A Dimensionless Parameter Describing Sea Cliff Erosion

Akira Mano [1]and Shigenori Suzuki [2]

Abstract

Macroscopic analysis based on the energy conservation on sea cliff erosion gives a dimensionless parameter including recession rate of the cliff, wave energy flux at the breaking point, Young's modulus of rocks composing the cliff, and cliff height. This parameter was examined through field data on the Fukushima Coast where a series of soft rock cliffs extends about 50 km. The modulus was obtained by the measurement of the propagation velocity of elastic waves in the rocks and the energy flux was evaluated by the wave ray method. The long-term recession rate had been obtained from the aerial photographs. The recession rate is shown to be proportional to the wave energy flux and inversely proportional to Young's modulus. These quantities prove that the dimensionless parameter has the constant value of 0.082.

1 Introduction

It is a common sense from the physical point of view that the rate of sea cliff erosion depends on both of the intensity of incoming waves and the strength of the rocks composing the cliff. However quantitative relationship to connect these parameters in the field was still unknown. This comes mainly from the difficulty of the observation. Significant cliff erosion occurs in the storm condition, often associated with massive collapse of the cliff. We cannot approach such site during the storm. Furthermore individual collapse may not represent the recession characteristics at the site, because the structural cracks would determine the position of the failure. Debris produced by the collapsed cliff would also affect the recession rate.

Thus we have only limited data on the sea cliff erosion in spite of the significant requirements on the protection works. To develop physical model on the recession characteristics at this stage, macroscopic approach using the energy conservation relationship would be appropriate.

[1]Disaster Control Research Center, Tohoku University, Aobayama 06, Sendai 980-8579, Japan
[2]Electric Power Development Co. Ltd., 6-15-1 Ginza, Tokyo, 104-0061, Japan

2 Energy Conservation Analysis

For the high cliffs made of soft rock, the erosion would take the following process: (1) erosion of the cliff base, (2) collapse of the overhanging, (3) successive fragmentation from large rocks into sands or mud, (4) longshore transport of the fine materials. However since the knowledge of (1) and (2) is unsatisfactory and the both process produce the debris, we will simplify them as one process. Let us consider a two dimensional configuration of the cliff erosion as illustrated in Fig. 1. Taking unit width in the longshore direction and denoting the cliff height and recession rate by L and q respectively, we have volume rate of the debris production from the continuity of the cliff material,

$$\dot{V}_1 = Lq. \tag{1}$$

Furthermore steady state assumption gives the volume rate of the debris removal equal to the production rate.

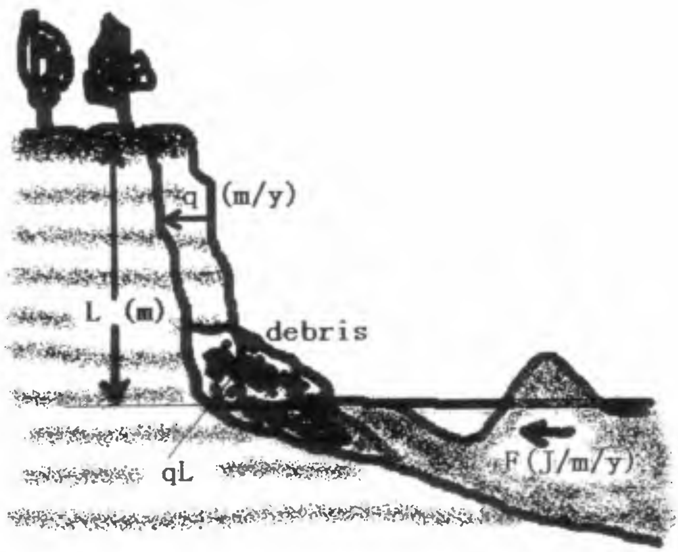

Fig. 1: Definition sketch.

The energy conservation equation for the cliff erosion could be written as,

$$F = \dot{W}_1 + \dot{W}_2 + \dot{W}_3 + \dot{R}. \tag{2}$$

Where F is the energy supply rate and represented by the onshore component of the wave energy flux in unit longshore length at the breaking point as,

$$F = E_b C_{gb} \cos^2 \theta_b. \tag{3}$$

Here subscript b denoting the breaking point, E_b is the wave energy density, C_{gb} group velocity, and θ_b wave angle. The powers $\dot{W}_1$, $\dot{W}_2$, and $\dot{W}_3$ are the rate of the work to detach the large rocks from the cliff, to fragmentize the large rocks of debris into sand or mud, and to transport these fine sediments in the longshore direction. The power $\dot{R}$ is the remainder such as energy dissipation rate in the surf zone.

