CHAPTER 355

Near-Field Measurements of a Buoyant Jet in Waves and Currents.

K.H. Kwan[1] & C. Swan[2].

Abstract.

The present paper concerns the near-field characteristics of a jet discharged into a wave environment, and contrasts the results of two new experimental studies with a modified integral solution based upon a quasi-Lagrangian description of the flow field. This solution differs from previous attempts to model near-field wave mixing (namely Koole and Swan, 1994) in that it is formulated within an absolute frame of reference, and therefore adequately incorporates the momentum flux associated with the entrained fluid. Having implemented this important modification, the Lagrangian model is shown to provide a good quantitative description of a buoyant hot-water jet discharged into either a steady current or an unsteady wave field. In particular, there is no uncertainty regarding the magnitude of the entrainment coefficients, and as such the model is suitable for design calculations. In addition, detailed comparisons between the temperature profiles ensemble-averaged with respect to the phase of the wave cycle hint at a new mixing mechanism. Experimental data is presented which suggests that the wave-induced oscillatory motion leads to the division of the jet at certain phases of the wave cycle. If this effect is indeed important, then it implies that the average dilution will be enhanced when a jet is discharged into the plane of the wave motion.

Introduction.

Under most practical conditions outfalls discharge their pollutants, whether it be unwanted heat or partially treated effluent, into coastal waters which are dominated by the effects of wave action. However, present design practice largely ignores the wave motion and applies a simplistic approach in which both the

[1] Research student & [2] lecturer, Department of Civil Engineering, Imperial College, London. SW7 2BU. United Kingdom.

concentrations and the velocities arising within the near-field are predicted using some form of integral solution (Morton et al, 1956) based upon the conservation of mass, momentum, and where appropriate species concentration. This approach is often justified on the basis that the wave motion will, at worst, produce additional mixing and thus the design calculations are conservative. Unfortunately, recent studies (Koole and Swan, 1995) have shown that this does not necessarily follow, and that under some circumstances the wave motion may produce local increases in the concentration levels. Furthermore, previous studies have also clearly demonstrated that the occurrence of any additional wave-induced mixing is critically dependent upon the flow conditions at the exit nozzle. In particular, the ratio of the exit velocity to the local wave-induced velocity is a significant parameter. This in itself has important design implications given the increasing tendency to build long-outfalls. These are constructed so that firstly, the source of the pollutant is far removed from the adjacent coastline; and secondly, it is discharged in deeper water so as to reduce the pollutant concentrations arising at the water surface. Although, at first sight, these objectives appear desirable, the construction of a long-outfall may inadvertently move the pollutant source to a location at which the effective wave-induced mixing is negligible. As a result, the increased construction costs may actually produce higher concentrations at the water surface, and therefore potentially increase the risk of pollutant contamination arising on an adjacent coastline.

However, in order to incorporate these effects within the design process, and thereby optimise the efficiency of a given outfall design, one first requires a quantitative model capable of predicting the additional wave effects within the near-field. Although previous studies (notably Chin, 1988 and Koole and Swan, 1994) have considered this problem, quantitative comparisons with laboratory data are at best inconclusive. The present paper addresses this point, and provides a brief description of a corrected model which is subsequently shown to be in good quantitative agreement with new laboratory data describing a buoyant hot-water jet discharged into both a steady current and an unsteady (or time-dependent) wave field.

Background.

Although previous studies have clearly demonstrated the potential importance of wave-induced mixing (Sharp (1986), Chyan et al. (1991) and Hwung et al. (1995)); the only papers to have considered an appropriate mixing model are Chin (1988), Koole and Swan (1994) and more recently Chu and Lee (1996). In the first two cases the models were written with the specific intention of incorporating wave effects, while the latter model has to date only been applied to the description of pollutants discharged in a steady current. However, each of these models are similar in that they apply a quasi-Lagrangian representation of the flow field in which elements of the pollutant jet are tracked within the ambient flow.

Detailed comparisons with laboratory data involving both two-dimensional plane jets and three-dimensional round jets suggest that the model outlined by Koole and Swan (1994 and 1995) provides a good qualitative description of a jet discharged in waves. In particular, this model provides a plausible explanation for the non-Gaussian characteristics of the time-averaged radial distributions (both in terms of velocities u(r) and concentrations C(r)), and suggests that the occurrence of a bi-peaked or flat-topped distribution is simply dependent upon the local ratio of the jet diameter to the wave-induced displacement of the jet axis. Furthermore, the multi-stage centre line decay (i.e. u(z) or C(z) at r=0) first noted by Chyan et al. (1991) may also be explained (at least qualitatively) by the wave-induced displacements, and is a clear consequence of applying a time-averaged representation of a cyclic process.

