PROBABILITY DISTRIBUTION OF WAVE HEIGHT IN FINITE WATER DEPTH

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Abstract

This paper presents probability density functions applicable to peaks, troughs and peak-to-trough excursions of coastal waves with finite water depth in closed form. It is found that for a non-Gaussian waves for which the skewness of the distribution is less than 1.2, the probability density function of peaks (and troughs) can be approximately represented by the Rayleigh distribution with a parameter which is a function of three parameters representing the non-Gaussian waves. The agreement between the probability density functions and the histograms constructed from data obtained by the Coastal Engineering Research Center is satisfactory.

Introduction

It has been known that waves in finite water depth (hereafter defined as coastal waves) are, in general, considered to be a nonlinear, non-Gaussian random process. The profile of wave peaks (positive side) is sharp as contrasted to the round profile of the troughs (negative side) as shown in Figure 1. The degree of difference in the positive and negative sides of the wave profile can be presented mathematically in terms of skewness. It is highly desirable that the statistical properties of peaks and trough amplitudes be presented separately, and then the properties of wave height (peak to trough excursion) may be obtained through the distribution function applicable to the sum of two independent

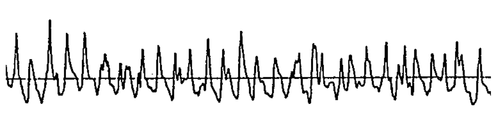

Figure 1: A portion of the time history of shallow water wave record obtained by CERC during ARSLOE Project

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random variables (peaks and troughs). Furthermore, it is requisite that the probability distributions of peaks and troughs be developed from the distribution representing a non-Gaussian wave profile; the same approach as considered for derivation of the Rayleigh probability distribution for Gaussian waves in deep water.

Only a limited number of studies has been carried out on the probability distribution of coastal wave height. It is often assumed in these studies that the wave profile is amplitude modulated and follows the Stokes expansion to the second or third components; Tayfun (1980), Arhan and Plaistad (1981), among others. There is some reservations, however, in applying Stokes theory for shallow water waves unless the expansion includes higher order terms. Furthermore, it is advisable not to assume any preliminary form of the wave profile for derivation of its probability distribution.

Another approach for derivation of the probability distribution of wave amplitudes is to apply the Gram-Charlier series distribution representing coastal wave profiles; Ochi and Wang (1984) for example. The results, however, are not promising since the Gram-Charlier series distribution is not given in closed form and the density function at times becomes negative for large negative displacements.

On the other hand, several empirical probability distributions applicable for coastal wave heights have been developed from analysis of observed data. These include Kuo and Kuo (1975), Goda (1975), Ochi, Malakar and Wang (1982) and Hughs and Borgman (1987), among others.

In the present paper, the probability function applicable to amplitudes of response of a nonlinear mechanical system (Ochi 1998) is applied to coastal waves. That is, the probability density function applicable to peaks and troughs of wave data are presented separately in closed form as a function of three parameters representing non-Gaussian waves. Since these three parameters have been presented as a function of water depth and sea severity (Robillard and Ochi, 1996), the probability density function of wave amplitudes of coastal waves can be evaluated for a specified water depth and sea severity.

Probability Distribution of Wave Peaks

For derivation of the probability distribution representing statistical properties of wave peaks, a peak envelope process, denoted by $\xi(t)$, shown in Figure 2 is considered.

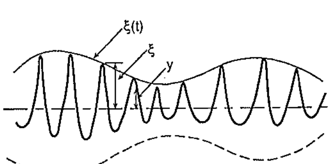

Figure 2
Definition of the envelope process of peaks of non-Gaussian wave

The non-Gaussian random waves y(t) are expressed as a function of normal random process and its square. That is,

$$y(t) = U(t) + a\{U(t)\}^2 \tag{1}$$

where "a" is a constant and U(t) is a Gaussian wave with mean μ_* and variance σ_*^2; all of these parameters can be evaluated from data. The parameter "a" represents the intensity of non-Gaussian wave characteristics; the larger the a-value the stronger the nonlinearity. We may consider Y and U to be random variables for a given time of y(t) and u(t), respectively, and the functional relationship given in Eq.(1) is inversely expressed as follows (Ochi and Ahn 1994):

$$U = \frac{1}{\gamma a}\left(1 - e^{-\gamma a Y}\right) \tag{2}$$

where γ = 1.28 for y > 0, 3.00 for y < 0.

