2DH NON-LINEAR DISPERSIVE WAVE MODELLING AND SEDIMENT TRANSPORT IN THE NEARSHORE ZONE

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Abstract

A 2D-horizontal sediment transport energetics model is developed in this work for the evaluation of the wave-induced sediment transport. The time dependent energy equation is incorporated into a nonlinear dispersive wave model in order to simulate breaking wave propagation in the surf zone. Total immersed weight transport is related, through an energetics approach, to the total dissipated fluid power. Both the dissipation due to bed friction and, inside the surf zone, due to the wave breaking are considered. The methodology is applied to predict the longshore transport rate and to simulate the coastline evolution in beach nourishment scenario assuming a trapezoidal beachfill.

Introduction

Long-shore and cross-shore sediment transport due to wave action play an important role in various engineering problems. One of the most important problems is the wave and the wave-induced sediment transport effects on coastal environment in terms of the bed morphology changes.

There exist two main approaches for the estimation of the sediment transport rate inside and outside surf zone: the deterministic and the energetics. The deterministic models are based on the description of both the wave induced mean flow and the concentration of suspended sediment, usually using quasi 3D models and linear wave theory (deVried and Stive 1987, Katopodi and Ribberink 1992, Briad and Kamphuis 1993). The energetics approach is based on the idea that the sediment transport is related to the rate of energy dissipation of the flow (Bailard 1981, Roelvink and Stive 1989). In the present work the second approach is adopted.

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A 2DH non-linear breaking wave model is developed based on the numerical solution of the time dependent wave energy equation which is incorporated into a Boussinesq model. The main advantages of the use of the Boussinesq type of equations are outlined below:

- A unified model, without the assumption of progressive waves, is used for non- linear wave refraction, shoaling, diffraction, reflection (presence of structures), breaking, dissipation after breaking, run-up.
- The equations are easily extended in deeper waters
- The propagation of irregular waves is modelled including LFW and non-linear wave-wave interactions.
- Breaking wave induced current are automatically incorporated. Thus there is no need for additional current and sediment transport model (i.e. coupling of 3 models).
- 3D effects (inclusion of a 'mean' undertow) are present.
- The models can be extended for the simulation of sediment transport in swash zone.

For the evaluation of the sediment transport rate (i.e. bed load and suspended load) the Bailard (1981) theory is used as in the 1D version of the model (Karambas et al. 1995). Following the sediment transport calculations, the morphological changes of the sea bed are updated in the model according to the conservation equation of sediment mass.

2DH non-linear wave breaking wave model

Karambas (1996) incorporated the time dependent energy equation into a nonlinear wave model based on the Boussinesq equations in order to simulate breaking wave propagation in the surf zone. Extending the analysis in two dimensions the continuity and the momentum equations are written (Peregrine 1972, Madsen and Svendsen 1979):

$$\frac{\partial \zeta}{\partial t}+\frac{\partial (Uh)}{\partial x}+\frac{\partial (Vh)}{\partial y}=0$$

$$\frac{\partial (Uh)}{\partial t}+\frac{\partial\left(\int_{-d}^{\zeta} u^2 dz\right)}{\partial x}+\frac{\partial\left(\int_{-d}^{\zeta} uv dz\right)}{\partial y}+gh\frac{\partial \zeta}{\partial x}=\frac{d^2}{2}\frac{\partial^3 (Ud)}{\partial x^2 \partial t}-\frac{d^3}{6}\frac{\partial^3 U}{\partial x^2 \partial t}+$$
$$\frac{d^2}{2}\frac{\partial^3 (Vd)}{\partial x \partial y \partial t}-\frac{d^3}{6}\frac{\partial^3 V}{\partial x \partial y \partial t}+\frac{\partial}{\partial x}\left(v_\tau h\frac{\partial U}{\partial x}\right)+\frac{\partial}{\partial y}\left(v_\tau h\frac{\partial U}{\partial y}\right)-\frac{\tau_{bx}}{\rho}$$

