RANDOM WAVE MODELLING APPROACH INCLUDED
IN A BEACH DEFORMATION MODEL

Ioan Nistor[1] and Tomoya Shibayama[2]

Abstract

The paper presents the results of a study dealing with the inclusion of wave randomness into a beach deformation and a sand transport model developed for the regular wave input. The effect of wave irregularity is considered by employing the significant wave characteristics method and a joint distribution of wave heights and wave periods. The theoretical framework and the practical method for obtaining the joint distribution together with its application procedure are presented. The validity of the random wave approach modelling is verified by applying it for two different models: a time-averaged beach deformation model and a time-dependent sand transport model. The time-averaged model is capable of computing the beach deformation profiles while the time-dependent turbulent sand transport model is capable of computing the total sand transport rate over the entire surf zone. The simulation results for both numerical models are compared with laboratory data in order to verify, analyze and discuss the validity of the random wave approach as presented.

1. Introduction

Over the last years, numerous studies have been carried out for modeling beach profiles evolution under various wave conditions. Most of these efforts, whether physical or numerical, have been conducted for regular waves. Since the theoretical assumptions for many of the regular wave input models are reasonable and based on detailed physical and numerical assumptions, it is therefore necessary to try to extend the results of such models for the case of random waves. In natural conditions, the state of the sea is irregular. The present theoretical models, with a few exceptions, are considering the wave input as a regular one. Despite their usually good results, when compared to laboratory

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[1]Doctoral Student, Dept. of Civil Engineering, Yokohama National University, 79-5 Tokiwadai, Hodogaya-ku, Yokohama 240-8501, JAPAN.
[2]Professor, ditto.

data dealing with regular waves, the practical purpose of such models is limited when it comes to practical applications since wave randomness, the true state of the ocean waves, is not included. The irregularity of the waves should be included in a theoretical model whose aim is – beyond its theoretical value - some level of practical application. Researchers opinioned that the relatively low level of understanding the sediment transport problems considerably slowed the introduction of the wave irregularity concept within sediment transport models. However, at this point, it is still difficult to incorporate a complicated random wave model coupled with the Navier-Stokes equations (even in their Reynolds averaged form) due to the very large computational effort required.

2. Random Wave Modelling

Random waves should be taken into account for a realistic modelling of the surf zone phenomena. For the practical application, Wu *et al.* (1994) discussed three basic approaches for the treatment of the random wave transformation in shoaling water and through the surf zone.

(1) Using a deepwater random wave time series as input and transforming each individual wave in the series as if it were a regular wave component with a distinct wave amplitude and period. The input deepwater time series could be generated from (a) real-time data series, (b) simulated from a given spectrum or (c) simulated from a given distribution function.

(2) Carrying out a spectral transformation, first from deepwater into shallow water. A time series is then created from the shallow water spectrum for further shoaling and surf zone transformation of each individual wave. The first step is equivalent to the transformation of Fourier components under the constraint of energy conservation. Some numerical models were used to accomplish this transformation.

(3) Conducting a "parametric" type transformation of deepwater random wave directly into the surf zone. The surf zone wave properties are then expressed as significant wave parameters and, sometimes, associated with distribution functions.

Each approach presented above can be considered as random wave input information. Approach (1) and (2) are conceptually the same. The third approach yields local wave information inside the surf zone but is difficult to reconstruct a continuous spatial variation of each wave which is required in some of the sand transport models. Many surf zone hydrodynamic or sand transport models focus on using individual wave analysis since this was proved to be more suitable for the surf zone. The probabilistic approach, which considers individual waves in the time domain, can be used to predict the distribution of wave heights and wave periods. In the surf zone, energy dissipation produces a strong coupling between components of the spectrum that is not well known. Therefore, a detailed description of the wave spectrum is not needed - in most of the practical cases - to compute wave-driven currents and sand transport. As a result, the researchers proposed simpler parametric approaches, which seek to reduce the computational effort - implicit in a fully discrete spectral model - by expressing the wave action balance in terms of a small number of characteristic parameters.

For the present case, a joint distribution of wave heights and wave periods is proposed. The method and theoretical framework for obtaining this distribution is further on presented.

