QUASI 3-D EFFECTS IN INFRAGRAVITY WAVES

A.R. Van Dongeren[1], I.A. Svendsen[2], and U. Putrevu[3]

ABSTRACT: *The present paper describes a numerical study of infragravity waves forced by obliquely-incident wave groups. In this study, the depth-integrated, shortwave-averaged nearshore circulation model SHORECIRC (Van Dongeren et al., 1994), which includes the quasi three-dimensional effects, is used. The governing equations that form the basis of the model, as well as the numerical model and the boundary conditions are described. The model is applied to the case of leaky infragravity waves. The magnitude of the quasi 3-D terms and their effect on the infragravity wave envelope is analyzed, and the velocity profiles of the infragravity waves are shown.*

INTRODUCTION

The depth-integrated, shortwave-averaged nearshore circulation model SHORECIRC (Van Dongeren *et al.*, 1994) belongs to the class of quasi-3D (Q3D) models, which combine the effect of the vertical structure with the simplicity of two-dimensional horizontal (2DH) models.

Several approaches to the development of Q3D models can be found in the literature. De Vriend & Stive (1987) split the current into primary and secondary flow profiles based on the assumption that the primary velocity profiles are the same in the cross-shore and longshore direction. In a different approach, Svendsen & Lorenz (1989) determined the vertically-varying longshore and cross-shore currents separately under the assumption of weak dependence. Svendsen & Putrevu (1990) formulated a steady state version of the nearshore circulation model using analytical solutions for the 3D current profiles in combination with a numerical solution of the depth-integrated 2D horizontal equations for a long straight coast. They split the current velocity into a depth-invariant component and a component with a vertical variation with zero mean flow integrated over the central layer. Sánchez-Arcilla *et al.* (1990, 1992) presented a similar concept.

Putrevu & Svendsen (1992) and Svendsen & Putrevu (1994) recognized that the current-current and current-wave interactions neglected in previous investigations induce a non-linear dispersion mechanism, analogous to the dispersion of solutes (Taylor, 1954). This mechanism significantly augments the lateral turbulent mixing

[1]Associate Researcher, Netherlands Centre for Coastal Research, Delft University of Technology, P.O. Box 5048, 2600 GA, Delft, The Netherlands. email: A.vanDongeren@ct.tudelft.nl

[2]Distinguished Professor, Center for Applied Coastal Research, University of Delaware, Newark, DE 19716, USA. email: ias@coastal.udel.edu

[3]Research Scientist, NorthWest Research Associates, Inc., 14508 NE 20th Street, Bellevue, WA 98007, USA. e-mail: putrevu@nwra.com.

and accounts for the difference in magnitude between the vertical and horizontal mixing in the case of a longshore current on a long, straight coast.

The time-dependent version of this model, called SHORECIRC, was presented in Van Dongeren *et al.* (1994) for the special case of longshore uniformity in both the bathymetric and hydrodynamical conditions. The generalized quasi 3-D governing equations were derived in Putrevu & Svendsen (1997a,b) and Van Dongeren & Svendsen (1997a).

Recently, SHORECIRC has been used to study a number of nearshore phenomena, such as shear instabilities over longshore-varying bathymetries (Sancho *et al.*, 1997, 1998) and rip currents over a bar (Haas *et al.*, 1998).

In the present paper SHORECIRC will be used to study infragravity waves forced by obliquely-incident wave groups and the associated quasi 3-D effects on a cylindrical coast.

GOVERNING EQUATIONS

The depth-integrated, time-averaged mass and momentum equations incorporated in SHORECIRC are derived from the Reynolds' equations which are integrated over depth and averaged over the shortwave period largely following the procedure outlined in Phillips (1977) and Mei (1983). In the present approach, which is given in more detail in Putrevu & Svendsen (1997a,b) and Van Dongeren & Svendsen (1997a), the total horizontal velocity of the long and short waves is further split into a depth-uniform long wave part $\tilde{V}_\alpha$, a depth-varying long wave part $V_{1\alpha}$ and a short-wave contribution $u_{w\alpha}$, so that

$$u_\alpha(x,y,z,t) = \tilde{V}_\alpha(x,y,t) + V_{1\alpha}(x,y,z,t) + u_{w\alpha}(x,y,z,t) \tag{1}$$

We define $\overline{u_{w\alpha}} \equiv 0$ below trough and define the shortwave-induced volume flux above the trough level as

$$Q_{w\alpha} \equiv \overline{\int_{\zeta_t}^{\zeta} u_{w\alpha}\, dz} \tag{2}$$

where ζ_t denotes the trough level of the short-wave motion. The total flux $\tilde{Q}_\alpha$ can then be written as

$$\tilde{Q}_\alpha \equiv \overline{\int_{-h_o}^{\bar{\zeta}} u_\alpha\, dz} = \tilde{V}_\alpha\, h + \int_{-h_o}^{\bar{\zeta}} V_{1\alpha}\, dz + Q_{w\alpha} \tag{3}$$

where h_o is the still water depth, $\bar{\zeta}$ is the surface elevation of the long (or IG) wave motion, and $h = h_o + \bar{\zeta}$ is the total depth. With this result, the depth-integrated shortwave-averaged conservation of mass can be written as

$$\frac{\partial \bar{\zeta}}{\partial t} + \frac{\partial \tilde{Q}_\alpha}{\partial x_\alpha} = 0 \tag{4}$$

where the index α represents the horizontal x, y directions, and z is the vertical coordinate, defined from the still water level (SWL) up.