$$\frac{\partial (Vh)}{\partial t}+\frac{\partial\left(\int_{-d}^{\zeta} uv\,dz\right)}{\partial x}+\frac{\partial\left(\int_{-d}^{\zeta} v^2 dz\right)}{\partial y}+gh\frac{\partial \zeta}{\partial y}=\frac{d^2}{2}\frac{\partial^3 (Vd)}{\partial y^2 \partial t}-\frac{d^3}{6}\frac{\partial^3 V}{\partial y^2 \partial t}+$$
$$\frac{d^2}{2}\frac{\partial^3 (Ud)}{\partial x \partial y \partial t}-\frac{d^3}{6}\frac{\partial^3 U}{\partial x \partial y \partial t}+\frac{\partial}{\partial x}\left(v_\tau h\frac{\partial V}{\partial x}\right)+\frac{\partial}{\partial y}\left(v_\tau h\frac{\partial V}{\partial y}\right)-\frac{\tau_{by}}{\rho} \tag{1}$$

in which ζ is the surface elevation, d is the still water depth, h is the total depth, u and v are the horizontal velocity components in the x and y directions respectively, U and V are the depth integrated velocities, v_τ is the eddy viscosity coefficient and τ_{bx} , τ_{by} are the bottom shear stresses.

The shear stresses are estimated from:

$$\frac{\tau_{bx}}{\rho}=\frac{f_w}{2}u_o\sqrt{u_o^2+v_o^2}$$

$$\frac{\tau_{by}}{\rho}=\frac{f_w}{2}v_o\sqrt{u_o^2+v_o^2} \tag{2}$$

where u_o, v_o are the components of the bottom velocity (adopting the velocity distribution given by Peregrine, 1972) and f_w a friction factor (Nairn and Southgate 1993):

$$f_w=\exp(5.2\ (r/\alpha)^{0.2} - 6.0) \tag{3}$$

in which α is the orbital amplitude at the bed and r is the bottom roughness:

$$r=170\sqrt{\theta_{2.5}-0.05}\,D_{50} \tag{4}$$

where $\theta_{2.5}$ is the Shields parameter for $f_{2.5}=\exp(5.2\ (2.5D_{50}/\alpha)^{0.2} - 6.0)$.

The equation of the conservation of the energy density E of the mean flow per unit horizontal area is written:

$$\frac{\partial E}{\partial t}+\frac{\partial E_{fx}}{\partial x}+\frac{\partial E_{fy}}{\partial y}=-D-u_o\tau_{bx}-v_o\tau_{by}+U\,BT+V\,BT \tag{5}$$

$$E_{fx}=\int_{-d}^{\zeta}\frac{1}{2}u^3\,dz+\int_{-d}^{\zeta}\frac{1}{2}uv^2\,dz+g\zeta Uh\ ,$$

$$E_{fy}=\int_{-d}^{\zeta}\frac{1}{2}v^3\,dz+\int_{-d}^{\zeta}\frac{1}{2}u^2v\,dz+g\zeta Vh$$

in which D is the dissipation of the mean energy (equal to minus the production of turbulent energy) and BT are the Boussinesq dispersion terms of the momentum equations (1).

The dissipation of the wave energy D is given by (Karambas, 1996):

$$D = \Omega \ (c - u_{ot})^3 \tag{7}$$

in which c is the wave celerity, $u_{ot} = \sqrt{u_o^2 + v_o^2}$ is the bottom velocity in the direction of the wave propagation and Ω a constant, Ω=0.03.

The eddy viscosity coefficient v_τ is calculated from the solution of the turbulent kinetic energy equation (Nwogu, 1996):

$$\partial k/\partial t + \mathbf{U}\nabla k = v_\tau \nabla^2 k + B\ D - C_d k^{3/2}/ l_t \tag{8}$$

in which $\mathbf{U}$=(U,V), k is the turbulent kinetic energy, C_d=0.08, and l_t is the turbulent length scale, l_t=0.15 d.

In the 1D case (Karambas, 1996) the constant Ω is equal to 0.015. In the present work a greater value (Ω=0.03) is adopted. This value (according to Morfett, 1995) is claimed to give a good fit to measured longshore transport rate. To compensate the increase of the value of Ω, the turbulent length scale l_t is taken equal to l_t =0.15 d, instead of l_t =0.3 d which has been adopted in the 1D case.

