EFFECT RESONANCE ON MORPHOLOGY OF TIDAL CHANNELS

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Abstract

Tidal basins with a length of the order of 1/4 of a wave length of the dominant tide, susceptible to resonance, occur on many sites on the world. This paper will show, that these basins can become morphologically unstable. The most primitive schematization possible has been investigated: a prismatic channel, closed at one end. Under some circumstances (for which dimensionless criteria are given), small increments in depth intensify the vertical tidal amplitude at the landward side of the basin, which in turn triggers increased erosion of the basin. Knowledge on this subject can lead to redirect sand fluxes in the basin by taking small measures just in time. Starting from the assumption of a power-law sand transport formula, it is found, that for prismatic channels, morphodynamic instabilities occur when a small increment dh of depth generates a larger value[2] of Z_0/h, where Z_0 is the amplitude of the vertical tide at the landward end of the channel. This criterium can be translated in relative length of the basin and in bottom roughness (fig.2).

Method

Only the Eulerian resultant current and the first harmonic of the tidal motion have been taken into account. The morphological model consists of 3 parts: "hydraulics"; "accretion/erosion" and "morphodynamic stability".

a. hydraulic computation (the water motion in the channel):

- The first harmonic is calculated analytically with the harmonical method of LORENTZ et al. (1926). If friction would have been neglected, the incoming tidal wave and the corresponding reflected wave would have resulted in a standing wave. However, friction makes the reflected wave much weaker than the incoming wave. Thus, the character of the wave shifts from "propagating" at the entrance of the basin (vertical tide and velocity are nearly in phase) to

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2 I.e.: the generated relative increase of Z_0 should be larger than the assumed relative increase of h

"standing" at the closed end (90° phase difference between vertical and horizontal tide). The analytical computations can be presented in a dimensionless way. Results depend on angle θ, indicating the rate of bottom friction: $\theta = 0°$ for zero- and $\theta = 45°$ for infinitely large friction.

- The tide-averaged Eulerian current below the LW-level is calculated in the following way. The harmonic Eulerian horizontal velocity in each point of the vertical cross-section above the LW-line is tide-averaged. During the part of the tide, when the level considered lies above the actual water level the velocity is taken equal to zero.
  Next, these values are integrated over the height (between the LW- and HW-line), thus giving a tide-averaged residual flow rate above the LW-level. The magnitude of this Eulerian residual strongly depends on phase difference between vertical and horizontal tide.
- At the (seaward) end of the channel, the tide-averaged flux of water above LW will be relatively large and landward-directed: velocities there are high anyway and the horizontal and vertical tides are approximately in phase. At the closed end, all velocities are zero. Continuity requests a seaward-directed tide-averaged velocity below LW, increasing towards the entrance. Hence, all along the channel there must be a tide-averaged flux from the upper to the lower level, down through the LW-line (fig.1).

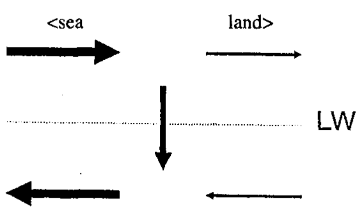


Fig 1. Rain in the drain

BAKKER & DE VRIEND (1995), consider a network instead of a single channel. They concentrate this flux in the nodes of the network and call it: "rain in the drain", because of the analogy with a drain system: water flushing in above the "pavement" (at LW-level) flushes out through the drain system, i.e. the tidal channels below LW. In the case presently considered, the "rain in the drain" is a continuous function of the horizontal co-ordinate x; its dimension is $[(m^3/s)/m^2]$. So it degenerates to a tide-averaged downward velocity (fig.1), called "w".

