

## Parameterizing Beach Erosion/Accretion Conditions

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### Abstract

A simple method to parameterize beach erosion and accretion processes is presented. The parameterization is accomplished by combining important findings from two studies on the initiation of sediment movement, with near-bottom velocities estimated from nonlinear wave theory. This synthesis allows the development of a simple schematization or model based on only two variables that provides considerable insight on cross-shore sediment movement. Once calibrated, the model shows surprising skill (0.94) in predicting erosional or accretionary profile changes on sand beaches and is general enough to predict observed onshore movement of gravel during storms. If linear wave theory is used, the ability to discriminate between erosional and accretionary events declines.

### Introduction and Background

Analysis by Ahrens and Hands (1998) showed clearly that shoreline erosion or accretion could be predicted quite well using the ratio of two velocities. Symbolically, we have:

$$U = u_{d \max} / u_{crit} \quad (1)$$

where  $u_{d \max}$  is the maximum near-bottom orbital velocity of the wave and  $u_{crit}$  is the critical velocity required to initiate sediment movement under the wave. Two versions of  $U$  were developed, denoted  $U_c$  and  $U_t$ , for the maximum near-bottom velocity under the crest and trough respectively. Near-bottom maximum velocities under waves were calculated using Dean's (1974) Stream Function Wave Theory (SFWT). Critical velocities for the initiation of sediment movement are calculated as follows:

$$u_{crit} = \sqrt{8\Delta g d_{50}}, \text{ for } d_{50} \leq 2.0 \text{ mm} \quad (2)$$

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$$u_{crit} = [0.46\Delta g T^{3/4} (\pi d_{50})^{3/4}]^{4/7}, \text{ for } d_{50} > 2.0 \text{ mm} \quad (3)$$

where  $g$  is the acceleration of gravity,  $d_{50}$  is the median sediment diameter,  $T$  is the wave period, and  $\Delta = (\rho_s - \rho) / \rho$ , where  $\rho_s$  is the density of the sediment and  $\rho$  is the density of water. Eqs. 2 and 3 are based on research by Hallermeier (1980) and Komar and Miller (1974), respectively.

Although not developed in Ahrens and Hands (1998), their analysis indicated that it was possible to make good approximations of  $U$  using only two variables. This was accomplished by generating an extensive synthetic data set using SFWT and Eqs. 2 and 3 above. Regression analysis and dimensional analysis were used to obtain the following equations:

$$U_c = 0.132 N_s^{0.466} (d/L_o)^{-0.0506} \exp[-3.523(d/L_o)] \quad (4)$$

$$R^2 = 0.970, N = 392,$$

$$\text{max. \% error} = 36.1\%,$$

$$\text{rms \% error} = 22.2\%,$$

and for the trough:

$$U_t = -0.519 N_s^{0.447} (d/L_o)^{0.569} \exp[-5.049(d/L_o)] \quad (5)$$

$$R^2 = 0.993, N = 392,$$

$$\text{max. \% error} = 29.4\%,$$

$$\text{rms \% error} = 16.3\%,$$

where  $N_s$  is the stability number commonly used to measure the stability of rubble structures,  $d$  is the water depth and  $L_o$  is the deep-water wave length. The negative sign in Eq. 5 is used to indicate an offshore directed velocity.

Hudson, et al. (1979) shows how the stability number evolves from dynamic and dimensional considerations from a combination of a Froude Number and a density ratio. The combination accounts for the most important forces on an armor unit, i.e., form drag and submersed weight. An investigation of rubble-mound stability might involve stability numbers in the range of 1.0 to 3.0, but for this study the range was about 1 to 60,000. For investigating sediment movement under waves, it is more appropriate to refer to  $N_s$  as a mobility number. The mobility number is defined:

$$N_s \equiv H / (\Delta d_{50}). \quad (6)$$

Madsen (1998) notes the similarity between the mobility number for shallow water wave conditions and the Shields parameter, i.e.,

$$N_s \equiv H / \Delta d_{50} = gH / g\Delta d_{50} \propto u_{-d_{\max}}^2 / \Delta g d_{50}$$

The Shields parameter is the ratio of the drag forces that initiates sediment movement and the resisting force of the submerged weight.