The rate of production of turbulent kinetic energy is taken equal to the dissipation D of the wave energy (equation 7). The parameter B (Nwogu, 1996) is introduced to ensure that turbulence is produced only when horizontal velocity at the wave crest u_{st} exceeds the celerity c.

$$B = \begin{cases} 0 & \text{in the region where } u_{st} < c \\ 1 & \text{in the region where } u_{st} > c \end{cases}$$

where u_{st}, is the velocity at the wave crest, $u_{st} = \sqrt{u_s^2 + v_s^2}$, and u_s and v_s are its components from Boussinesq theory (Peregrine, 1972).

The eddy viscosity is given by:

$$v_\tau = k^{1/2}\ l_t \tag{9}$$

The two integrals in the equations (1) are estimated form (Karambas 1996):

$$\int_{-d}^{\zeta} \frac{1}{2} u^2 dz = E - \int_{-d}^{\zeta} \frac{1}{2} v^2 dz - \frac{1}{2} g\zeta^2$$

$$\int_{-d}^{\zeta}\frac{1}{2}v^2dz = E - \int_{-d}^{\zeta}\frac{1}{2}u^2dz - \frac{1}{2}g\zeta^2 \tag{10}$$

The integrals in equations (1) and (6), containing the terms u^3, v^3, uv, u^2v, uv^2, are estimated numerically adopting the following horizontal velocity distribution in the turbulent region of a breaking wave (Karambas 1996):

$$u(z)=u_o+u_d\, f(\sigma)$$

$$v(z)=v_o+v_d\, f(\sigma) \qquad \text{for } \zeta-\delta<z<\zeta \tag{11}$$

where δ is the depth of the turbulent region and

$$u_d = u_s - u_o\ , \quad v_d = v_s - v_o, \quad f(\sigma)=-A\sigma^3+(1+A)\sigma^2, \quad \sigma=(d+z)/h, \quad A=1.4$$

Using the definition of the mean velocity U and equation (11) the turbulent region depth δ can be estimated from (Karambas, 1996):

$$\delta = \frac{U-u_o}{u_d}\frac{h}{0.45} \tag{12}$$

In the non-turbulent region, D=0, $E=0.5hU^2+0.5g\zeta^2$ and the system reduces to the classical Boussinesq equations.

An important result of the above model is the prediction of the nonlinear instantaneous bottom velocity which also includes the mean motion (mainly responsible for the sediment transport).

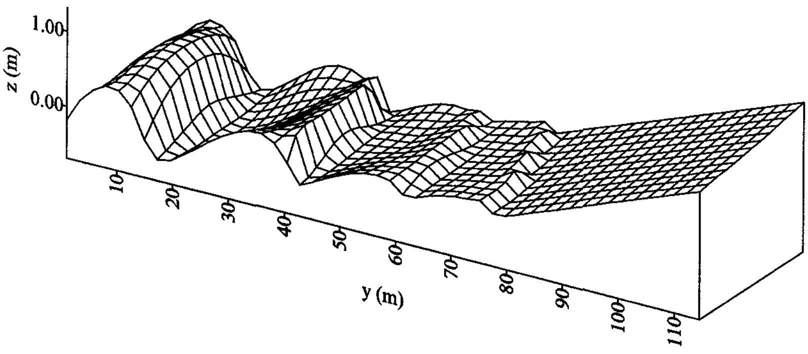

Figure 1. Perspective view of the wave field for oblique wave incidence. Wave height H=1m, period T=6 secs and slope 1:30.

Numerical solution

The system of equations (1) and (5) is simultaneously solved as in the 1-D model (Karambas, 1996) in the following way:

1. Calculation of U^{n+1}, V^{n+1}, ζ^{n+1} (at time level $(n+1)\Delta t$) from the solution of the Boussinesq equations (1).
2. Calculation of E^{n+1} *explicitly* from the energy equation (5) using exactly the same Finite Differences approximations (both new time step $(n+1)\Delta t$ and previous time step nÄt values of the variables are employed). The integrals containing the terms u^3, v^3, u^2v, uv^2, are estimated numerically (equation 6).
3. Calculation of the integral in the momentum Boussinesq equations (1) using equation (10). The integrals containing the terms uv are estimated numerically.
4. Next time step: replacement in the non linear terms of the momentum Boussinesq equations (1) the values of the integrals of step 3.

