

Numerical Investigation of Sediment Transport for Combined Waves and Currents

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Abstract

This paper presents a numerical investigation of sediment transport under the combined effect of waves and current motion. Comparison of predictions to experimental measurements showed good agreement. The presence of a second peak over the decelerating and flow reversal was predicted by the numerical model due to the inclusion of an extended bed reference concentration and a turbulence model that is able to predict a sudden increase of the turbulent properties of the flow over this stage. Evidence suggests that the second peak is caused by an inflection point in the velocity profile leading to high diffusivities near the wall.

Introduction

Sediment transport in a coastal environment occurs under the action of a number of hydrodynamic processes. These cover flow situations ranging from waves to currents while passing through a variety of combined wave-current situations. In order to describe these complex mechanisms it is rather common to simplify them into ideal cases as pure waves or pure currents, but in most real situations we are confronted with rather complex situations, where waves and currents interact. The purpose of this work is to perform a numerical experiment of sediment transport under the combined effects of waves and currents

Model Description

The temporal description of the suspended sediment can be expressed as a transport equation where particles are advected by means of their own settling velocity and

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diffused by the natural turbulence of the flow. This, for the one-dimensional (in vertical) case, can be mathematically expressed as:

$$\frac{\partial c}{\partial t} = \frac{\partial(cw_s)}{\partial z} + \frac{\partial}{\partial z} \left(\epsilon_s \frac{\partial c}{\partial z} \right) \quad (1)$$

where c is the concentration of sediment particles, w_s the settling velocity of the particles and M_s the turbulent diffusion of sediment particles.

This equation can be solved numerically if the value of the settling velocity, the turbulent diffusion of the sediment particles and an appropriate reference concentration or sediment flux are known. The description of the flow is carried out by solving the momentum equation in the x-direction

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{\partial}{\partial z} \left(\nu \frac{\partial u}{\partial z} - \overline{u'w'} \right) \quad (2)$$

where $\overline{u'w'}$ is the effective component of the Reynolds-stresses and ν is the kinematic viscosity.

Since the case to be simulated is associated with high Reynolds numbers the viscous stresses are negligible. The turbulent stresses, on the other hand, have to be expressed as function of the mean quantities. A turbulence model has to be applied then to express the correlation between the turbulent velocities as a function of the mean flow variables. Adopting the Boussinesq assumption, the effective shear stress, arising from the cross-correlation of fluctuating velocities, can be replaced by the product of the mean velocity gradient and a turbulent viscosity; the turbulent stresses can then be expressed for the one-dimensional case as:

$$\overline{u'w'} = \nu_t \frac{\partial u}{\partial z} \quad (3)$$

where ν_t , unlike the molecular viscosity ν , is not a property of the fluid. Under this assumption, the Reynolds stresses are defined in terms of only one unknown quantity, the eddy viscosity, therefore it is necessary to express the value of the turbulent eddy viscosity in terms of the known mean-flow quantities therefore the k -M model was applied (Rodi, 1984).

This model is based on two transport equations, one for the turbulent kinetic energy k and a second one to compute the rate of turbulent energy dissipation M . These two equations are expressed for the one-dimensional case as:

$$\frac{\partial k}{\partial t} = \frac{\partial}{\partial z} \left(\frac{\nu_t}{\sigma_k} \frac{\partial k}{\partial z} \right) + \nu_t \left(\frac{\partial u}{\partial z} \right)^2 - \varepsilon \quad (4)$$

$$\frac{\partial \varepsilon}{\partial t} = \frac{\partial}{\partial z} \left(\frac{\nu_t}{\sigma_\varepsilon} \frac{\partial \varepsilon}{\partial z} \right) + c_\varepsilon \frac{\varepsilon}{k} \nu_t \left(\frac{\partial u}{\partial z} \right)^2 - c_{\varepsilon 2} \frac{\varepsilon^2}{k} \quad (5)$$

where the turbulent eddy viscosity is defined as:

$$\nu_t = c_1 \frac{k^2}{\varepsilon} \quad (6)$$

The value of the constants considered for this type of flow are expressed in Table 1

c_1	c_{M1}	c_{M2}	L_k	L_M
0.09	1.44	1.92	1.00	1.30

Table 1 Values of the constants applied in the *k-M* model

In the case of combined waves and current it can be assumed that the ambient pressure can be obtained as a combination of the oscillatory component added to the pressure gradient that drives the current, therefore the 'total' ambient pressure can be obtained as the sum of the individual components. In oscillatory boundary layers it is assumed that the ambient pressure penetrates the entire boundary layer undisturbed. The pressure gradient can be obtained by applying the momentum equation in the *x*-direction to an outer boundary layer where the turbulence-representing quantities are assumed to vanish and the velocity approaches the free stream velocity u_∞ . In the case of a pure current the pressure gradient is then related to the slope of the energy line and the pressure gradient of both effects can be expressed as follows:

$$\frac{1}{\rho} \frac{\partial p}{\partial x} = - \frac{\partial u_\infty}{\partial t} - gHS \quad (7)$$