Unfortunately, quantitative comparisons between the laboratory data and the model predictions proved more difficult. In particular, Koole and Swan (1995) demonstrated that good agreement along the time-averaged jet axis could only be obtained if the coefficient of forced entrainment (Winiarski and Frick, 1976) was set at β=1.0; and, in addition, the coefficient of radial entrainment was increased three-fold with respect to the value typically adopted in a stagnant ambient (i.e. α=3×0.055). The first of these points is consistent with previous studies, and stresses the importance of forced entrainment in non-stagnant ambients. However, the three-fold increase in the coefficient of radial entrainment is perhaps more difficult to explain. Koole and Swan (1994) argued that this change was justified by the large increase in the turbulent shear stresses ($\rho u'v'$) measured along the boundary of the jet discharged in a wave environment. However, no attempt was made to correlate the Reynold's stresses with the increased entrainment, and consequently it remains unclear whether a large increase in the coefficient of radial entrainment represents a genuine increase in the rate of mixing, or merely arises due to some inadequacy within the model formulation.

To help clarify this point the Lagrangian model proposed by Koole and Swan (1994) was applied to the case of a vertical jet discharged in a steady cross-flow (i.e., the ambient fluid flow is perpendicular to the initial jet axis). Although a Lagrangian model is equally applicable to this case, or indeed any other, there is a clear distinction between the cross-flow and waves cases in that previous researchers (Lee and Cheung, 1990) have already confirmed that for the case of a jet discharged into a steady cross-flow the appropriate entrainment coefficients are given by β=1.0 and α=0.055. In other words, in this case it is well known that the ambient flow has no effect upon the coefficient of radial entrainment.

If (x,y,z) represent the usual Cartesian coordinates in which (x,y) defines a horizontal plane and z is measured vertically upwards; the trajectory of a vertical jet subject to a cross-flow orientated along the x axis may be represented on the (x,z) plane. Figure 1 concerns four such cases corresponding to u_j/u_a=45.1, 20.0, 16.2 and 11.7; where u_j is the exit jet velocity and u_a is the ambient cross-flow velocity. In each of these cases laboratory data describing the jet trajectory (Patrick, 1967) is

compared with the results of the Lagrangian model proposed by Koole and Swan (1994) in which the entrainment coefficients are defined as α=0.055 and β=1.0. The agreement between the measured data and the model predictions is clearly very poor, and suggests (at least in this case) that this Lagrangian description of the flow field is fundamentally incorrect.

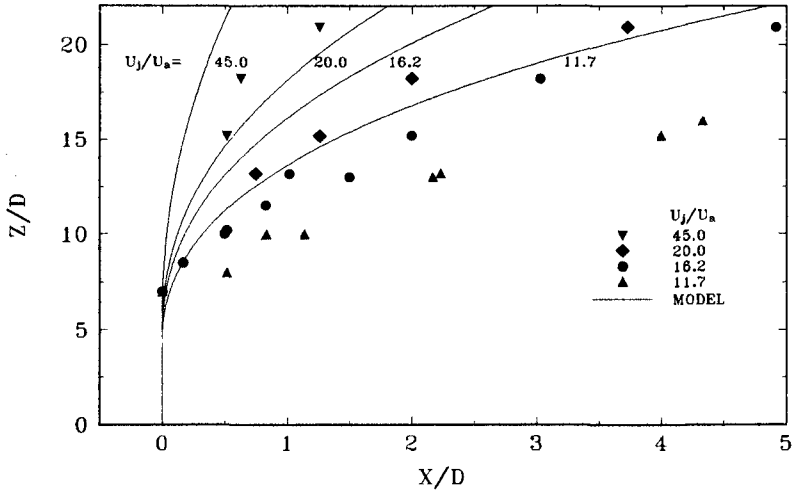


Figure 1. Jet trajectory: comparisons with a Lagrangian model formulated in a relative frame of reference.

Improved Integral Formulation.