It is noted that the random variable U defined in Eq.(2) is normally distributed with sample space $(-\infty, 1/\gamma a)$ instead of $(-\infty, \infty)$ as originally defined. However, this restriction does not affect the distribution of U, in practice, since the value of $(\mu_* + 3\sigma_*)$ where the density function of the normal variate U becomes almost zero, is much smaller than $1/\gamma a$. Hence, the sample space $(-\infty, 1/\gamma a)$ is essentially equivalent to $(-\infty, \infty)$.

For the random variable U associated with the peak envelope process, the mean value is μ_* but the variance is that affiliated with the positive (peak) side only which can be evaluated by

$$\begin{aligned}\sigma_1^2 &= 2\int_0^\infty y^2 f(y)\,dy \\ &= \frac{2}{\lambda_1^2}\left\{\frac{1}{2}\left(\lambda_1\sigma_*\right)^2 + \sqrt{2/\pi}\left(\lambda_1/\sigma_*\right) + \frac{3}{2}\left(\lambda_1/\sigma_*\right)^4\right\}\end{aligned} \tag{3}$$

where $\lambda_1 = a\gamma$, γ = 1.28 for the positive y.

The probability density function f(y) in Eq.(3) represents a non-Gaussian random wave profile and is given by

$$f(y) = \frac{1}{\sqrt{2\pi}\sigma_*}\exp\left\{-\frac{1}{2\left(\gamma a\sigma_*\right)^2}\left(1 - \gamma a\mu_* - e^{-\gamma a y}\right)^2 - \gamma a y\right\} \tag{4}$$

Next, we may define the random variable V by subtracting the mean value μ_* from the random variable U. That is,

$$V = U - \mu_* = \frac{1}{\lambda_1}\left(1 - e^{-\lambda_1 Y}\right) - \mu_* \tag{5}$$

We may write the cosine and sine components of V as V_c and V_s, respectively, and the joint probability density function of V_c and V_s is given by a statistically independent bi-variate normal distribution with zero mean and a common variance σ_1^2. That is,

$$f\left(v_c, v_s\right) = \frac{1}{2\pi\sigma_1^2}\exp\left\{-\frac{1}{2\sigma_1^2}\left(v_c^2 + v_s^2\right)\right\} \tag{6}$$

By applying the change of random variable technique and by using the relationship given in Eq.(5), the joint probability density function $f(V_c, V_s)$ is transformed to the joint probability density function $f(y_c, y_s)$. Furthermore, we may write Y_c and Y_s as a function of amplitude and phase. That is,

$$\begin{aligned} Y_c &= \xi \cos\tau \\ Y_s &= \xi \sin\tau \end{aligned} \tag{7}$$

where ξ = amplitude (peak), τ = phase.

Then, we can derive the joint probability density function of ξ and τ as follows:

$$\begin{aligned} f(\xi, \tau) = \frac{\xi}{2\pi\sigma_1^2}\exp&\left\{-\left(\frac{1-\lambda_1\mu_*}{\lambda_1\sigma_1}\right)^2 - \lambda_1\xi(\cos\tau + \sin\tau)\right. \\ &+ \frac{1-\lambda_1\mu_*}{\lambda_1^2\sigma_1^2}\left(e^{-\lambda_1\xi\cos\tau} + e^{-\lambda_1\xi\sin\tau}\right) \\ &\left.- \frac{1}{2\lambda_1^2\sigma_1^2}\left(e^{-2\lambda_1\xi\cos\tau} + e^{-2\lambda_1\xi\sin\tau}\right)\right\}, \\ &0 \le \xi < \infty, \quad 0 \le \tau \le 2\pi. \end{aligned} \tag{8}$$

In order to obtain the marginal density function of ξ by integrating Eq.(8) with respect to τ, the exponential parts of the 3rd and 4th term of Eq.(8) are expanded to a series of $\lambda_1\xi$ and we may take terms up to $(\lambda_1\xi)^2$. We can then derive the following probability density function of peaks $f(\xi)$.

$$f(\xi) = \frac{(1+\lambda_1\mu_*)\xi}{\sigma_1^2}\, e^{-\left\{\frac{(\lambda_1\sigma_1^2-\mu_*)^2}{(1+\lambda_1\mu_*)\sigma_1^2} + \frac{1+\lambda_1\mu_*}{2\sigma_1^2}\xi^2\right\}}\, I_o\left\{\sqrt{2}\left(\lambda_1 - \frac{\mu_*}{\sigma_1^2}\right)\xi\right\}$$

$$0 \le \xi < \infty. \qquad (9)$$

The detailed description of the derivation of Eq.(9) is given in the reference Ochi (1998).