The first two terms of $\dot{W}_1$,$\dot{W}_2$ are modeled by the elastic theory. If external force is applied to compress the rock sample with unit length and unit cross section from zero to the yield stress, the work done by the force is given by

$$w = \frac{1}{2}\sigma_y \epsilon_y = \frac{1}{2}\frac{\sigma_y^2}{E}. \tag{4}$$

Where σ and ϵ are the stress and strain respectively, E is Young's modulus, and the subscript y indicates yield point. Among various kinds of metals, the yield stresses of the materials are directly proportional to their Young's moduli as stated in Kobayashi (1993). This corresponds to the physical state that fracture occurs when the relative dislocation of atoms exceeds a certain time of the interatomic distance. Introducing this relationship also for rocks, we can rewrite the above equation as,

$$w \propto E. \tag{5}$$

Thus the maximum strain energy that the rock of unit volume can hold is proportional to its Young's modulus. Then if cracks generate in the rock, a part of the strain energy transforms into the surface energy of the newly generated cracks. Therefore the energy consumed in the processes from the erosion to the fragmentation could be interpreted to change the surface energy. Thus, with coefficient C_1

$$\dot{W}_1 + \dot{W}_2 = C_1 L q E. \tag{6}$$

The third term is modeled by employing Komar and Inman(1970) with the coefficient K,

$$\dot{W}_3 = KF \tan\theta_b. \tag{7}$$

The last term is expressed by the efficiency factor β as,

$$\dot{R} = (1-\beta)F. \tag{8}$$

By substituting all these model into Eq.(2) and dividing by F, it follows,

$$1 = C_1 \Pi_c + K\tan\theta + (1-\beta). \tag{9}$$

Where

$$\Pi_c \equiv \frac{qEL}{F}. \tag{10}$$

Equation (9) indicates the dimensionless parameter, Π_c is constant if the wave angle θ_b is small and the efficiency coefficient β is constant.

$$\Pi_c = const. \tag{11}$$

In the following sections, we will examine this parameter by collecting field data on the Fukushima Coast.

3 Outline of the Coast

The Fukushima Coast locates in the East of the main land of Japan and faces directly toward the Pacific Ocean as shown in Fig.2. The coast extends about 50 km in the north direction and is composed of a series of cliffs and pocket beaches. The coast had suffered from serious erosion, reaching 10 m/y at most. The geographical situation that the coast is subject to high wave attack is one reason of the severe erosion.

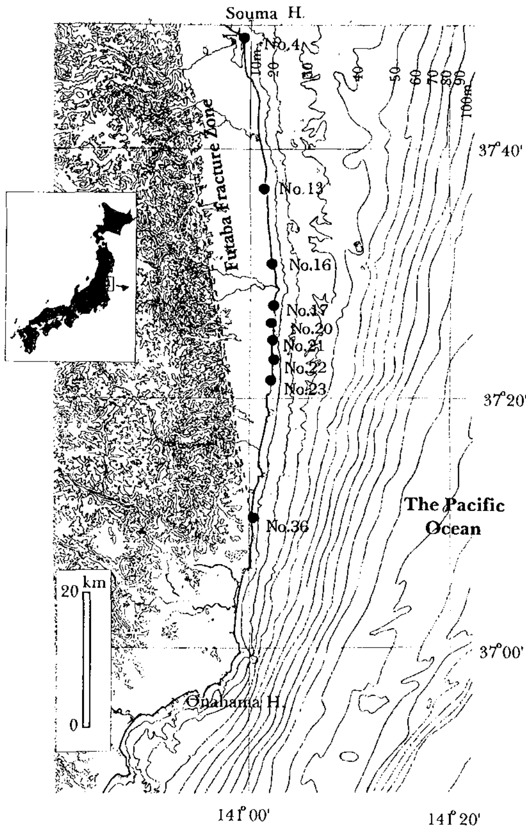

Fig. 2: Topography of the Fukushima Coast and sampling points.