In attempting to explain the apparent inadequacy of the previous Lagrangian models (namely Chin 1988 and Koole and Swan 1994) the recent formulation proposed by Chu and Lee (1996) was noted. Although there are many similarities between these three models, the latter solution is quite distinct in that the integral equations are formulated in an absolute frame of reference. The distinction between this and the relative frame of reference, used in the former models, is shown to be very important and in particular accounts for discrepancies within the momentum conservation equation. Figure 2 concerns a simple jet element of area A_j which has a velocity u_j measured relative to the ambient fluid, which itself has a velocity u_a. The vector sum of u_j and u_a defines the instantaneous jet axis such that u_j and u_a are respectively at angles of γ and δ to the ξ-axis. In a relative frame of reference, or one moving with the ambient flow, the conservation equations relating to mass and momentum are given by:

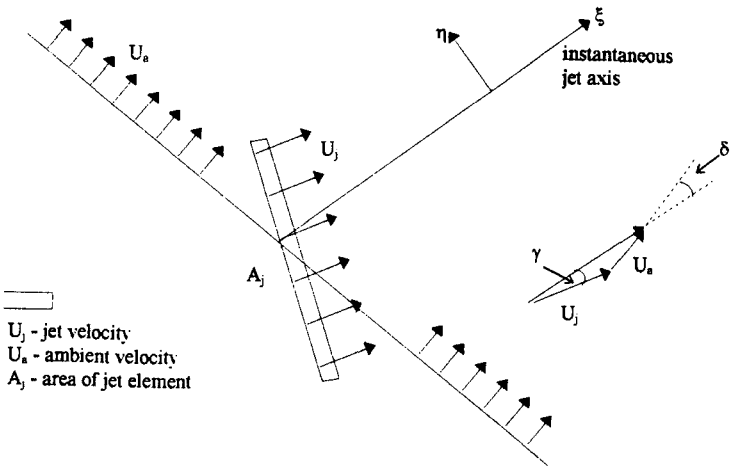


Figure 2. Definition sketch.

$$\frac{\partial}{\partial \xi}\int_{A_j} u_j \cos\gamma d\eta = \alpha u_j 2\pi b_j \cos\gamma + 2b_j\beta\left|u_j \sin\gamma\right| \tag{1a}$$

$$\frac{\partial}{\partial \xi}\int_{A_j}\left(u_j \cos\gamma\right)^2 d\eta = 0 \tag{1b}$$

where again α is the coefficient of radial entrainment, β is the coefficient of forced entrainment and b_j is related to the radius of the jet element. In contrast, the same conservation equations written within an absolute frame of reference are given by:

$$\frac{\partial}{\partial \xi}\int_{A_j} u_j \cos\gamma d\eta = \alpha u_j 2\pi b_j \cos\gamma + 2b_j\beta\left|u_j \sin\gamma\right| \tag{2a}$$

$$\frac{\partial}{\partial \xi}\int_{A_j}\left(u_j \cos\gamma + u_a \cos\delta\right)^2 d\eta = u_a E \tag{2b}$$

where E corresponds to the total mass of fluid entrained into the jet element, and is given by the right hand side of equation 2a (or 1a). Comparisons between these equations immediately confirm that while the mass conservation equation is unaltered (since the entrainment of fluid is dependant upon a relative velocity effect), the equations defining the conservation of momentum (1b and 2b) differ significantly. In particular, equation 2b incorporates the momentum of the entrained fluid.

In light of these differences a new Lagrangian model was written in which the integral equations were formulated in an absolute frame of reference. A complete description of this model is beyond the scope of the present paper (see Kwan and Swan, 1997), but comparisons with the previously noted cross-flow data (Patrick, 1967) suggest that this modification is sufficient to explain the inadequacy of the previous model highlighted in figure 1. Indeed, figure 3 shows a similar set of comparisons with the new model, and confirms that a good description of the jet trajectory can indeed be achieved with the previously recorded entrainment coefficients (α=0.055 and β=1.0).