Joint Distribution of Wave Heights and Wave Periods

In the last few years, few reliable experiments involving irregular wave action were performed in different laboratories around the world. In spite of the fact that not all-complete data parameters are available to public use (sometimes they are still being processed or incompletely reported), part of them can be used for the kind of analysis which the present work is dealing with.

The experiment involving irregular waves which is used for the present study is the Beach Deformation Research Project - BDRP – which was performed at the Central Research Institute for Electric Power Industry (CRIEPI), Japan in October 1995. All the cases involved in the BDRP experiment have several common features:

- were performed in Large Wave Flumes;
- all involved random waves action on a sloping sandy beach;
- all cases investigated the cross-shore hydrodynamics, sand transport and beach profile changes processes;

Two approaches were considered for including the irregularity of the input wave data (1) using significant wave parameters such as the significant wave height, $H_{1/3}$, and significant wave period, $T_{1/3}$, and (2) using a joint distribution of wave heights and wave periods.
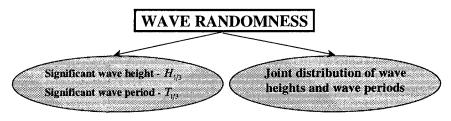

Fig. 1: Wave randomness input for the present study.

In order to obtain the above-mentioned elements, the time-history of water surface elevation was necessary. For the case of the BDRP, Japan, the time-history of water surface elevation was available, so that it was possible to calculate the joint distribution of wave heights and wave periods. The details of the experimental set-up are not described in detail in the present material. A full description of the experimental facility and of the three cases analyzed in the present paper can be found in the paper of Shimizu *et al.* (1996).

The time-history of the water surface elevation throughout the entire surf zone was available for the case of the present experiment as recorded by a number of wave gages. For this latter case, the zero-down crossing method was employed, obtaining thus the time series of waves which were generated for all the three cases performed during the

laboratory experiments. For all cases, the number of raw data from the time-history of water surface elevation was 16384 (approximately 13.6 minutes duration) while the sampling frequency was 20 Hz. A Fortran program was developed to smooth the raw water surface data by using a low passing filter with a cut frequency of 5 Hz. Then, the zero-down crossing method was employed. The final outputs of zero-down crossing method were the wave characteristics (wave heights and periods) for all the waves in the series. Figure 2 explains the procedure used to obtain the joint distribution of wave heights and wave periods.
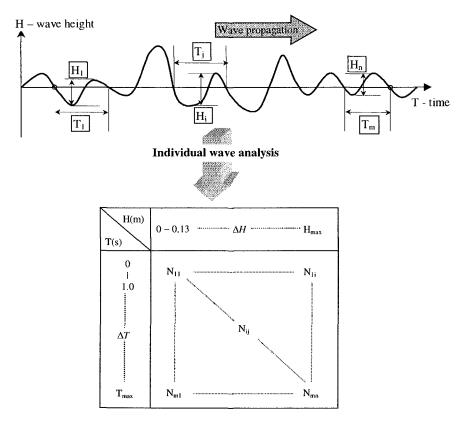

Fig. 2: Calculation of the joint distribution of wave heights and wave periods.

The columns represented equal wave period intervals $(\Delta T = 1.0 \sec)$ while the rows represented wave height intervals $(\Delta H = 0.13 \, cm)$. The size of the intervals was defined in correlation with the maximum recorded wave height, H_{max}, and maximum recorded wave period, T_{max} Then, all waves in the three series were considered in order to

determine the number (or percentage of the total number of waves) of waves corresponded to each matrix location (e.g., what number (or percentage) of the total number of waves have the period in the interval between 5.0 and 6.0 seconds and the height in the interval between 0.40 and 0.53 m). All the waves in the time series were thus analyzed so that a particular distribution was obtained for all the three cases.

For these cases, 1a, 1b and 2 of the BRDP experiment, the wave series were analyzed and the results of the joint distribution are presented in Figs. 3, 4 and 5.
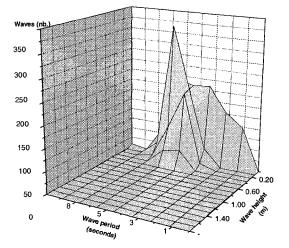

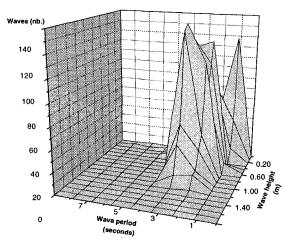Fig. 3: Joint distribution of wave heights and wave periods for Case 1a (BDRP, Japan).