The other reason comes from geological condition. The coastal area together with the hinter land and offshore region is subject to the uplift, the rate of which is estimated by Oka et al. (1981) as 0.2 to 1.6mm/y , varying temporary and spatially. The bed layer of the cliff in the northern part of the coast is marine mudstone or sandstone, formed in the Upper Pliocene, while the southern part is widely covered with green tuff formed in the Miocene, the Tertiary. Weakly consolidated rocks of geologically young age are easily eroded by wave attacks.

In the West of the coastline, Futaba Fracture zone parallels the coast. The West of the fracture zone uplifted about 200 m relative to the East in the Lower Pleistocene and is now called Abukuma Plateau. Many rivers originated in the plateau run east by eroding the Pliocene sediments on the bed layer and made pocket beaches at the coast. The submarine contour lines of 20 to 40m are significantly winding in the northern part. It is also estimated that the submarine valleys were made by the fluvial erosion during the regression of seawater in the Pliocene. The valleys would affect the refraction of incident waves and then the distribution of wave energy.

Nine sampling points for the examinations of the rock and wave properties described in the next section were selected so as to cover the wide area of the coast and to include wide range of the recession rate. Fig.2 illustrates also the points by the gray circles with numbers.

Figure 3 shows the cliff at No.4 point. The headland is eroded vertically at the tip and has a hollow near the base at the side. In order to mitigate the erosion, numbers of concrete block were put at the shoreline and demonstrated to mitigate the erosion. The other places on the coast are also in the similar circumstances. To guard the railway, roads, and houses in the hinterland from the erosion, protection works were done in 1960s and 1970s. Now the protection works such as the block at the shoreline or detached breakwater has covered major part of the coast.

Figure 4 shows the cliff at No.20. The cliff surface exhibits many layers stretching horizontally. At the base of the cliff, there scatter numerous large rocks of debris. The cliff height is about 18 m. Fig. 5 shows the cliff at point No. 36, which is formed by the hardest rocks in our study. The layers run nearly vertically. Thus, nine samples were taken from various conditions of cliff.

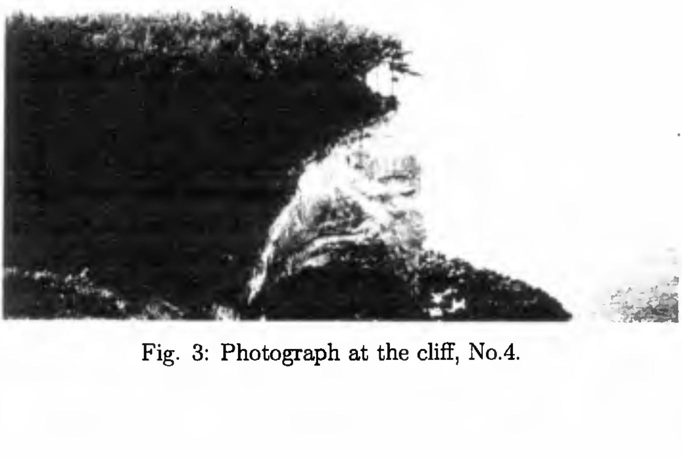

Fig. 3: Photograph at the cliff, No.4.

Fig. 4: Photograph at the cliff, No.20.

Fig. 5: Photograph at the cliff, No.36.

4 Evaluation of the Parameters

Young's modulus E of the rock composing cliffs was evaluated by the measurement of the propagation velocity of the elastic wave. First, we collected rock samples of the size of about 30cm by detaching them from cliffs near the sea water level. Then, after cutting them into the rectangular solids and drying them in the electric furnace at 200°C for 24 hours, we measured the propagation velocity of the elastic primary wave, V_p by attaching transducers on the surface of the formed samples. Young's modulus is given by the following equation,

$$E = \rho V_p^2 \frac{(1+\nu)(1-2\nu)}{1-\nu}. \tag{12}$$

Where ν is Poisson's ratio and the standard value 0.25 was used. The obtained values of Young's modulus together with the other quantities are listed in Table 1.

Table 1 Parameters for the sampled cliffs.