Using the same splitting procedure, the horizontal conservation of momentum can be expressed as

$$\frac{\partial \bar{Q}_\beta}{\partial t} + \frac{\partial}{\partial x_\alpha}\left(\frac{\bar{Q}_\alpha \bar{Q}_\beta}{h}\right) + \frac{\partial}{\partial x_\alpha}\overline{\int_{\zeta_t}^{\zeta} (u_{w\alpha} V_{1\beta} + u_{w\beta} V_{1\alpha})\, dz} +$$

$$\frac{\partial}{\partial x_\alpha}\int_{-h_o}^{\bar{\zeta}} V_{1\alpha} V_{1\beta}\, dz = -g h \frac{\partial \bar{\zeta}}{\partial x_\beta} - \frac{1}{\rho}\frac{\partial}{\partial x_\alpha}\left[S_{\alpha\beta} - \overline{\int_{-h_o}^{\zeta} \tau_{\alpha\beta}\, dz}\right] + \frac{\tau_\beta^S - \tau_\beta^B}{\rho} \quad (5)$$

where β is index notation for the horizontal x and y directions, $\tau_{\alpha\beta}$ represents the turbulent shear stresses, τ_β^S is the surface stress and τ_β^B is the bottom stress. The radiation stress is defined as[1]

$$S_{\alpha\beta} \equiv \overline{\int_{-h_o}^{\zeta} (p\,\delta_{\alpha\beta} + \rho u_{w\alpha} u_{w\beta})\, dz} - \delta_{\alpha\beta}\frac{1}{2}\rho g h^2 \quad (6)$$

The governing equations (4) and (5) would be readily solvable numerically if the depth variation of the long wave velocity $V_{1\alpha}$ were known. However, this would require a three-dimensional grid and therefore a large computional time. In order to reduce the computational time, the depth-dependent terms can be replaced by semi-analytical solutions, which are functions of the depth-integrated terms. In this so-called quasi 3-D approach only a two-dimensional numerical model is needed. This substitution can be achieved by using the local (i.e., *not* depth-integrated), time-averaged momentum equations (see e.g., Svendsen & Lorenz, 1989), which after some manipulations can be written as (Putrevu & Svendsen, 1997a,b; Van Dongeren & Svendsen, 1997a)

$$\frac{\partial V_{1\beta}}{\partial t} - \frac{\partial}{\partial z}\left(\nu_t \frac{\partial V_{1\beta}}{\partial z}\right) = -\beta_\beta + \frac{1}{\rho h}\frac{\partial}{\partial x_\alpha}\left(S_{\alpha\beta} - \overline{\int_{-h_o}^{\zeta} \tau_{\alpha\beta}\, dz}\right) - \frac{\tau_\beta^S - \tau_\beta^B}{\rho h}$$

$$- V_{1\alpha}\frac{\partial \tilde{V}_\beta}{\partial x_\alpha} - \tilde{V}_\alpha \frac{\partial V_{1\beta}}{\partial x_\alpha} - W\frac{\partial V_{1\beta}}{\partial z} \quad (7)$$

where

$$\beta_\beta = \frac{\partial}{\partial x_\alpha}\left(\overline{u_{w\alpha}u_{w\beta}} - \overline{w_w^2}\right) + \frac{\partial \overline{u_{w\beta}w_w}}{\partial z} - \frac{\partial}{\partial x_\alpha}\left(\nu_t\left(\frac{\partial \tilde{V}_\alpha}{\partial x_\beta} + \frac{\partial \tilde{V}_\beta}{\partial x_\alpha}\right)\right) \quad (8)$$

In this expression w_w denotes the vertical short-wave velocity and ν_t is the turbulent eddy viscosity.

Eq. (7) can be solved more easily if we split the depth-varying velocity into two parts

$$V_{1\beta} = V_{1\beta}^{(0)} + V_{1\beta}^{(1)} \quad (9)$$

[1]This definition is symbolically similar to Mei (1983), who uses a different definition of $u_{w\alpha}$, however. He requires $\overline{\int_{-h_o}^{\zeta} u_{w\alpha}\, dz} = 0$.

where the first part is primarily the (slowly time-varying) component generated by the local external forcing, which are the first five terms on the RHS of (7), while the second, smaller contribution is generated by the advective terms (the last three terms on the RHS) in (7). Hence, we can solve for $V_{1\beta}^{(0)}$, which represents the first approximation to the depth-varying part of the infragravity velocity profiles.

In the present paper we further assume a quasi steady-state. This means that we can solve the first part of (7) by integration twice over depth with two boundary conditions: a slip boundary condition at the bottom and a conservation of mass condition over the vertical. If we define

$$f_\beta \equiv \beta_\beta - \frac{1}{\rho h}\frac{\partial}{\partial x_\alpha}\left(S_{\alpha\beta} - \overline{\int_{-h_o}^{\zeta} \tau_{\alpha\beta}\, dz}\right) + \frac{\tau_\beta^S - \tau_\beta^B}{\rho h} \tag{10}$$

the solution of (7) becomes

$$V_{1\beta}^{(0)} = \frac{f_\beta}{2\,\nu_t}\,\xi^2 + \frac{\tau_\beta^B}{\rho\,\nu_t}\,\xi - \left(\frac{f_\beta}{6\,\nu_t}\,h^2 + \frac{\tau_\beta^B}{\rho\,\nu_t}\,\frac{h}{2} + \frac{Q_{w\beta}}{h}\right) \tag{11}$$

where the vertical coordinate z has been transformed to a new coordinate ξ, under the transformation $\xi = z + h_o$, which means that $\xi = 0$ at the local bottom and $\xi = h = h_o + \bar{\zeta}$ at the mean surface elevation.