- the model has been checked (BAKKER & DE VRIEND (1995); fig.6) with the fully non-linear DUFLOW computer model (SPAANS et al.,1989).

b. calculation accretion/erosion rate:
- The sand transport rate is assumed proportional to the third power of the velocity, the latter being represented by its zeroth and first harmonic. Thus the tide-averaged gradient of the sand transport (=erosion/accretion rate) is calculated analytically as a function of those zero- and first harmonics and their gradients in x-direction. The gradients in the tide-averaged current are caused by the rain in the drain. Dimensionless presentation is possible again.

c. Morphodynamic stability analysis:
- The stability analysis reveals, how hydraulic conditions change, when the water depth changes and whether this can be a reason for enhanced erosion. Rate of change of depth dh/dt as function of depth h (cf.ad "b") renders a differential equation, resulting in depth as function of time. Thus (in)stability can be determined.

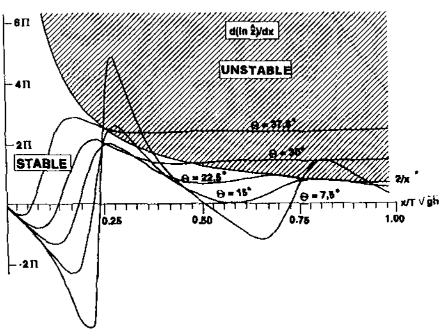
Results

For derivations refer to "Details". Morphodynamic stability depends on the ratio $L^* = L/L_0$, where L is the length of the channel and $L_0 = T\sqrt{gh}$ would be the tidal wave length, if friction would be neglected. Criterium for instability appears to be (cf.eqn. (9)) that a small increase of depth dh increases Z_0/h, where Z_0 is the amplitude of the vertical tide at the closed end of the channel. Relative growth of Z_0 should surpass the one of h. If friction is very low ($\theta = 7.5°$) instability would occur for values of $\frac{1}{4} < L^* < \frac{3}{8}$ and $\frac{3}{4} < L^* < \frac{7}{8}$ (standing-wave effects). If friction is higher, near the entrance no effect of reflection is found: here the tidal wave is a propagating, attenuating wave. After some initial deepening, reflection may reach the entrance: this will increase the tidal motion and trigger further deepening. This kind of instability occurs for $L^* > \frac{1}{2}$ ($\theta = 30°$) or $L^* > \frac{1}{4}$ ($\theta = 37.5°$).

Figure 2, Explanation:
The following criterion will be derived for instability of the channel (cf. eqn.(12)):

$$\left[\frac{d(\ln \hat{z}^*)}{dx^*}\right]_{x=L} > 2/L^* \qquad (1)$$

where x is the horizontal coordinate of the prismatic channel, starting at the closed end of the channel and pointing seaward; $\hat{z}$ is the local amplitude of the vertical tidal wave related to its value at the closed end.
The figure depicts the lefthand side of equation (1) as function of $x/T\sqrt{gh}$. Thus, for a given case, determined by L^* and friction factor θ the value of the lefthand side of (1) is determined by a point in the plot of the figure. If this point is in the gray area, the case is unstable.



Relation to other investigations; discussion of assumptions

Classifying estuary models as empirical models, semi-empirical models and dynamical models, the present model should be earmarked as a one-dimensional analytical dynamical initial morphological model. Equilibrium conditions are not assumed a priori, contrarily to many morphological models, like BRUUN & GERRITSEN, 1960, which start from assumptions that a profile adjusts itself until a certain maximum shear stress is reached.

It is a core model with a large mathematical rigidity, aiming at the demonstration of easy to understand input-output relations. Where on one hand matters as (for instance) interaction between shoal-channel interaction remain (as yet) out of scope, on the other hand no further unknowns like diffusion coefficients are introduced.

The present model is "initial": difference between absolute instability and metastability is not made. The present investigation is a part of a joint Dutch effort to improve the insight in the morphodynamics of Wadden Sea and estuaries. This concerns as well the tidal basins itself as its interaction with the adjacent outer delta's.

