EXPERIMENTAL STUDY ON NON-LINEAR WAVE BOUNDARY LAYERS

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Abstract

A mechanical generation method of an asymmetric oscillatory flow has been proposed by the authors (Tanaka et al., 1997, 1998), though, its applicability to an actual experiments has not been thoroughly examined yet. In the present study, laboratory experiments are carried out for an asymmetric oscillatory flow in a U-shaped oscillating tunnel to make comprehensive investigation on the validity of the generation method under both laminar and turbulent condition using air and water respectively as working fluid.

Introduction

In numerous experimental studies on oscillatory boundary layers, a U-shaped oscillating tunnel has been effectively utilized in place of a wave flume (e.g., Jonsson, 1963). Use of this type of facility enables us to generate oscillating motion with the period almost similar to prototype wave motion. Most of the existing experimental facilities are, however, restricted to sinusoidal oscillation, equivalent to fluid motion induced by linear waves. In reality, waves are generally asymmetric more or less due to non-linear effect. In order to generate corresponding asymmetric oscillatory flow in a U-tunnel, it requires a highly expensive sophisticated equipment to control the piston movement, the reason being for scarcity of the experiments in this regard. For example, Nadaoka et al.(1994, 1996) and Ribberink and Al-Salem (1994) performed some experiments by using a computer-controlled piston system to study the characteristics of asymmetric oscillatory boundary layers.

Recently, the authors have proposed a rather simple and inexpensive piston mechanism, by which an asymmetric oscillation simulating fluid motion under non-linear waves is produced mechanically, and preliminary experiments have already been done using this generation method (Tanaka et al., 1997, 1998). However, number of experimental cases performed using this system is too limited to judge whether or not it works in actual experiments under various conditions. In the present study, the applicability of this generation method is examined at low and high Reynolds numbers.

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Mechanical Generation of Asymmetric Oscillatory Flow

The schematic description of the piston mechanism to generate asymmetric oscillatory motion is shown in Fig.1. According to Tanaka et al.(1997,1998), time-variation of the velocity induced by the piston system is

$$\frac{U}{U_c} = \frac{(b_* - 1)(b_* \cos \omega t - 1)}{(b_* - \cos \omega t)^2}$$
(1)

where U is the horizontal velocity, U_c : peak velocity of U during wave crest, ω is the angular frequency (=2 π / T , T: wave period), t is the time, $b_*=b/a$ and the definitions of b, and a are found in Fig.1.

Time-variation of the velocity given by Eq.(1) is plotted in Fig.2, where A_s denotes the asymmetry of the velocity variation defined as (Dibajnia and Watanabe 1992, Ribberink and Al-Salem 1994)

$$A_s = \frac{U_c}{U_c + U_t} \tag{2}$$

and U_t is the magnitude of the trough velocity. Since the velocity variation of Eq.(1) is symmetric about the crest, only the first half of the wave cycle is shown in Fig.2. The dotted lines with open circles in Fig.2 denotes the first-order solution of cnoidal wave theory given by

$$\frac{U}{U_c} = \operatorname{cn}^2 \left(\frac{2Kt}{T}; k \right) - \overline{\operatorname{cn}^2}$$
(3)

where cn is the Jacobian elliptic function with modulus k, K is the complete elliptic integral of the first kind, and the bar denotes time average. In the above equation, the asymmetry of the velocity is more predominant with the increase of k, whereas the asymmetry of Eq.(1) is dependent on b_* . These two parameters governing the asymmetry can be correlated as follows (Tanaka et al., 1997, 1998). According to Eq.(3), A_s defined by Eq.(2) is

$$A_s = \frac{1}{k^2} \left(1 - \frac{E}{K} \right) \tag{4}$$

where E is the complete elliptic integral of the second kind of modulus k. On the other hand, the expression for A_s induced by the present generation system is obtained from Eq.(1).

$$A_s = \frac{b_* + 1}{2b_*} \tag{5}$$

Thus, by equating Eqs.(4) and (5), we obtain

$$\frac{1}{k^2} \left(1 - \frac{E}{K} \right) = \frac{b_* + 1}{2b_*}$$
(6)

Figure 3(a) shows how A_s varies with the change of k^2 calculated from Eq.(4), whereas Fig.3(b) denotes the relationship between b_* and k^2 obtained from Eq.(6). Figure 3(c) is drawn to show the relationship between q and k^2 .