In the case of Gaussian waves, μ_* as well as λ_1 are both zero, and the variance σ_1^2 reduces to σ^2. Hence, Eq.(9) becomes

$$f(\xi) = \frac{\xi}{\sigma^2}\, e^{-\frac{\xi^2}{2\sigma^2}} \qquad (10)$$

which is the Rayleigh probability density function applicable for amplitudes of narrow-band Gaussian waves.

In Eq.(9), let us write

$$\frac{\sigma_1^2}{1+\lambda_1\mu_*} = s_1^2$$

$$\frac{\sqrt{2}(\lambda_1\sigma_1^2 - \mu_*)}{1+\lambda_1\mu_*} = c_1 \qquad (11)$$

then, we have the probability density function of the positive amplitude as

$$f(\xi) = \frac{\xi}{s_1^2}\exp\left\{-\frac{1}{2s_1^2}\left(c_1^2 + \xi_1^2\right)\right\}\cdot I_o\left(c_1\xi / s_1^2\right) \qquad (12)$$

The above equation is the probability density function applicable for the sum of two statistically independent random processes; one being a narrow-band Gaussian random process with zero mean and variance s_1^2, the other a sine wave with amplitude c_1, both having the same frequency (Rice 1945). This implies that the probability density function of the envelope of the positive side of the non-Gaussian waves given in Eq.(9) is equivalent to the probability density function of the envelope of a random process consisting of the sum of a narrow-band Gaussian wave with zero-mean and variance $\sigma_1^2/(1+\lambda_1\mu_*)$ and a sine wave with amplitude $\sqrt{2}(\lambda_1\sigma_1^2-\mu_*)/(1+\lambda_1\mu_*)$. This result provides insight as to the structure of the probability distribution function of wave height in finite water depth.

It is further possible to simplify Eq.(12) for non-Gaussian waves for which the skewness of the distribution is less than approximately 1.2. That is, we may approximate the modified Bessel Function in Eq.(12) as

$$I_o(z) \sim \exp\left\{z^2/5\right\}, \qquad \text{where } z < 3.0 \tag{13}$$

A comparison of the approximate formula and $I_o(z)$ is shown in Figure 3. By applying the approximation given in Eq.(13), the probability density function given Eq.(12) can be expressed in the following form:

$$f(\xi) = K \cdot \left(\frac{1}{s_1^2}\exp\left\{c_1^2/2s_1^2\right\}\right) \cdot \xi \exp\left\{-\frac{1}{2s_1^2}\left(1-\frac{2c_1^2}{5s_1^2}\right)\xi^2\right\} \tag{14}$$

$$0 < \xi < \infty$$

where K is a normalization factor determined from the condition that the integration of Eq.(14) in the sample space be unity. Then we have

$$f(\xi) = \frac{1}{s_1^2}\left(1-\frac{2c_1^2}{5s_1^2}\right)\xi \cdot \exp\left\{-\frac{1}{2s_1^2}\left(1-\frac{2c_1^2}{5s_1^2}\right)\xi^2\right\} \tag{15}$$

$$0 < \xi < \infty$$

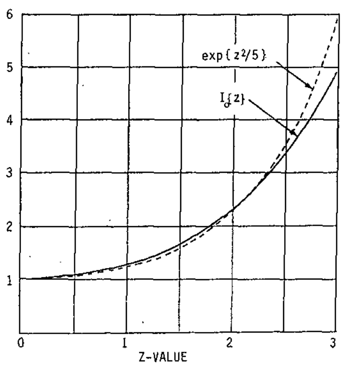

Figure 3
Comparison between the modified Bessel function $I_o(z)$ and its approximate formula

This is the Rayleigh probability density function. Thus, it is found that amplitudes of the positive part of non-Gaussian waves may be approximately distributed following the Rayleigh probability distribution with the parameter $(2s_1^2)/\{1-(2c_1^2/5s_1^2)\}$.