BAKKER & DE VRIEND, 1995 report on the general approach of this problem. One of the constituents is: modeling the behaviour of the outer delta under the influence of a combination of sources and sinks (DE VRIEND et al.,1994). Source is the sand output of ebb-channels in the outer delta, a sink (for the outer delta) is the influx of sand in the tidal basin through a flood channel.
Another constituent (one of the other subjects of BAKKER & DE VRIEND, 1995) is the quantification of such a "source": an embouching ebb-channel as most seaward channel of a network of channels in the tidal basin. Furthermore, BAKKER & DE VRIEND (1995) treat the morphological behaviour of those tidal channels itself, using the results of the present paper.
The power-law transport formula presumes immediate adjustment of sand transport to bed shear; the transport rate equals the transport capacity.
In the present model all sediment is transported below LW. This excludes the effect of fines, which explains why near the closed end of the estuary hardly any bottom change is found; partly because of low velocities, partly because horizontal and vertical tide are 90° out of phase. In practice, those areas will silt up quickly. Possibly, combination of present theory with the theory of KROL (1990), (assuming a uniform sediment concentration over the depth) could shed any light on this matter. Water motion being investigated only up to first order of accuracy, effects of imported or internally generated higher harmonics are overlooked. In practice, those might lead to sedimentation of (short) basins (VAN DONGEREN & De VRIEND, 1994). On the other hand, V.D.KREEKE & ROBACZEWSKA (1993), investigating for the Ems estuary interactions of M_0..M_6, find the tide-averaged sediment transport dominated by interactions between M_0 and M_2. Thus, for longer channels the used schematization might be appropriate.
BLIEK points out (cf. BAKKER et al. (1998)) that also in well-mixed estuaries mostly a longitudinal salt gradient will counteract the rain in the drain. At a smaller space scale the horizontal tide-averaged circulation in ebb- and flood channels will interact with the "rain in the drain" effect.
The "closed" end of the channel does not have to be strictly "closed". An extensive underwater sill (artificial? sand? silt?) with (about) the same reflection coefficient for the tide as the closure dam could react morphodynamically about in the same way[3].
Refer furthermore to ch.7 ("Discussion") of BAKKER & de VRIEND (1995).

Details.

Hydraulic calculation

Concerning the first harmonic of the vertical and horizontal tidal motion, the relevant equations of the Lorentz method are summarized in Appendix A.
Let $\zeta(x,t)$ denote the elevation of the water surface (in reference to the still-water level) and $v(x,t)$ the horizontal motion, which is assumed uniform over the depth. The positive x-axis, originating at the closed end of the channel points seaward ; t denotes time. Given channel length and depth, amplitudes $\hat{\zeta}$ and $\hat{v}$ depend upon the bottom friction, in the Lorentz method determined by the friction angle θ.
In the following, dimensionless variables (cf."Method") will be indicated with a star.

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3 This under the restriction of zero (or slightly seaward) tide-averaged drift at the site of the sill