The ranges of variables in the synthetic data set were:

$$0.002 \leq d/L_o \leq 0.2$$

$$H_b/4 \leq H \leq H_b$$

$$0.1 \leq d_{50} \leq 100 \text{ mm}, \rho_s = 2.65 \text{ gr/cm}^3$$

$$\rho = 1.000 \text{ gr/cm}^3 \text{ (fresh water) and } \rho = 1.025 \text{ gr/cm}^3 \text{ (sea water).}$$

Eqs. 4 and 5 have captured, in a simple formulation, some of the essential information about sediment movement under waves from three important studies, Dean (1974), Komar and Miller (1974) and Hallermeier (1980). The equations are a function of just two dimensionless variables which in turn are a function of six dimensional variables. None of the dimensional variables appears in both dimensionless variables, i.e. the dimensionless variables are independent and unusually effective in describing erosional and accretionary processes. The profile adjustment variables used here are the height and period of the wave, size and density of the sediment, and depth and density of the water. These variables seem like the minimum set required to predict profile response to waves and at the same time in many situations they may well be all the information available. This discussion is quite simplified because it assumes that sediment movement responds only to the near-bottom velocity produced by wave motion. Obviously, many other variables could be important at the selected site of application, such as the slope of the bed, the presence of other currents, and the bedform.

## Findings

### a. The Big Picture

Fig. 1 provides perspective on the implications of Eqs. 4 and 5. Fig. 1 shows  $U_c$  and  $U_t$  for three values of relative depth as a function of mobility number. The following features of Fig. 1 are noted:

- 1.)  $U_c$  and  $U_t$  are small for small mobility numbers and large for large mobility numbers, indicating increasing sediment movement under waves with increasing mobility numbers.
- 2.)  $U_c$  and  $U_t$  are quite asymmetrical about  $U = 0$  for  $d/L_o = 0.02$  and  $0.002$ , suggesting the accretional properties of long waves, i.e. because for a given mobility number the absolute value of  $U_c$  is much greater than  $U_t$ .
- 3.)  $U_c$  and  $U_t$  are almost symmetrical about  $U = 0$  for deeper water, e.g.,  $d/L_o \geq 0.2$ , suggesting the erosional characteristics of short-period waves because the absolute values of  $U_c$  and  $U_t$  are about the same, but the offshore flow under the trough lasts longer than the landward directed flow under the crest.
- 4.) Because sediment characteristics are contained only in the mobility number, the figure shows how wave conditions that might be erosive for fine-grained sediment could be accretive for large sediment: e.g. sediment with a high mobility

could move shoreward under the crest and offshore under the trough, whereas sediment with a lower mobility number could be below the threshold for offshore movement and be constrained to move shoreward under the crest.

All of the characteristics noted above are consistent with current understanding of sediment movement under waves.

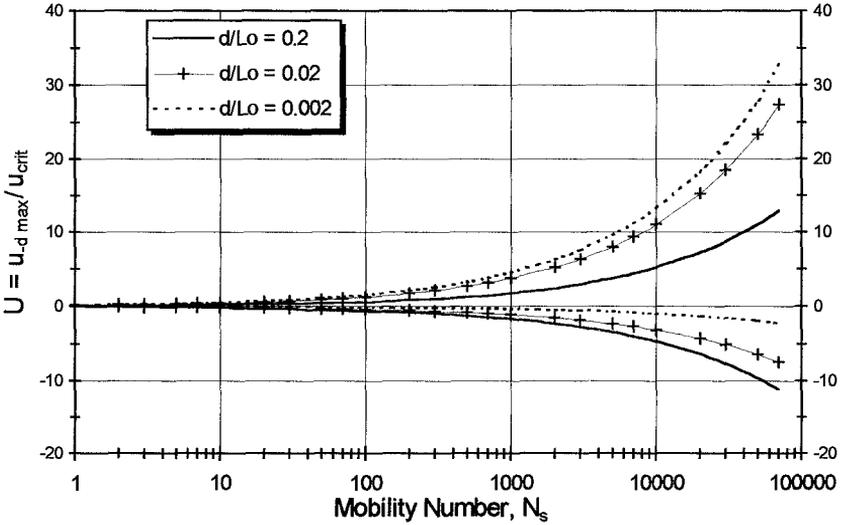


Fig. 1  $U_c$  and  $U_t$  as functions of mobility number, for three values of relative depth, from Eqs. 4 and 5.