Figure 4 shows a comparison of the exact (Eq.9) and the approximate (Eq.15) probability density functions with the histogram constructed from data shown in Figure 1. The data shown in the

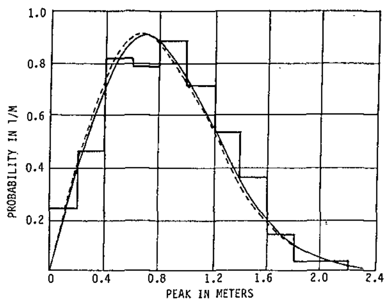

Figure 4
Comparison of exact (solid line) and approximate (dashed line) probability density functions applicable for peaks of a non-Gaussian wave with the histogram constructed from data figure are obtained at Duck, North Carolina, by the Coastal Engineering Research Center during the ARSLOE Project. As seen, the difference between the exact and approximate density functions is very small and they represent well the histogram of peaks.

Derivation of Probability Distribution of Wave Troughs

The probability density function applicable to troughs of non-Gaussian waves, denoted by $f(\eta)$, has essentially the same form as Eq.(9). The parameter λ_1 and variance σ_1^2 in Eq.(9), however, should be replaced by λ_2 and σ_2^2, respectively, which are appropriate for troughs. The variance σ_2^2 for the trough envelope process can be evaluated by subtracting σ_1^2 from twice the data variance σ^2 computed including both positive and negative displacements from the mean value. That is, the probability density function is given by

$$f(\xi) = \frac{(1+\lambda_2\mu_*)\eta}{\sigma_2^2}\, e^{-\left\{\frac{\left(\lambda_2\sigma_2^2-\mu_*\right)^2}{\left(1+\lambda_2\mu_*\right)\sigma_2^2}+\frac{1+\lambda_2\mu_*}{2\sigma_2^2}\eta^2\right\}}\; I_o\left\{\sqrt{2}\left(\lambda_2-\frac{\mu_*}{\sigma_2^2}\right)\eta\right\}$$

$$0 \le \eta < \infty \qquad (16)$$

where $\sigma_2^2 = 2E\left[y^2\right] - \sigma_1^2$

$\lambda_2 = a\gamma$ with $\gamma = 3.00$

As in the case for the probability distribution of peaks, the probability distribution of envelope of troughs is equivalent to that of a random process consisting of the sum of a narrow-band Gaussian wave with zero-mean and variance $\sigma_2^2/(1+\lambda_2\mu_*)$ and a sine wave with amplitude $\sqrt{2}\,(\lambda_2\sigma_2^2-\mu_*)/(1+\lambda_2\mu_*)$. These may be denoted by s_2^2 and c_2, respectively. Furthermore, the probability density function may be expressed approximately by the following Rayleigh probability distribution:

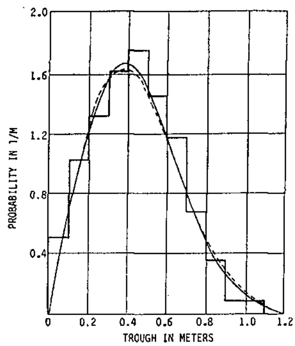

Figure 5
Comparison of exact (solid line) and approximate (dashed line) probability density functions applicable for troughs of a non-Gaussian wave with the histogram constructed from data

$$f(\eta) = \frac{1}{s_2^2}\left(1 - \frac{2c_2^2}{5s_2^2}\right)\eta \cdot \exp\left\{-\frac{1}{2s_2^2}\left(1 - \frac{2c_2^2}{5s_2^2}\right)\eta^2\right\}$$

$$0 < \eta < \infty \tag{17}$$

Figure 5 shows a comparison between the exact (Eq.16) and approximate (Eq.17) probability density functions applicable for the troughs of non-Gaussian waves and the histogram constructed from data. As seen, the difference between the exact and approximate density functions is extremely small and the overall agreement between the histogram and density functions is satisfactory.