Fig. 4: Joint distribution of wave heights and wave periods for Case 1b (BDRP, Japan).
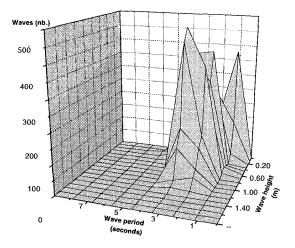
Fig. 5: Joint distribution of wave heights and wave periods for Case 2 (BDRP, Japan).

Basically, the irregularity of the wave series is reduced to a number of regular waves having their characteristics (heights and periods) comprised within certain selected intervals. As a result, it is possible to input the joint distribution output to any numerical model, which was developed for the regular wave input.

The values $N_{i,j}$ represented on the vertical axes of the Figs. 3, 4 and 5 are the numbers of waves having the wave height comprised between (H_i, H_{i+1}) and the wave period within the interval (T_j, T_{j+1}). For each matrix element, $N_{i,j}$, the mean value of wave height and wave period, $(\overline{H_i}, \overline{T_j})$ is calculated as

$$\overline{H_i} = \frac{H_{i+1} - H_i}{2} \qquad (1)$$

$$\overline{T_j} = \frac{T_{j+1} - T_j}{2} \qquad (2)$$

The pairs of wave characteristics, $(\overline{H_i}, \overline{T_j})$, are taken as input data for the beach deformation model. The total duration of action for each of the pairs was obtained as

$$T_{ij} = N_{ij} \times \overline{T_j} \qquad (3)$$

However, the input data for the duration of the simulation is the entire duration of the experiment, T_d. The corresponding weight for each of the matrix elements is then calculated as

$$p_{ij} = \frac{T_{ij}}{\sum\limits_{i=1}^{n} \sum\limits_{j=1}^{m} T_{ij}} = \frac{T_{ij}}{T_d} \qquad (4)$$

The number of output data - i.e., final beach deformation profiles or sand transport rates – is thus equal to the number of matrix elements. The elements are characterized by each combination of wave heights and wave periods. Then, the values of water depth in each location, h_{ij}, (the vertical distance between the bottom and the mean water level which reflects the changes in the beach profile) are multiplied with the value of each weight, p_{ij}, obtaining thus the final beach profile. A Fortran program was developed to compute the final beach profile.

$$h = \sum_{i=1}^{n} \sum_{j=1}^{m} \left(h_{ij} \, p_{ij} \right) \tag{5}$$

The joint distribution is further verified using a numerical beach deformation model initially developed by Winyu and Shibayama (1996) and a turbulent sand transport model developed by the authors of the present paper. The beach deformation model is based on the time-averaged concept when computing the flow field and sediment concentration and the final output is represented by the beach profile change. The turbulent sand transport model is based on the time-dependent calculation of the hydrodynamic and sand concentration fields and its final result is the sand transport rate integrated over the entire water thickness and also, over one wave period. Further on, a brief description of the framework of the numerical models and their output as a result of applying the random wave approaches is presented.

3. Time-averaged Beach Deformation Model (Winyu and Shibayama, 1996)

The modified time-averaged beach deformation model of Winyu and Shibayama (1996) is composed of a sand transport model driven by a hydrodynamic model. The cross-shore change of local water depth, h, can be calculated by solving the equation of conservation of sediment mass.

Assuming a steady concentration above the bed within the control volume, conservation of sand mass can be expressed as

$$\frac{\delta h}{\partial t} = -\frac{1}{1-\lambda} \frac{\partial q_T}{\partial x} \tag{6}$$

where t : the time, x : the horizontal coordinate in cross-shore direction, λ : the sand porosity and q_T : the total sand transport rate per unit width. In order to estimate the water depth change, the sand transport rate has to be evaluated at each local point.

