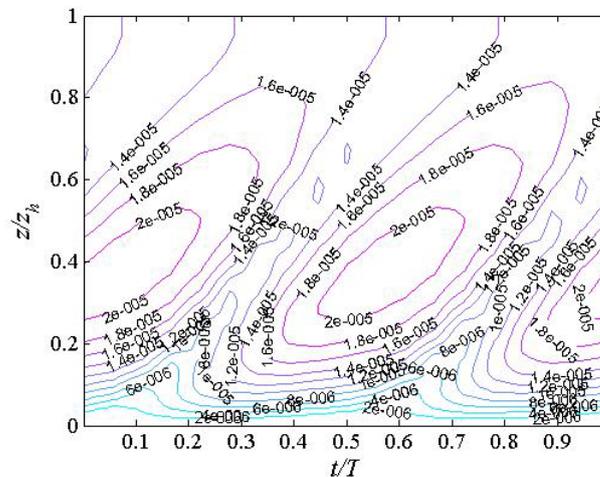


Figure 6. Temporal and Spatial Variation of dimensionless eddy viscosity for a sinusoidal wave obtained by the BSL k- ω model, Flow conditions : $U_0=3,63$ m/s ; $a_m=1,73$ m ; $T=3$ s ; $k_s=1,5$ cm, $Re=437000$.

Figure (6) presents temporal and spatial variation of dimensionless eddy viscosity for a sinusoidal wave given by $U(t) = U_0 \sin(\omega t)$. Figure (7) shows comparison between period-averaged eddy viscosity obtained from the BSL k- ω model (symbols) and analytical profile of Eq. (2) (dashed line). Even if the eddy viscosity is highly time-dependent (figure 6), the period-averaged dimensionless eddy

viscosity (Figure 7) has a shape which is well described by the *exponential-linear* analytical profile given by Eq. (2) for $z/z_h < 0.6$ (figure 7) where z_h is the water depth or the distance from the wall to the axis of symmetry (for oscillatory water tunnel experiments) or free surface.

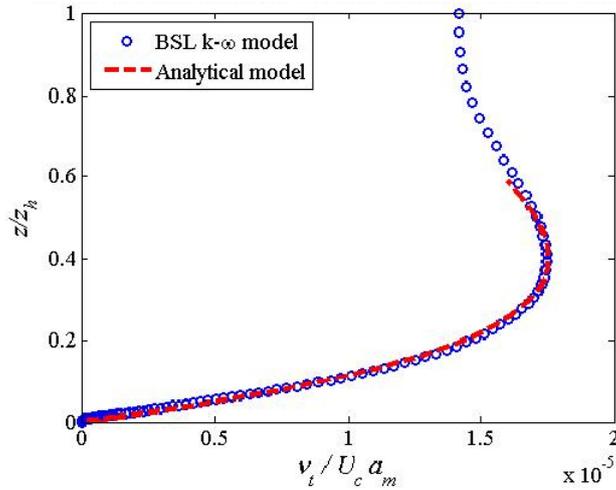


Figure 7. Period-averaged dimensionless eddy viscosity for a sinusoidal wave.

At the present stage, the *exponential-linear* profile (Eq. 2) is validated by both eddy viscosity profiles for steady plane channel flow (figure 4) and with period-averaged eddy viscosity obtained by the BSL $k-\omega$ model for sinusoidal wave for $a_m/k_s = 115$ (figure 7). However, it is important to provide a validation through comparisons for different wave conditions.

VALIDATION FOR SINUSIDAL WAVES FOR DIFFERENT FLOW CONDITIONS

Figure (8) shows that the *parabolic-uniform* profile (Eq. 1) allows a good description of eddy viscosity for $a_m/k_s \geq 500$.