b. Depth of Breaking Calculations

In order to test the ability of Eqs. 4 and 5 to predict erosional or accretional events on beaches, it is necessary to select a depth of application. The following procedure is used to select this depth:

The maximum or breaking wave height,  $H_b$ , consistent with SFWT can be estimated by:

$$H_b/d \approx 0.171(L_o/d) \{ \tanh[0.73(2\pi d/L_o)] \} \tag{7}$$

$R^2 = 0.9997, N = 10$   
 max. % error = 1.16%  
 rms % error = 0.72%  
 over the range,  $0.002 \leq d/L_o \leq 2.0$

Eq. 7 was developed using regression analysis. To estimate  $H_b$  from deep water conditions the breaker height index formula of Kaminski and Kraus (1993), is used:

$$H_b/H_o = 0.46(H_o/L_o)^{-0.28} \quad (8)$$

Combining Eqs. 7 and 8 gives a relative depth of breaking as:

$$d_b/L_o = 0.109 \ln[(1+x)/(1-x)] \quad (9)$$

where  $x = 2.69(H_o/L_o)^{0.72}$ .

Fig. 2 shows a plot of Eq. 9 with the relative depth at breaking as a function of deep-water wave steepness. The figure indicates that almost any deep-water steepness will yield a relative depth at breaking within the range required for Eqs. 4 and 5, i.e.  $0.002 \leq d/L_o \leq 0.2$ . For convenience, the relative depth at breaking calculated using Eq. 9 will be referred to as the reference depth.

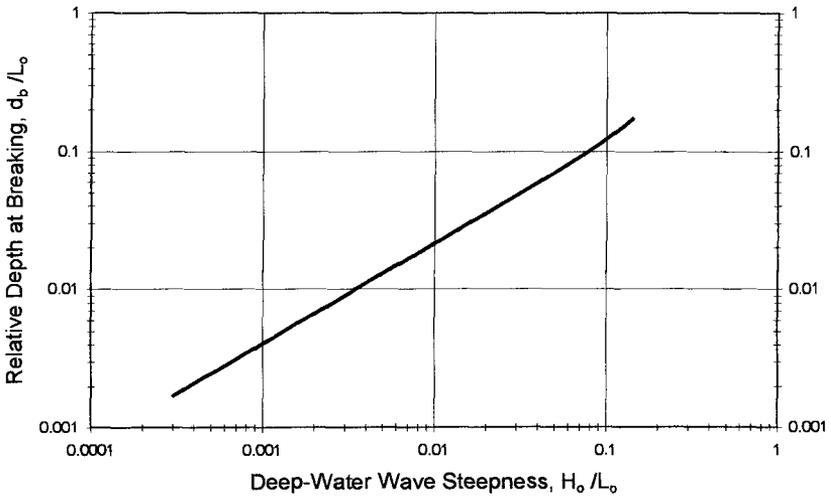


Fig. 2 Relative depth at breaking as a function of deep-water wave steepness

### c. Field Beach Profiles

Kraus, et al. (1991) used a field data set to develop criteria to discriminate between erosional and accretionary type beach profiles; these data are tabulated in Kraus and Mason (1991). Data are from beaches all over the world, collected by many researchers, and include observations of 99 distinct wave conditions. Deep-water significant wave heights were in the range  $0.08 \leq H_o \leq 7.90$  m, wave periods were in the range  $2.0 \leq T \leq 15.3$  sec, and sediment sizes were in the range  $0.17 \leq d_{50} \leq 3.5$  mm for

these observations. Wave periods are associated with either the deep-water significant wave height or deep-water spectral peak. For the field data set, the deep-water significant wave height is used in Eqs. 8 and 9; this approach is used to adapt equations developed for monochromatic waves to the irregular wave conditions of nature. The procedure yields relative depths (reference depth) near the seaward limit of the surf zone. Erosional profiles were defined as having no berm and at least one pronounced bar; accretionary profiles were defined as having a prominent berm and no bar formations.