Putrevu & Svendsen (1997a,b) show that this expression is the first approximation to the time-dependent solution of (7). Hence, in the quasi-steady approximation the velocity profiles are quadratic and known so that the quasi 3-D coefficients (12) - (15) can be expressed in terms of the coefficients of the velocity profiles.

Following the derivation of Putrevu & Svendsen (1997) and Van Dongeren & Svendsen (1997a), which is omitted here for brevity, we can define the coefficients

$$D_{\alpha\gamma} \equiv \frac{1}{h}\int_{-h_o}^{\bar{\zeta}} V_{1\alpha}^{(0)} \int_z^{\bar{\zeta}} \frac{1}{\nu_t} \int_{-h_o}^{z} V_{1\gamma}^{(0)}\,(dz)^3 \tag{12}$$

$$M_{\alpha\beta} \equiv \int_{-h_o}^{\bar{\zeta}} V_{1\alpha}^{(0)}\, V_{1\beta}^{(0)}\, dz + V_{1\alpha}^{(0)}(\bar{\zeta})\, Q_{w\beta} + V_{1\beta}^{(0)}(\bar{\zeta})\, Q_{w\alpha} \tag{13}$$

$$\begin{aligned} A_{\alpha\beta\gamma} \equiv & -\int_{-h_o}^{\bar{\zeta}} V_{1\alpha}^{(0)} \int_z^{\bar{\zeta}} \frac{1}{\nu_t}\left(\frac{\partial}{\partial x_\gamma}\int_{-h_o}^{z} V_{1\beta}^{(0)}\, dz - V_{1\beta}^{(0)}\frac{\partial h_o}{\partial x_\gamma}\right)(dz)^2 \\ & -\int_{-h_o}^{\bar{\zeta}} V_{1\beta}^{(0)} \int_z^{\bar{\zeta}} \frac{1}{\nu_t}\left(\frac{\partial}{\partial x_\gamma}\int_{-h_o}^{z} V_{1\alpha}^{(0)}\, dz - V_{1\alpha}^{(0)}\frac{\partial h_o}{\partial x_\gamma}\right)(dz)^2 \end{aligned} \tag{14}$$

and

$$\begin{aligned} B_{\alpha\beta} \equiv & \frac{1}{h}\int_{-h_o}^{\bar{\zeta}} V_{1\alpha}^{(0)} \int_z^{\bar{\zeta}} \frac{1}{\nu_t} \int_{-h_o}^{z} V_{1\beta}^{(0)}\,(dz)^3 - \frac{1}{h}\int_{-h_o}^{\bar{\zeta}} V_{1\alpha}^{(0)} \int_z^{\bar{\zeta}} \frac{1}{\nu_t} V_{1\beta}^{(0)}\,(h_o + z)\,(dz)^2 \\ & -\frac{1}{h}\int_{-h_o}^{\bar{\zeta}} V_{1\beta}^{(0)} \int_z^{\bar{\zeta}} \frac{1}{\nu_t} V_{1\alpha}^{(0)}\,(h_o + z)\,(dz)^2 \end{aligned} \tag{15}$$

These expressions appear when the solution for $V_{1\alpha}^{(0)}$ is substituted into the depth-dependent integrals in the conservation of momentum equation (5), which - after some algebra - becomes

$$\frac{\partial \bar{Q}_\beta}{\partial t} + \frac{\partial}{\partial x_\alpha}\left(\frac{\bar{Q}_\alpha \bar{Q}_\beta}{h} + M_{\alpha\beta}\right) - \frac{\partial}{\partial x_\alpha}\left[h\left(D_{\beta\gamma}\frac{\partial \tilde{V}_\alpha}{\partial x_\gamma} + D_{\alpha\gamma}\frac{\partial \tilde{V}_\beta}{\partial x_\gamma} + B_{\alpha\beta}\frac{\partial \tilde{V}_\gamma}{\partial x_\gamma}\right)\right]$$
$$+ \frac{\partial}{\partial x_\alpha}\left[A_{\alpha\beta\gamma}\tilde{V}_\gamma\right] = -g\,h\frac{\partial \bar{\zeta}}{\partial x_\beta} - \frac{1}{\rho}\frac{\partial}{\partial x_\alpha}\left(S_{\alpha\beta} - \overline{\int_{-h_o}^{\zeta} \tau_{\alpha\beta}\,dz}\right) + \frac{\tau_\beta^S - \tau_\beta^B}{\rho} \quad (16)$$

where an eddy viscosity closure for the turbulent shear stresses (e.g., Rodi, 1980) is used.

NUMERICAL MODEL

In the version of SHORECIRC used in this paper, eqs. (4) and (16) are solved using a central finite difference scheme on a fixed spatial grid with an explicit second-order Adams-Bashforth predictor and a third-order Adams-Moulton corrector time-stepping scheme. Additionally, the program evaluates (11) in order to calculate the quasi 3-D coefficients (12) - (15).

At the seaward boundary an absorbing-generating boundary condition which is capable of simultaneously generating and absorbing obliquely-incident long waves with a minimum of reflection is implemented, see Van Dongeren & Svendsen (1997b) for details.