The resulting one-dimensional equation for the mean flow is then expressed as:

$$\frac{\partial u}{\partial t} = \frac{\partial u_\infty}{\partial t} - gHS - \frac{\partial}{\partial z} \left(\nu_t \frac{\partial u}{\partial z} \right) \quad (8)$$

Boundary conditions at the free boundary must be also introduced. Here the distribution of kinetic energy and the rate of dissipation of kinetic energy are defined as a symmetric condition.

$$\frac{\partial k}{\partial z} = 0 \qquad \frac{\partial \varepsilon}{\partial z} = 0 \qquad (9)$$

On the wall, a non-slip condition applies, so that the velocity is considered to be zero

$$u = 0 \qquad (10)$$

The behaviour of the wall can be considered hydraulically rough, so that the friction velocity can be obtained by assuming a logarithmic velocity distribution

$$u = \frac{u_f}{\kappa} \ln \left(\frac{z}{z_0} \right) \qquad (11)$$

where u_f is the friction velocity, κ is the von Karman constant and $z_0 = k_n/30$, k_n being the Nikuradse equivalent roughness.

In this area close to the wall, the Reynolds stresses are nearly constant so that the convection and diffusion of turbulence can be considered negligible and local equilibrium prevails. Accordingly, the production and dissipation of kinetic energy balance out and applying the assumption of a logarithmic velocity distribution, the values of k and ε at the wall boundary can be expressed as follows:

$$k = \frac{u_f^2}{\sqrt{c_1}} \qquad \varepsilon = \frac{u_f^3}{\kappa z_0} \qquad (12)$$

The solution of the transport equation requires also the definition of the boundary conditions for the bed and the free surface. In the case of the free surface, no flux of sediments is possible; therefore:

$$w_s c + \varepsilon_s \frac{\partial c}{\partial z} = 0 \qquad (13)$$

In the case of the bed the definition of the concentration is more complicated. In this case the ratio between the stabilizing and agitating forces, defined as Π' , is related to the value of the concentration at the bottom, so that a relationship between a bed concentration and Π' can be established. Zyserman and Fredsøe (1994), proposed an empirical formulation which related the bed concentration at $z = 2 d_{s0}$ to the Shields parameter, Π' . This formulation, which will be applied in the present model, is expressed as follows:

$$c_b = \frac{A(\theta' - \theta'_c)^m}{1 + A/c_m(\theta' - \theta'_c)} \qquad (14)$$

where $\Pi' > \Pi'_c = 0.045$ and $A = 0.331$, $c_m = 0.46$ and $m = 1.75$

Figure 1 shows a comparison of the bed reference concentration as obtained from this empirical equation with measured data.

In the case of a flow which is reduced from a value of Π' which is larger than the critical value to zero, the reference concentration drops to zero as soon as the value Π' is less or equal to the critical value, and this does not seem physical correct. For this reason Justesen et al. (1986) proposed a so-called flux boundary condition where the value of the bed concentration, c_b , is also a function of the settling velocity of the particles, expressed as:

$$c_b = \max\{c_b(\theta'), c(w_s)\} \tag{15}$$

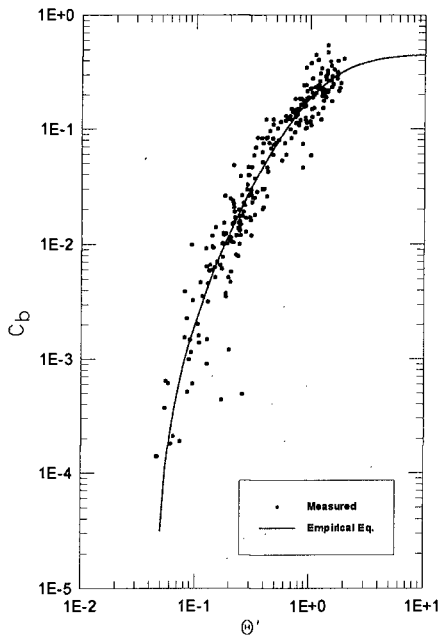


Figure 1. Comparison of Zyserman and Fredsøe's empirical formulation with measured data