Probability Distribution of Wave Height

Wave height is defined as a peak-to-trough excursion. As stated earlier, peak and trough envelope processes are independently considered for probability distribution of peaks and troughs of non-Gaussian waves. The results of analysis have shown that large peak envelopes and large trough envelopes do not occur simultaneously, in general. Correlation between the two envelopes is rather small; hence, it is assumed that peaks and troughs are statistically independent and thereby the probability density function of peak-to-trough excursions may be obtained as a convolution integral of the two probability density functions, $f(\xi)$ and $f(\eta)$. That is

$$f(\zeta) = \int_0^\infty f_\xi(\xi)\, f_\eta(\zeta - \xi)\, d\xi \tag{18}$$

where $f_\xi(\ )$ and $f_\eta(\ )$ represent the probability density functions given in Eqs.(9) and (16), respectively, for the exact density functions, and Eqs.(15) and (17), respectively, for the approximate density functions.

The integration given in Eq.(18) cannot be analytically carried out for the exact density functions because of the product of two modified Bessel functions involved. The probability density function of ζ, therefore, is numerically evaluated. The convolution integral for the two approximate probability density functions given in Eqs.(15) and (17), however, can be analytically carried out. It is the sum of two independent Rayleigh distributions. That is, by writing the parameters of the Rayleigh distributions applicable for the peaks and troughs as

$$\begin{aligned} R_1 &= \left(2s_1^2\right) / \left\{1 - \left(2c_1^2 / 5s_1^2\right)\right\} \\ R_2 &= \left(2s_2^2\right) / \left\{1 - \left(2c_2^2 / 5s_2^2\right)\right\} \end{aligned} \qquad (19)$$

the probability density function of the sum of the Rayleigh distribution, denoted by $f(\zeta)$, becomes

$$\begin{aligned} f(\zeta) &= \frac{2\xi}{R_1} e^{-\frac{\xi^2}{R_1}} \cdot \frac{2(\zeta - \xi)}{R_2} e^{-\frac{(\zeta-\xi)^2}{R_2}} d\xi \\ &= \frac{2\zeta}{(R_1 + R_2)^2} \left(R_1 e^{-\frac{\zeta^2}{R_1}} + R_2 e^{-\frac{\zeta^2}{R_2}} \right) \\ &\quad + \frac{2\sqrt{\pi}}{R_1 + R_2} e^{-\frac{\zeta^2}{R_1 + R_2}} \sqrt{\frac{R_1 R_2}{R_1 + R_2}} \left(\frac{2}{R_1 + R_2} \zeta^2 - 1 \right) \\ &\qquad \times \left\{ \Phi\left(\sqrt{\frac{2R_2}{R_1(R_1 + R_2)}}\, \zeta \right) - \Phi\left(-\sqrt{\frac{2R_1}{R_2(R_1 + R_2)}}\, \zeta \right) \right\} \\ &\qquad\qquad 0 < \zeta < \infty \end{aligned} \qquad (20)$$

where Φ = cumulative distribution function of the standardized normal distribution.

Figure 6 shows a comparison between the exact (Eq.18) and approximate (Eq.20) probability density functions for the peak-to-trough excursions and the histogram constructed from the same wave data as used for the histograms of peaks and troughs. The difference between the exact and approximate density functions is negligibly small, and the theoretical density functions agree reasonably well with the histogram. Included also in the figure is the Rayleigh probability density function applicable for Gaussian waves commonly considered for analysis of deep water waves. The probability density function for the Gaussian wave assumption substantially deviates from the histogram.

The cumulative distribution function of the wave height can be evaluated by integrating Eq.(20) with respect to ζ. The derivation of the distribution function, however, may be much easier using the

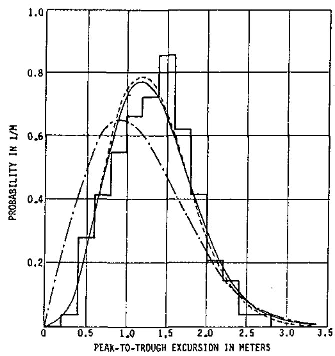

Figure 6
Comparison between exact (solid line) and approximate (dashed line) probability density functions developed based on non-Gaussian concept, Gaussian concept (chain line) and histogram constructed from data

following approach since the distributions of the peaks and troughs are assumed to be statistically independent. That is

$$F(\zeta) = \iint\limits_{\xi+\eta\le\zeta} f(\xi,\eta)d\xi\, d\eta = \int_o^{\zeta} \left(\int_o^{\zeta-\xi} f(\eta)\, d\eta \right) f(\xi)\, d\xi \tag{21}$$