At the shoreline, an inundation-drainage procedure, as described in Van Dongeren & Svendsen (1997a), is used which allows for time-varying run-up and run-down of the shortwave-averaged shoreline. At the lateral (shore-normal) boundaries a periodicity condition is used.

QUASI 3-D EFFECTS IN LEAKY INFRAGRAVITY WAVES

In this analysis the model is applied to the case of leaky infragravity waves forced by obliquely-incident wave groups. We will focus on the effect of the quasi 3-D coefficients defined in (12) - (15) and the corresponding quasi 3-D terms in the momentum equation (16).

In our investigation we will choose a very simple bathymetry consisting of a plane beach rising from an offshore shelf, see Fig. 1a for a definition sketch. On the offshore shelf we assume that the depth varies so gently that the waves stay in local equilibrium and that at the toe of the coastal slope the set-down wave corresponds to the equilibrium bound wave found by Longuet-Higgins & Stewart (1962, 1964). This avoids the difficulty encountered by e.g. Lippmann *et al.* (1997) in justifying the use of long wave theory in the deep water part of their plane slope. It also enables us to specify the conditions at the toe as a simple boundary condition for the inflow to the coastal slope. In the present study we limit the analysis to a plane coastal slope to facilitate a comparison to analytical results, but it should be emphasized that the model can be run on an arbitrary bottom topography.

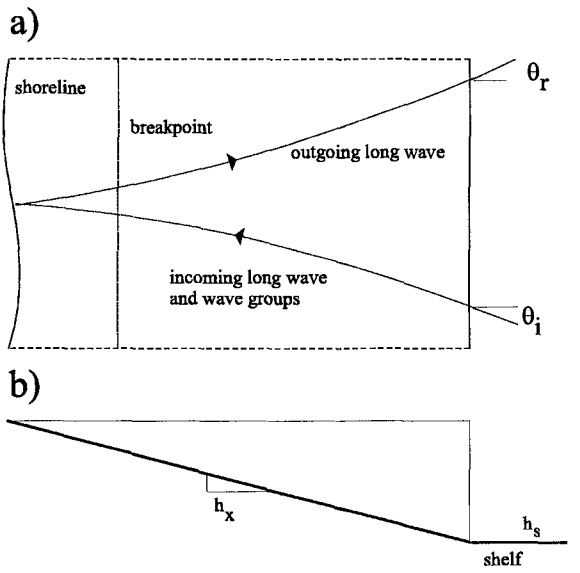

Figure 1: Definition sketch of obliquely-incident and obliquely-reflected infragravity waves. a) plan view; b) cross-section.

The incident wave groups consist of two sinusoidal short wave components that have a slightly different frequency but have the same direction of propagation as a given depth.

Given this short-wave forcing, the radiation stress $S_{\alpha\beta}$ can be written as (generalizing from Schäffer (1994))

$$S_{\alpha\beta} = \rho g P_{\alpha\beta} \begin{cases} H_1^2 \, (1 + 2\delta \cos(2\vartheta)) \,, & h \geq h_b \\ \gamma^2 h_o^2 \, (1 + 2\delta \, (1 - \kappa) \cos(2\vartheta)) \,, & h \leq h_b \end{cases} \tag{17}$$

where H_1 is the height of the carrier wave and the wave height modulation $\delta = H_2/H_1$ is the ratio of the wave heights of the secondary wave to the primary wave in the group. In this formulation it is assumed that δ is small. κ is the breaking parameter, as defined by Schäffer (1994). The breaking criterion is $H_1 = \gamma h_o$, $P_{\alpha\beta}$ is the nondimensional shape parameter for the short waves, and ϑ is the phase function defined following Schäffer (1994).

As these wave groups propagate towards shore at the group speed c_g, they refract towards the shore-normal direction whereby it is assumed that both short wave components refract in the same way and that they do not diverge. The incoming bound infragravity wave that propagates with the groups in the same direction will

also refract in. As the wave groups shoal onto the beach and the short waves are dissipated, this incoming IG wave is modified by the wave group transformation and is released. It will then reflect off the shore and propagate and refract seawards as a free long wave, see Fig. 1 for a definition sketch.

In the following case the input parameters are: shelf depth $h_s = 3\,m$ and mean short-wave frequency $\omega = 1.8\,s^{-1}$, so that the wavenumber $k_s = 0.397\,m^{-1}$. The wave height of the primary short wave on the shelf $H_{1,s} = 2\,a_{1,s} = 0.6\,m$ and the wave height modulation $\delta = 0.1$. The frequency modulation is chosen as $\epsilon = 0.1$. The breaking index is $\gamma = 0.75$, and we choose first to consider a fixed breakpoint $\kappa = 0$. The beach slope is chosen as $h_x = 1/20$. Finally, the angle of incidence on the shelf is chosen as $\theta_{i,s} = 22.37^o$ with the normal, which corresponds to an alongshore wave length of the infragravity wave of 150 meters. The angle of incidence is less than the limiting angle of incidence $\theta_{i,s}^{max} = 37.07^o$, so that the infragravity wave motion is "leaky," i.e., the outgoing long waves reach the shelf and are not trapped.

On the shelf we will assume an incoming equilibrium bound long wave (Longuet-Higgins & Stewart, 1962, 1964) which propagates in the direction of the wave groups. The longshore domain length is chosen equal to the longshore projection of the infragravity wave length.