Figure 2 shows a comparison between Eq.(1) and Eq.(2) thus correlated. In Fig.2, it is seen that Eq.(1) and the exact solution of the cnoidal wave theory, Eq.(3), show surprisingly excellent agreement, especially at smaller values of A_s . Theoretical treatment of Tanaka et al. (1997, 1998) revealed that Eq.(1) can be derived from Eq.(3) using an approximate formula for the elliptic function in the form of infinite product in terms of nome, $q (= \exp(-\pi K'/K), K'(k) = K(\sqrt{1-k^2}))$.



Fig.1 Mechanical generation method of an asymmetric oscillatory movement proposed by Tanaka et al. (1997, 1998)



Fig.2 Time-variation of free-stream velocity



Fig.3 Relationship between A_s , b_* , q and k^2

Furthermore, Tanaka et al. (1997,1998) correlated b_* with Ursell number, U_r , as follows:

for
$$U_r < 90$$
: $b_* = \frac{32\pi^2}{3U_r} + \tanh^{4/3} \{ (U_r / 70)^{3/4} \}$ (7)

for
$$U_r > 90$$
: $b_* = \frac{1}{1 - \frac{8}{\sqrt{3U_r}}}$ (8)

in which the definition of Ursell number is

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$$U_r = \frac{HL^2}{h^3} \tag{9}$$

where H, L and h are the wave height, the wave length and the water depth, respectively. Figure 4 shows the accuracy of the approximate formulae, Eqs.(7) and (8), in which the exact relationship is obtained from Eq.(10). (Shuto, 1974)

$$k^2 K^2 = \frac{3}{16} U_r \tag{10}$$

For designing an actual oscillating facility, Eqs.(7) and (8) can be used to determine b_* of piston system for a given Ursell number. Meanwhile, if A_s value is given instead of U_r as an experimental condition, corresponding b_* value is easily computed from Eq.(5).

 $b_* = \frac{1}{-1}$



Fig.4 Relationship between b* and Ursell number

(11)

Experimental Set-up and Experimental Condition

The present experiments were performed in a U-shaped oscillating tunnel with smooth walls seen in Fig.5 and Photo 1. The vertical risers of the tunnel were connected to the piston movement system described in the previous section. The velocities were measured with the help of a one-component fiber-optic laser doppler velcimeter (LDV) in forward scatter mode.

	working fluid	T(s)	$U_{c}(cm/s)$	$U_t(cm/s)$	A_s	R _s
Case 1	air	2.15	161.3	88.0	0.64	345
Case 2	water	2.38	89.3	56.9	0.61	824

Table 1 Experimental condition

Table 1 shows the experimental conditions for the experiments presented herein. In this table, R_{δ} is the Reynolds number defined in terms of the crest velocity and the Stokes layer thickness.

$$R_{\delta} = \frac{U_c \sqrt{2\nu/\omega}}{\nu} \tag{12}$$

where ν is the molecular viscosity of the fluid.

As to the present days, the critical Reynolds number in case of asymmetric oscillatory boundary layer has not been studied in detail. As a first approximation, the value for purely sinusoidal oscillatory boundary layers may be considered to be applicable. According to an experimental study of Hino et al. (1976), transition from laminar to turbulence occurs around $R_d = 550$ in a sinusoidal wave boundary layer. It may be thus anticipated that Case 1 may be in laminar regime, whereas Case 2 may be turbulent, although the representative velocity in the definition is different between sinusoidal and asymmetric oscillatory flow.

Experimental Results and Discussions

Case 1

Figure 6(a) shows the temporal variation of velocity at the axis symmetry in the wind tunnel. It is seen that the measurement and the theory, Eq.(1), shows excellent agreement.

The open and closed circles in Figs.6(b) and 6(c) show the profile of measured mean velocity in horizontal direction. The solid lines denote an analytical solution for laminar asymmetric oscillatory flow expressed in terms of Fourier series (Schäffer and Svendsen, 1986).

$$u = \sum_{n=1}^{\infty} \left[a_n \left\{ \sin n\omega t - \exp(-\beta_n z) \sin(n\omega t - \beta_n z) \right\} + b_n \left\{ \cos n\omega t - \exp(-\beta_n z) \cos(n\omega t - \beta_n z) \right\} \right]$$
(13)

where a_n , b_n are the coefficients obtained by Fourier analysis of the measured free stream velocity, β_n is the Stokes layer thickness for each harmonics,

$$\beta_n = \sqrt{n\omega/2\nu} \tag{14}$$

and z is the vertical coordinate taken positive upward from the wall surface. In the







Photo 1 Piston system



Fig.6 Velocity profile in a wind tunnel

present computation, the total number of terms in Eq.(13), m, is 5 to achieve sufficient accuracy. It is seen that the agreement between the measurement and the theory in Fig.6 is excellent, suggesting the validity of the present generation system.