By applying Eqs.(15) and (17), and with the parameters given in Eq.(19), we have

$$F(\zeta) = 1 - \frac{1}{R_1 + R_2}\left[R_1 e^{-\frac{\zeta^2}{R_1}} + R_2 e^{-\frac{\zeta^2}{R_2}} \right.$$

$$\left. + 2\zeta\sqrt{\frac{\pi R_1 R_2}{R_1 + R_2}}\, e^{-\frac{\zeta^2}{R_1+R_2}} \left\{ \Phi\left(\sqrt{\frac{2R_2}{R_1(R_1+R_2)}}\,\zeta \right) - \Phi\left(-\sqrt{\frac{2R_1}{R_2(R_1+R_2)}}\,\zeta \right) \right\} \right]$$

$$0 < \zeta < \infty \tag{22}$$

It may be of interest to see how the shape of the probability distribution of amplitudes of coastal waves changes when wave energy propagates from deep to shallow water. Figure 7 shows an example of the probability density functions of peaks and troughs along with wave records simultaneously measured at four locations (60 m, 151 m, 456 m and 12 km offshore) during the ARSLOE Project. The waves obtained at the 12 km offshore are considered to be Gaussian at the time of measurement. As seen, the mode of the Rayleigh distribution applicable to non-Gaussian waves shifts to the smaller values as waves approach the shoreline. In particular, the rate of change of the probability density function of troughs is much faster than that

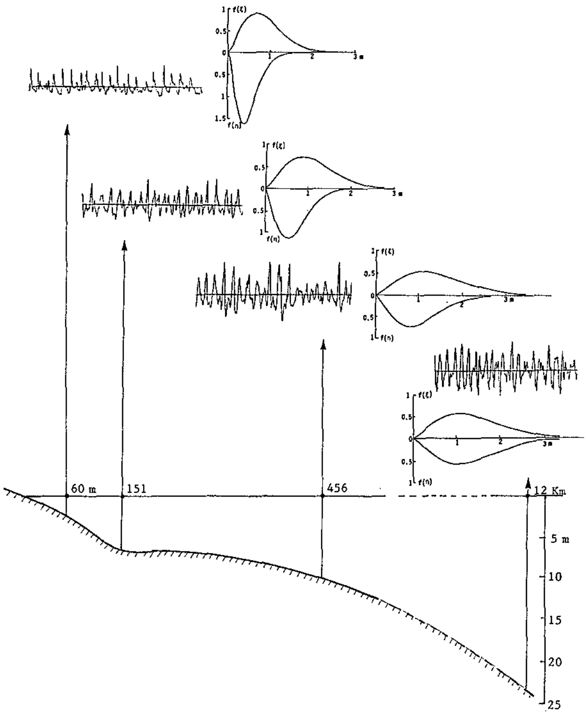

Figure 7 Probability density functions of wave peaks and troughs obtained from data at various locations in the nearshore zone

of peaks. This is understandable since wave troughs are much more susceptible to bottom effect.

Significant Wave Height

Significant wave height defined as the average of the highest one-third wave heights, denoted by H_s, is most commonly used for representing the severity of random waves. It can be evaluated by

$$H_s = 3\int_{\zeta_*}^{\infty} \zeta f(\zeta)\, d\zeta \tag{23}$$

where ζ_* = the value of wave height ζ for which $F(\zeta) = 2/3$,

$f(\zeta)$ = probability density of wave height.