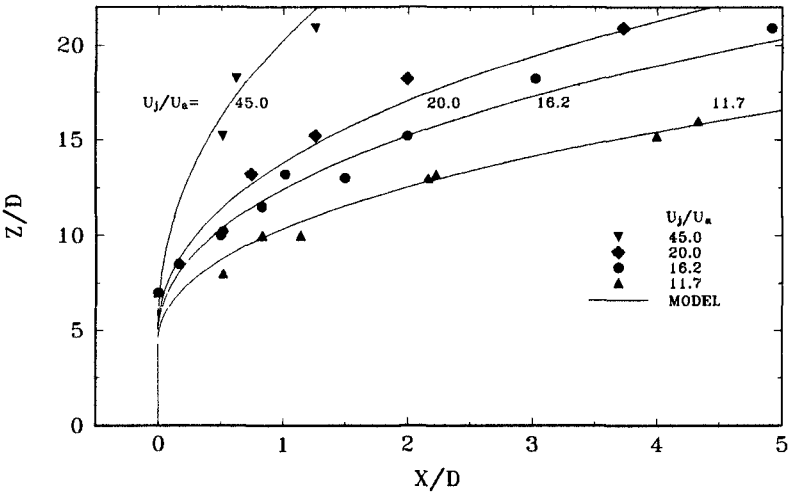
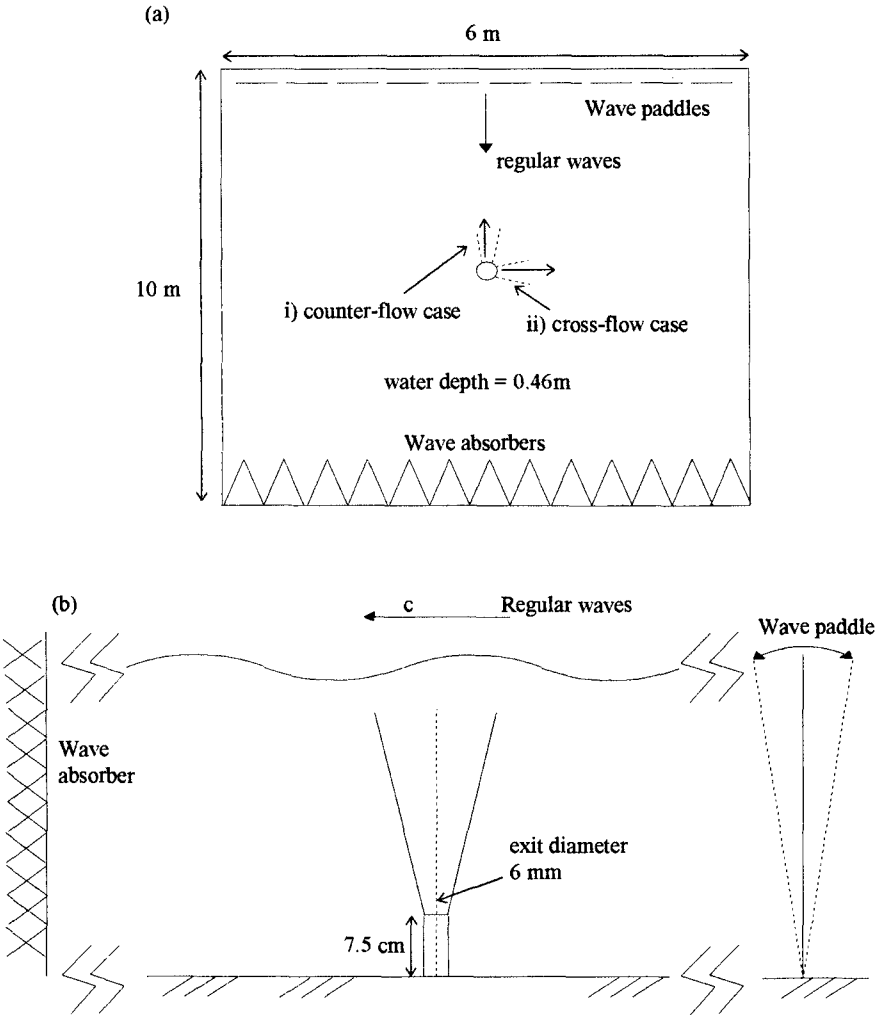


Figure 3. Jet trajectory: comparisons with a Lagrangian model formulated in an absolute frame of reference.

Experimental Work.