This expresses that the concentration is obtained as the maximum value of the predicted concentration based on the instantaneous shear stresses or the one obtained by considering the sediment settling down by it falling velocity. This was further developed, as presented by Savioli and Justesen (1997), by including the effect of the turbulent

diffusion in the movement of the sediment particles. The boundary condition is then expressed as:

$$c_b = \{c_b(\theta'), c(w_s, \varepsilon_s)\} \quad (16)$$

In this case the bed concentration is obtained as the maximum value obtained for the concentration based on the instantaneous shear stresses, or by considering the sediment settling down and being also influenced by the turbulent diffusion generated by the flow. A detailed description of the implementation of this boundary condition is presented in Savioli and Justesen (1997). It should be remarked that this boundary condition has been applied in order to investigate the behaviour of sediment for waves and current but it is not readily applicable in practical engineering cases.

Comparison of Results and Discussion

In order to compare the model with measured data a test case was simulated. The test case has been obtained from the experimental work carried out by Katopodi et al. (1994). This was performed at the Large Oscillating Water Tunnel of Delft Hydraulics where waves, currents and wave-current flow can be simulated. Natural uniform sand was used for the tests where sediment concentrations were measured. The mean grain size of the sediment particles was $d_{50} = 0.21$ mm with a settling velocity of 2.6 cm/s. Test case E1 has been chosen for the comparison which is based on a sinusoidal wave combined to a current. Table 2 presents the values of the mean current velocity U_m , the wave amplitude velocity U_o and the period T for the test case.

Case	U_m (m/s)	U_o (m/s)	T (s)
E1	0.15	1.65	7.2

Table 2. Characteristics of the flow for test case

Figure 2 shows the time evolution of the measured and calculated sediment concentrations at different heights from the bed: $z = 1.45$ cm, 2.35 cm and 3.65 cm. At the top the undisturbed velocity is also presented.

Two major concentration peaks occur when the flow velocity is at its maximum, these are not equal in magnitude due to the combined effect of the wave and current motion. Very good agreement is obtained between the predicted and the measured data, especially close to the bed at $z = 1.45$ cm. The model predictions show remarkable agreement in phase and value of the concentration, during the first peak. The second peak, on the other hand, is slightly underestimated. As we move up from the bed, to $z = 2.35$ cm and $z = 3.65$ the difference between the predicted and measured values of concentration becomes more significant.

It is also observed a peak in the sediment concentration at flow reversal in the measured data. This has been captured by the numerical model due to the inclusion of an extended formulation which makes sediment available at flow reversal when the flow velocity is nearly zero and therefore the Shield's parameter value is smaller than the critical value Π_c

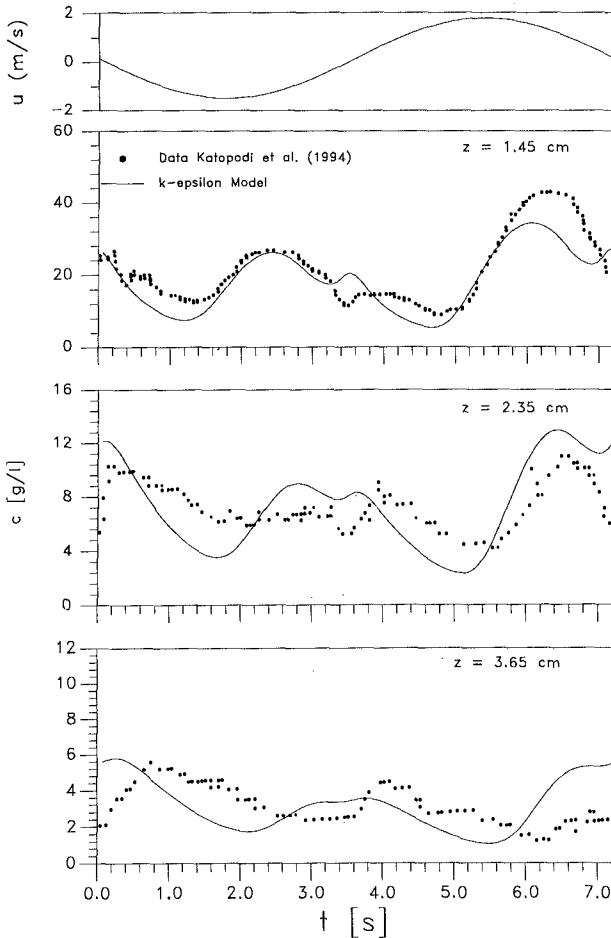


Figure 2 Observed and predicted sediment concentrations during a whole wave period