Cliff point	Recession rate q (m/y)	Young's modulus $E(10^9\times$ N/m$^2)$	Cliff height L (m)	Energy flux $F(10^{11}\times$ J/m/y)	$1/E$	$1/L$
4	0.35	1.08	20	1.55	0.93	0.0050
13	2.12	0.56	18	2.43	1.79	0.0056
16	1.56	0.65	20	3.44	1.54	0.0050
17	3.22	0.74	20	4.79	1.35	0.0050
20	1.86	0.56	18	2.75	1.79	0.0056
21	1.08	0.97	20	2.09	1.03	0.0050
22	0.92	0.92	20	2.81	1.09	0.0050
23	2.06	0.53	25	2.83	1.89	0.0040
36	1.03	2.31	30	3.55	0.43	0.0330
Mean $\bar{z}$				2.92	1.31	0.048
Standard deviation Sd				0.883	0.457	0.00677
Relative range $Sd/\bar{z}$				0.303	0.348	0.140

Energy flux was obtained by the wave ray method of Mano and Sawamoto (1997). That is, the wave ray equation of Mei(1983) and wave ray density equation of Munk and Arthur(1952) together with the dispersion relation are solved on the sea bottom topography approximated as a set of triangular panels. The breaking points are determined by Goda's criterion (1974).

Examples of wave rays are shown in Figs. 6 and 7 for respective wave directions ENE and ESE of wave period, 12.0s. The numbers and vertical lines at the bottom of the figures indicate the sample numbers of cliffs specified in Fig. 2 and cliff extension respectively. Wave rays refract remarkably by the submarine valleys and then result in the periodic distribution of energy concentration and dispersion. In order to evaluate the wave energy flux from wave rays that include crossing, the

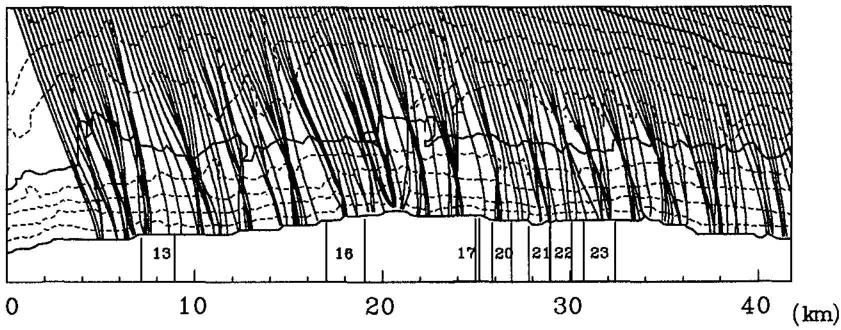

Fig. 6: Wave rays for the incident waves with the period, 12s, and direction, ENE.

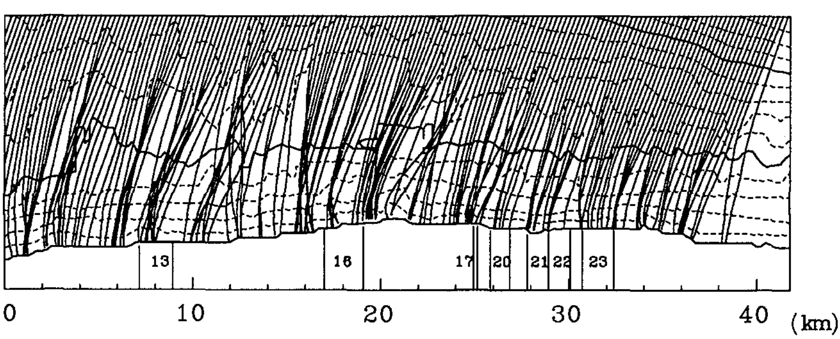

Fig. 7: Wave rays for the incident waves with the period, 12s, and direction, ESE.

following definitions were adopted,

$$F = \frac{1}{N}\sum_{j=1}^{N}\frac{f_j}{n_j}\sum_{i=1}^{n_j}(E_b C_{gb} cos^2\alpha_b)_i. \tag{13}$$

where N is the number of the data sets in a year, $f_j = n_j\Delta S/\ell$, ΔS is the ray ejection spacing in the offshore boundary, n_j is the number of wave rays approaching the cliff, ℓ is the cliff extension projected to the offshore boundary. The coefficient f_j is introduced to take into account of the wave ray gathering. Here we adopted the data sets of daily mean values of the significant wave height, period and direction observed in 1991 off the Souma Harbor located at the north end of the coast with the total days, $N = 364$.