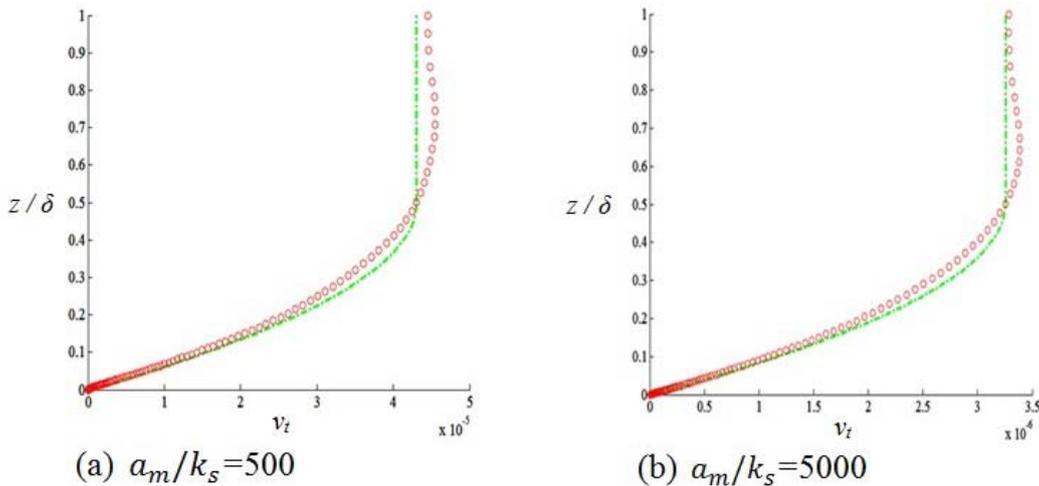


Figure 8. Validation of analytical eddy viscosity profile for $a_m/k_s \geq 500$. Symbols: period-averaged eddy viscosity obtained from the BSL $k-\omega$ model for a sinusoidal wave; curves: *parabolic-uniform* eddy viscosity profile (Eq. 1).

However, figure (9) shows that the *exponential-linear* profile (Eq. 2) allows accurate description of eddy viscosity for $a_m/k_s < 500$ and a distance from the bottom $z/\delta < 0.8$.