To further clarify the near-field behaviour of a jet discharged into a wave environment a new experimental programme was undertaken. In the first stage a buoyant hot-water jet was discharged horizontally beneath a series of regular waves having a period of T=1s and a wave amplitude of a=0.032m. These measurements were undertaken within a small wave basin located in the hydraulics laboratory within the Civil Engineering Department at Imperial College. This facility has a plan area of 6m x 10m, and has an effective working depth of 0.46m. At the upstream end of the basin the waves were generated by 10 individually controlled random wave paddles each 0.6m wide; while at the downstream end the wave energy was dissipated on a one in ten sloping beach, with further passive absorption provided by several large blocks of poly-ether foam. The hot-water jet was introduced in the centre of the wave basin through a horizontal nozzle of diameter D=6mm, with its centre line

located 150mm above the bottom boundary. Although several orientations of the jet relative to the direction of wave propagation were considered, the present paper only addresses one case in which the jet was discharged perpendicular to the plane of the wave motion. In this case the exit characteristics of the jet were such that its temperature was 17°C above the ambient water temperature, and its velocity was 0.76m/s. These conditions correspond to a densimetric Froude number of F_{ro}=45 and a velocity ratio of u_j/u_a=10.9, where u_a now represents the maximum wave-induced ambient velocity at the elevation of the discharge nozzle. A sketch describing the layout of this test case is given on figure 4a.



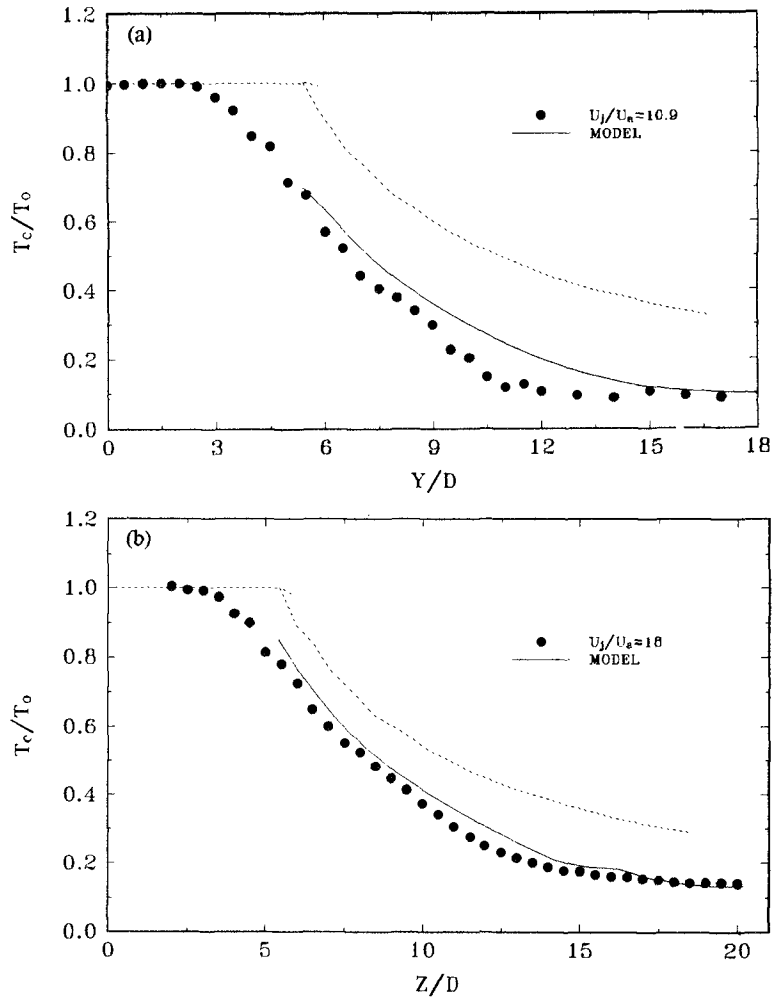
Figures 4a-4b. Experimental apparatus: (a) horizontal jet, and (b) vertical jet.

In a second series of tests a vertical hot-water jet was discharged from a similar nozzle located 75mm above the bed of a two-dimensional wave flume. This facility, which is again located in the Civil Engineering Department at Imperial College, is 25m long, 0.3m wide, and (for the duration of the present tests) had a working depth of 0.62m. The wave conditions again consisted of a regular wave train having a period of T=1.0s and wave amplitude of a=0.3m. Likewise, the exit temperature of the jet was again 17°C above the ambient water temperature and the exit velocity was 0.76m/s. These conditions are similar to those occurring in the previous test case, and correspond to a densimetric Froude number of F_{ro}=37.8 and velocity ratio of u_j/u_a=18. A sketch showing the layout of this case is given on figure 4b.