In this manner energy equation is numerically solved simultaneously with the Boussinesq equations and its effects are introduced explicitly in the momentum equation. The main advantage of the above procedure is that it can be easily introduced in the existing models without the need for changing their numerical scheme.

Waves propagating out of the domain are artificially absorbed using sponge layer technique. The Orlanski open boundary condition is also applied, since it is more efficient for the absorption of the generated currents and long waves.

The 'dry bed' boundary condition is used to simulate runup (Figure 1). Consider the one-dimensional case the condition is written:

$$\text{if}\,(d+\zeta)<0.00001 \qquad \text{then} \qquad z=-d \tag{13}$$

The above conditions has been successfully used in long wave runup modelling (Karambas et al., 1991).

The Finite Differences numerical scheme is described in Karambas et al. (1990).

An energetics sediment transport formula

The energetics approach is based on the Bagnold's original idea that the sediment transport load is proportional to the time averaging energy dissipation of the stream (Bailard 1981).

In an energetics approach the submerged weight transport rates, i_{xt} in the x direction and i_{yt} in the y direction, are given by Bailard:

$$<i_{xt}> = <\left[\frac{\varepsilon_b}{\tan\phi}\left(\frac{u_o}{u_{ot}}+\frac{d_x}{\tan\phi}\right)\omega_b+\varepsilon_s\frac{u_{ot}}{w}\left(\frac{u_o}{u_{ot}}+\varepsilon_s d_x\frac{u_{ot}}{w}\right)\omega_t\right]>

$$< i_{yt} >=< \left[\frac{\varepsilon_b}{\tan\varphi}\left(\frac{v_o}{u_{ot}}+\frac{d_y}{\tan\phi}\right)\omega_b+\varepsilon_s\frac{u_{ot}}{w}\left(\frac{v_o}{u_{ot}}+\varepsilon_s d_y\frac{u_{ot}}{w}\right)\omega_t \right]> \tag{14}$$

where the angled brackets represent (numerical) time-averaging, w is the sediment fall velocity, φ is the angle of internal friction, ε_b and ε_s are the bed and suspended load efficiency factors respectively, ω_t is the local rate of energy dissipation given by:

$$\omega_t=\omega_b+C_d\, k_b^{3/2} / l_t\ h \tag{15}$$

where k_b is the turbulent kinetic energy at the bottom and ω_b is the dissipation at the bottom:

$$\omega_b= f_w/2\ u_{ot}^3 \tag{16}$$

The above sediment transport formula has been derived directly form the primitive equations (equation 7 of Bailard 1981 paper) without the assumption that the only dissipation mechanism is the bed friction. This is the most important limitation of the Bailard theory and precludes the use of the original formula within the surf zone, where the dissipation of energy associated with the process of wave breaking is largely dominant.

Since a Boussinesq model automatically includes the existence of the mean wave-induced current there is no need to separate the bottom velocities $u_o(t)$ and $v_o(t)$ into a mean and a oscillatory part as in most of the previous works which adopt only period-mean approaches. This is another significant advantage of the present model since the approach is based on the original energetics formula without the above simplification. In addition there is no need for the decomposition of the moments by assuming that the bottom wave velocity is larger than the mean current velocity. This assumption (Roelvink and Stive, 1989) is not always valid, especially in complicated wave fields near coastal structures (behind a detached breakwater there is a strong mean current without significant wave motions).Finally the present model includes a quasi 3D structure of the motion (mean and oscillatory) predicting in this way the undertow effects on the cross-shore sediment transport rate.

According to the original Bagnold estimations (from river data) the bed and suspended load efficiency factors ε_b and ε_s take the values ε_b=0.13 and ε_s=0.01.

Bailard (1981) first calibrated the energetics approach based on laboratory and field measurements. Least square estimates of ε_b and ε_s resulted in values 0.21 and 0.025 respectively. However, the Bailard's value of ε_s has to compensate for increased turbulence due to breaking in the surf zone. Since the breaking wave-induced dissipation has already been incorporated, the Bailard's value is not valid. Here the value ε_s=0.01 is used, considering the following exponential, over the depth, decay of the turbulence kinetic energy k (according to Roelvink and Stive 1989):

$$k_b=k[(\exp(d/H)-1]^{-1} \tag{17}$$

in which H is the wave height.