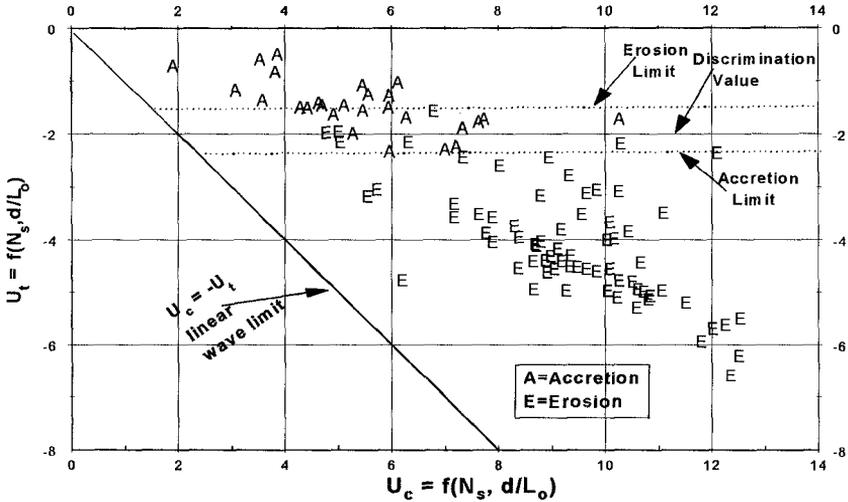


Fig. 3 Field beach data of Kraus and Mason (1991)

Accretionary or erosional type profiles are denoted in Fig. 3 as functions of  $U_c$  and  $U_t$ , calculated using Eqs. 4 and 5 respectively. Surprisingly,  $U_t$  discriminates well between erosional and accretionary profiles, without the need to use  $U_c$ , at a value around -2. Interestingly if  $U_t \leq -2$  erosion profiles will occur even if  $U_c$  is quite large. If a value of  $U_t = -2.0$  is used as a threshold level, see Fig. 3, then there are three miscategorized erosion and three miscategorized accretion observations in a data set of 99, for a prediction skill of 0.94. Skill is the ratio of correct predictions to total observations, Seymour and Castel (1989). If linear wave theory is used to calculate  $u_{d,max}$ , the ability to predict erosional and accretionary beach conditions declines to a skill of 0.88. The difference in skill may not seem great, but consider that the  $U_t$  approach corrects 6 of the 12 miscategorized observations using linear theory.

Fig. 3 shows a line for  $U_t = -2.0$ , as the best discrimination value between erosional and accretionary conditions. Above the discrimination line is a line given by  $U_t = -1.51$ , which is the smallest value which has only accretionary conditions above it, i.e. it is the limit for erosion. There is also a line for  $U_t = -2.27$ , which is the smallest

value which has only erosional conditions below it, i.e. it is the limit for accretion. These lines have been transferred to Fig. 4, using Eq. 5, which shows erosion/accretion conditions as a function of  $N_s$  and  $d_b/L_o$ . Above the upper limit curve is a region on the plane where erosion would be expected, below the lower limit curve is a region where accretion would be expected, and between the two is a transition region where both accretionary and erosional profiles are observed. The transition region is somewhat weighted toward accretionary profiles with 10 observations, as opposed to 6 observations of erosional profiles. If linear wave theory is used to establish a transition region, for the data of Kraus and Mason (1991), it contains 42 observations, 26 erosional and 16 accretionary. Clearly, the  $U_t$  approach is more effective in defining a transition region.

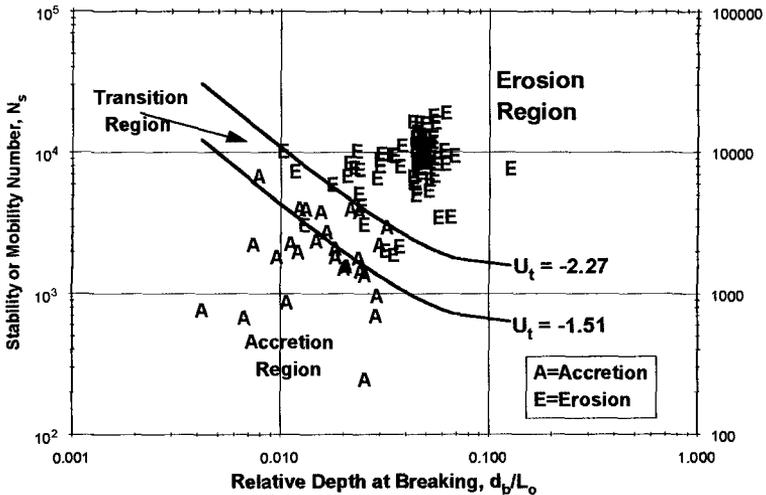


Fig. 4  $N_s$  vs.  $d_b/L_o$  plane showing beach condition regions defined by  $U_t$  values