Figure 7 depicts the time-variation of the wall shear stress from the experiment and the laminar theory,

$$\tau_0(t) = \rho \sum_{n=1}^m \sqrt{n\omega \nu} \left\{ a_n \sin(n\omega t + \frac{\pi}{4}) + b_n \cos(n\omega t + \frac{\pi}{4}) \right\}$$
(15)

In the experiment, $\tau_0(t)$ is evaluated by multiplying the measured near-wall velocity gradient to the kinetic viscosity of the fluid. It is seen that the time-variation of the measurement shows reasonable agreement with the theory, although small difference is seen near the peak. As may be recalled, in case of purely sinusoidal wave boundary layer in laminar flow regime, shape of the wall shear stress is also sinusoidal, but there exit a phase difference of $\pi/4$ between free-stream velocity and τ_0 . It may be noted that from Eq.(15) that following the same basic principle, every component of the wall stress has a phase lead of $\pi/4$ from the corresponding velocity component. But after the addition of these components with different frequency, the shape of the resultant wall shear stress profile is altogether different from the corresponding free-stream velocity over the wave cycle.

Judging from Figs.6 and 7, it can be concluded that the present generation system works very well for the use of wave boundary layer experiment under non-linear waves.



Fig.7 Time-variation of bottom shear stress

Case 2

During the experiment with water as working fluid, it was observed that the temporal free-stream velocity showed slight deviation from Eq.(1). Therefore, it was necessary to open both the valves on top of the risers in order to get the velocity in close agreement with the theory (Fig.8(a)). The velocity profile thus obtained is shown in Fig.8 at selected phases. The laminar solution, Eq.(13), is also plotted in Figs.8(b) and 8(c) along with numerical solution based on $k - \epsilon$ model. Among various versions of k = m model, Jones and Launder's (1972) low-Reynolds number model is employed in the present study, based on Sana and Tanaka's (1996) comparative study of the existing models. Detailed computation method is already described elsewhere (Tanaka and Sana, 1994, Sana and Tanaka, 1996).

In contrast to Fig.6, the experimental result in Figs.8(b) and 8(c) shows distinct difference from the laminar solution. The velocity profile at the beginning of deceleration phase (t/T=0.0) shows better agreement with k- ϵ model prediction, especially where the velocity overshooting occurs. But during the course of deceleration, it seems that the model fails to cope with the flow situation. During deceleration, both of the experiment and model show logarithmic behavior, although the boundary layer thickness is different each other. As the flow proceeds to acceleration in the trough phases, the model predictions begin to conform to the data very well. In the next deceleration phase during the trough phase (t/T=0.5-0.7), the model prediction is satisfactory. During the next acceleration phase (t/T=0.8-0.9), an excellent agreement is found between prediction of the model and the experimental data.

Figure 9(a) depicts turbulence intensity measured in an oscillating tunnel. As may be observed around t/T=0.5, turbulence is generated with time near the wall, yet with a lower rate as compared to that in crest period around t/T=0.1. During the trough period, the contours are almost horizontal, suggesting small variation in the turbulence intensity in that region. This is distinct difference from sinusoidal boundary layer and in accordance with what might be anticipated on the basis of the flat free-stream velocity during trough period, where the behavior similar to a steady flow should be observed. The contour plot obtained from the model prediction is shown in Fig.9(b). Since the model provides the total turbulent kinetic energy k, x-direction fluctuating component u' is obtained using Nezu's (1977) expression to correlate k and u' in a steady open channel flow. It is seen that the k- ε turbulence model can very well reproduce the fluctuating velocity near the wall.

Conclusions

The following conclusions can be drawn from this experimental study on nonlinear wave boundary layers:

- (1) It is confirmed that the generation method of an asymmetric oscillatory flow proposed by Tanaka et al. (1997, 1998) can be effectively utilized in a laboratory experiment for both air and water as working fluid.
- (2) The low-Reynolds number k- ε model by Jones and Launder (1972) shows good performance to predict mean velocity profile except decelerating phases under wave crest. As for turbulence intensity in x-direction, the overall agreement is satisfactory between the model prediction and experimental data.



Fig.8 Velocity profile in an oscillating tunnel



(b) k- ϵ model computation Fig.9 Turbulence intensity in an asymmetric wave boundary layer

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