For typical values of the ratio d/H inside surf zone the dissipation of k (i.e the term $C_d k_b^{3/2} / l_t h$) is finally multiplied by a factor 0.002, which is similar to the value proposed by Morfett (1995).

Morphology module

The morphological changes are calculated by solving the conservation of sediment transport equation:

$$\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} = \frac{\partial z_b}{\partial t} \tag{18}$$

where z_b is the bed elevation above arbitrary datum and q_x, q_y are the volumetric sediment transport rate related to the immerged weight sediment transport through:

$$q_{x,y} = \frac{i_{x,y}}{(\rho_s - \rho) g N} \tag{19}$$

in which N is the volume concentration of solids of the sediment (N= 0.6) and ρ_s and ρ are the sediment and fluid densities.

Applications

In two previous work of the author (Karambas et al, 1995, and Karambas et al., 1997) the 1D version of the model has been successfully used to predict cross-shore sediment transport and bed evolution. In the present model the simulation of the runup leads also to the prediction of the sediment transport in swash zone. In this work two applications are presented: the prediction of the longshore transport rate and the simulation of the coastline evolution in a beach nourishment scenario.

In the computations of the longshore transport rate it is assumed that the shoreline and depth-contour lines are straight and parallel to each other, the incident waves are regular and uniform in the alongshore direction and that the sediment grain size is spatially uniform. The following four parameters are varied in the numerical experiments: breaking wave height H_b, incident angle θ_b, beach slope tana and grain size D_{50}. Wave period T is assumed constant, T=9 secs. For certain values of the three of the above parameters and different values of the fourth, the longshore transport rate Q, from the swash zone across the surf zone to deep water, is calculated by the cross-shore integration of $q_y(y)$:

$$Q = \int_{Ru}^{\infty} q_y(y)\,dy \tag{20}$$

where Ru is the location of the run-up point.

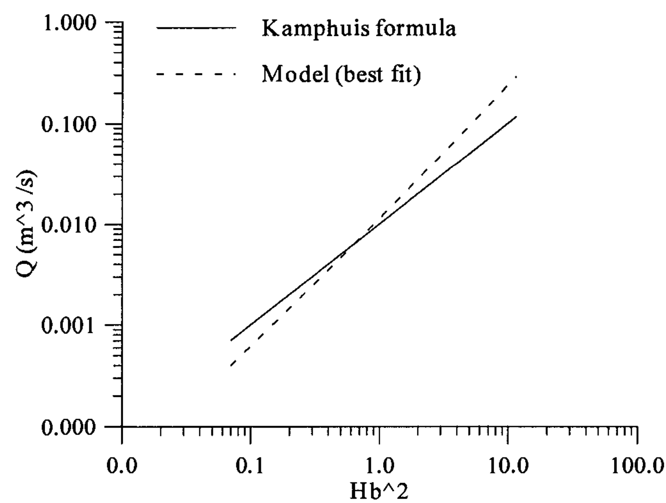

Figure 2. Comparison of longshore transport rate between Kamphuis formula and present model in different breaking wave heights H_b (T=9 secs, D_{50}=0.0003 m, θ_b=0.3 rad, slope=0.015).

The Kamphuis longshore transport formula is used for comparison with the present model. The improved Kamphuis formula (Schoones and Theron, 1996) is written as follows:

$$Q_{Kamphuis} = 63433 \frac{1}{(1-p)\rho_s} (\rho / T) L_o^{1.25} H_b^2 (\tan a)^{0.75} (1/D_{50})^{0.25} (\sin 2\theta_b)^{0.6} \quad (m^3/\text{year}) \tag{21}$$

where p is the porosity, L_o is the deep-water wavelength, H_b is the wave height, θ_b is the incident angle, tana is the beach slope and D_{50} the grain size.

Model predictions are plotted against H_b^2, $(\sin 2\theta_b)^{0.6}$, $\tan a^{0.75}$ and $(1/D_{50})^{0.25}$ in Figures 2 to 5. The longshore transport rate Q which is predicted by the model is generally close to the values obtained by Kamphuis formula. In general a more strong dependence on breaking wave height, incident angle and grain size is predicted by the model. However, for the verification of the model, comparisons with experimental data and field measurements are required.