In both test cases the near-field mixing was quantified using an array of thermocouples. These were supported in a two-dimensional traverse having a positional accuracy of ±0.5mm in both the horizontal and the vertical directions. Each thermocouple was constructed from k-type thermocouple wires having an external diameter of 3 microns, and fused at one end to produce a measuring volume with a diameter of $\phi \leq 0.2$mm. Although probes of this type were difficult to handle, their size was essential to ensure the required accuracy and response (0.1°C in 0.01s). After careful calibration this measuring apparatus was able to resolve the temperature fluctuations within an individual wave cycle, and was thus deemed appropriate to determine the nature of the mixing processes.

Discussion of Results.

Figures 5a and 5b respectively concern the horizontal and vertical jets discussed above, and describe the decay in the time-averaged temperatures measured on the central axis of the jet. In both cases the measured data is compared with two models. The first, indicated by a dashed line, represents a standard integral solution based upon the assumption that the jets were discharged into a stagnant ambient; while the second, indicated by a solid line, represents the results of the new Lagrangian model outlined above. In this latter case it is important to note that although the Lagrangian model is not in perfect agreement with the laboratory data, it provides a significant improvement over the stagnant ambient model. Furthermore, unlike the previous formulations proposed by Koole and Swan (1994), there is no uncertainty with regard the entrainment coefficients (α, β). Indeed, the calculations presented on figures 5a and 5b are based upon the widely accepted entrainment coefficients commonly applied in both stagnant and steady flows (i.e. α=0.055 and β=1.0).



Figures 5a-5b. Centre-line temperature decay: (a) horizontal jet and (b) vertical jet.

To further examine the success of the Lagrangian solution figure 6 concerns the time-averaged temperature profiles at five elevations above the vertical jet, and contrasts the measured data with the model predications. These comparisons clearly suggest that the modified Lagrangian solution is, in general, in good quantitative agreement with the laboratory data. However, on closer inspection it is apparent that the temperature profiles become increasingly asymmetric at higher elevations above the discharge orifice. This pattern has been observed in several other test cases and is, at first sight, very difficult to explain since one would assume that the cyclic nature of the wave motion effects both sides of the jet in an identical manner. However, if one considers that the central core of the jet, at least in the near-field region, provides