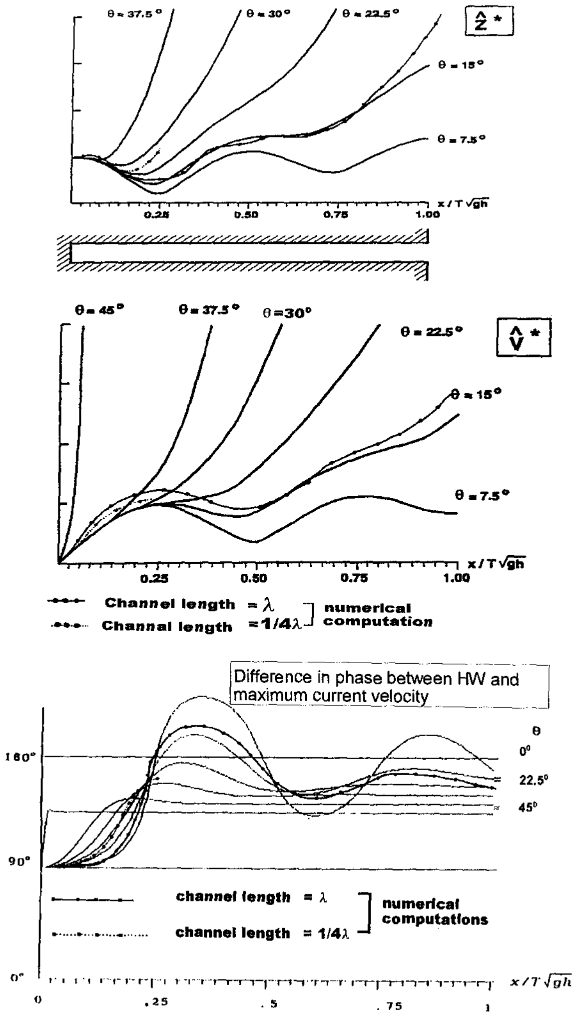

fig.3. Amplitude vertical tide and horizontal tide along the channel;
phase difference vertical tide and horizontal tide;
comparison numerical and analytical computation.
a: Vertical tide; b: plan view channel; c: horizontal tide; d: Phase difference

As elucidated in appendix A, fig.3a shows the dimensionless amplitude $\hat{\zeta}^*$ of ζ as function of x^*. Curves are given for various values of θ. In the same way, fig. 3b shows the velocity amplitude $\hat{v}^*$. For zero friction, curves would have the shape of a | cos |- and a | sin |-function, respectively.

Important for the calculation of the tide-averaged resultant current is the phase difference of the first harmonic between ζ and v, called ϕ. Fig. 3c depicts the phase difference ϕ between ζ and v. Phase difference is about 180°-θ near the seaward end and is 90° at the landward end. This indicates, that the wave has the character of a propagating wave at the seaside (moving in negative x-direction) and of a standing wave at the closed side.

Checking the values with a non-linear computation is difficult, as - because of non-linear friction - a constant Chezy- friction value does not result in a constant θ-value over the channel. Furthermore, for friction values which are physically real the reflected wave under normal conditions of real estuaries practically always will be attenuated after half a wave length or less: values of θ of 7.5° or 15° are not very likely in practice.
In order to check the mathematics, some tests with physically unrealistic values of the Chezy -value have been performed. Two cases have been checked, in which the channel length was L_0 and $L_0/4$ respectively. The channel length in either case was equal (86.4 km), but the depth h was different: .408 m, resp. 16*.408=6.41 m. The tidal wave period T was taken: 12 hours. Chezy-values C_h were: 120, resp. 60 $\sqrt{m}/s$ and the amplitude $\hat{\zeta}_s$ at the seaward boundary was taken $h/10$.
When relating C_h to θ, the velocity amplitude $\hat{v}$ enters the formula (see (A3)). In the case of a channel with a closed end, neglecting friction, one finds a channel-averaged value of $\hat{v}$:

$$\hat{v} \approx \frac{2}{\pi}.\frac{\hat{\zeta}_0}{h}.\sqrt{gh} \tag{2}$$

Here $\hat{\zeta}_0$ denotes the vertical wave amplitude in the antinodes.
Then from (A3) the following relation between relation between C_h and θ can be derived:

$$\tan(2\theta) = \frac{8}{3\pi^3}.\frac{\hat{\zeta}_0}{h}.\frac{L_0}{h}.\frac{g}{C_h^2} \tag{3}$$

For the $L_0/4$-channel a value $\theta = 25,44°$ is found. For the L_0-channel, the fact that friction is neglected in the derivation of (2) gives some ambiguity: in fact $\hat{v}$ in the most landward node (at $x \approx L_0/4$) will be much less than the one in the most seaward node ($x \approx 3L_0/4$). If for $\hat{\zeta}_0/h$ in (3) the seaward boundary value of .1 is taken,

the same value of θ results as for the $L_0/4$-channel as (3) shows[4]. However, if one takes for $\hat{z}_0/h$ the value at the landward side of the channel, which is .01377/.408, a value of 11.37° is found. Thus, for the L_0-channel θ should be in the range
$11.4° < \theta < 25.4°$
Fig.3[a,b,c] shows the correspondence between the given ranges of θ, for the amplitudes of the vertical and horizontal tide, as well as for the phase differences.