Fig. 2 shows the normalized envelope of the IG wave as a function of the cross-shore coordinate h_o/h_s, where the origin is at the still water shoreline and unity is at the toe of the slope. The dash-dotted lines indicate linear solution by Schäffer & Svendsen (1988). The dashed lines indicate the envelope computed by the present model *without* the quasi 3-D terms. It can be seen that the nonlinear terms in the equations shift the (quasi) nodes shoreward and have some effect on the amplitude of the anti-nodes. The solid lines indicate the solution computed by the full quasi 3-D model. The quasi 3-D terms change the horizontal distribution of the IG wave amplitudes, especially around the breakpoint and in the surf zone. This will be analyzed in more detail below.

In order to calculate the quasi 3-D terms in the momentum equations, we have to determine the vertical variation of the infragravity wave particle velocities from (11). It is illustrative to use this numerical output and plot the IG particle velocity profiles at various locations and at various time-instances.

Fig. 3 shows the IG particle velocity profiles for three different locations (h_o/h_s = 0.42, 0.17 and 0.07) and at five time intervals of the infragravity wave period. The motion is a result of the forcing by the obliquely-incident wave groups, and of the incoming IG wave and the obliquely-reflected IG wave. It is important to notice that the steady part of the short-wave forcing drives a steady longshore current and a steady undertow, which are both included in the figure, and that the time-varying part of the forcing cause a variation of the velocity profiles over an IG period, so in essence the infragravity wave motion is riding on top of a relatively strong current.

Since the breakpoint is located at $h_o/h_s = 0.3$, the location $h_o/h_s = 0.42$ is outside the surf zone. The IG wave velocity profiles at that location show a little

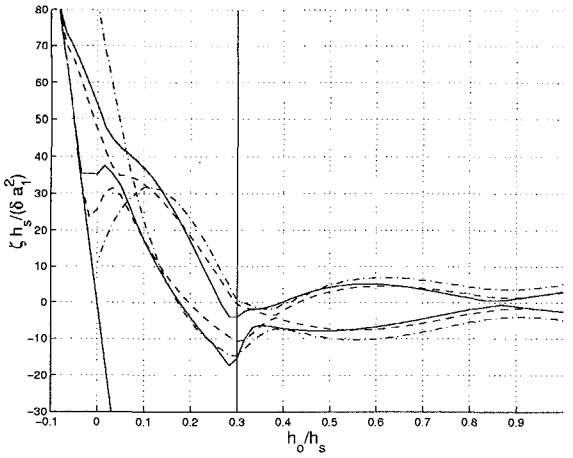

Figure 2: Envelope of the surface elevation of an IG wave vs. cross-shore distance: linear analytical solution by Schäffer & Svendsen (1988) (dash-dot line), SHORECIRC without Q-3D terms (dashed line), and SHORECIRC with Q-3D terms (solid line). The breakpoint is located at $h_o/h_s = 0.3$.

bit of curvature in the cross-shore direction and essentially vary linearly with depth in the longshore direction. The two locations inside the surf zone show much more variation. In particular the cross-shore velocity profiles vary significantly over one infragravity wave period, especially at $h_o/h_s = 0.07$.

The details of the variation of the velocity profiles can better be seen in Fig. 4, which shows the projections of the profiles in the longshore and cross-shore direction. Fig. 4 (a) shows the cross-shore velocity (commonly called the "undertow") normalized by the local long-wave celerity $c_o = \sqrt{g\,h_o}$ versus normalized depth at $h_o/h_s = 0.42$, which is located well outside the surf zone, for ten intervals per infragravity wave period. It can be seen that the profiles are slightly curved and also that the vertical gradient varies substantially with time. This is due to the (time-varying) forcing f_x in (11), which is a function of the radiation stress gradients, the pressure gradient and the gradients in the short-wave velocities.

Fig. 4 (b) shows the longshore velocity V at the same location. Due to the relatively small angle of incidence of the short-wave groups, the forcing induced by the short waves in the y direction is also small. This leads to the profiles with only a slight curvature. The profiles exhibit a non-zero mean over depth due to the momentum that has advected out of the surf zone mostly due to the dispersive mixing (Svendsen & Putrevu, 1994).

The cross-shore profiles in Figs. 4 (c) and (e) exhibit the typical characteristic time-

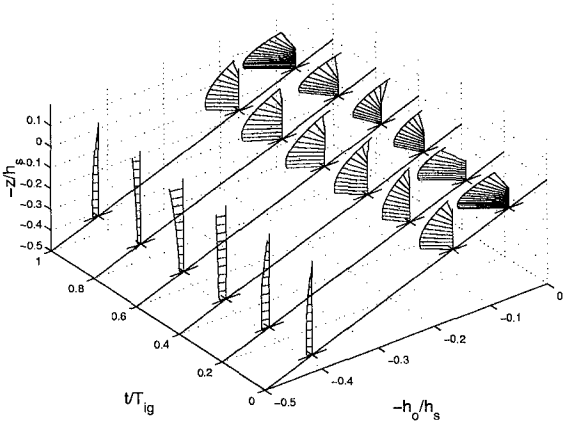

Figure 3: Infragravity wave profiles for three locations ($h_o/h_s = 0.42$, 0.17 and 0.07) and for five time instances of the infragravity wave period. The breakpoint is located at $h_o/h_s = 0.3$.

varying undertow profile in the surf zone that was previously shown by Putrevu & Svendsen (1995). The longshore profiles in Figs. 4 (d) and (f) are slightly more tilted than the longshore current profile in Fig. 4 (b) because inside the surf zone a strong mean forcing is present due to the difference between the radiation stress gradient and the pressure gradient. The time variation of the longshore profiles is not very large because the short-wave groups have refracted to near normal incidence inside the surf zone.