A two-layer sand transport model is considered. In the upper layer, the transport rate is computed as a product of time-averaged suspended sand concentrations and time-averaged particle velocities at respective elevations. Integrating the product over the upper layer one can obtain the suspended load. The transport rate in the bottom boundary layer (bedload) is calculated in the form of an empirical formula. Therefore, the total load at the local point of the cross-shore sand transport rate, q_T, is expressed as

$$q_T = q_S + q_B \tag{7}$$

where q_S : the suspended sand transport rate and q_B : the bedload transport rate. In order to compute the total sand transport rate, q_T, the sand concentration, fluid velocity and

bed load should be known first. Figure 6 shows the functional diagram of the 2-DV time-averaged beach deformation model.
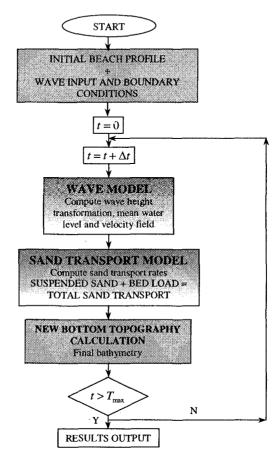

Fig. 6: Beach deformation model - functional diagram.

The Wave Model

In order to calculate the sand transport rate and beach profile change, wave height at each location must be computed. Wave height transformation in cross-shore direction is calculated using the energy flux conservation method in the form

$$\frac{\partial Ec_g}{\partial x} = -D_B \tag{8}$$

where, E : the wave energy density, c_g : the wave group velocity and D_B : the energy dissipation rate which is zero outside of the surf zone. The energy dissipation rate is

calculated assuming its proportionality with the difference between the local energy flux and the stable energy flux.

$$D_B = \frac{K_d}{h}\left[Ec_g - \left(Ec_g\right)_s\right]$$ (9)

where all the variables are computed using linear wave theory, so that $E = \rho g H^2/8$: the wave energy, $K_d = 0.15$: a constant, h : the local water depth while subscript s means "stable". The advantage of this procedure is that it is able to reproduce the pause (or stop) in the wave breaking process at a finite wave height on a horizontal bed or in the recovery zone.

The Sand Model

A two-layer sand transport model is considered. The upper layer includes the suspended sand while the lower one defines the bedload. The transport mechanisms for the two layers are essentially different concerning all phases of sand transport: initial movement, displacement of sand particles and deposition.

For calculating the sand transport rate in the case of suspended load, the employed approach is the *time-averaged concept*. That is, calculating the time-averaged sand concentration and velocity profiles for each section of the beach in cross-shore direction, multiplying their values and integrating the product over the time and space. The time-averaged concentration profiles are calculated using the simplified diffusion equation, neglecting the horizontal convection and diffusion.

$$\overline{c}w_s + \varepsilon_s\frac{\partial c}{\partial z} = 0$$ (10)

where $\overline{c}$: the time-averaged sand concentration, w_s : the falling velocity of the sand grain and ε_s : the diffusion coefficient. To solve the diffusion equation, the concentration at the reference level should be given as boundary condition and the distribution of the diffusion coefficient should also be known. The reference concentration is calculated for both the cases of breaking and non-breaking waves. The diffusion coefficient is calculated using a new empirical formula developed by Winyu and Shibayama (1996), based on a large number of laboratory data and is given as

$$\varepsilon_s = 0.21u_*a_b\left(\frac{w_s}{u_*}\right)^2\left(\frac{\eta}{d}\right)d_*^{-1.5}$$ (11)

where u_* : the maximum bed shear velocity, a_b : the orbital amplitude of fluid particle just above the bottom boundary layer, w_s : the falling velocity of the sand grain, η : the ripple height, d : the mean sand particle diameter and d_* : the dimensionless parameter of sand grain diameter.