Another perspective on the information shown in Fig. 4 can be obtained by using Eq. 9 to transform the relative depth at breaking to deep-water wave steepness. This transformation is shown in Fig. 5 for the limit curves and a curve for the approximate limit of sediment movement. The limit for sediment movement was calculated by setting  $U_c = 1.0$  in Eq. 4; this limit helps add scale to the figure.

d. Offshore Dumping of Gravel for Beach Nourishment

Zenkovich and Schwartz (1987) discuss offshore dumping of gravel in the Black Sea. Gravel was dumped by barge in depths of 4 to 6 m and storm wave conditions brought this material up to the shoreline over the next year or so to provide effective

beach nourishment and shore protection. Findings of Zenkovich and Schwartz were consistent with a limited number of wave tank tests at CERC (Ahrens and Camfield 1989).

Typically the deep-water wave steepness in a storm is in the range,  $0.03 \leq H_0/L_0 \leq 0.06$ . Eqs. 7 and 8 can be used to calculate breaker heights, which for this range of steepness is  $H_b/d \approx 0.76$  and in these water depths give  $3.04 \leq H_b \leq 4.56$  m. The size of the gravel was not given in Zenkovich and Schwartz (1987), but sediment with median diameters in the range of  $5 \leq d_{50} \leq 76$  mm is normally regarded as gravel. This range of breaker heights and sediment sizes gives mobility numbers in the range  $24 \leq N_s \leq 553$ . Fig. 5 shows that this range of mobility numbers and wave steepness generally fall into the accretionary region, which is consistent with the on-shore movement of the gravel observed by Zenkovich and Schwartz. The curve for the limit to sediment movement suggests that coarse gravel might have remained at depths of 4-6 m rather than move onshore. If fine sand, say  $d_{50} = 0.2$  mm, had been dumped at these depths mobility numbers in the range  $9,152 \leq N_s \leq 13,818$  would have been obtained and Fig. 5 indicates this is an erosional condition, i.e., the sand would have moved offshore during

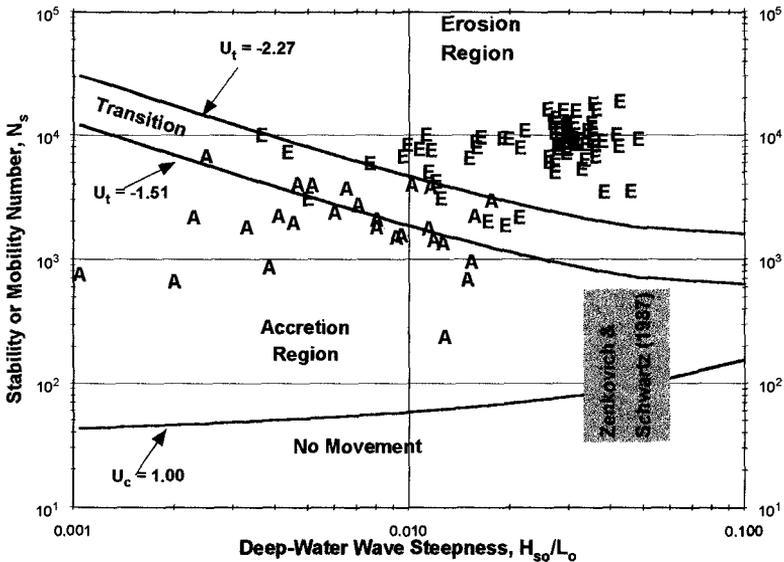


Fig. 5  $N_s$  vs.  $H_0/L_0$  plane with beach condition regions defined by  $U_t$  and  $U_c$

storms.

Fig. 5 provides a rather comprehensive view of erosion/accretion processes on beaches and is easier to interpret than Fig. 4. Fig. 5 shows that when the deep-water waves are steep, erosional conditions dominate, but there is a window of accretional conditions for gravel that was confirmed by the research of Zenkovich and Schwartz (1987). When deep-water steepness is quite small, say  $H_o/L_o = 0.001$ , accretional conditions dominate. At this steepness erosion will occur only for median sediment diameters smaller than used in the development of this model, i.e.,  $d_{50} = 0.1$  mm.