Calculation accretion/erosion rate

Let S_1 be (in magnitude) the transport (per unit of width), when the velocity is 1 m/s. Writing $S = S_1 v^3$, the appropriate unit for S_1 is [sec^2/m], since the transport is expressed in m^2/sec. The calculated transport rate is expressed in terms of deposited sediment volume. In order to obtain the solid volume, one should multiply by 1- the pore content of the bed.

Denoting tide-averaged values by <>, the third-power sand transport formula yields for the tide-averaged local erosion rate, $< \partial h/\partial t >$:

$$< \frac{\partial h}{\partial t} > = \frac{3}{2} \frac{S_1 \hat{v}^2}{h} (w + 2\delta\omega\hat{z} \sin\phi) \tag{4}$$

This equation is derived in Appendix B. Here ω is the angular velocity of the dominant tidal constituant (mostly M_2) and δ and w are defined by:

$$\delta = < v > / \hat{v} \qquad w = h \frac{\partial < v >}{\partial x} \tag{5a,b}$$

w being the rain in the drain. Starting from the definitions (5a,b) w can be expressed as:

$$w = h\left(\hat{v}\frac{d\delta}{dx} + \delta\frac{d\hat{v}}{dx} \right) \tag{6}$$

The second term inside the brackets of (4) shows the effect on the erosion $< \partial h/\partial t >$ of the phase coupling between vertical tide ζ and horizontal tide v: $\sin\phi = 1$ for a standing wave, where ζ and v are out of phase and $\sin\phi = 0$ when ζ and v are in phase).
For the derivation of (4), it has been assumed, that δ is much smaller than 1.
Remembering that gS_1 is dimensionless one may write (4) in a dimensionless form, yielding:

$$\frac{\partial h^*}{\partial(\omega t)} = -\frac{g S_1}{(h^{*2})(h^* - 1)} f(x^*, \theta) \tag{7}$$

4 for the long channel $\hat{z}_0/h$ remains the same; L_0 is divided by 4 and h is divided by 16; C_h is multiplied by 2

Function $f(x^*, \theta)$ (of which the exact expression is derived in appendix C) follows from (4) to (7) and the harmonical tidal theory (LORENTZ et al. (1926); THIJSSE;(1965)). It is depicted in fig.4.

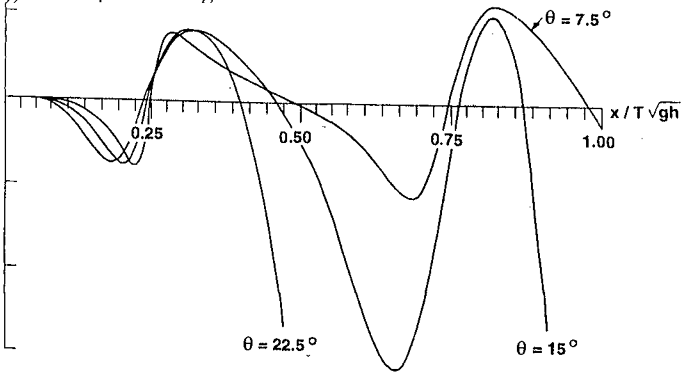

Fig.4. Dimensionless accretion/erosion
as function $f(x^*, \theta)$ of site and bottom friction

Morphodynamic stability analysis

With the aid of (7), the morphological instability of a channel can be traced. Here instability will be defined as an acceleration of erosion ($\partial^2 h/\partial t^2 > 0$).
Mechanism will be: increased h gives increased tidal wave length, whence the ratio "channel length/ wave length" becomes less. When originally the channel was somewhat larger than 1/4 of a wave length, a subsequent decrease of the relative channel length enhances the wave amplitude Z_0 at the closed end of the channel. This may enhance the erosion process in the channel, etc.
In the following some simplifying assumptions will be made: the effect of changes in friction (in angle θ and thus in $f(x^*, \theta)$) will be neglected. As θ tends to decrease during the erosion process described, the probability of instability will be underestimated.
Let x_s^* be the dimensionless channel length and let $\bar{f}$ be the mean value of f between 0 and x_s^*.
The tide-averaged equivalent of eqn. (7) shows, that if x_s^* is such, that $\bar{f} > 0$ no instability has to be expected.
Investigate now the case: $\bar{f} < 0$. Considering eqn. (7) as a differential equation in a dimensionless depth and a dimensionless time, it seems logical[5] to incorporate the dimensionless $gS_1\bar{f}$ in the reference time scale, call t_0:

$$t_0 = -T/(2\pi \bar{f} g S_1) \tag{8}$$

5 Thus switching from the hydraulic time scale to the morphological time scale,

Note that in the present case ($\bar{f} < 0$) t_0 will be larger than zero.