The vertical distribution of time-averaged velocity profiles is calculated based on the assumption of the eddy viscosity model. By considering the time-averaged values, the eddy viscosity model can be expressed as

$$\overline{\tau} = \rho v_t\frac{\partial\overline{u}}{\partial z}$$ (12)

where $\overline{\tau}$: the time-averaged shear stress, ρ : the fluid density, v_t : the eddy viscosity coefficient, $\overline{u}$: the time-averaged velocity and z : the upward coordinate from the bed. Finally, the suspended sand transport rate is

$$q_S = \frac{1}{T} \int_0^T \int_{z_m}^h \overline{c}(z) \overline{u_s}(z) dz dt \tag{13}$$

where $\overline{c}(z)$: the time-averaged vertical distribution of sand concentration, $\overline{u_s}(z)$: the time-averaged vertical distribution of sand grain velocity, T : the wave period, z_m : the level bellow which there is no movement of suspended sand particles, z : the vertical coordinate measured upward from the bed. The bedload is calculated using an empirical formula calibrated with a large set of experimental data.

4. Time-dependent Turbulent Sand Transport Model (Integrated Model)

The present numerical model is a 2-DV (two-dimensional vertical) model, named here as "Integrated Model", which is divided into two distinct parts: (1) the upper model comprising the description of the upper layer (suspended sediment) and (2) the lower model (sediment as bedload), concerned with the bottom boundary layer.

The structure of the numerical model ensures continuity in the final calculation of the sediment transport rate over the entire water depth. Also, the hydrodynamic field and the sand concentration values are also obtained as equi-phase mean values. At first, the upper hydrodynamic model of Shibayama and Duy (1994), which is based on the averaged form of the Navier-Stokes equations (Reynolds equations), is used to calculate the velocity field in the area above the boundary layer. At the same time, the free stream velocity at the upper edge of the bottom boundary layer is obtained. The model assumes a time-dependent eddy viscosity coefficient depending on the wave breaking characteristics. The time series of the horizontal and vertical velocity at the upper limit of the bottom boundary layer which represent the "forcing factor" driving the flow inside the area close to the bed are the output of the upper hydrodynamic model. These values are imposed to the next model, which is the hydrodynamic and sediment model for the bottom boundary layer. This model is also based on the averaged Navier-Stokes equations (computing the velocity vector) and on the two-dimensional convection-diffusion equation (computing sand concentration). The output of the model include the equi-phase mean values of the horizontal and vertical velocity, of the sediment concentration and the sediment transport rate integrated over one wave period and over the thickness of the bottom boundary layer is calculated. The last element of the "Integrated Model" is the suspended sediment model for the area above the bottom boundary layer. This is a modified version of the suspended sediment model of Duy and Shibayama (1997) based on the same mechanism of diffusion combined with convection. Unlike the original model, which included a time-dependent pick-up function as bottom boundary condition, the modified model uses the time series of the concentration at the upper edge of the bottom boundary layer as the lower boundary condition. Then, the sediment transport rate is calculated for the upper layer using the time series of velocities and the time series of suspended sand concentration. Finally, the total sediment transport rate averaged over the entire surf zone and over one wave period is the summation of the

sediment transport rate in the upper and lower layer. The diagram of the "Integrated Model" is presented in Fig. 7.
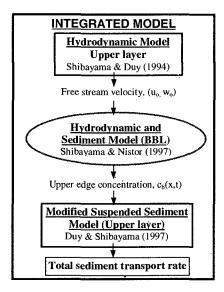

Fig. 7 Diagram of the Integrated Model

5. Comparison with Laboratory Data

Since the random wave input is the first step for the research processes of the present paper, it is necessary to evaluate the ideas proposed. The validity of the random wave input (as presented here) is evaluated by verifying it with laboratory data.

The Time-averaged Beach Deformation Model

The computed final beach profiles and the laboratory measured ones are shown in Fig. 8 (a, b and c). The calculated beach profile for all the three cases (Case 1a, 1b and 2) were determined using as input data both the significant wave characteristics and the joint distribution of wave heights and wave periods. As shown in the previously mentioned figures, each case involved different input data in terms of either the significant wave characteristics or the experiments duration. Using the joint distribution of wave heights and wave periods led to better results comparing to using the significant wave characteristics, both in terms of the horizontal location and vertical amplitude of beach profile deformations. For all the three cases shown in Fig. 8, using the significant wave height and wave period as wave input led to an overestimation of the beach deformation patterns. For a short duration of the experiment, using the joint distribution of wave heights and wave periods underestimates the laboratory data.