#### e. A Further Simplification and Application

Fig. 5 suggests that a further simplification could be made in predicting erosional and accretionary conditions by determining the equations of the discrimination curve, the accretion limit curve, erosion limit curve, and the limit of movement curve directly as  $N_s = f(H_{so}/L_o)$ . This functional relation was determined using regression analysis, with the following results:

for  $U_c = 1.00$ , approximate limit of movement,

$$N_s = 93.2(H_{so}/L_o)^{0.113} \exp[7.89(H_{so}/L_o)] \quad (10)$$

$$R^2 = 0.9998, N = 99, 0.1 \leq H_{so}/L_o \leq 0.001$$

and for  $U_t$ ,

$$N_s = C_o(H_{so}/L_o)^{-0.854} \exp[10.1(H_{so}/L_o)] \quad (11)$$

$$R^2 = 0.9999, N = 99, 0.1 \leq H_{so}/L_o \leq 0.001$$

and where:  $C_o = 30.8$ ,  $U_t = -1.51$ , erosion limit

$C_o = 62.1$ ,  $U_t = -2.00$ , best discrimination value

$C_o = 83.0$ ,  $U_t = -2.27$ , accretion limit

Eq. 11 discriminates between erosional and accretionary conditions exactly the same way as Eq. 5 and has exactly the same 16 observations in the transition region as Eq. 5. However, Eq. 11 is easier to use than Eq. 5 because there is a direct link between the mobility number and wave steepness. Fig. 6 illustrates an application of Eq. 11.

In Fig. 6, beach condition regions are shown as a function of  $H_{so}$  and  $T$  for two beaches, one with  $d_{50} = 0.2$  mm and one with  $d_{50} = 0.3$  mm, and using  $\Delta = 1.65$ . Using the 0.2 mm beach as a reference, it can be seen that the transition region moves up for the 0.3 mm beach expanding the accretion region and reducing the erosion region. The figure provides a very clear idea of what wave conditions cause erosion or accretion and provides a simple way to compare the response of beaches with different size sand to various wave conditions. Fig. 6 could be used to assess the value of nourishing a 0.2 mm beach with somewhat coarser sand. The limit on wave steepness is  $H_{so}/L_o = 0.08$ .

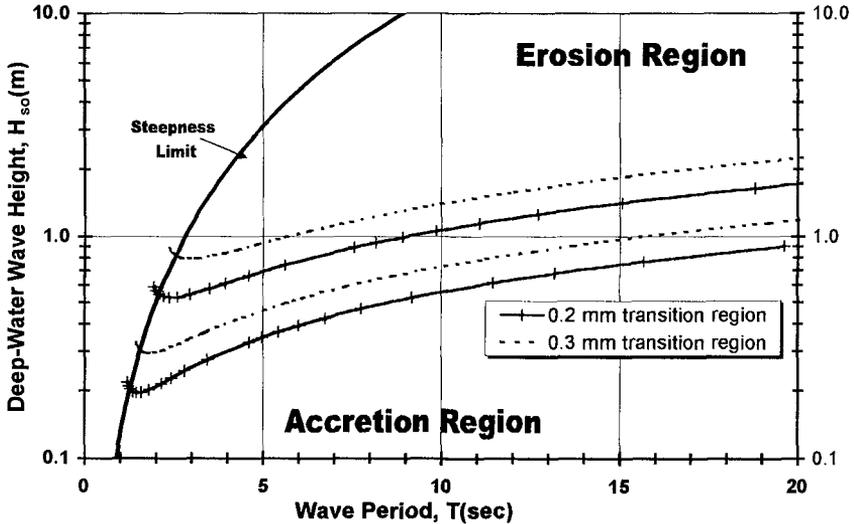


Fig. 6 Beach condition regions for 0.2 and 0.3 mm sand as a function of  $H_{so}$  and  $T$

A question and a comment made at the conference can be addressed in this section. The question in essence was, can this model predict erosion of gravel beaches? The related comment observed, in part, that gravel beaches do in fact suffer erosion, however, they recover very quickly, Nicholls (1998). Fig. 7 shows the beach condition regions for a beach composed of small gravel, i.e.,  $d_{50} = 5$  mm. The figure indicates that small gravel will erode, but that  $H_{so}$ 's would probably have to be in the range of 10 – 15 m. There is a very large accretion region in Fig. 7 indicating that most wave conditions are accretionary for a 5 mm beach and supporting Nicholl's observation that gravel beaches recover from erosion very quickly. Fig. 7 also shows a small region where no onshore/offshore sediment movement would be expected. For the 0.2 mm and 0.3 mm beaches shown in Fig. 6, there is no region of no movement shown because wave heights for this region were always less than 0.1 m.