For convenience, the dependent variable h^* in (7), equal to h/Z_0, will be replaced by $h' = h/\hat{z}_s$, in which $\hat{z}_s$ is the amplitude at the seaward boundary, For h' only changes in time if h changes, where in h^* a change of h affects also the reference, i.e., the vertical tide Z_0 at the closed end of the channel. Denoting t/t_0 by t', and $\hat{z}_s/Z_0$ by $\hat{z}_s^*$, the non-dimensional depth-evolution equation becomes:

$$\frac{\partial h'}{\partial t'} = \left[(h'\hat{z}_s^*)^2 (h'\hat{z}_s^* - 1)\right]^{-1} \tag{9}$$

where $h'\hat{z}_s^*$, equal to h/Z_0 can be assumed much larger than 1. Instability occurs, if the righthand part of eqn.(9) increases with h'. This implies, that h/Z_0 should decrease as h' (or h) increases[6]. In other words: instability occurs when $\partial(h'\hat{z}_s^*)/\partial h'$ is negative. Thus the stability criterion is:

$$\hat{z}_s^* + h\frac{\partial \hat{z}_s^*}{\partial h} > 0 \quad \Rightarrow stable \tag{10a}$$

$$1 + h\frac{\partial(\ln \hat{z}_s^*)}{\partial h} > 0 \Rightarrow stable \tag{10b}$$

Here one finds $\partial(\ln \hat{z}_s^*)/\partial h$ as:

$$\frac{\partial(\ln \hat{z}_s^*)}{\partial h} = \frac{\partial(\ln \hat{z}_s^*)}{\partial x^*} \cdot \frac{\partial x^*}{\partial h} \tag{11}$$

Differentiation of $x^* = x/T\sqrt{gh}$ to h gives $-x^*/(2h)$ as result. Thus the stability criterion becomes:

$$\frac{\partial(\ln \hat{z}_s^*)}{\partial x^*} < \frac{2}{x^*} \quad \Rightarrow stable \tag{12}$$

Here x^* denotes the channel length, expressed in $T\sqrt{gh}$. In fig. 2, both $\partial(\ln \hat{z}_s^*)/\partial x^*$ and $2/x^*$ are plotted. All the points above the line $2/x^*$ (which corresponds with a certain dimensionless channel length and a certain friction angle θ) refer to potential unstable channels, provided that $\bar{f}$ (cf. fig 4) is negative.
When the channel length is less than about a quarter of a wave length, no instabilities are found. However, longer channels with a moderate bottom friction should theoretically become often unstable.

6 This was, mentioned in "Results"

Acknowledgements.

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Appendix A. The Lorentz theory

The Lorentz theory starts from the linearized equation of motion and the continuity equation, viz.:

$$\frac{\partial v}{\partial t}+g\frac{\partial \zeta}{\partial s}+(\omega \tan 2\theta)v=0 \qquad (A1)$$

$$\frac{\partial \zeta}{\partial t}+h\frac{\partial v}{\partial t}=0 \qquad (A2)$$

Here, $\omega \tan 2\theta$ is a linearized friction coefficient, related to the Chezy coefficient C_h in the following way:

$$\omega \tan(2\theta)=\frac{8g}{3\pi C_h^2}\frac{\hat{v}}{h} \qquad (A3)$$

For the case of a prismatical channel, closed on one side, the following solution is found, which can be presented in a dimensionless way.