If the bed reference concentration would have been related only to the instantaneous bed shear stresses, $c_b = f(\theta')$, the concentration increase at flow reversal would not have been obtained since the value of the shear stresses over this period of

time is smaller than the critical value. In fact the inclusion of an extended bed reference concentration allowed this increase in the concentration at flow reversal to occur.

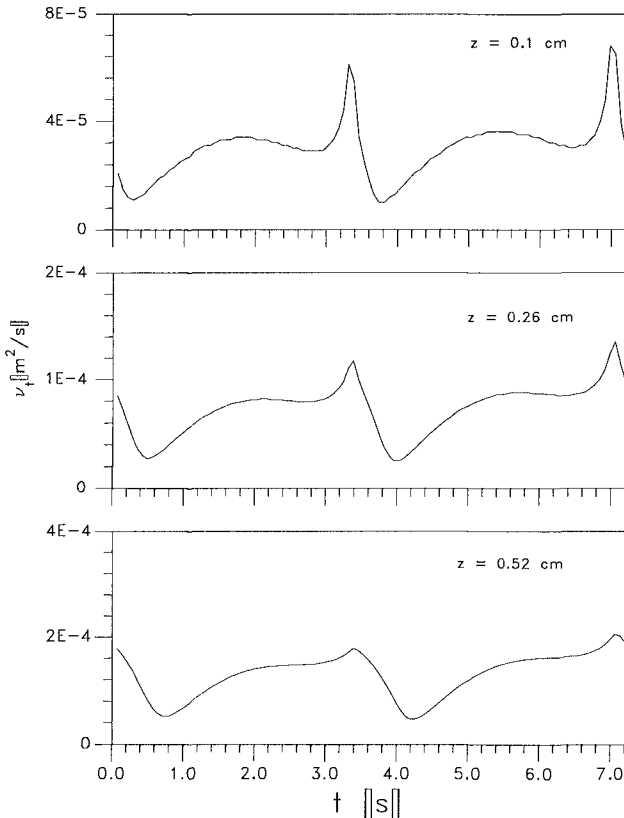


Figure 3 Predicted value of the turbulent eddy viscosity at different distances from the bottom during a whole wave period

In order to observe the behaviour of this extended boundary condition, the values of the turbulent eddy viscosity have been extracted at three different elevations above the bottom. Figure 3 shows the temporal variation of the turbulent eddy viscosity at $z=0.1$, 0.26 and 0.52 cm. It can be observed that the value of the turbulent eddy viscosity follows closely the flow velocity so that it is at its maximum when the velocity is also peaking. Nevertheless, at flow reversal, a sudden and large increase of the turbulent eddy viscosity occurs generating a period of high diffusivity that promotes the spreading of sediment particles if the sediment particles are available, as introduced in the extended boundary condition.

As it has already been introduced, the value of the turbulent eddy viscosity is dependent on the turbulent kinetic energy to the power 2 and the rate of dissipation of the turbulent kinetic energy. If we now follow the evolution of the instantaneous value of k^2 and M over a wave period, as presented in Figure 4, it is clear that the rate of growth of both of these during the acceleration stage is rather similar. On the other hand, over the deceleration phase, the rate of decay of M is much larger than and a singular point is observed at flow reversal. At this particular moment there is an inflexion point in the instantaneous velocity profile, the velocity gradient is at its maximum and there is a large production of turbulence (the production term in the k and M equations is dependent on the velocity gradient to the power 2).