Table 1
Comparison between computed and measured significant wave heights

Water Depth (M)	Significant Wave Height in meters		
	Non-Gaussian Concept	Gaussian Concept	Measured
2.32	1.97	1.86	1.92
6.53	2.56	2.48	2.52
10.07	3.72	3.69	3.70

Equation (23) yields a very simple result for deep water : $4\sqrt{m_o}$ where m_o is the area under the spectral density function. For non-Gaussian waves, however, the computation of Eq.(23) is rather complicated. After some mathematical manipulations, we can derive

$$H_s = 3\left[\left(\frac{R_1}{R_1+R_2}\right)^2\left[\zeta_* e^{-\frac{\zeta_*^2}{R_1}} + \sqrt{\pi R_1}\left\{1-\Phi\left(\sqrt{2/R_1}\,\zeta_*\right)\right\}\right] + \sqrt{\pi}\sqrt{\frac{R_1R_2}{R_1+R_2}}\,e^{-\frac{\zeta_*^2}{R_1+R_2}}\left(\frac{2}{R_1+R_2}\zeta_*^2 + 1.1\right)\right] \tag{24}$$

Comparisons of significant wave heights computed by Eq.(24) and those evaluated from data obtained at three water depths is shown in Table 1. Included also in the table are those computed by using the formula applicable for deep water. As seen in the table, significant wave heights computed based on non-Gaussian and Gaussian concepts do not differ more than 6 per cent, and the significant wave height obtained from measured data is between the two computed values for a given water depth. As shown in Figure 6, the probability density function of wave height for non-Gaussian waves intersects that of Gaussian waves at a large wave height. This results in the centers of gravity of the highest one-third of these two probability density functions (significant wave heights) may not be too far apart.

In order to supplement the above-mentioned statement, Figure 8 is prepared. The figure shows the cumulative distribution function of wave heights computed at two water depths based on Gaussian and non-Gaussian concepts. As seen, the two cumulative distribution functions slowly approach each other with increase in wave height and intersect at a certain high wave height. The value of the significant wave height is slightly greater than the wave height at

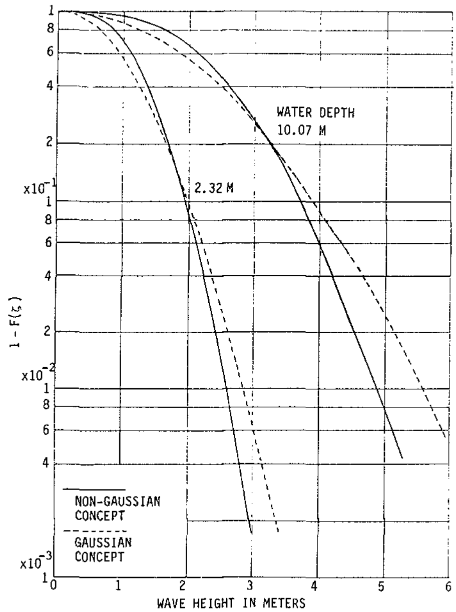

Figure 8
Comparison of cumulative distribution functions of wave heights computed based on Gaussian and non-Gaussian concepts

the point of crossing, but much less than the height where the two distribution functions start separating widely. Since the computation of significant wave height based on the Gaussian concept is quite simple, and since computed significant wave height is close to that computed using the formula for non-Gaussian waves, it may be concluded that the formula to evaluate significant wave height in deep water may also be applied approximately to non-Gaussian waves as far as the evaluation of significant wave height is concerned.

Conclusions

Probability density functions applicable to peaks, troughs and peak-to-trough excursions of coastal waves with finite water depth are presented separately in closed form. It is found that the probability density function applicable to peaks (and troughs) consists of the sum of narrow-band Gaussian waves and sine waves having the same frequency. It is also found that for non-Gaussian waves for which the skewness of the distribution is less than 1.2, the probability density function of peaks (and troughs) can be represented approximately by the Rayleigh distribution with a parameter which is a function of three parameters representing the non-Gaussian waves. Since these three parameters have been presented as a function of water depth and sea severity, the probability density function of amplitudes of coastal waves can be evaluated for a specified water depth and sea severity. The agreement between the probability density functions and the histograms constructed from data obtained by the Coastal Engineering Research Center during the ARSLOE Project is satisfactory.

The significant wave height of non-Gaussian coastal waves is analytically derived. The results of the computations show that computed significant wave height is close to that evaluated by applying the formula for waves in deep water (Gaussian waves). Therefore, since the computations based on the Gaussian concept are quite simple, it may be used for non-Caussian waves as far as the evaluation of significant wave height is concerned.

Acknowledgments

The author is grateful to Ms. Laura Dickinson for typing the manuscript.

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