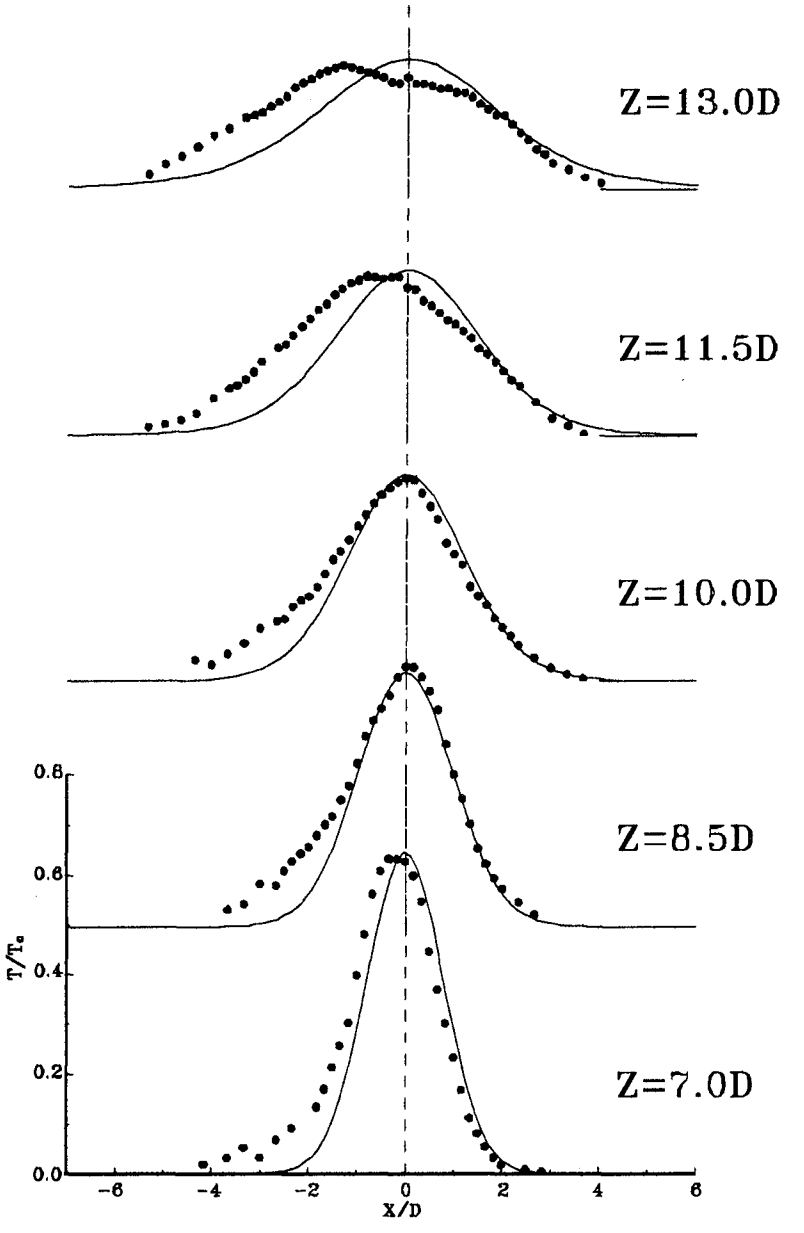


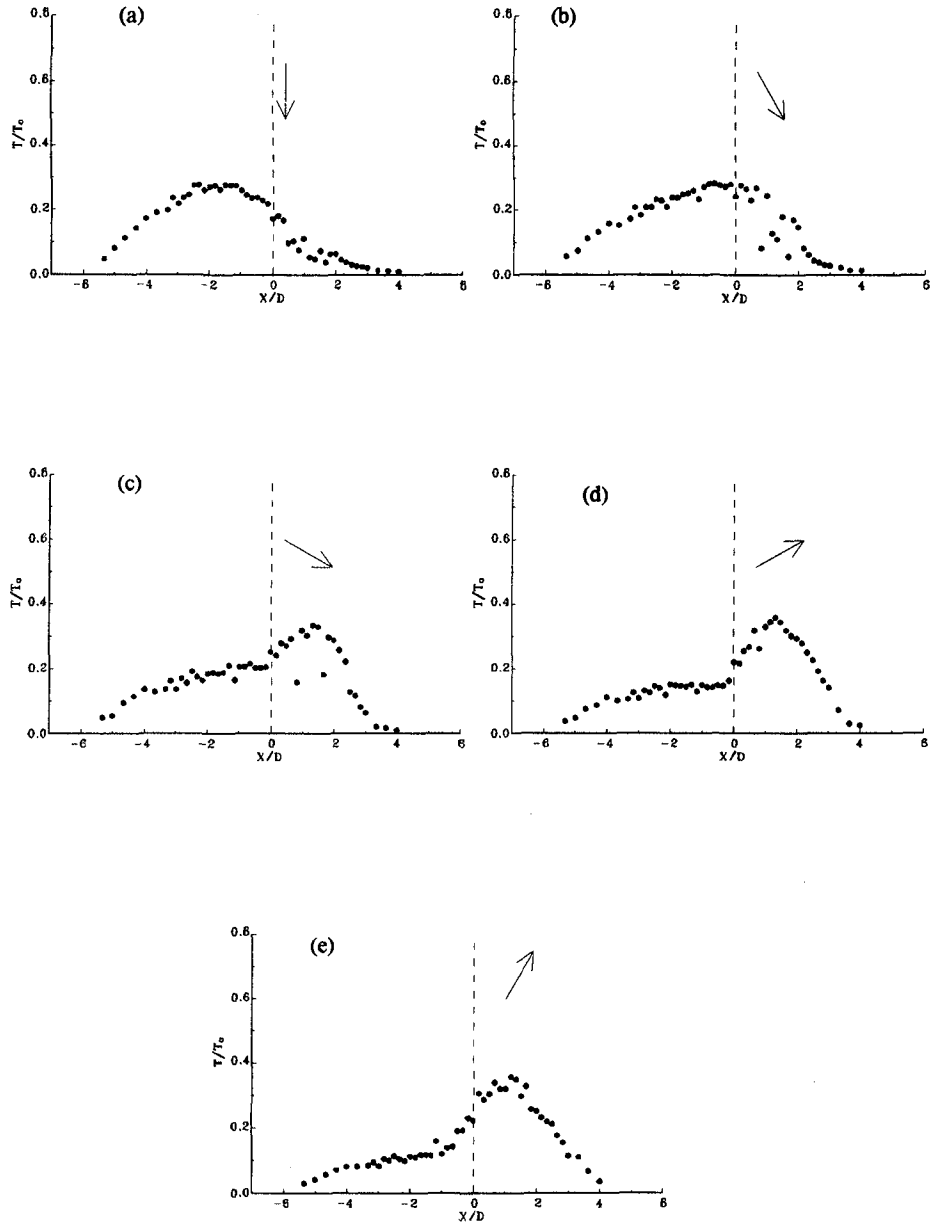
Figure 6. Time-averaged temperature profiles measured above the vertical jet. • laboratory data, ——— Lagrangian model.