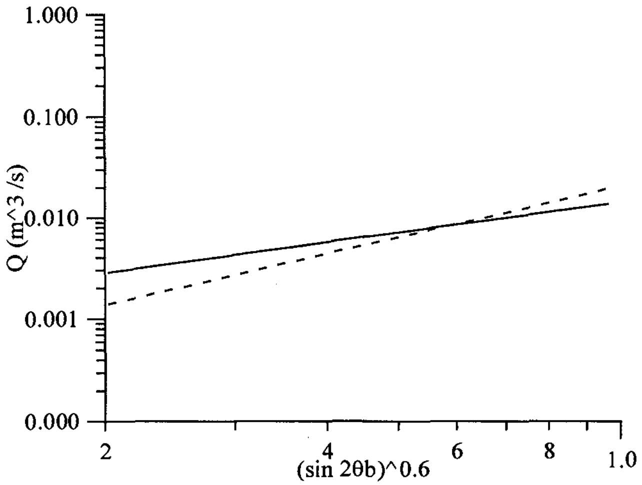

Figure 3. Comparison of longshore transport rate between Kamphuis formula (solid line) and present model (dashed line) in different wave directions θ_b (H_b=1.0 m, T=9 secs, D_{50}=0.0003 m, slope=0.015).

The model is also applied to simulate the coastline evolution in a beach nourishment scenario assuming normal wave incidence on a trapezoidal beachfill (Figure 6). In Figure 7 the predicted shoreline change after nondimensional time T=0.5 is compared with an analytical solution of the Pelnard-Considere equation (one-line model, Work and Rogers, 1997). The nondimensional time T is defined as: $T=4(Gt)^{0.5}/l_l$, where G is 'longshore diffusivity' parameter (Work and Rogers, 1997) and l_l is the longshore length (l_l=20m in the present case). The depth at the toe of the beachfill is h_t=1.5m, the slope 1:15, the incident wave height H=1m, the period T=6 secs and the fall velocity w=0.03 m/s.

In an one-line model changes in shoreline position are assumed to be produced by spatial differences in the **longshore** sand transport rate. However present model is also able to simulate cross-shore transport (Karambas et al., 1995, Karambas et al., 1997). Thus the predicted shoreline change is expected to include the effects of the cross-shore (offshore or onshore) transport, i.e. erosion or accretion. In Figure 7 the difference between the present model and the analytical solution is the shoreline displacement due to offshore transport. Under the applied conditions (H=1m, T=6 secs and w=0.03 m/s) the nondimensional fall speed N=H/(wT)=5.55, also known as the Dean number, is greater than the critical value N_c=3.2 and consequently the direction for the cross-shore sediment transport is expected to be offshore (erosion).

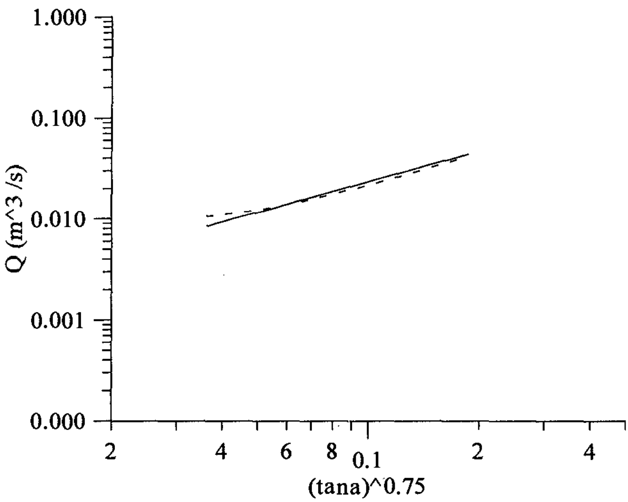

Figure 4. Comparison of longshore transport rate between Kamphuis formula (solid line) and present model (dashed line) in different slopes tana (H_b=1.0 m, T=9 secs, D_{50}=0.0003 m, θ_b=0.3 rad).

Conclusions

The 2DH energy and the Boussinesq equations are simultaneously solved for the simulation of breaking wave propagation in the surf zone. Model results are used in an energetics sediment transport model based on the Bailard formula.