The relative magnitude of the quasi 3-D coefficients can be calculated directly from the model results using the definitions (12) - (15). For brevity we will only present the results for the coefficients $B_{\alpha\beta}$ and $D_{\alpha\beta}$.

Fig. 5 shows the variation of the $B_{\alpha\beta}$ and $D_{\alpha\beta}$ coefficients versus the cross-shore coordinate for five time intervals of the infragravity wave period. In the figures $h_o/h_s = 0$ corresponds to the still water shoreline and $h_o/h_s = 1$ to the toe of the beach.

We see that the B and D coefficients exhibit a quite large variation over an IG wave period, which indicates that the local time-varying forcing is very important. It can also be seen that the magnitude of all coefficients is significantly larger than the magnitudes which would have been found for the case of no groupiness ($\delta = 0$), which is indicated by the thick solid line. The values increase significantly across the breakpoint, since the "undertow" profiles become much more curved inside the surf zone due to the increased forcing. Because of the simple short-wave modeling, this transition in curvature occurs very rapidly, which increases the cross-shore gradients of the quasi 3-D coefficients.

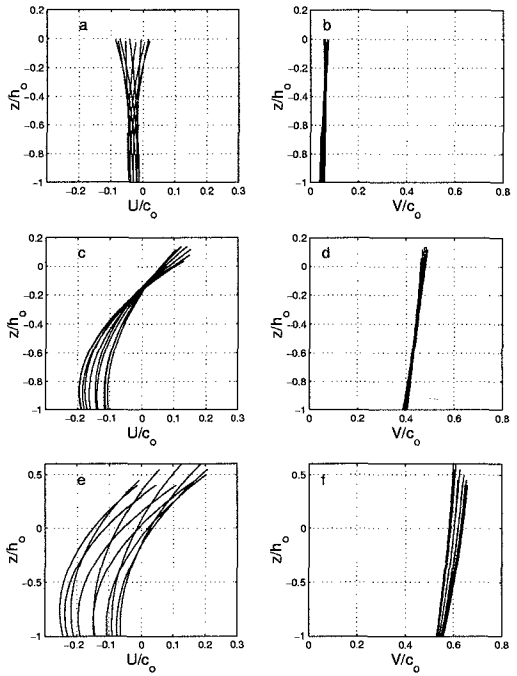

Figure 4: IG wave particle velocities in the cross-shore and longshore direction normalized by the longwave celerity c_o vs. normalized depth for ten intervals per IG wave period: (a) Cross-shore velocity U at $h_o/h_s = 0.42$. (b) Longshore velocity V at $h_o/h_s = 0.42$. (c) U at $h_o/h_s = 0.17$. (d) V at $h_o/h_s = 0.17$. (e) U at $h_o/h_s = 0.07$. (f) V at $h_o/h_s = 0.07$.

We see that the D_{xx} and B_{xx} coefficients are larger than the D_{xy} and B_{xy} coefficients, which are in turn larger than the D_{yy} and B_{yy} coefficients. This is because the short-wave groups refract towards the shorenormal, which means the forcing f_x in (11) becomes dominant over the forcing in the longshore direction. Eq. (11) also implies that the cross-shore velocities are more curved than the longshore velocities which could already be seen in Fig. 4. This curvature of the velocity profiles directly influences the magnitude of the dispersive coefficients.

An assessment of the quasi-3D effects is obtained by looking at the magnitude of the terms in the momentum equations in which these quasi 3-D coefficients appear. The analysis is performed at an arbitrary time after the periodic state of the IG waves has been reached and is strictly speaking only valid for this particular time. However, the magnitude of the terms in the equations at this time instance are

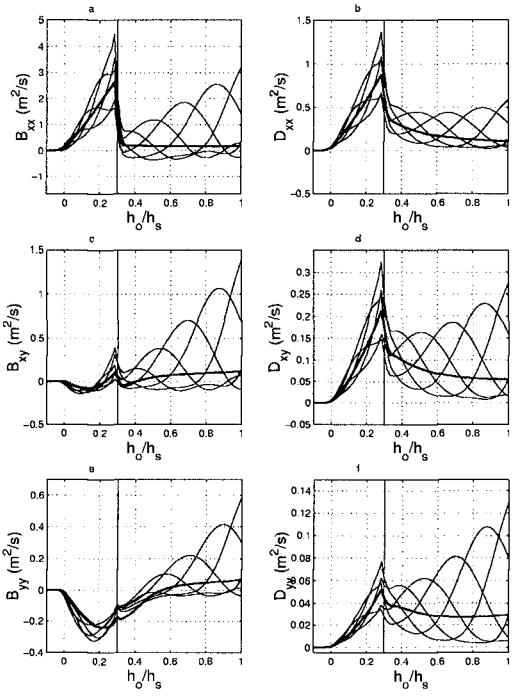

Figure 5: Magnitude of B and D coefficients vs. cross-shore distance h_o/h_s for five intervals per IG wave period: (a) B_{xx}. (b) D_{xx}. (c) B_{xy}. (d) D_{xy}. (e) B_{yy}. (f) D_{yy}. The stillwater shoreline is located at $h_o/h_s = 0$ and the breakpoint at $h_o/h_s = 0.3$. Also shown as the thick solid line is the magitude of the terms for the case of no groupiness $\delta = 0$.

characteristic for their magnitudes at any time in the periodic state. The conclusions that are drawn from this analysis are therefore considered representative for the case in question.