The Fukushima Prefecture(1993) had performed comprehensive research on the coastal erosion, including the recession rate evaluation of all cliffs on the Fukushima Coast and histories of the protection works. The aerial photographs taken in 1963 to 1991 were used for the evaluation. Thus the rate is the mean values in time span of 15 to 30 years. We took these values but modified them by assuming no recession after the protection works. The cliff height is obtained also in the same report.

5 Results and Discussion

Equation (11) for the dimensionless parameter Π_c constituted by four quantities deduces further several relations with the additional conditions that two quantities among four are chosen with keeping the other quantities constant;

$$q \propto F, \tag{14}$$

$$q \propto 1/E, \tag{15}$$

$$q \propto 1/L. \tag{16}$$

Before getting into the individual relationship, let us examine the characteristics of our data set in Table 1. The lowest three rows of the table indicate the mean values, standard deviation and relative range, which is defined by the standard deviation normalized by the mean value, among samples. In the three quantities, F, $1/E$, and $1/L$, the first two quantities have wide range, while the last quantity narrow. In other words, our data set is appropriate to examine Eqs. (14) and (15), and is rather poor to Eq.(16).

Figure 8 examines the relationship between the recession rate q and the wave energy flux F. The recession rate is linearly proportional to the energy flux as shown by the regression line but with a certain data scatter. This corresponds to Eq. (14). Similar relationship has been obtained by Gelinas and Quigley (1973) on the cliff erosion of the north shore of the Great Lake, Erie. As the parameter of wave intensity, they used the total energy flux, not the onshore component, however the difference is generally small, because waves approach perpendicularly to the shore. They made $q-F$ plot and got the regression line but without passing the origin. If we ignore the intercept, which is not so unnatural judging from the scatter of their data, it also becomes Eq. (14).

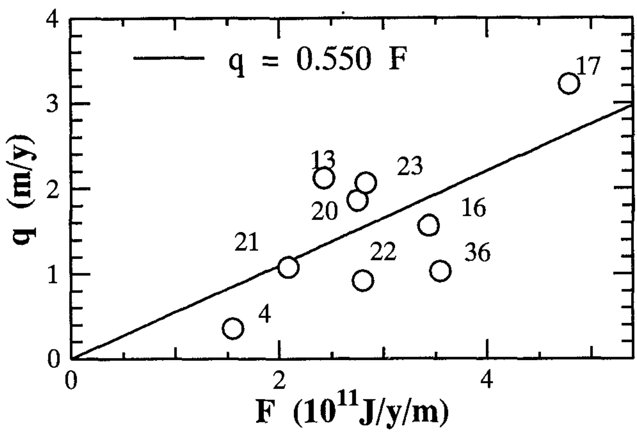

Fig. 8: Relationship between the recession rate and the wave energy flux.

Figure 9 examines the relationship between the recession rate and Young's modulus. The recession rate is inversely proportional to Young's modulus. Although there is also data scatter, referring to the previous figure, we could find some reasons for the scatter. For example, No 17 far above the regression line indicating excessive erosion for the strength of the rock is reasoned by the high energy flux shown in Fig.8. Therefore the data scatter of these figures does not always mean the poorness of the models (14) and (15). Fig. 9 is also supporting Eq.(15).

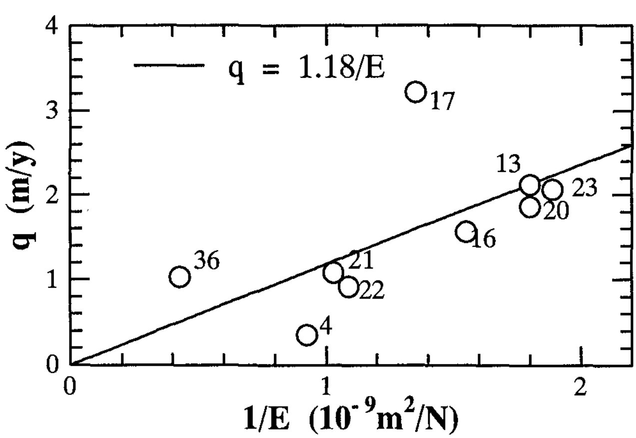

Fig. 9: Relationship between the recession rate and the inverse of Young's modulus.