### Summary and Conclusions

This paper uses nonlinear wave theory to predict cross-shore sediment movement under waves in shallow water. This synthesis of wave theory and sediment movement initiation criteria allows much of the present understanding of cross-shore sediment movement to be schematized or modeled using only two dimensionless variables, i.e., a mobility number and relative depth. When calibrated, the model shows surprising skill (0.94) in predicting erosional or accretionary beach profiles. The model is sensitive enough to distinguish between strong and weak erosional/accretionary tendencies. A wide range of sediment sizes was included in the development, which allows consideration of not only sand, but also gravel-sized particles. The model also provides

an interesting and useful perspective on cross-shore sediment movement under waves as functions of a mobility number and relative depth or a mobility number and deep-water wave steepness.

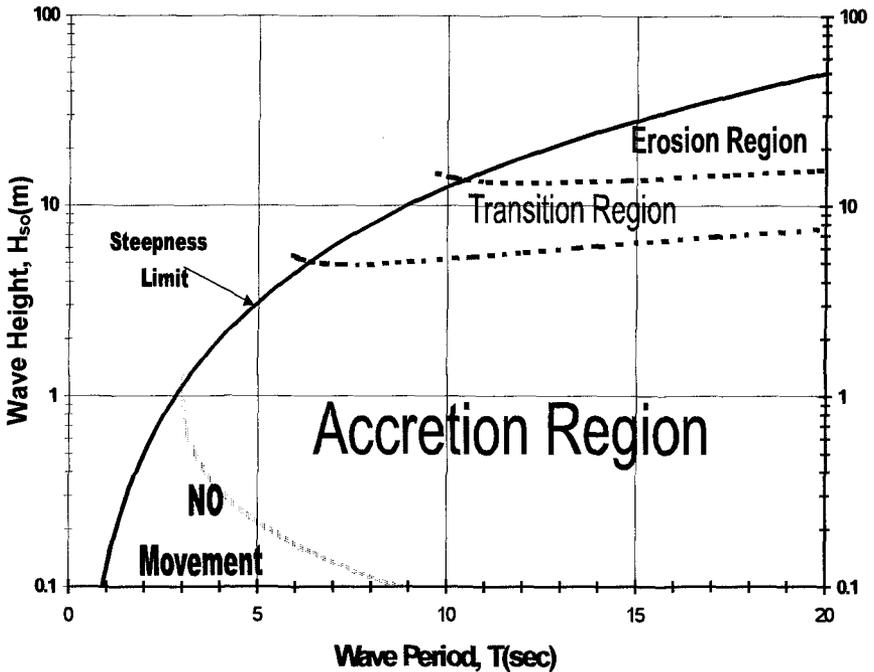


Fig. 7 Beach conditions for  $d_{50} = 5.0$  mm as a function of  $H_{50}$  and  $T$

It is the characteristics of nonlinear waves and specifically, the near-bottom velocities under the troughs, that accounts for the high skill of the model in predicting erosion/accretionary beach conditions. If linear wave theory is used to predict near-bottom velocities, the ability of the model to discriminate between erosional and accretionary beach events declines from a skill of 0.94 to 0.88.

The reference depth used to apply the model is near the outer limit of the surf zone. Surprisingly, the dimensionless version of the reference depth is easy to calculate, Eq. 9, as function of only deep-water wave steepness. When near-bottom velocities under a trough are more than twice the velocity required to initiate sediment movement, at this depth, the beach experiences erosion. If this ratio of velocities is less than two, then accretionary or possibly static profile conditions exist on the beach. Prediction of erosion or accretion is not improved by knowledge of the near-bottom fluid movement under the crest.

Simplifications were possible at three stages in the development of this model: First, the recognition that much of the physics of sediment movement under waves in shallow water could be summarized using only two variables, i.e., a mobility number and either relative depth or deep-water wave steepness. Second, the discovery that erosional or accretionary conditions, or a transition region could be discriminated using only Eq. 5, related to movement under the trough. Third, that a direct relationship between the mobility number and deep-water wave steepness could be developed, Eq. 11, that could be used to define beach conditions.