Define the complex function $Z(kx)$ in such a way, that

$$\zeta(x,t)=Z_0 Re\{Z(kx)e^{i\omega t}\} \qquad (A4)$$

and the complex function V such, that

$$v(x,t)=T\sqrt{g/h}\,Re\{V(kx)e^{i\omega t} \qquad (A5)$$

where the wave number k equals:

$$k=\frac{\omega}{c}(1-i.\tan\theta) \qquad (A6)$$

and where the propagation velocity c of a wave component (incoming or reflected) equals:

$$c=\sqrt{gh(1-\tan^2\theta)} \qquad (A7)$$

In the paper, the horizontal coordinate is dimensionless (x^*), where in (A4) and (A5) still the original coordinate "x" is used; however, kx is dimensionless and can be written as "k^*x^*", where k^* can be derived from (A6) and (A7).
For the given boundary conditions the solution of (A1) and (A2) is:

$$Z=\cos kx \qquad V=i\sqrt{\cos 2\theta}\,.\sin kx \qquad (A8)$$

$\hat{v}$ and $\hat{z}$ are found as the modulus of Z and V and are shown in fig 3[a] and 3[c].
The difference in argument between Z and V is depicted in fig.3[d].

Appendix B. Derivation of equation (4)

The local erosion according equation (4) can be derived from the transport equation. Splitting into means and harmonics gives (indicating first harmonics by a tilde):

$$\frac{\partial S}{\partial x} = 3S_1(\tilde{v} + < v >)^2 \left(\frac{\partial \tilde{v}}{\partial x} + \frac{\partial < v >}{\partial x} \right) \qquad (B1)$$

Neglecting the effect of bottom changes, the equation for momentaneous continuity of water gives:

$$-h\frac{\partial \tilde{v}}{\partial x} = \frac{\partial \zeta}{\partial t} \qquad (B2)$$

According to (B2) $\partial\tilde{v}/\partial x$ is proportional to $\partial\zeta/\partial t$, of which the phase is 90° ahead with respect to ζ. Denoting the phase difference between $\tilde{v}$ and ζ by ϕ, the phase difference between $\tilde{v}$ and $\partial\tilde{v}/\partial x$ will be $90° - \phi$.

Choose $t = 0$ in this way, that $\tilde{v} = \hat{v} \sin \omega t$. Use of (B2), (5a) and sand continuity transfers (B1) into:

$$\frac{\partial h}{\partial t} = 3\frac{S_1 \hat{v}^2}{h} < (\sin \omega t + \delta)^2 \left\{ \omega \hat{z} \sin\left(\omega t + \frac{\pi}{2} - \phi \right) + w \right\} > \qquad (B3)$$

Neglect of δ^2-terms leads to (4), as all odd powers of $\sin \omega t$ and combinations of $\sin \omega t$ and $\cos \omega t$ cancel during the time-averaging process.

Appendix C. THE FUNCTION $f(x^*, \theta)$

The function $f(x^*, \theta)$ has been derived from (4). The equation has been made dimensionless by dividing by $-\omega Z_0$; thus emerges the lefthand side of (7).

Call $\hat{z}^*$ the dimensionless local amplitude of ζ.

For the flux above the LW-line (calculated as described in "Method") is found: $(\hat{z}\hat{v} \cos\phi)/2$.

Because of continuity, this flux should pass (in opposite direction) as well between the LW-level and the bottom. It is assumed, that the latter flux is stationary and uniform over the depth. Approximating further this separation plane between seaward and landward flux as horizontal, i.e. everywhere equal to the LW-level at the closed end of the channel, one finds for δ:

$$\delta = \frac{(\hat{z}/Z_0)\cos\phi}{2\left(\frac{h}{Z_0} - 1\right)} \qquad (C1)$$

Here $\hat{z}$ indicates the local tidal amplitude and Z_0 the amplitude at the closed end of the channel.