Sunamura (1992) examined the relationship between the recession rate and compressible yield stress, by collecting related data on the Fukushima Coast by three researchers. He plotted these two parameters in the log-linear scale plane and got regression relationship by the logarithmic function. However, recalling the relationship used in Section 2 that the yield stresses are linearly proportional to Young's moduli, we can expect inversely linear relationship between the two quantities with the help of Eq. (15). Fig.10 is the modification of Sunamura (1992)'s Fig. 5.13, by taking the linear scale for the inverse of the yield stress. Here the numbers by the circle indicate different sites. The expectation is realized with small data scatter in this figure.

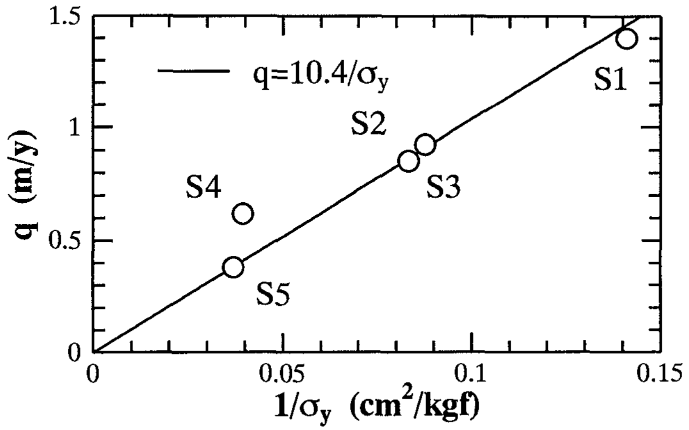

Fig. 10: Relationship between the recession rate and inverse of the yield stress, obtained by modifying Sunamura (1992)'s Fig. 5.13.

Figure 11 shows the correlation between the recession rate and cliff height. The largest data scatter would come from the narrowness of the relative range of $1/L$. We have shown the significant effects of the energy flux and Young's modulus on the recession rate, which was enabled by the wideness of the relative range of these parameters and the narrowness of the remaining parameter, because Eqs. (14) to (16) require the constancy of the other parameters when one parameter among $F, 1/E$ and $1/L$ is chosen.

As for the effect of the cliff height on the recession rate of the sea cliff, there have been conflicting arguments that high cliff is effective to retard the erosion in the short time span but ineffective in the long time span in Sunamura (1992). Detailed analysis on the effect of cliff height and discussions on the above arguments will appear in Mano and Suzuki.

Figure 12 shows the correlation between the recession rate and the combined parameter F/EL. The recession rate is proportional to the parameter with small data scatter. We have the regression line,

$$q = 0.082F/EL, \tag{17}$$

identically,

$$\Pi_c = 0.082 = const. \tag{18}$$

Thus, Eq.(11) is proofed.

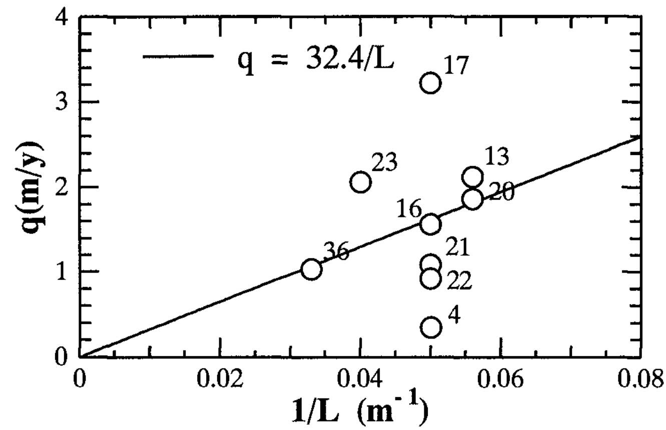

Fig. 11: Relationship between the recession rate and the inverse of cliff height.