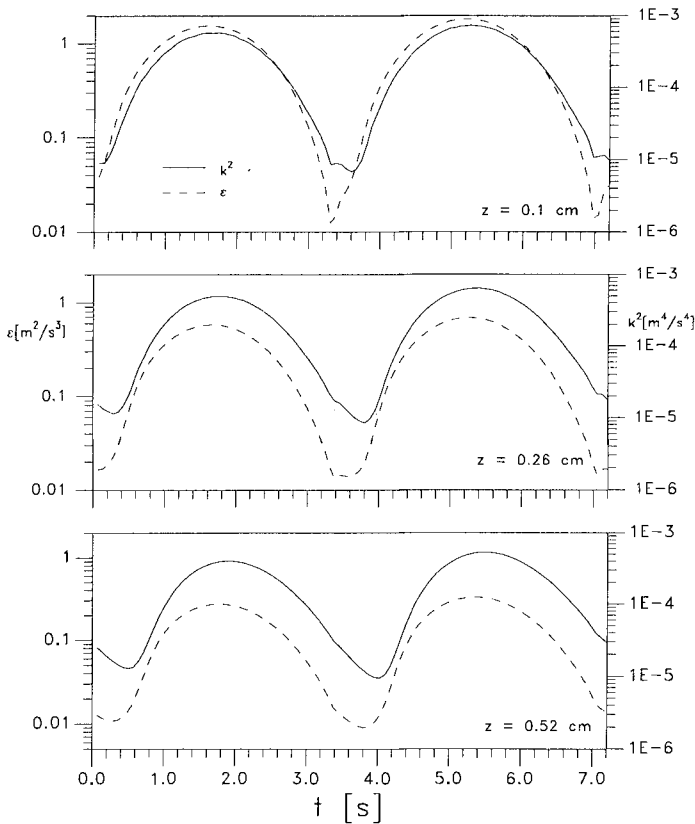


Figure 4 Evolution of k^2 and M at different distances from the bottom during a whole wave period

By analogy from the theory of gases the turbulent diffusion can be expressed as $v_t \propto u' l$ where u' is related to k and M to the mixing length l . What is visible from the results is that, at maximum velocity, u' is dominant while, at flow reversal, l predominates.

One of the explanations regarding this behaviour was presented by Smith in 1977 where it was expressed in the form that the non-slip condition at the bed causes the fluid close to the bed to have a smaller inertia. It will consequently respond more rapidly to the free stream pressure gradient prior to the fluid both in the upper part of the boundary layer and in the free stream. Thus, the boundary layer leads the free stream flow, resulting in an inflection point (see Figure 5) in the bottom boundary profile during flow deceleration, reversal and subsequent acceleration.

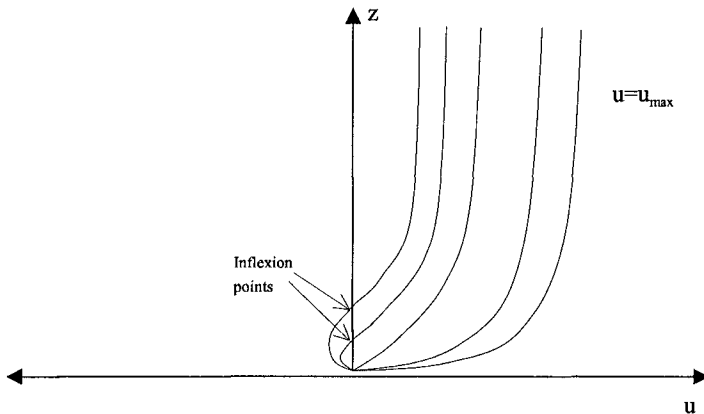


Figure 5. Schematization of the flow velocity profiles during deceleration and flow reversal

Foster, in 1984, presented a theoretical study that suggested that flow in the bottom leads that of the free stream, resulting in an inflection point (a necessary condition for an instability to occur) in the vertical profile of cross-shore velocity during flow deceleration and reversal. During this period small perturbations may grow exponentially to breaking, leading to increased levels of turbulence. The existence of shear instabilities during the period of flow reversal was modelled by a simple linear stability analysis on a time varying bottom boundary layer. Predicted growth rates were large enough to yield an order of magnitude increase in initial perturbation amplitudes.

Conclusions

A numerical investigation has been performed in order to predict sediment transport in a combined wave-current flow. The predictions showed good agreement with the experimental measurements. A second peak near flow reversal has been predicted due to the use of an extended boundary condition and a turbulence model that is able to predict a sudden increase of the turbulence over the deceleration stage and at flow reversal. The results suggest that there is an inflection point, a necessary condition for a shear instability. Nevertheless further analysis should be carried out to investigate the flow behaviour and the bed reference concentration in this type of flow. Soon the advance of computer technology will allow to carry out Direct Numerical Simulations which resolve all the flow features avoiding the use of turbulence models based on average quantities.

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