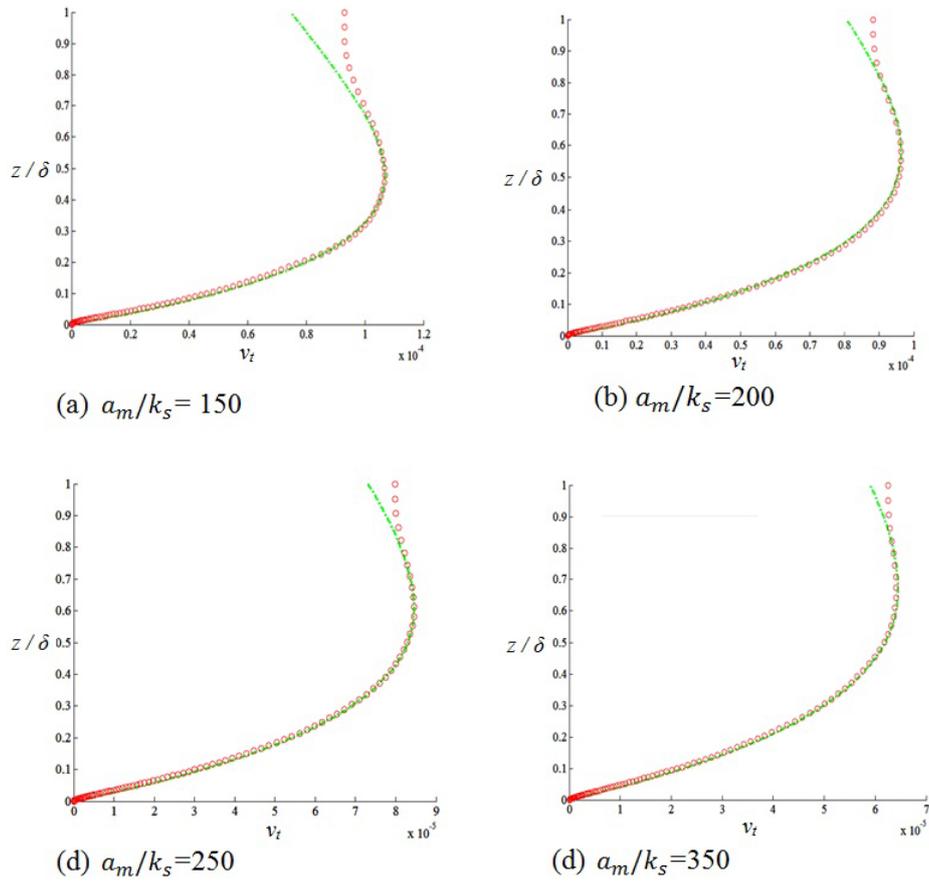


Figure 9. Validation of analytical eddy viscosity profile for $a_m/k_s < 500$. Symbols: period-averaged eddy viscosity obtained from the BSL $k-\omega$ model for a sinusoidal wave; curves: *exponential-linear* eddy viscosity profile (Eq. 2).

CALIBRATION OF THE TWO ANALYTICAL EDDY VISCOSITY PROFILES

Our study shows that the coefficients of the two analytical eddy viscosity profiles (*parabolic-uniform* and *exponential-linear*) are not constants and should depend on wave conditions through the parameter a_m/k_s .

From the analysis presented in figure (10), we propose the following equations for the different coefficients:

$$C_L = 0.4 (a_m/k_s)^{-1.1} \tag{8}$$

$$\alpha = 0.2 (a_m/k_s)^{-0.97} \tag{9}$$

$$C_A = 29.7 (a_m/k_s)^{-0.52} \tag{10}$$

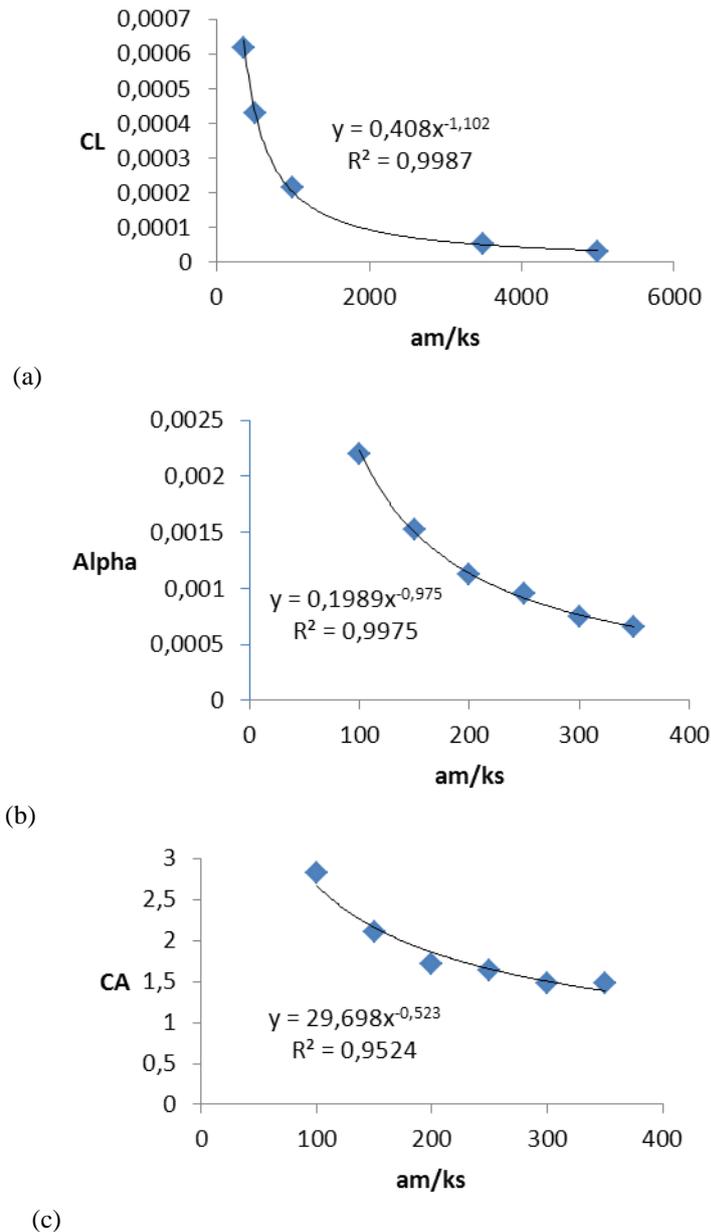


Figure 10. Calibration of the two eddy-viscosity profiles

THE CASE OF AN ASYMMETRIC WAVE

Figure (11.a) presents temporal and spatial variation of dimensionless eddy viscosity for asymmetric waves given by $U(t) = U_1 \sin(\omega t) - U_2 \cos(2\omega t)$. Figure (11.b) shows comparison between period-averaged eddy viscosity obtained from BSL $k-\omega$ model (symbols) and analytical profile of Eq. (2) (dashed line). Even for the case of asymmetric wave, the period-averaged dimensionless eddy viscosity has a shape which is well described by Eq. (2) for $z/z_h < 0.5$ (figure 11.b). Figures (7) and (11.b) shows that the period-averaged eddy viscosity profile for sinusoidal wave is different from the profile of asymmetric wave. This indicates that the period-averaged eddy viscosity profile should depend on the wave non-linearity parameter given by $N_i = (U_1 + U_2)/(2U_1)$ or $N_i = U_c/\hat{u}$, where U_c is the velocity at wave crest and \hat{u} is the total velocity amplitude. We need therefore, for asymmetric waves, a specific calibration for parameters of Eq. (2).