For reasons of clarity Fig. 6 only shows the most important terms in the equations. We first analyze the terms in the x momentum equation (16) versus h_o/h_s, see Fig. 6 (a). The terms shown are the pressure gradient, the radiation stress gradient $\frac{\partial S_{xx}}{\partial x}$ and the local acceleration. Of the quasi 3-D terms, the $\frac{\partial M_{xx}}{\partial x}$ term is large only locally at the breakpoint. This is due to the choice of $\kappa = 0$, which causes the cross-shore velocity profiles to undergo a rapid change over a short distance around the break point. (Notice that M_{xx} is equivalent to the momentum correction factor in hydraulics.) The term is positive, which leads to a negative pressure gradient in

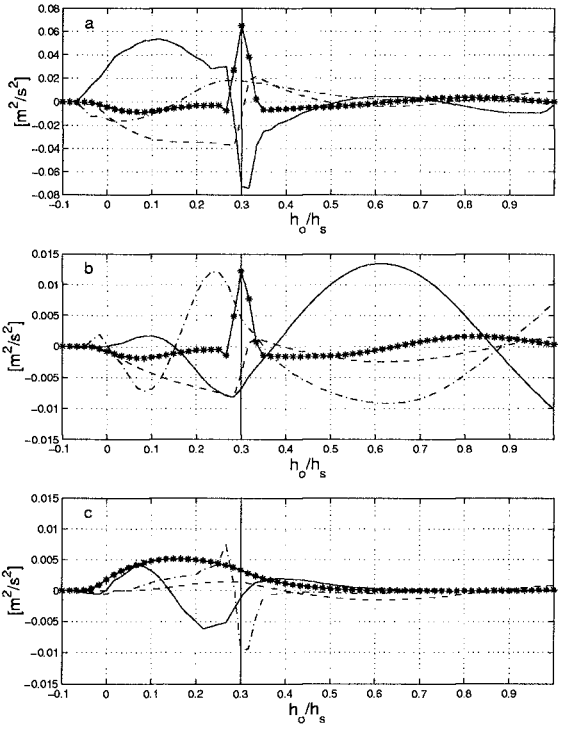

Figure 6: Magnitude of significant terms in the momentum equations vs. cross-shore distance h_o/h_s: (a) x-momentum equation: pressure gradient (solid), radiation stress gradient $\partial S_{xx}/\partial x$ (dashed), local acceleration (dash-dotted) and $\partial M_{xx}/\partial x$ (stars). (b) y-momentum equation: pressure gradient (solid), radiation stress gradient $\partial S_{xy}/\partial x$ (dashed), local acceleration (dash-dotted) and $\partial M_{xy}/\partial x$ (stars). (c) y-momentum equation: advective acceleration $\frac{\partial}{\partial x}\left(\frac{\bar{Q}_x \bar{Q}_y}{h}\right)$ (solid), $\partial S_{yy}/\partial y$ (dashed), $-\frac{\partial}{\partial x}\left(h\, D_{xx} \frac{\partial \bar{V}}{\partial x}\right)$ (dash-dotted) and bottom friction(stars). The stillwater shoreline is located at $h_o/h_s = 0$ and the breakpoint at $h_o/h_s = 0.3$.

the balance. In the inner surf zone, this term is found comparable to other terms, and is negative, which causes an increase in the pressure gradient and explains the difference between the envelopes of Fig. 2. In general, however, the dispersive mixing terms are of minor importance in the cross-shore momentum balance.

The dominating terms in the y-component of (16) are shown in Fig. 6 (b) and (c). Those terms are the local acceleration, the pressure gradient, the radiation shear stresses $\frac{\partial S_{xy}}{\partial x}$ and $\frac{\partial S_{yy}}{\partial y}$, the advection term $\frac{\partial}{\partial x}\left(\frac{\bar{Q}_x \bar{Q}_y}{h}\right)$, and the bottom friction. However, we also see that two Q3D terms, $\frac{\partial M_{xy}}{\partial x}$ and $\frac{\partial}{\partial x}\left(h\, D_{xx} \frac{\partial \bar{V}}{\partial x}\right)$ are important. The first term is significant around the breakpoint for the same reason that the $\frac{\partial M_{xx}}{\partial x}$ was in the x momentum equation. The second term is of the same order of magnitude as the 2-DH terms inside the surf zone and is the same that was found to be in the dispersion of momentum in the case of a steady longshore current (Svendsen & Putrevu, 1994). In this case where the shear in the longshore current is also large, this term is again important.

CONCLUSIONS

A numerical study of the forcing of leaky infragravity waves by obliquely-incident wave groups is performed using the SHORECIRC model which incorporates all quasi 3-D terms. It is shown that some of these terms have a significant effect on the envelope of the infragravity waves. The magnitude of the quasi 3-D terms is analyzed and compared to the size of the terms which are retained in conventional nonlinear shallow water models. This shows that the quasi 3-D terms have a significant size around the break point and in the surf zone. The velocity profiles of the infragravity waves inside the surf zone exhibit a large curvature and time variation in the cross-shore direction, and a small - but essential - depth variation in the longshore direction. Outside the surfzone the velocities in the longshore direction are small, while in the cross-shore direction only the upper part of the profile is curved.