The unified model is capable of predicting:

- 2DH breaking wave propagation
- Longshore transport rate from the swash zone to shoaling region
- Coastline evolution in a beach including cross-shore sediment transport

The comparison of longshore transport rates between present model and Kamphuis formula shows close agreement.

Coastline evolution is also predicted well in comparison with an analytical solution of the one-line equation.

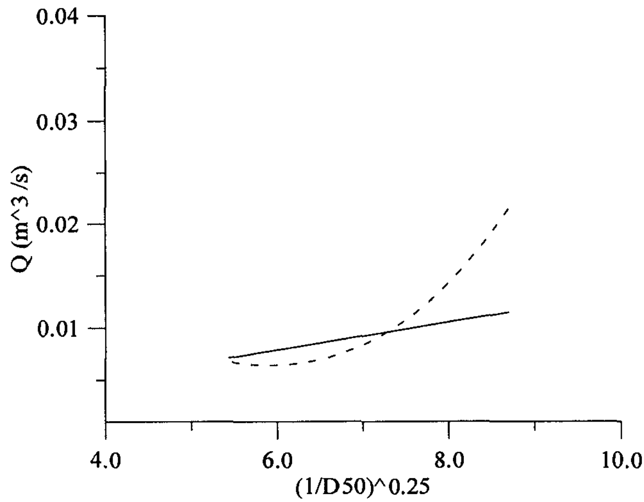

Figure 5. Comparison of longshore transport rate between Kamphuis formula (solid line) and present model (dashed line) in different median grain size D_{50} (H_b=1.0 m, T=9 secs, θ_b=0.3 rad, slope=0.015).

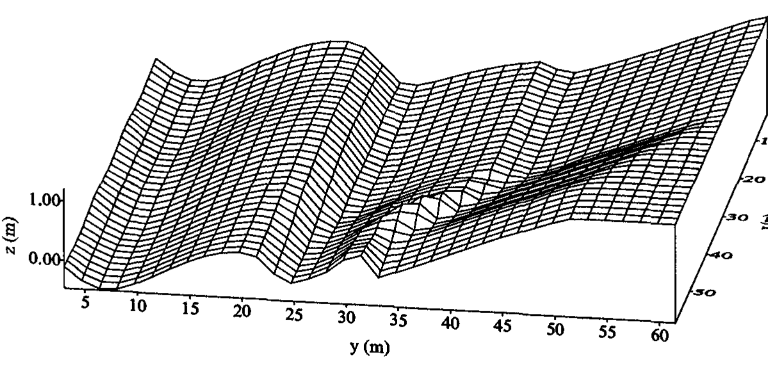

Figure 6. Perspective view of the wave field for normal wave incidence on a trapezoidal beachfill. Wave height H=1m, period T=6 secs, depth at toe h_t=1.5m and slope 1:15.

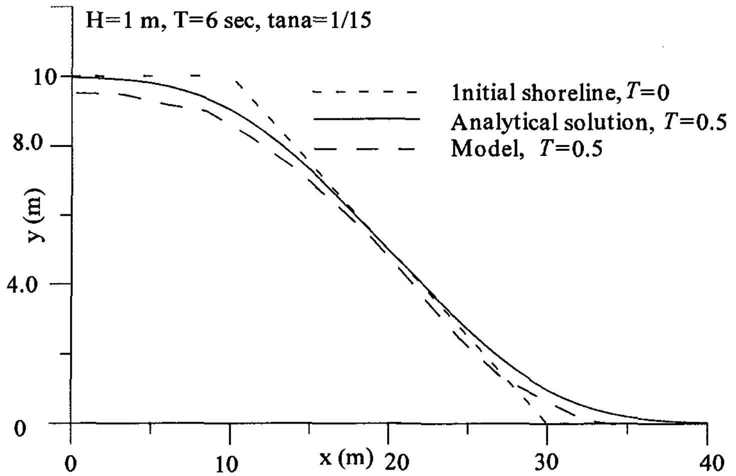

Figure 7. Shoreline change of a trapezoidal beachfill. Comparison between analytical solution (Work and Rogers, 1997) and numerical model.

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