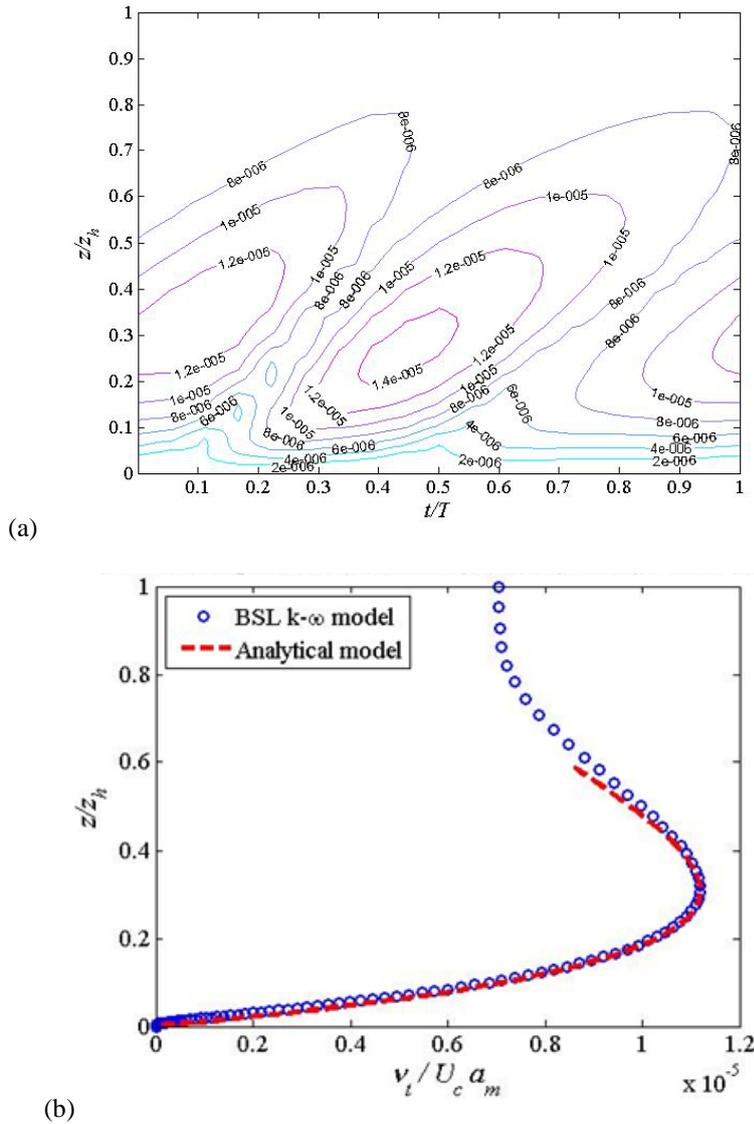


Figure 11. Dimensionless eddy viscosity for asymmetric wave $N_f=0.67$; (a) Temporal and Spatial Variation obtained by BSL $k-\omega$ model; (b) Period-averaged dimensionless eddy viscosity

CONCLUSIONS

The aim of this study was to provide simple analytical tools for practical use in coastal engineering. We validated and calibrated two analytical eddy viscosity profiles by numerical results of a two-equation $k-\omega$ model for the case of sinusoidal wave for different wave conditions (figure 12). Our study shows that:

- For practical applications the period-averaged eddy viscosity could be described by simple analytical formulations
- The *parabolic-uniform* profile allows a good description of eddy viscosity for $a_m/k_s \geq 500$.
- The *exponential-linear* profile allows accurate description of eddy viscosity for $a_m/k_s < 500$ for a distance from the bottom $z/\delta < 0.8$.
- The *parabolic-uniform* profile needs an adjustment parameter C_L
- Coefficients in the two analytical eddy viscosity profiles (*parabolic-uniform* and *exponential-linear*) are not constants and depend on parameter a_m/k_s , we proposed equations for these coefficients

- For asymmetric waves, eddy viscosity profile depends on the wave non-linearity parameter and requires therefore a specific calibration.

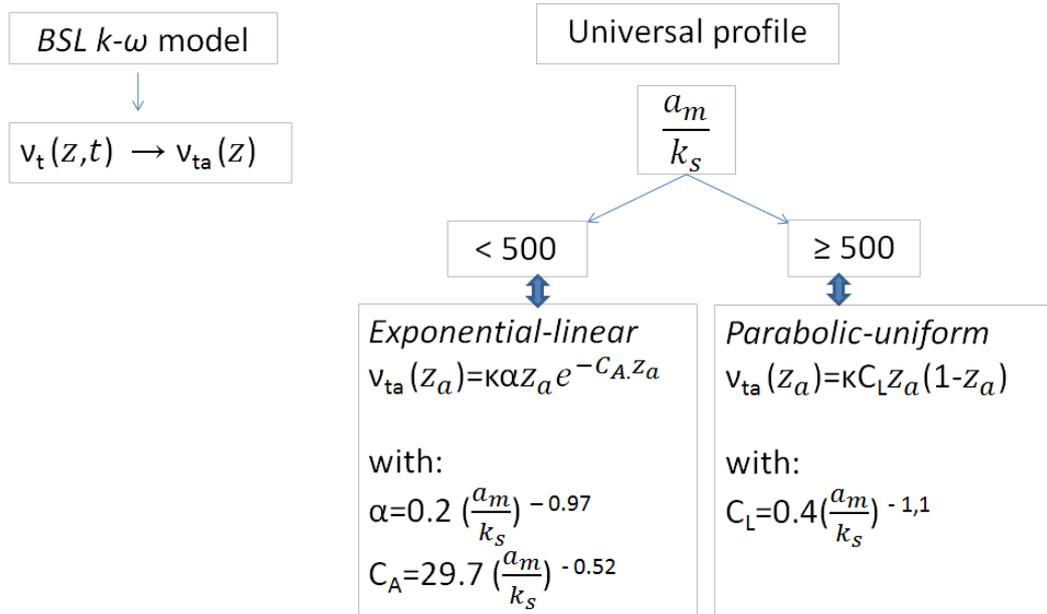


Figure 12. Summary of the present study

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