ACKNOWLEDGMENTS

This work is a result of research sponsored by NOAA Office of Sea Grant, Department of Commerce, under Award No. NA 56 RG 0147 (Project No. R/OE-17) and by the U.S. Army Research Office, University Research Initiative under Contract No. DAAL03-92-G-0116. Additional funding has been provided by the Netherlands Centre for Coastal Research, Delft University of Technology, and WL|Delft Hydraulics. The U.S. Government is authorized to produce and distribute reprints for government purposes notwithstanding any copyright notation that may appear herein.

REFERENCES

De Vriend, H.J. and M.J.F. Stive (1987). "Quasi-3D modelling of nearshore currents." *Coastal Engineering, 11*, pp. 565-601.

Haas, K.A., I.A. Svendsen and R.A. Dalrymple (1998). "Numerical modeling of nearshore circulation on a barred beach with rip channels." *Proc. of the 26th International Conference on Coastal Engineering*, this volume.

Lippmann, T.C., R.A. Holman and A.J. Bowen (1997). "Generation of edge waves in shallow water." *J. of Ceophys. Res., 112, C4*, pp. 8863-8879.

Longuet-Higgins, M.S. and R.W. Stewart (1962). Radiation stress and mass transport in gravity waves with application to 'surf-beats'. *Journal of Fluid Mechanics, 8*, pp. 565-583.

Longuet-Higgins, M.S. and R.W. Stewart (1964). Radiation stress in water waves, a physical discussion with applications. *Deep Sea Research, 11*, pp. 529-563.

Mei, C.C. (1983). *The applied dynamics of ocean surface waves.* John Wiley and Sons, New York, 740 pp.

Phillips, O.M. (1977). *The dynamics of the upper ocean.* Cambridge University Press, 336 pp.

Putrevu, U. and I.A. Svendsen (1992). "A mixing mechanism in the nearshore region." *Proc. of the 23rd International Conference on Coastal Engineering*, pp. 2758-2771.

Putrevu, U. and I.A. Svendsen (1995). "Infragravity velocity profiles in the surf-zone." *Journal of Geophysical Research, 100, C8*, pp. 16,131-16,142.

Putrevu, U. and I.A. Svendsen (1997a). "Dispersive mixing in the nearshore." *Proc Coastal Dynamics '97*, pp. 207-216.

Putrevu, U. and I.A. Svendsen (1997b). "Shear dispersion of momentum in the nearshore." Submitted.

Rodi, W. (1980). *Turbulence models and their application in hydraulics.* Int'l Assoc. for Hydraulic Res., Delft, The Netherlands, 104 p.

Sánchez-Arcilla, A., F. Collado, C. Lemos and F. Rivero (1990). "Another quasi-3D model for surf-zone flows." *Proc. of the 22nd International Conference on Coastal Engineering*, pp. 316-329.

Sánchez-Arcilla, A., F. Collado and A. Rodriguez (1992). "Vertically varying velocity field in Q-3D nearshore circulation." *Proc. of the 23rd International Conference on Coastal Engineering* , pp. 2811-2824.

Sancho, F.E.P., I.A. Svendsen and U. Putrevu (1997). "Modeling of longshore currents over alongshore nonuniform topographies: Effects of second order terms." Submitted to *J. of Geophys. Res.*

Sancho, F.E.P. and I.A. Svendsen (1998). "Shear waves over longshore nonuniform barred beaches." *Proc. of the 26th International Conference on Coastal Engineering*, this volume.

Schäffer, H.A. and I.A. Svendsen (1988). Surf heat generation on a mild slope beach. *Proc. of the 21st International Conference on Coastal Engineering*, pp. 1058-1072.

Schäffer, H.A. (1994). "Edge waves forced by short-wave groups." *J. Fluid Mech., 259*, pp. 125-148.

Svendsen, I.A. and R.S. Lorenz (1989). "Velocities in combined undertow and longshore currents." *Coastal Engineering, 13*, pp. 55-79.

Svendsen, I.A. and U. Putrevu (1990). "Nearshore circulation with 3-D profiles." *Proc. of the 22nd International Conference on Coastal Engineering*, pp. 241-254.

Svendsen, I.A. and U. Putrevu (1994). "Nearshore mixing and dispersion." *Proc. Roy. Soc. Lond. A., 445*, pp. 1-16.

Taylor, G.I. (1954). "The dispersion of matter in a turbulent flow through a pipe." *Proc. of the Royal Society of London. Series A*, 219, pp. 446-468.

Van Dongeren, A.R., F.E. Sancho, I.A. Svendsen and U. Putrevu (1994). "SHORECIRC: A quasi 3-D nearshore model." *Proc. of the 24th International Conference on Coastal Engineering*, pp. 2741-2754.

Van Dongeren, A.R. and I.A. Svendsen (1997a). "Numerical modeling of quasi 3-D nearshore hydrodynamics." Ph.D. Dissertation. Research report No. CACR-97-04, Center for Applied Coastal Research, University of Delaware, Newark, 243 p.

Van Dongeren, A.R. and I.A. Svendsen (1997b). "An absorbing-generating boundary condition for shallow water models." *Journal of Waterway, Port, Coastal and Ocean Engineering, 123*, no. 6, pp. 303-313.