Concluding, the comparisons of the numerical and laboratory data seem to show a better performance when using the joint distribution compared to the use of the significant wave characteristics.

Fig. 5.25 Comparison of computed and laboratory data for Case 2.
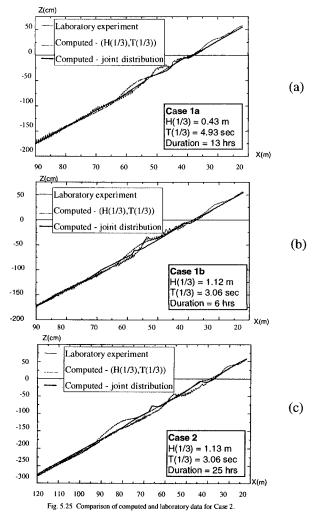
Fig. 8: Comparison of computed and laboratory data for BDRP – Japan (1995).

More than that, the differences between laboratory data and the computation using the significant wave characteristics are increasing with the duration of the experiment. The differences between computation and laboratory data for the case of the joint distribution have an opposite trend and tend to get closer in terms of deformation pattern and location.

The Time-dependent Turbulent Sand Transport Model (Integrated Model)

The measured sand transport rates (calculated from the beach profile change) are compared with the sand transport rates as computed by the Integrated Model. For the case of the Integrated Model, the sand transport rate is computed after a relatively small number of waves (6-8 waves for stable computational conditions), *assuming a uniform beach profile without any small structure.* Therefore, the computed sand transport rate does not take into consideration any change of the beach profile and remains uniform for any duration of the wave action. At the same time, it does not include the slope effect on the transported sand.

In Fig. 9 (a, b and c), the total measured sand transport rates together with the computed ones for both irregular wave approaches are compared. For all the three cases, a general characteristic is that the total sand transport rate computed by using significant wave characteristics has similar magnitude and space location as the ones resulted from the application of the joint distribution of wave heights and wave periods.
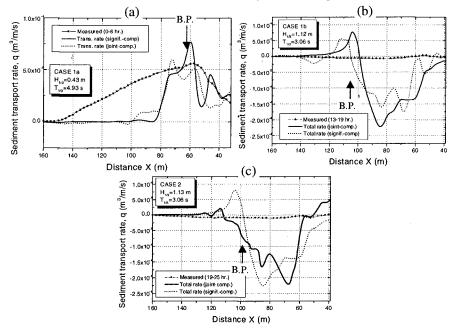

Fig. 9: Total sand transport rate for BDRP – Japan (using joint distribution of wave heights and wave periods and significant wave characteristics).

For Case 1a and Case 1b and to some extent for Case 2, the volume of sand transported as a result of applying the joint distribution of wave heights and wave periods appears to be smaller than the volume of sand computed by using the significant wave. The observation confirms previous conclusion of other studies, which suggested that, the

volume of sand transported under the effect of an irregular wave train, is smaller than the one transported by a regular wave train having the same significant dimensions (wave height and wave period) as the irregular wave train.

7. Concluding Remarks

The aim of the present study is to analyze the inclusion of wave irregularity through two approaches: (1) the significant wave characteristics and (2) the joint distribution of wave heights and wave periods. The two approaches were applied for two different models: (1) a time-averaged beach deformation model, based on the linear wave theory, which is capable of simulating the beach profile changes for the entire duration of wave action and (2) a time-dependent, high computationally demanding turbulent sand transport model, based on the Navier-Stokes equations and on the in turbulent convection-diffusion equation, which is capable of computing the sand transport rate for a relatively short duration of the wave action.

For both models, using the joint distribution of wave heights and wave periods led to better results than for the case of using the significant wave characteristics. For the case of the beach deformation model, the advantage of using the joint distribution becomes clearer with a longer duration of the irregular wave action. For the case of the turbulent sand transport model, the application of the joint distribution of wave heights and wave periods reflects into slightly reduced sediment transport rates than when employing the significant wave heights and periods. At the same time, the sand transport rate is more uniformly distributed over the surf zone area when applying the joint distribution.

Yet, the results found for the present stage are not evident enough to draw a clear-cut conclusion so that more experimental and numerical results should be analyzed before a well-endorsed and solid statement to be made.

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