an effective barrier to the wave motion in the vicinity of the jet, it becomes apparent that the upstream and downstream sides of the jet (measured relative to the direction of wave propagation) experience very different flow regimes. Adopting the usual notation whereby a co-flow corresponds to the movement of ambient fluid in the same direction as the jet velocity, and a counter-flow opposes the jet velocity; it can easily be shown that the upstream side of the jet is subject to a transition from a co-flow to a cross-flow, while the downstream side of the jet experiences a transition from a counter-flow to a cross-flow. These changes occur between phase angles $0 \leq \theta \leq 0.25$ and $0.5 \leq \theta \leq 0.75$ respectively, where $\theta=0$ corresponds to a zero up-crossing. The difference between these regimes provides a plausible explanation for the jet asymmetry observed in figure 6.

Indeed, if one further considers the temperature profiles in a phase-averaged sense (or ensemble-averaged at specific phases of the wave cycle) there is evidence of a new wave-induced mixing mechanism which specifically effects the downstream side of the evolving jet. Figures 7a-7e describe the ensemble-averaged temperature profiles at $\theta=0.5$, 0.6, 0.7, 0.8 and 0.9 respectively, where $\theta=0.5$ corresponds to a zero down-crossing and $\theta=0.75$ is in phase with the wave trough. During this portion of the wave cycle, the vertical jet is first subject to a counter-flow ($\theta=0.5$), followed by an increasing cross-flow which is orientated towards the downstream side of the jet. In each of these five figures the large arrow positioned above the temperature data indicates the approximate direction of the ambient flow. In this case the effect of the ambient motion appears to divide the jet so that although a large proportion of the jet is swept upstream (in the positive x direction), a significant proportion also appears to remain almost static. This apparent division of the jet only arises during this stage of the wave cycle, and in particular does not occur 180° later, when the jet is subject to a co-flow ($\theta=0$) followed by an increasing cross-flow orientated towards the upstream face. In this latter case the entire jet is swept downstream (in the negative x direction), and thus the pattern looks very different to that presented on figures 7a-7e.

Concluding Remarks.

The present study has addressed the case of a buoyant hot-water jet discharged in a wave environment, and has sought a quantitative description of the near-field mixing based upon a quasi-Lagrangian formulation of the integral equations. Previous attempts at a solution of this type have provided a good qualitative representation of the flow field, but detailed quantitative comparisons have been limited due to the apparent uncertainty in the entrainment coefficients. In particular, a three-fold increase in the coefficient of radial entrainment was necessary to model the centre-line decay. To investigate this point an existing Lagrangian model, formulated in a relative frame of reference, was used to describe a vertical jet discharged in a steady cross-flow. Although in this case there is no ambiguity regarding the entrainment coefficients, the agreement between the model predictions



Figures 7a-7e. Phase-averaged temperature profiles at z=13D.
(a) θ=0.5, (b) θ=0.6, (c) θ=0.7, (d) θ=0.8, (e) θ=0.9.

and laboratory data were poor. On closer inspection, it has been shown that a Lagrangian model formulated in a relative frame of reference neglects the momentum flux associated with the entrained fluid. In contrast, an alternative model, formulated in an absolute frame of reference, is shown to be in good agreement with all cases involving buoyant jets discharged in both steady currents and unsteady wave fields.

To further validate this latter model, new experimental studies were undertaken in which both horizontal and vertical jets were discharged within a wave environment. Laboratory data describing the near-field mixing is shown to be in good agreement with the modified Lagrangian solution, and perhaps most importantly there is no ambiguity with regard the entrainment coefficients. As a result, the proposed solution is appropriate to design applications. Furthermore, detailed consideration of the phase-averaged temperature data measured within the near-field provides evidence of a new wave-induced mixing mechanism. This effect provides a plausible explanation for the development of non-symmetric time-averaged temperature profiles. More importantly, it provides the first quantitative evidence which suggest that the average dilution will be enhanced if a pollutant is discharged in the plane of the wave motion.

Acknowledgements.

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