Furthermore δ will be replaced in this Appendix by a more concise variable δ^*:

$$\delta^* = \frac{\hat{z}^*}{2}\cos\phi \qquad (C2)$$

and thus, according to (C1), $\delta = \delta^*/(h^* - 1)$. Thus (6) reads, in a dimensionless way:

$$\frac{w}{\omega Z_0} = \frac{1}{2\pi(h^* - 1)} \left(\hat{v}^* \frac{\partial \delta^*}{\partial x^*} + \delta^* \frac{\partial \hat{v}^*}{\partial x^*} \right) \qquad (C3)$$

Substitution of (C3) into (4) gives, in a dimensionless shape:

$$\frac{\partial h^*}{\omega t}=\frac{gS_1}{h^{*2}(h^*-1)}\cdot\frac{3}{2}\hat{v}^{*2}\cdot\left[\frac{1}{2\pi}\left(\hat{v}^*\frac{\partial\delta^*}{\partial x^*}+\delta^*\frac{\partial\hat{v}^*}{\partial x^*}\right)-2\delta^*\hat{z}^*\sin\phi\right] \tag{C4}$$

Using (7) and (C4), and after after substitution of (C2), one can express $f(x^*,\theta)$ in values, to be derived from the Lorentz theory (1926; app.A):

$$f(x^*,\theta)=\frac{3}{2}\hat{v}^{*2}\cdot\left[\frac{-1}{4\pi}\left(\hat{v}^*\hat{z}^*\sin\phi\frac{d\phi}{dx^*}+\hat{v}^*\cos\phi\frac{d\hat{z}^*}{dx^*}+\hat{z}^*\cos\phi\frac{d\hat{v}^*}{dx^*}\right)+\hat{z}^{*2}\cos\phi\sin\phi\right] \tag{C5}$$

This function $f(x^*,\theta)$ has been depicted in fig.4. It is memorized, that in the Lorentz theory vertical and horizontal tide are found as the real part of the complex values z^* and v^* (made dimensionless according to (A1a,b) and to be multiplied with $\exp(i\omega t)$):

$$Z=\cos kx \qquad V=i\sqrt{\cos 2\theta}\,.\sin kx \tag{C6}$$

with:

$$k=\frac{\omega}{c}(1-i\quad\tan\theta) \qquad c=\sqrt{gh(1-\tan^2\theta)} \tag{C7a,b}$$

The difference in argument between Z and V is depicted in fig.3[d]; $\hat{z}$ and $\hat{v}$ are found as the modules of Z and V.

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Symbols

c	propagation velocity of a sinusoidal wave
C_h	Chezy coefficient
f	local accretion in the channel, expressed in a dimensionless way
g	acceleration of gravity
h	water depth
i	$\sqrt{-1}$
k	wave number (cf.(A6))
h'	$h/\hat{z}_s$
h^*	h/Z_0
L_0	wave length (if friction could be neglected): $T\sqrt{gh}$
n	the sand transport is assumed to be proportional to the n-th power of the velocity. In the paper, $n = 3$ is assumed.
t	time
t'	t/t_0
t_0	reference time scale as given by (7)
T	tidal period
v	instantaneous water velocity in seaward direction in the channel
v^*	$v/(Z_0\sqrt{g/h})$
V	complex, dimensionless presentation of v (cf. (A5))
w	rain in the drain (cf."Method; hydraulics" and eqn. (4b)
x	horizontal coordinate; the origin is at the closed side of the channel
x^*	$x/(T\sqrt{gh})$
z	vertical coordinate (positive upward)
$\hat{z}$	amplitude of ζ
$\hat{z}^*$	$\hat{z}/Z_0$
$\hat{z}_s$	amplitude vertical tide at the seaward end of the channel
$\hat{z}_s^*$	$\hat{z}_s/Z_0$
Z	complex, dimensionless presentation of ζ (cf.(A4))
Z_0	amplitude vertical tide at the closed end of the channel
δ	$<\mathrm{v}>/\hat{\mathrm{v}}$
δ^*	$\delta(h^* - 1)$
θ	friction angle (cf.(A3))
ϕ	phase difference between horizontal tide v and vertical tide ζ
ζ	elevation of water surface (function of x and t
ω	$2\pi/T$