### Appendix, Definition of Symbols

$d$  - water depth

$d_b$  - water depth at wave breaking

$d_{50}$  - median grain diameter

$d_b/L_o$  - relative depth at breaking, referred to as reference depth

$g$  - acceleration of gravity

$H$  - wave height

$H_o$  - deep-water monochromatic wave height or significant deep-water wave height for field wave conditions, depending on context

$H_b$  - maximum or breaking wave height

$L_o$  - deep water wave length =  $gT^2/2\pi$

$N_s$  - mobility number, Eq. 6

% error = [(predicted - observed)/observed]\*100

rms - root mean square

SFWT - Stream Function Wave Theory

$T$  - monochromatic wave period or characteristic wave period for field wave conditions, depending on context

$u_{d\ max}$  - maximum near bottom velocity due to waves

$u_{crit}$  - critical velocity required to initiate sediment movement

$U$  - ratio  $u_{d\ max}/u_{crit}$ , in general

$U_c$  - value of  $U$  under crest

$U_t$  - value of  $U$  under trough

$\rho_s$  - density of sediment

$\rho$  - density of water

$\Delta$  -  $(\rho_s - \rho)/\rho$

### References

Ahrens, J.P., and Camfield, F. (1989). "Dynamic Revetments," The CERCular, Information Exchange Bulletin, Vol. CERC-89-2, Coastal Engineering Research Center, Vicksburg, MS.

Ahrens, J.P. and Hands, E.B., (1998). "Velocity Parameters for Predicting Cross-Shore Sediment Movement," ASCE Jour. of Waterway, Port, Coastal and Ocean Engineering, Vol.124, No.1, 16-20.

- Dean, R.G.(1974). "Evaluation and Development of Water Wave Theories for Engineering Application; Volume I - Presentation of Research Results; Volume II - Tabulation of Dimensionless Stream-Function Variables," Coastal Engineering Research Center Special Report No. 1, Fort Belvoir, VA.
- Dean, R.G. (1987). "Coastal Sediment Processes: Toward Engineering Solutions," Proc. Coastal Sediments '87, ASCE, 1-24.
- Hallermeier, R.J.(1980). "Sand Motion Initiation by Water Waves: Two Asymptotes," ASCE Jour. of Waterway, Port, Coastal, and Ocean Engineering, Vol. 106, No. WW3, 299-318.
- Hudson, R.Y., Herrmann, F.A., Sager, R.A., Whalen, R.W., Keulegan, G.H., Chatham Jr., C.E., and Hales, L.Z., (1979). "Coastal Hydraulic Models," Coastal Engineering Research Center Special Report No. 5, Ft. Belvoir, VA.
- Kaminski, G.M., and Kraus, N.C.(1993). "Evaluation of Depth-Limited Wave Breaking Criteria," ASCE Proceedings Ocean Wave Measurement and Analysis Conf., ASCE, New Orleans, LA, 180 - 193.
- Komar, P.D., and Miller, M.C.(1974). "Sediment Threshold under Oscillatory Waves," Proc. 14th International Conference Coastal Engineering, ASCE, Copenhagen, Denmark, 756-775.
- Kraus, N.C., Larson, M., and Kriebel, D.L.(1991). "Evaluation of Beach Erosion and Accretion Predictors," Proceedings Coastal Sediments '91, ASCE, Seattle, WA, 572-587.
- Kraus N.C., and Mason, J.M.(1991). "Field Data Set for Testing Beach Erosion and Accretion Predictive Criteria," Memorandum for Record, Coastal Engineering Research Center, U.S. Army Eng. Waterways Experiment Station, Vicksburg, MS, 28 May 1991.
- Madsen, O.S. (1998) personal communication.
- Nicholls, R.J. (1998) personal communication.
- Seymour, R.J., and Castel, D.(1989). "Modeling Cross-Shore Transport", in *Nearshore Sediment Transport*, R.J. Seymour, Editor, Plenum Press, New York.
- Zenkovich, V.P. and Schwartz, M.L.(1987). "Protecting the Black Sea-Georgian S.S.R. Gravel Coast," J. of Coastal Research, 3 (2), 201-211.