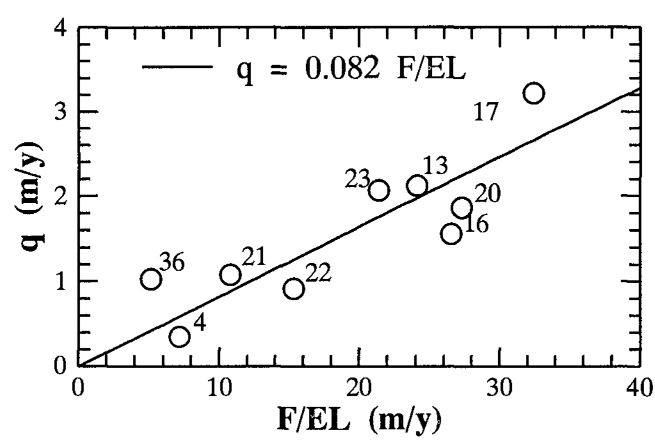

Fig. 12: Relationship between the recession rate and the combined parameter F/EL.

6 Conclusions

The macroscopic analysis based on the energy conservation relation on the sea cliff erosion produces a dimensionless parameter $\Pi_c \equiv qEL/F$ and expects this parameter is constant in the first approximation. Four quantities constituting the dimensionless parameter were evaluated and collected on the Fukushima Coast which had been suffered from severe erosion through the dual reasons of the high wave attack and weakly consolidated rocks. Young's modulus was evaluated by the measurement of the propagation velocity of the elastic waves, while the energy flux through the wave ray analysis. The recession rate and the cliff height had been evaluated through the measurement by the aerial photographs and by the geographical maps, respectively, by Fukushima Prefecture.

Equation (11) on the constancy of the dimensionless parameter includes Eqs. (14) to (16) as special cases. Eq. (14) on the dependency of the recession rate on the energy flux and Eq. (15) on Young's modulus are satisfied by our data set and are shown to be consistent with the foregoing studies by Gelinas et al (1973) and Sunamura (1992). However for the last quantity, the cliff height, our data set is poor to proof Eq. (16), directly. Finally, as for the whole four quantities, our data set gives the relationship $q = 0.082F/EL$ with small data scatter. This is equivalent to $\Pi_c = 0.082$ and the constancy of the dimensionless parameter is proofed.

Acknowledgements
The authors are indebted to Professors Masaki Sawamoto, who introduced us this exciting subject, Tsuguo Sunamura, whose comprehensive review guided us to the important problems on this subject, Shigeru Niizeki, who allowed us to use the measurement system of the elastic wave, and Satoshi Kanisawa, who helped us to study the geological circumstance on the Fukusima Coast. Thanks are also due to Fukushima Prefecture for the permission to use the data and to the Ministry of Education, Science, Sports and Culture in Japan for the Grant-in-Aid.

References

[1] Fukushima Prefecture(1993):Research on the Coastal Erosion in 1993 – Reading from the aerial photograph, as referendum, 62p.(in Japanese).

[2] Gelinas,P.J. and R.M.Quigley(1973):The influence of geology on erosion rates along the north shore of Lake Erie, Proc. 16th Conference Great Lakes Research, pp.421-430.

[3] Goda, Y. (1974): New wave pressure formulae for composite breakwater, 14 th Int. Conf. Coastal Engineering, pp.1702-1720.

[4] Kobayashi, H. (1993): Fracture Mechanics, Kyoritu Publishing Co., 193p.(in Japanese).

[5] Komar, P. D. and D. L. Inman(1970): Longshore sand transport on beaches, Journal of Geophysical Research, Vol. 75, No. 20, pp.5914-5927.

[6] Mano A. and M. Sawamoto(1997): Interactions between a sand barrier and flood terrace at the Abukuma river mouth, Proc. 25th Inter. Conf. Coastal Engineering, pp.4505-4516.

[7] Mano A. and S. Suzuki: Erosion characteristics of sea cliff on the Fukushima Coast, Coastal Engineering Journal, (to be published).

[8] Mei, C.C.(1983):The Applied Dynamics of Ocean Waves, John Wiley & Sons, 740p.

[9] Munk, W.H. and R.S. Arthur (1952):Wave intensity along a refracted ray, Natl. Bur. Stand., U.S. , Circ., 521, Gravity Waves, pp.95-108.

[10] Oka, S., S. Katsurajima, T. Kakimi, T. Aihara and A. Unosawa (1981): Deformatio of a terrace plain along the eastern coast of Fukushima Prefecture,Bulletin of the Geological Survey of Japan, Vol. 32, pp.275-292(in Japanese).

[11] Sunamura, T.(1992): Geomorphology of Rocky Coasts, John Wiley & Sons, 302p.